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SCHREYER HONORS COLLEGE

DEPARTMENT OF FINANCE

THE IMPACT OF UTILIZING OPTION IN RETIREMENT INVESTING

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ABSTRACT

As people are retiring early in order to add more values in their later lives, the concept of retirement investing is, today, a great concern for millions and billions of people around the world. Common retirement funds provided in companies, such as, Vanguard, Fidelity and Blackrock, focus their main allocation on market indices and diversify their portfolios with international stocks and bonds. The financial crisis in 2008 inflicted a significant recession in the U.S. economy and a greater uncertainty towards investing in market indices, such as the S&P 500 index. Past academic research reports that a safer investment strategy can be achieved by combining two financial instruments: call options and treasury bills. This paper examines various comparisons between the new investment method and other retirement strategies. I use simulation of hypothetical S&P 500 index prices to compare the value of investment from different strategies. The results from the simulation and its analysis show that the use of treasury bills and call options provides downside protection at the expense of slightly reduced upside potential. Regarding the riskiness, this new investment method for retirement becomes an attractive strategy for risk-averse investors.
# TABLE OF CONTENTS

LIST OF FIGURES .................................................................................................................iii

LIST OF TABLES ........................................................................................................................iv

Chapter 1: Hypothesis (Z. Bodie) ...........................................................................................1

Chapter 2: Literature Review ...............................................................................................3

Chapter 3: Simulation and Analysis .......................................................................................5

Chapter 4: Conclusion .........................................................................................................15

Appendix A: Black-Scholes Model ......................................................................................16

Appendix B: Data ....................................................................................................................18
LIST OF FIGURES

Figure 1-1: Comparison of Strategies ................................................................. 9
Figure 1-2: Comparison of Strategies (Focused) .................................................. 11
Figure 1-3: Percentile Graphs ............................................................................ 13
LIST OF TABLES

Table 1-1: Summary of Strategies ................................................................. 6
Table 1-2: Percentage of Insurance ................................................................. 7
Table 1-3: 0.05 Percentile Values ................................................................. 14
Table 1-4: Summary of Black-Scholes Model ............................................. 16
Chapter 1

Hypothesis (Z. Bodie)

Despite the huge downturn of U.S. stock market in 2008, investors are venturing back into equities again. The Federal Reserve has announced that it tends to keep its interest rate low through 2014 to induce investing in stocks. Since market indices, such as the S&P 500 index provides superior returns, investors who experienced huge losses in 2008 are eager to “catch up” by investing heavily on equities. People are lulled into thinking that buying and holding stocks for a long period of time greatly reduces the risk. However, stocks are still risky no matter how long the holding period is. No one can predict the market’s movement, and one severe downturn has the ability to destroy an individual’s investment that has been growing for years. Therefore, it is essential for an individual to strictly define how much money he or she must have for his or her retirement. And, protecting and ensuring that money becomes extremely important. As explained before, heavy allocation on equities exposes the portfolio to relatively huge risk and cannot guarantee the safety of the investment.

In his paper, Z. Bodie, Professor of Finance at Boston University, proposes a new retirement savings product that guarantees the safety of investor’s money- the combination of treasury bills and call options. Through history, treasury bills are proved to be one of the safest securities backed by the government. By investing heavily on treasury bills, investors are able to protect their money in times of economic downturn. The role of call options is to participate in market indices when the stock market is bullish. The combination of the two financial instruments allows the investor to possess downside protection at the expense of slightly reduced upside potential. In terms of allocation, a large portion must be in treasury bills to guarantee a certain amