MODELING OF A HIGHLY DAMPED NATURAL RUBBER ELASTOMERIC BEARING
FOR SEISMIC BASE ISOLATION PURPOSES

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Abstract
Seismic base isolation is widely used as a method for limiting the damage caused to buildings, bridges, and other structures during a seismic event. Currently, one of the most popular methods of achieving seismic base isolation is to insert elastomeric bearings at the foundation of a structure. Several types of elastomeric bearings exist, including low damping natural rubber, high damping natural rubber, and lead plug bearings. These bearings achieve seismic base isolation by allowing the structure to move horizontally while maintaining the vertical load carrying capacity needed to support the weight of the structure. Several parameters are important when selecting an elastomeric bearing for various applications. This research project aims at analyzing the most important geometric parameters for the high damping natural rubber bearing. These parameters were analyzed using ANSYS Workbench and their effects on the vertical and horizontal natural frequency of the elastomeric bearing were determined. The goal of this modeling is to determine the optimum parameters necessary to obtain the most successful elastomeric bearing. The effects of the isolation system were further analyzed through modeling of a five story building in ANSYS Workbench. After modeling an isolated and un-isolated building, the effects of the elastomeric bearings were clearly evident. The isolated building was clearly able to move horizontally at a much lower frequency while still maintaining a vertical frequency close to that of the un-isolated building.
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Finally, I would like to thank my family for their support throughout this entire project and throughout my entire academic career. They have always been there for me and have provided me with the motivation and encouragement that I needed to successfully complete my research project.
Chapter 1: Introduction to Seismic Base Isolation

Across the globe, earthquakes have been occurring for thousands of years. As the population continues to increase and the world continues to industrialize the amount of damage done by earthquakes, to human built structures, will continue to grow. As a result of this, significant research has been performed, over the past century, to develop a method of limiting the damage caused to large structures during a seismic event. One of the most successful and widely used of these methods is seismic base isolation.

There are several types of seismic base isolation systems, all of which have one common objective. This objective is “to decouple the building structure from the damaging components of the earthquake input motion [1].” This is achieved by making the base of the structure more flexible in the horizontal direction while maintaining the necessary vertical strength. The two most common types of seismic base isolation systems are the elastomeric bearing and the sliding bearing (also known as the friction pendulum bearing). Some other methods of seismic base isolation include rollers, sliding plates, cable suspension, rocking foundation, sleeved piles, air cushions, and coil springs [2]. Each of these types of base isolation use one of two methods to achieve seismic isolation. These two methods include shifting the period of the structure and cutting off the load transmission path [3]. The elastomeric bearing reduces the forces transmitted by an earthquake by changing the fundamental period of the structure to avoid resonance with the dominant frequency of the earthquake [3]. The sliding bearing, on the other hand, reduces earthquake forces with the use of discontinuous sliding interfaces [3]. Between these interfaces the transmitted forces are limited by the maximum friction forces [3].

Due to the increasing use of seismic base isolation systems by design engineers, it is important to understand the theory behind which they operate. The following chapters focus on a brief background of the elastomeric bearing, as well as some in depth modeling that has been performed to better understand the this type of base isolation system.

1.1 Motivation

Seismic base isolation is currently the most effective and most widely used form of mitigating damage caused by a seismic event. Significant research is currently being performed to expand
the use of seismic isolation from buildings and bridges to other applications such as the isolation of storage racks. In order for a seismic base isolation system to be most effective, in a wide variety of applications, it must first be properly understood. The goal of this thesis is to identify and analyze the important design parameters of a highly damped natural rubber elastomeric bearing.

1.2 A Historical Overview of Seismic Base Isolation

1.2.1 Early Seismic Base Isolation

Although base isolation was not officially used in the United States until 1986, it is believed that ideas for seismic base isolation date back to the early 20th century [4]. The first application for a patent related to seismic base isolation was filed in 1906 by Jacob Bechtold in Munich, Germany [2]. He had the idea for "an earthquake proof building consisting of a rigid base-plate to carry the building and a mass of spherical bodies of hard material to carry the said base-plate freely [2]." A similar patent application was filed by a British medical doctor from Scarborough, England in 1909 [4]. His proposal was to separate a building from its foundation with a layer of talc [4]. A third patent attempt was made by Robert deMontalk of Wellington, New Zealand in 1929 [5]. His idea involved using “a bed which is placed and retained between the base of a building and its solid foundation, the bed being composed of a material which will absorb or minimize shock thereby saving the building [5].” There is no evidence that any of these devices were actually used to seismically isolate a building. The most likely reason for this is these designs were quite impractical for the time period.

1.2.2 Proof of Concept

The first actual evidence of a seismically isolated structure dates back to the 1920’s. In 1921 Frank Lloyd Wright designed the Imperial Hotel in Tokyo [4]. His building was built on an eight foot layer of good soil with a soft mud layer below that [4]. During a severe earthquake in Tokyo in 1923 the Imperial Hotel performed exceptionally well, suffering only minor damage [4]. To this day, it is not clear whether Wright intentionally had the building built for this purpose or if it was just a coincidence. Regardless, this incident provided proof that seismic base isolation could in fact protect a building in the event of an earthquake.
Similar incidents in the mid 20th century provided additional proof for the concept of seismic base isolation. During the Long Beach earthquake in 1933 multiple masonry buildings experienced only minor damage [2]. Another masonry building survived the 1976 Tangshan earthquake [2]. It is believed that these buildings suffered only minor damage because they were able to slide on their foundations [2].

1.2.3 Current Use of Seismic Base Isolation

Currently, there are hundreds of structures across the globe that use seismic base isolation. The most prominent locations for seismic base isolation systems are Italy, New Zealand, Japan, and the United States [5]. The first United States building to use seismic base isolation was the Foothill Communities Law and Justice Center in San Bernardino County, California. Located only 12 miles from the San Andreas Fault, the building is four stories tall and was completed in March of 1986 [4]. The building contains a sub-basement where 98 isolators are used as an isolation system [4]. These isolators are multilayer rubber bearings reinforced with steel plates [4]. Other seismically isolated structures in the U.S. include the Evans and Sutherland Building, the Salt Lake City and County Building, the Sierra Point Overhead Bridge, and the Santa Ana River Bridge [2].

More recently the concept of seismic base isolation has been applied to industrial steel storage racks. In most warehouse settings, tall steel storage racks are used to store heavy equipment, supplies, merchandise, and other important resources. Much concern has arisen regarding the potential for these items to fall from the storage rack in the event of an earthquake. This has the potential to cause severe injury, or even death. In addition to this, the fallen resources will likely be damaged, costing companies a great deal of money. Ridg-U-Rak, a company from Erie, Pennsylvania has spent several years researching types of base isolation systems to be used to prevent storage racks from shedding material. They have developed systems that have been claimed to dissipate up to 85% of seismic energy and are currently being distributed across the globe [6].
Chapter 2: Background of Elastomeric Bearings

2.1 Elastomeric Bearing Overview

Elastomeric bearings are currently the most widely used system for seismic base isolation worldwide. Most elastomeric bearings consist of alternating laminated layers of elastomer and steel shims [5]. Typically, either natural rubber or neoprene is used to construct the elastomeric layer of the bearing [7]. The goal of this design is to produce a vertically stiff but horizontally flexible isolator. The horizontal flexibility lengthens the fundamental period of a structure while also reducing the seismic forces transmitted to the structure whereas the vertical stiffness provides the support needed to withstand the large static loads caused by the weight of the structure [8].

As mentioned, most elastomeric bearing consists of alternating layers of elastomer and steel. This being said, the number of alternating elastomer and steel layers can vary depending on the type of application. A bearing without any steel layers will have a significantly lower compression stiffness than a bearing with steel layers and thus will have a much lower compression load carrying capacity [5]. The following paragraphs provide a description of several types, applications, and recent performance of elastomeric bearings.

2.2 Types of Elastomeric Bearings

Several types of elastomeric bearings are currently being used as seismic isolators today. These include lead-plug bearings, low-damping natural and synthetic rubber bearings, and high-damping natural rubber systems [7]. There are also some systems that include both elastomeric bearings and sliding bearings [7].

2.2.1 Lead-Plug Bearings

The first lead-plug elastomeric bearing was invented in New Zealand in 1975 and is being widely used in countries such as New Zealand, Japan, and the United States [7]. Currently the lead-plug bearing is the most commonly used isolation system. This bearing is typically made from a low-damping elastomer and contains one or more lead cores that can range in size from 15% to 33% of the overall diameter of the bearing, depending on the application [9]. The elastomer isolates the structure whereas the lead plug provides the necessary damping [9]. The
lead plug allows for significantly higher damping levels than any other type of elastomeric bearing [5]. The lead plugs are inserted into holes in the elastomeric bearing, as seen in Figure 1. To ensure that the lead plug fits tightly in the bearing, they are typically made slightly larger than the hole and are forced in [7]. The typical shear-strain range of this type of bearing is between 125% and 200% [9]. The lead-plug bearing proved its effectiveness during the 1994 Northridge and 1995 Kobe Earthquakes [7]. During these earthquakes, several buildings that were isolated with lead plug bearings performed exceptionally well [7].

### 2.2.2 Low-Damping Natural and Synthetic Rubber Bearings

The next type of elastomeric bearing is the low-damping natural and synthetic rubber bearing. This type of bearing has been widely used in countries such as Japan and France [7]. The two types of rubber that are typically used to construct this type of bearing are natural rubber and neoprene [7]. These bearings typically consist of two thick steel endplates with several alternating rubber and steel shim layers [7]. The steel shims allow for a high vertical stiffness but do not affect the horizontal stiffness [7]. Low-damping elastomeric bearings are typically easy to manufacture and easy to model, but need to be accompanied by another type of supplementary damping system such as a lead plug [7]. The function of the supplementary damping system is to prevent the rubber layers from bulging under the extremely high static loads. Bearings of this type can typically withstand shear-strain loads up to and exceeding 100% [10].

### 2.2.3 High-Damping Natural Rubber Systems

The third type of elastomeric bearing is the high-damping natural rubber system. This type of bearing was first created in 1982 by the Malaysian Rubber Producers’ Research Association of the United Kingdom [7]. Bearings that fit this classification are typically made out of a natural
rubber compound that doesn’t require any sort of supplementary damping as mentioned with the low-damping rubber bearings [7]. One example of a highly damped natural rubber currently used in this type of bearing is butyl [5]. Typically the maximum shear-strain range for high-damping rubber bearings is between 200% and 350%, but varies as a function of compound and manufacturer [9]. This range is quite higher than that of the lead plug bearing and nearly three times higher than that of the low damping natural rubber bearing. This type of elastomeric bearing will be the primary focus of this thesis.

2.2.4 Elastomeric and Sliding Bearing Systems
The first elastomeric and sliding bearing combination system was developed and tested at the Energy & Environmental Research Center [7]. This system consisted of Teflon on stainless steel sliding elements and low-damping natural rubber bearings [7]. The sliding elements were used to support the interior columns of the structure and the rubber bearings were used to support the exterior columns of the structure [7]. The sliding elements provided the necessary damping whereas the elastomeric bearings re-centered the structure and controlled torsion [7]. Systems similar to this one have been used at the Mackay School of Mines at the University of Nevada and at the Martin Luther King Jr.-C.R. Drew Diagnostics Trauma Center in Willowbrook, California [7].
2.3 Applications of Elastomeric Bearings

Elastomeric bearings are most commonly used to isolate buildings, but they can be used in some other applications. These other applications include bridges, storage racks, and even some automobiles. The following paragraphs describe these different applications in further detail.

2.3.1 Buildings

By far the most common application of an elastomeric bearing is the isolation of a building during a seismic event. Elastomeric bearings were first used in this application, in 1969, to isolate the Pestalozzi School in Skopje, Macedonia [7]. The technology that went into developing this early design was much less advanced than the current technology. The rubber blocks that were used in this application were unreinforced and, as a result, the weight of the building caused the blocks to bulge laterally [7]. More recent applications include some type of steel reinforcement or highly damped natural rubber in order to support the tremendous weight of the building.

2.3.2 Bridges

The second most common application of elastomeric bearings is the isolation of bridges during seismic events. This application was first developed more than 40 years ago in Great Britain, and was first used in the United States on a bridge in Texas [11]. The use of elastomeric bearings for bridge isolation is much similar to that used in buildings. The bridge bearings are designed to support high compressive loads while allowing horizontal movement [11].

Figure 3: Elastomeric Bearing used in a bridge in New Zealand [5]
2.3.3 Automotive

In addition to the seismic isolation of buildings and bridges, elastomeric bearings have also been used in several automotive applications. Since the 1960's, countless patents have been filed for the use of elastomeric bearings in helicopter rotors, suspension systems, railway trucks, constant velocity joints, and many other similar applications. One of the first ever patents for an elastomeric bearing helicopter rotor system was approved on August 17, 1965 [12]. The primary goal of this invention was to increase the beneficial effects of centrifugal forces, while eliminating the centrifugal force bearing friction effects in a helicopter rotor [12]. In 1979 Robert C. Rybicki and Brian Cuerden were granted a similar patent [13]. This patent was designed as an improvement to the previous design, in that, it aimed to increase the life of the elastomeric bearings used in helicopter rotors [13].

2.4 Current Performance Evaluation

Much research has been performed over the past twenty years regarding the performance of existing seismically isolated structures to various earthquakes. The Foothill Communities Law and Justice Center, the Los Angeles Fire Command and Control Building, the Los Angeles USC Hospital, and the Seal Beach Building are several examples of seismically isolated buildings that have been researched. The following sections will focus on performance evaluations that have been performed on these isolations systems after various earthquakes. The earthquakes include the Northridge earthquake and the Landers earthquake. The Northridge earthquake, which occurred in January 1994, was a magnitude 6.7 earthquake and was centered approximately 20 miles northwest of Los Angeles, CA [14]. More than sixty people were killed and over 7,000 people were injured during this violent earthquake [14]. In addition to this, over 40,000 buildings sustained damage [14]. The Landers earthquake, which occurred in June 1992, was a magnitude 7.3 earthquake and was located in Landers, CA [15]. Three people died during this earthquake and more than 400 were injured [15]. Due to the severity of these earthquakes, a significant amount of base isolation research followed.

2.4.1 Foothill Communities Law and Justice Center

As mentioned earlier, the Foothill Communities Law and Justice Center (FCLJC) was the first building in the United States to use seismic base isolation. As a result, it is one of the most
popular sites for research on the performance of seismic base isolation. The FCLJC is four stories tall and has a single level basement [16]. The isolation system present in this building consists of 98 high-damping elastomeric bearings, which are 76 cm in diameter and 46 cm in height [16]. With the use of this isolation system, the building is said to be capable of withstanding a magnitude 8.3 earthquake [17]. Results obtained from the Landers earthquake indicate that the isolators successfully lengthened the fundamental period of this structure [18]. The fixed base period of the FCLJC in the longitudinal direction is around .61 s, whereas the longitudinal period of the FCLJC during the Landers earthquake was found to be .76 s [18]. Because of this, the isolators were able to minimize the damage caused to the FCLJC during the Landers earthquake in 1992.

2.4.2 Los Angeles Two-Story Fire Command and Control Building
The Los Angeles Fire Command and Control Building (FCC Building) is a 2 story building with no basement located in Los Angeles California [16]. The building sits on 32 high damped rubber bearings, which provides the necessary seismic isolation [16]. The isolation system present in this building is much similar to that used in the FCLJC. During the Northridge earthquake this building remained fully functional, but did suffer minor damage [19]. Several tiles in the entryway of the building were damaged as a result of several high frequency spikes [19]. This entryway was designed to allow free motion during an earthquake but was constructed incorrectly [20]. As a result of this, the entryway was not allowed to move freely and thus sustained minor damage [20].

2.4.3 Los Angeles Seven-Story USC Hospital
The Los Angeles USC Hospital is a seven story building that contains a partial basement [16]. The isolation system contained in this building is made up of lead-rubber and natural rubber elastomeric bearings [16]. The isolation system contains 68 lead-rubber bearings and 81 natural rubber bearings [21]. The evaluation that was performed after the Northridge earthquake found that this building performed well during this earthquake. The isolation system successfully prevented the building and its contents from being damaged in any way [22]. In addition to this only 10 % of the capability of these isolators was used [22]. This suggests that the isolation system would have held up to a more serious earthquake.
2.4.4 Seal Beach 8-Story Building

The Seal Beach Building is an eight-story building with a single level stepped basement [16]. Of the four buildings, this is the only one that has been retrofitted with a base isolation system. The building was built in 1967 and later retrofitted with seismic isolators in 1990 [18]. This system consists of isolators placed in the columns between the first and second floor [16]. Results from the 1992 Landers earthquake show that this isolation system successfully lengthened the fundamental period of the structure. The longitudinal period of the Seal Beach Building during the Landers earthquake was found to be 1.36 s [18]. In comparison, the fixed base period in the longitudinal direction was determined to be 1.1 s [18]. This isolation helped to minimize the damage that was caused to the Seal Beach Building during the Landers earthquake.

Chapter 3: Initial Elastomeric Bearing Modeling

The following modeling was performed to achieve a better understanding of how an elastomeric bearing works. ANSYS Workbench was used to analyze a three-dimensional model of an elastomeric bearing. Mathcad was then used to verify the results obtained from the ANSYS model. This modeling focuses on two types of elastomeric bearings. These include the high damping natural rubber bearing and the lead plug bearing. The high damping natural rubber bearing will be the major focus for the majority of this paper. The modeling of the lead plug bearing was performed in order to provide a comparison to the highly damped natural rubber bearing, but will not be the primary focus. The low damping natural rubber bearing was excluded since it is similar in geometry to the highly damped rubber bearing and is a less practical design.

3.1 Behavior of Rubber

The behavior of rubber, from a material standpoint, is very important when attempting to produce an accurate finite element model. The primary concern is evident when analyzing the stress-strain relationship of a typical rubber compound. Rubber behaves quite differently when it is loaded under tension versus compression. The compressive behavior of rubber will be the primary focus for the models in this paper, since all loading will be in compression. The compressive modulus of rubber is highly dependent on the shape factor.
Figure 4 shows the effect of shape factor on the compressive stress strain curve for rubber. The shape factor is defined as the ratio of loaded area to the total force-free area [23]. For a circular section the shape factor can be represented by the following equation:

\[ S = \frac{\pi d^2}{4\pi dt} = \frac{d}{4t} \]

In this equation, \( d \) represents the diameter of the cross section and \( t \) represents the thickness. The compressive modulus can then be calculated using the following relationship:

\[ E_c = E(1+2kS^2) \]

In this equation, \( E \) represents the elastic modulus of the rubber, \( k \) represents the modification factor of the rubber, and \( S \) represents the previously defined shape factor. The modification factor is dependent on the hardness of the rubber compound. Throughout the modeling presented in the following sections, it is important to pay close attention to the compression modulus and its dependence on shape factor. A secondary concern in regards to the material properties of rubber is the nonlinear relationship between stress and strain at high deformations.
For simplification purposes, it will be assumed that all models in this paper are experiencing low deformations. By using this assumption, the linear range of the stress strain curve will be sufficient for modeling the material properties of rubber.

### 3.2 Modeling of a Highly Damped Rubber Bearing

The first model that was built was of a highly damped natural rubber bearing. The initial design parameters were obtained from the Earthquake Engineering Handbook and were used as a guideline for future models [24]. The following sections describe the model geometry and the different types of analyses that were performed.

#### 3.2.1 Model Geometry and Material Properties

The first model consisted of a round elastomeric bearing with twelve rubber layers and eleven steel layers. A diameter of .7 m was used as well as a thickness of .01 m for the rubber and .002 m for the steel. A base plated is located at the top and bottom of the bearing and has a thickness of .025 m. The mass of 137356 kg, which was applied in all of the models, was based on the amount of mass that an individual bearing would need to support. Figure 5 shows the initial geometry that was built using the ANSYS Workbench solid modeling application. The majority of the rubber material properties were obtained from Table 2 which shows material properties for rubber with various hardness values. An elastic modulus of 4.45 MPa and a Poisson's Ratio of .49999 were selected. Due to the unique behavior of rubber, described above, separate shear and vertical models had to be built in order to accurately model the behavior of the bearing. The shear modulus from Table 2 could not be used for this model since it results in a Poisson's Ratio that is greater than .5. ANSYS Workbench will not allow for a Poisson's Ratio that is greater than .5 so the shear modulus of rubber was calculated using the following relationship:
Equation 3: Relationship between Shear Modulus, Elastic Modulus, and Poisson's Ratio

\[ G = \frac{E}{2(1 + \nu)} \]

In this equation \( E \) represents the elastic modulus of rubber and \( \nu \) represents the Poisson's Ratio of the rubber. The resulting shear modulus was calculated to be 1.483 MPa. This value and Poisson's Ratio will be used when defining the material properties of rubber for the shear loading case. Since the elastic modulus doesn't apply when the rubber is under compressive loading, a compressive modulus was calculated using Equation 2 shown above in the previous section. The resulting compressive modulus was calculated to be 1.558 GPa. This value, along with Poisson's Ratio, will be used to define the material properties of rubber for the compressive loading case. A summary of the properties used for this model is contained in Table 1 shown below.

<table>
<thead>
<tr>
<th><strong>Parameter</strong></th>
<th><strong>Value</strong></th>
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<td>Rubber Thickness</td>
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<td>Steel Thickness</td>
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<td>Base Plate Thickness</td>
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<td>Number of Rubber Layers</td>
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<td>Building Mass</td>
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<tr>
<td>Rubber Elastic Modulus</td>
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<td>Rubber Compression Modulus</td>
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<tr>
<td>Rubber Shear Modulus</td>
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<tr>
<td>Rubber Poisson's Ratio</td>
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<td>Steel Elastic Modulus</td>
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Table 2: Hardness and Elastic Modulus [25]

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<td>0.81</td>
<td>0.64</td>
<td>1090</td>
</tr>
<tr>
<td>60</td>
<td>4.45</td>
<td>1.06</td>
<td>0.57</td>
<td>1150</td>
</tr>
<tr>
<td>65</td>
<td>5.85</td>
<td>1.37</td>
<td>0.54</td>
<td>1210</td>
</tr>
<tr>
<td>70</td>
<td>7.35</td>
<td>1.73</td>
<td>0.53</td>
<td>1270</td>
</tr>
<tr>
<td>75</td>
<td>9.40</td>
<td>2.22</td>
<td>0.52</td>
<td>1330</td>
</tr>
</tbody>
</table>

3.2.2 Static Modeling of Vertical Stiffness

The first analysis that was performed in ANSYS Workbench was a static model and it was used to determine the vertical stiffness of the elastomeric bearing. Since the bearing will be under a compressive load, the compressive modulus discussed above will be used to define the material properties of the rubber. An equivalent mass of a building was distributed over the top base plate of the bearing and a standard earth gravity load was applied. The bottom surface of the bottom base plate was fixed to prevent any motion. The vertical deflection of the bearing was determined and used to calculate the effective vertical spring constant. This vertical spring constant was then used to calculate the theoretical vertical natural frequency of the bearing using the following equation:

\[
 f_V = \frac{1}{2\pi} \sqrt{\frac{K_V}{M_B}}
\]

In this equation \(K_V\) represents the effective vertical spring constant of the bearing and \(M_B\) represents the equivalent mass of the building. The resulting vertical natural frequency was calculated to be 114.651 Hz. When comparing this value to the calculated vertical natural
frequency of the bearing, one important observation can be made. The following equation represents the approximation of vertical natural frequency of an elastomeric bearing:

\[ f_V = \frac{1}{2\pi} \sqrt{\frac{E_c \pi d^2}{4N R t_R M_B}} \]

The resulting vertical natural frequency calculated from this equation is 30.356 Hz. At first glance, this appears to be significantly different from that obtained from the ANSYS simulation. The key difference between these two numbers is that the equation does not account for the bonding between each layer of rubber and steel. Whereas in the ANSYS model and in real applications, each rubber layer will be bonded to their respective steel layers. In order to check this, another similar ANSYS simulation was performed. All parameters were left the same, only this time, the rubber was allowed to slide apart from the steel layers. This analysis provided a vertical natural frequency of 30.043 Hz, thus confirming this theory.

### 3.2.3 Static Modeling of Horizontal Stiffness

The second analysis that was performed in ANSYS Workbench was also a static model and was used to determine the horizontal stiffness of the elastomeric bearing. Since this bearing will be under a shear load, the shear modulus will be used to define the material properties of the rubber.

The same mass used in the first model was applied to the top base plate of the bearing and the bottom base plate was fixed to prevent any motion. This time an acceleration load equal to gravity was applied in the horizontal direction. The horizontal deflection of the bearing was determined and used to calculate the effective horizontal spring constant. This horizontal spring constant was then used to calculate the theoretical horizontal natural frequency of the bearing using the following equation:

\[ f_H = \frac{1}{2\pi} \sqrt{\frac{K_H}{M_B}} \]
In this equation $K_H$ represents the effective horizontal spring constant of the bearing and $M_B$ represents the equivalent mass of the building. The resulting horizontal natural frequency was calculated to be .92182 Hz. This value was then compared to that obtained from the equation for horizontal natural frequency of an elastomeric bearing. The following equation was used for this calculation:

\[
    f_H = \frac{1}{2\pi} \sqrt{\frac{G\pi d^2}{4N_R t_R M_B}}
\]

This equation is very similar to that for vertical natural frequency, with the exception that it uses the shear modulus of elasticity. Unlike the equation for vertical natural frequency, this one does accurately model the natural frequency of an elastomeric bearing. The horizontal natural frequency that was calculated from this equation was determined to be .93663 Hz. This value is very close to the horizontal natural frequency obtained from the ANSYS simulation. The small discrepancy is likely a result of the coarse mesh size that was used in the ANSYS model.

### 3.2.4 Modal Analysis

The third analysis that was performed in ANSYS Workbench was a modal analysis that was used to determine the natural frequencies of the same elastomeric bearing. The same mass was applied to the top base plate and the bottom base plate was fixed to prevent any motion. To ensure that the horizontal and vertical frequencies were obtained the top base plate was constrained to prevent any rotation. This was used to prevent ANSYS from picking up a rocking mode shape. The analysis was set to calculate the first six modes of the bearing. The first two modes showed the natural frequencies of the bearing in the two horizontal directions. These frequencies came out to be .92140 Hz and .92187 Hz. The third mode showed the natural frequency of the bearing in the vertical direction. The resulting vertical frequency was 114.64 Hz. When comparing the frequencies obtained in the modal analysis to that calculated in both static analyses, it is clear that they are very similar. The calculated horizontal natural frequency was only very slightly off of the modal analysis and the calculated vertical natural frequency was identical to that obtained in the modal analysis. These values were also used as a way to double
check that the ANSYS model was working properly. All the frequency values obtained through this analysis are summarized in Table 3 below. The remaining three modes were significantly higher and showed internal modes that were considered less important for this analysis. The fourth mode, at 647.74 Hz, showed the expansion and contraction of the bearing in the radial direction. Modes five and six both showed the rubber vibrating, while the top and bottom base plates remained stationary.

Table 3: First Six Modes for the Highly Damped Rubber Bearing

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Description of Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.92140</td>
<td>Shear</td>
</tr>
<tr>
<td>2</td>
<td>0.92187</td>
<td>Shear</td>
</tr>
<tr>
<td>3</td>
<td>114.64000</td>
<td>Vertical</td>
</tr>
<tr>
<td>4</td>
<td>647.74000</td>
<td>Breathing</td>
</tr>
<tr>
<td>5</td>
<td>2335.60000</td>
<td>Breathing</td>
</tr>
<tr>
<td>6</td>
<td>2337.70000</td>
<td>Breathing</td>
</tr>
</tbody>
</table>

3.2.5 Harmonic Response

The fourth analysis that was performed on the highly damped rubber bearing was a harmonic response. This was used to obtain a frequency response for the bearing over a set range of frequencies. The same model was used for this analysis as in the modal analysis, with one exception. In the harmonic response model, a vertical excitation force of 100 N was applied to the top surface of the bearing. This analysis was performed twice. The first analysis was used to capture the horizontal frequency using the shear modulus and the second was used to capture the vertical frequency using the compression modulus. The first analysis was performed over the range of 0 to 2 Hz and the second analysis was performed from 100 to 120 Hz. Figure 6 and Figure 7 show the horizontal and vertical frequency response spectrums that were obtained from this analysis. As expected, there are two clear peaks in amplitude. These two peaks occur at approximate frequency values of .92 Hz and 114.5 Hz. These results also match the results obtained in the previous static and modal analyses.
3.3 Modeling of a Lead Plug Bearing

The second model that was built was of a lead plug bearing. As in the first example, the initial design parameters were obtained from the Earthquake Engineering Handbook [24]. The following sections describe the model geometry and four types of analysis that were performed.

3.3.1 Model Geometry

The second model consisted of a round elastomeric bearing with a lead plug in the center. The bearing consisted of 42 rubber layers and 41 steel layers. A diameter of .7 m was used as well as a thickness of .01 m for the rubber and .002 m for the steel. The diameter of the central lead plug was .13 m and the height of the lead plug spanned the entire height of the bearing. A base plated is located at the top and bottom of the bearing and has a thickness of .025 m. The mass of 137356 kg, which was applied in all of the models, was based on the amount of mass that an individual bearing would need to support. Figure 8 shows the initial solid model of the lead plug.
bearing that was built using the ANSYS solid modeling application. The rubber material properties used for this model were the same as that used in the previous models. The compression modulus was used for all vertical load cases and the shear modulus was used for all of the horizontal load cases. The lead material properties used for this model were obtained from online research [26]. An elastic modulus of 14 GPa and a Poisson’s ratio of .42 were selected. From this a shear modulus of 4.93 GPa was calculated using Equation 3. A summary of the properties used in this model is shown below in Table 4.

Table 4: Geometric Properties and Material Properties for Lead Plug Bearing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.7 m</td>
</tr>
<tr>
<td>Rubber Thickness</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Steel Thickness</td>
<td>0.002 m</td>
</tr>
<tr>
<td>Base Plate Thickness</td>
<td>0.025 m</td>
</tr>
<tr>
<td>Lead Plug Diameter</td>
<td>0.13 m</td>
</tr>
<tr>
<td>Number of Rubber Layers</td>
<td>42</td>
</tr>
<tr>
<td>Building Mass</td>
<td>137,356 kg</td>
</tr>
<tr>
<td>Rubber Elastic Modulus</td>
<td>4.45 MPa</td>
</tr>
<tr>
<td>Rubber Compression Modulus</td>
<td>1,558 MPa</td>
</tr>
<tr>
<td>Rubber Shear Modulus</td>
<td>1.483 MPa</td>
</tr>
<tr>
<td>Rubber Poisson’s Ratio</td>
<td>0.49999</td>
</tr>
<tr>
<td>Lead Elastic Modulus</td>
<td>14,000 MPa</td>
</tr>
<tr>
<td>Lead Poisson’s Ratio</td>
<td>0.42</td>
</tr>
<tr>
<td>Steel Elastic Modulus</td>
<td>200,000 MPa</td>
</tr>
</tbody>
</table>

3.3.2 Static Modeling of Vertical Stiffness

As with the highly damped rubber model, the first analysis that was performed in ANSYS Workbench was a static model and it was used to determine the vertical stiffness of the lead plug bearing. The compression modulus was used to define the material properties of the rubber. An equivalent mass of a building was distributed over the top base plate of the bearing and a standard earth gravity load was applied. The bottom surface of the bottom base plate was fixed.
to prevent any motion. The vertical deflection of the bearing was determined and used to calculate the effective vertical spring constant. This vertical spring constant was then used to calculate the theoretical vertical natural frequency of the bearing using Equation 4 shown in the above sections. The resulting theoretical vertical natural frequency was calculated to be 51.396 Hz. Due to the complexity of the lead plug bearing, no simple calculations could be performed to check this value. Instead this value will later be compared to the vertical natural frequency obtained from a modal analysis of the same bearing.

### 3.3.3 Static Modeling of Horizontal Stiffness

A second analysis using ANSYS Workbench was performed to determine the horizontal stiffness of the lead plug bearing. In this case the shear modulus was used to define the rubber material properties. The same mass used in the first model was applied to the top base plate of the bearing and the bottom base plate was fixed to prevent any motion. This time an acceleration load equal to gravity was applied in the horizontal direction. The horizontal deflection of the bearing was determined and used to calculate the effective horizontal spring constant. This horizontal spring constant was then used to calculate the theoretical horizontal natural frequency of the bearing using Equation 6 shown in the above sections. The resulting theoretical horizontal natural frequency was calculated to be 2.04954 Hz. As with the vertical model of the lead plug, no simple calculations could be performed to check this value. Instead this value will later be compared to the horizontal natural frequency obtained from a modal analysis of the same bearing.

### 3.3.4 Modal Analysis

The third analysis that was performed in on the lead plug bearing was a modal analysis to determine the bearing’s natural frequencies. The same mass was applied to the top base plate and the bottom base plate was fixed to prevent any motion. To ensure that the horizontal and vertical frequencies were obtained the top base plate was constrained to prevent any rotation. This was used to prevent ANSYS from picking up a rocking mode shape. The analysis was set to calculate the first six modes of the bearing. The first two modes showed the natural frequencies of the bearing in the two horizontal directions. The frequencies for these two modes were 2.0438 Hz and 2.052 Hz. The third mode showed the natural frequency of the bearing in
the vertical direction. The resulting vertical frequency was 51.417 Hz. Once again, the frequency values obtained from the modal analysis are very similar to that obtained from both the vertical and horizontal static analyses. The remaining three modes obtained from this analysis occurred at much higher frequencies. They each showed internal vibration of the rubber. Table 5 shows a summary of all the frequency values that were obtained from the modal analysis.

### Table 5: First Six Modes for the Lead Plug Bearing

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Description of Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.04380</td>
<td>Shear</td>
</tr>
<tr>
<td>2</td>
<td>2.05200</td>
<td>Shear</td>
</tr>
<tr>
<td>3</td>
<td>51.41700</td>
<td>Vertical</td>
</tr>
<tr>
<td>4</td>
<td>635.64000</td>
<td>Breathing</td>
</tr>
<tr>
<td>5</td>
<td>636.47000</td>
<td>Breathing</td>
</tr>
<tr>
<td>6</td>
<td>667.64000</td>
<td>Breathing</td>
</tr>
</tbody>
</table>

### 3.3.5 Harmonic Response

The fourth and final analysis that was performed on the lead plug bearing was a harmonic response. This was used to obtain a frequency response over a set range of frequencies for both the vertical and horizontal natural frequencies of the lead plug bearing. The same model used in the modal analysis was used for the harmonic response analysis except that a 100 N excitation force was applied to the top surface of the bearing. The analysis was performed twice to obtain the frequency response in the horizontal direction and in the vertical direction. The first analysis was performed over the range of 1.5 to 2.5 Hz and the second was performed over the range of 50 to 55 Hz. Figure 9 and Figure 10 show the frequency response spectrums that were obtained from this analysis. The two clear peaks match the horizontal and vertical natural frequency values that were obtained from the previous modal and static analyses. The horizontal peak is located around 2.05 Hz and the vertical peak is located around 51.4 Hz.
3.4 Comparison of Results

When comparing the results of the highly damped rubber bearing and the lead plug bearing it is important to note the differences in geometry. Both bearings have the same diameter, same rubber thickness, and same steel thickness. The major difference is in the number of layers. The lead plug bearing consists of three and a half times more layers than the highly damped rubber bearing. When adding more layers to the highly damped natural rubber bearing it is expected that both the horizontal and vertical natural frequencies would decrease significantly. The lead plug bearing does have a vertical natural frequency that is nearly half that of the highly damped rubber bearing, but it also has a horizontal natural frequency that is more than double that of the highly damped rubber bearing. This suggests that the lead plug provides a large amount of horizontal stiffness that is not present with only the rubber and steel layers.
Chapter 4: Analysis of Design Parameters for the HDR Bearing

The following section describes analysis that was performed using ANSYS Workbench to determine the effects of several design parameters on the vertical and horizontal natural frequency of the highly damped natural rubber elastomeric bearing. The purpose of this analysis was to determine what design parameters would have the most significant impact on the vertical and horizontal natural frequency of the bearing. As mentioned previously, the goal of an elastomeric bearing is to produce a vertically stiff and horizontally flexible isolator. This analysis will then be used to determine which parameters will create an isolator that is horizontally flexible and vertically stiff. In order to meet this requirement, a target horizontal natural frequency below .5 Hz and a target vertical natural frequency above 50 Hz will be used to categorize a successful isolator. The parameters that will be focused upon throughout this analysis include bearing diameter, rubber thickness, steel thickness, and number of layers. The same approach that was used in the previous vertical and horizontal static analyses will be used for the extent of this analysis. In addition to this, the reference geometry and material properties used throughout this analysis will be the same as that used above. Table 6 shows the current parameters that were obtained before the optimization analysis was performed.

<table>
<thead>
<tr>
<th>Original Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.7 m</td>
</tr>
<tr>
<td>Rubber Thickness</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Steel Thickness</td>
<td>0.002 m</td>
</tr>
<tr>
<td>Base Plate Thickness</td>
<td>0.025 m</td>
</tr>
<tr>
<td>Number of Rubber Layers</td>
<td>12</td>
</tr>
<tr>
<td>Building Mass</td>
<td>137,356 kg</td>
</tr>
<tr>
<td>Rubber Elastic Modulus</td>
<td>4.45 MPa</td>
</tr>
<tr>
<td>Rubber Compression Modulus</td>
<td>1,558 MPa</td>
</tr>
<tr>
<td>Rubber Shear Modulus</td>
<td>1.483 MPa</td>
</tr>
<tr>
<td>Rubber Poisson's Ratio</td>
<td>0.49999</td>
</tr>
<tr>
<td>Steel Elastic Modulus</td>
<td>200,000 MPa</td>
</tr>
<tr>
<td>Horizontal Frequency</td>
<td>0.92182 Hz</td>
</tr>
<tr>
<td>Vertical Frequency</td>
<td>114.651 Hz</td>
</tr>
</tbody>
</table>
4.1 Effect of Diameter on Natural Frequency

The first parameter that was analyzed for the highly damped rubber bearing was diameter. The ANSYS Workbench parameter application was used to create design points which varied the diameter of the bearing from .5 m to 1 m in .05 m increments. It is important to note that as the diameter is varied, the compression modulus of the rubber will also vary. This will have an impact on the vertical frequency of the bearing but will not affect the horizontal frequency. All other parameters remained constant. These data points were then used to construct a scatter plot of the data. Figure 11 shows the plot that was created. The left vertical axis shows the vertical natural frequency and the right vertical axis shows the horizontal natural frequency. The horizontal axis shows the diameter of the bearing.

![Effect of Diameter on Vertical and Horizontal Natural Frequency](image)

**Figure 11: Plot Showing Effect of Diameter on Vertical and Horizontal Frequency**

From this plot, it is clear that as the diameter of the bearing increases, the vertical and horizontal frequency both increase following a roughly linear pattern. When looking to obtain a target horizontal natural frequency below .5 Hz it is clear that the optimal diameter will be below .6 m. The entire range of vertical natural frequencies shown on this graph does meet the target vertical frequency requirement of 50 Hz, but it is suggested that the largest vertical frequency possible
should be desired. As the vertical natural frequency decreases, the vertical compression stiffness of the bearing will also decrease. This may result in a bearing that is incapable of supporting the high compressive loads generated by the building. This may be of particular concern when selecting a diameter below the range shown on the graph.

4.2 Effect of Rubber Thickness on Natural Frequency

The next parameter that was analyzed for the HDR bearing was rubber thickness. The ANSYS Workbench parameter application was once again used to create eleven design points for various rubber thicknesses. The rubber thickness was varied from .0025 mm to .0275 m in .0025 m increments. As with the first case, as the rubber thickness is varied, the compression modulus of the rubber will also vary. This will once again have an impact on the vertical frequency of the bearing but will not impact the horizontal frequency. All other design parameters were held constant. These data points were then used to construct a scatter plot shown in Figure 12 below. The left vertical axis shows the vertical natural frequency and the right vertical axis shows the horizontal natural frequency. Rubber thickness is shown on the horizontal axis.

![Effect of Rubber Thickness on Vertical and Horizontal Frequency](image)

Figure 12: Plot Showing Effect of Rubber Thickness on Vertical and Horizontal Frequency
From this plot, it is clear that varying rubber thickness has a very significant impact on both vertical and horizontal natural frequency. As the rubber thickness is increased, the vertical and horizontal natural frequencies both decrease significantly. As the thickness of the rubber approaches zero, it is expected that the stiffness of the steel will dominate the overall stiffness of the bearing. This increase in stiffness will lead to an increase in both vertical and horizontal natural frequencies as shown in the plot above. Out of all the design parameters that were considered, rubber thickness seemed to have the most dramatic impact on the natural frequencies of the bearing. In this case, to obtain a target horizontal natural frequency of .5 Hz, a rubber thickness that is outside the upper range of the plot should be selected. The lowest horizontal natural frequency shown on the above graph is around .55 Hz for a rubber thickness of .0275 m. Although this is close to the target horizontal frequency, it appears that selecting a rubber thickness larger than .03 m (not shown on the graph) would better meet the target frequency. The graph above also shows that a rubber thickness of .02 m or smaller will meet the target vertical frequency of 50 Hz. Based on the graph, it is impossible to select a rubber thickness that meets both the target horizontal and target vertical natural frequency that was selected. This suggests that varying rubber thickness alone is not the best way to achieve the desired vertical and horizontal natural frequency for this elastomeric bearing.

4.3 Effect of Steel Thickness on Natural Frequency

The third parameter that was analyzed for the HDR bearing was steel thickness. As in the previous cases, the ANSYS Workbench parameter application was used to create eleven design points using various steel thicknesses. The steel thickness was varied from .001 m to .0035 m in .000125 m increments. All other design parameters were held constant. Using these design points a scatter plot was created to analyze the effects of steel thickness on vertical and horizontal frequency. Figure 13 shows the scatter plot that was created. The left vertical axis shows the vertical natural frequency and the right vertical axis shows the horizontal natural frequency. The horizontal axis shows the range of steel thicknesses that was selected for this bearing. From this graph, it is clear that varying steel thickness has a minimal effect on both vertical and horizontal natural frequency. Both the vertical and horizontal frequencies increase as steel thickness increases, but by a minimal amount. Over the selected range of steel
thicknesses, the horizontal frequency of the bearing only varies from a minimum value of .9211 Hz to a maximum value of .9222 Hz. Similarly, the vertical frequency only varies from a minimum value of 100 Hz to a maximum value of 125 Hz. Out of the four parameters selected, steel thickness had the smallest effect frequency.

![Effect of Steel Thickness on Vertical and Horizontal Natural Frequency](image)

*Figure 13: Plot Showing Effect of Steel Thickness on Vertical and Horizontal Frequency*

When analyzing the graph, it is clear the entire range of steel thicknesses provides a vertical frequency that is above the target frequency of 50 Hz. On the other hand, none of the selected steel thicknesses will provide a horizontal frequency below the target frequency of .5 Hz. It doesn't appear that any reasonable values for steel thickness will meet the target horizontal frequency. As a result, steel thickness does not appear to be the most effective parameter for obtaining the target frequency values for this bearing.

### 4.4 Effect of Number of Layers on Natural Frequency

The final parameter that was analyzed for the HDR bearing was number of layers. Eleven design points were created using the ANSYS Workbench parameter application. The number of steel
layers was varied from five layers to fifteen layers in one layer increments. All other design parameters were once again held constant. This data was then used to construct a scatter plot shown in Figure 14 below. The left vertical axis shows the vertical frequency of the bearing and the right vertical axis shows the horizontal frequency of the bearing. The horizontal axis shows the number of steel layers. When looking at the graph, it is clear that the number of layers has a significant impact on both vertical and horizontal frequency. As the number of layers is increased, both the vertical and horizontal frequencies decrease following a slightly non-linear trend. The vertical frequency varies from a maximum value of 180 Hz to a minimum value of 95 Hz. Similarly the horizontal frequency varies from a maximum value of 1.3 Hz to a minimum value of .8 Hz.

![Graph showing effect of number of layers on vertical and horizontal frequency](image)

**Figure 14:** Plot Showing Effect of Number of Layers on Vertical and Horizontal Frequency

It appears that none of the horizontal frequency values in this range will meet the target frequency of .5 Hz. It does appear, however, that the horizontal frequency will eventually meet the target frequency of .5 Hz if the number of layers were increased further. The entire range of vertical frequencies shown in the graph does meet the vertical natural frequency requirement of
50 Hz. As long as the trend shown in the graph continues, it appears that there will be an optimum number of layers that will meet both the horizontal frequency requirement and the vertical frequency requirement.

4.5 Effects of Multiple Parameters on Natural Frequency

The final step in the analysis of the HDR elastomeric bearing was to determine the effects of multiple parameters on natural frequency. Three out of the four parameters shown above were selected, in combinations of two, and their effects were analyzed. Steel thickness was excluded due to its minimal impact on both frequencies. The first two parameters selected were rubber thickness and number of layers. The rubber thickness was varied from .005 m to .025 m in .05 m increments and the number of rubber layers were varied from six layers to fourteen layers in two layer increments. This allowed for a total of 25 design points. These design points were then used to create a three dimensional surface plot showing the effects of both of these parameters.

![Effect of Rubber Thickness and Number of Rubber Layers on Horizontal Natural Frequency](image)

Figure 15: Plot Showing Effects of Rubber Thickness and # of Layers on Horizontal Frequency
Figure 15 above shows the surface plot that was created. The vertical axis shows horizontal frequency, the horizontal axis shows rubber thickness, and the depth axis shows the number of layers. From this graph similar effects can be seen as in the two dimensional analysis. As the rubber thickness is increased the horizontal frequency decreases. Similarly as the number of layers is increased, the horizontal frequency also decreases.

This same process was used for the next combination of parameters. The diameter of the bearing was varied from .5 m to .9 m in .1 m increments and the number of rubber layers were varied from six layers to fourteen layers in two layers increments. This once again allowed for a total of 25 design points. These design points were then used to construct the surface plot shown in Figure 16.

![Figure 16: Plot Showing Effects of Diameter and # of Layers on Horizontal Natural Frequency](image)

The vertical axis shows horizontal frequency, the horizontal axis shows the bearing diameter, and the depth axis shows the number of rubber layers. The graph shows that as diameter is increased the horizontal frequency also increases. As the number of layers is increased the
horizontal frequency decreases. These results are consistent with those found from the two dimensional analysis.

Once again, the same process was used for the final combination of parameters. The rubber thickness was varied from .005 m to .025 m in .05 m increments and the diameter of the bearing was varied from .5 m to .9 m in .1 m increments. The 25 design points obtained were then used to construct the surface plot shown in Figure 17.

![Effect of Rubber Thickness and Diameter on Horizontal Natural Frequency](image)

The vertical axis shows horizontal frequency, the horizontal axis shows rubber thickness, and the depth axis shows the diameter of the bearing. As the rubber thickness is increased, the horizontal frequency decreases following a non-linear trend. As the diameter is increased, the horizontal frequency increases following what appears to be a linear trend. These results are once again consistent with that obtained from the two dimensional analysis.
In summary, the following observations were made from the optimization analysis for the HDR bearing:

- Varying bearing diameter has a significant impact on both the vertical and horizontal natural frequencies of the highly damped rubber bearing. As the diameter is increased, the vertical and horizontal natural frequencies both increase following a linear trend.
- Varying rubber thickness also has a significant impact on both vertical and horizontal natural frequencies of a HDR bearing. As the rubber thickness is increased, the vertical and horizontal natural frequencies both decrease following a nonlinear trend.
- Varying steel thickness has a very minimal impact on the vertical and horizontal natural frequencies of a HDR bearing. As the steel thickness is increased, the vertical and horizontal natural frequencies both increase by a very small amount.
- Varying number of layers has a significant impact on both vertical and horizontal natural frequency of an HDR bearing. As number of layers is increased, the vertical and horizontal natural frequency decrease following a nonlinear trend.

Chapter 5: Proof of Concept

Using the optimized results in the previous section, the un-isolated and isolated seismic response of a building was modeled in ANSYS Workbench. The purpose of this model is to gain a better understanding of the effects of the elastomeric bearing isolation system in a real life application. The un-isolated model will be used to determine the fixed base natural frequencies of a building. Then the optimized parameters will be inserted into the same model to determine the effects of the isolation system on the natural frequency of the building. The following sections describe the modeling that was performed and the results that were obtained.

5.1 Model Parameters

In order to model the response of an un-isolated and isolated building, a two dimensional modal analysis was performed in ANSYS Workbench. A five story building was used with the following parameters. Each floor was 3.073 m tall, 6.096 m wide, and had a mass of 40,000 kg. The mass of each floor was obtained from an example in the Earthquake Engineering Handbook [24]. Each floor was held up by beam element connections. These beams act as the columns for each floor and were given a radius of .127 m. Cross beam elements with a radius of .0337 m
were also used to add stiffness to the structure in the horizontal direction. Figure 18 shows the two dimensional model that was built. The mass of each floor is represented by the sphere placed at the center of each floor. The floors are represented by the blue horizontal lines. The diagonal lines show the cross bracing that was added and the vertical lines show the columns that connect each floor.

Figure 18: Two Dimensional ANSYS Model of a Five Story Building

5.2 Un-isolated Results

The un-isolated model that was built in ANSYS Workbench was used to determine the fixed base frequencies a five story building. The two frequencies of interest were the horizontal and vertical natural frequencies. The model shown in Figure 18 was used and the bottom floor was fixed to simulate a fixed base. A modal analysis was then performed to
solve for the six lowest natural frequencies. Out of these six frequencies, the two frequencies of interest were analyzed. The horizontal natural frequency of the building was determined to be 1.9945 Hz and the vertical natural frequency of the building was determined to be 17.615 Hz. The horizontal frequency represented in Figure 19 shows the building swaying back and forth. The horizontal period of the structure was then calculated to be .5014 s.

This value was then compared to the fundamental period shown in Figure 20. This figure shows the approximate fundamental period of several buildings of various heights. For the building used in this model (approximately 50 feet tall) the figure shows that the approximate horizontal fundamental period should be around .5 Hz. This is consistent with the fundamental horizontal period obtained from the ANSYS model. This model will later be compared to the results obtained from the isolated building. A harmonic response analysis was also performed to analyze the amplitude of displacement for both the vertical and horizontal frequencies. Figure 21 and Figure 22 show the frequency response plots that were obtained from this analysis. For the vertical frequency response, a 100 N vertical excitation force was applied to the model and the frequency response was obtained over the range of 17 Hz to 18 Hz. The frequency peak, which occurred around 17.4 Hz has a max displacement amplitude of .000321 mm.
Figure 21: Un-isolated Vertical Frequency Response

For the horizontal frequency response, a horizontal excitation force of 100 N was applied to the model and the frequency response was obtained over the range of 1.5 Hz to 2.5 Hz. The frequency peak which occurred around 2 Hz has a max displacement amplitude of 1.18 mm.

Figure 22: Un-isolated Horizontal Frequency Response

5.3 Isolated Results

The isolated model that was built in ANSYS Workbench was used to determine the impact of an isolation system on the natural frequencies of a five story building. The same model used in the un-isolated case was used in this case with one exception. In this case, the base of the structure was not fixed. Instead vertical and horizontal springs were attached at the base of the structure to provide support and to seismically isolate the structure. The vertical and horizontal spring constants were determined based on the optimized parameters selected from the analysis performed in Chapter 4. As discussed in Chapter 4 Section 4.1, the optimum vertical and horizontal frequencies would be obtained from a diameter of .6 m. This should create an isolator
that allows the base of the building to be horizontally flexible and vertically stiff. Based on the horizontal natural frequency of this isolator a horizontal stiffness of 3,378,116 N/m should be inserted at the base of the structure. Similarly, based on the vertical natural frequency of the isolator a vertical stiffness of 45,146,352,078 N/m should be inserted at the base.

Figure 23 shows the model with the vertical and horizontal springs at the base of the building. Then, using a modal analysis, the horizontal and vertical natural frequencies of the isolated structure were determined. The resulting horizontal natural frequency was .77431 Hz and the resulting vertical natural frequency was 16.46 Hz. The horizontal frequency showed the building sliding uniformly side to side. This is important because the swaying motion seen in the un-isolated analysis is no longer evident. This is desirable, and a clear advantage of using elastomeric bearings, because it significantly reduces the stress in the columns due to bending. In addition, the horizontal frequency is significantly lower than that of the un-isolated building. The vertical frequency, on the other hand, is reasonably close to that obtained from the un-isolated model. This shows that the selected vertical stiffness does in fact create an isolator that is vertically stiff. In order to further analyze the effects of the isolators, a harmonic response analysis was performed and compared to the un-isolated building. Figure 24 and Figure 25 show the frequency response plots that were obtained from this analysis. As in the un-isolated case a 100 N vertical excitation force was applied to the model to obtain the vertical frequency response. The harmonic response was performed over the frequency range of 16 Hz to 17 Hz and the frequency peak occurred around 16.475 Hz. The max amplitude of vertical displacement in this case was .000506 m, which is slightly higher than that obtained from the un-isolated analysis.
For the horizontal frequency response the same 100 N horizontal force was applied to the model, as in the un-isolated case. The harmonic response was performed over the range of 0 Hz to 1 Hz and the frequency peak occurred around .76 Hz. The max amplitude of horizontal displacement was found to be 1.07 mm, which is slightly lower than that obtained from the un-isolated case.
Chapter 6: Conclusion

The goal of this thesis was determine the effects of several geometric parameters on the natural frequencies of a highly damped natural rubber elastomeric bearing and to evaluate the effects of this elastomeric when used in application. It was determined, through extensive modeling in ANSYS workbench, that the diameter, rubber thickness, steel thickness, and number of layers have various effects on the vertical and horizontal natural frequencies of an elastomeric bearing. Each of these parameters had a significant impact on the natural frequencies with the exception of steel thickness, which had very little impact. Using the graphs obtained from this analysis, one could select the optimum parameters necessary to meet specific vertical and horizontal frequency requirements.

The benefits of this optimization method were clearly evident when comparing the un-isolated and isolated five story building. By selecting the optimum parameters, the isolated building response showed significant improvements over the un-isolated building response. The isolated building response showed a significantly lower horizontal frequency, which eliminated the swaying of the building. In addition to this, the vertical frequency of the isolated building was very similar to that of the un-isolated one. These results confirm the goal of an elastomeric bearing which is to reduce the horizontal natural frequency of the structure, while maintaining the vertical stiffness needed to support the weight of the structure.

Further research should be performed to analyze the effects of a wider range of parameters. These parameters could include rubber material properties such as elastic modulus, shear modulus, and other damping effects. Similar analysis could also be used to determine the effects of various types of steel and steel properties. In addition, it would be beneficial to include the damping effects of the rubber in the ANSYS model. Damping effects were not included in this modeling due to the added complexity, but may provide a more realistic model.
References


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- Designed an efficient vertical axis wind turbine.
- Used Matlab/Simulink to determine a building's response to an earthquake.
- Utilized the Bernoulli Equation and the Navier Stokes Equations to design an irrigation system for a garden.
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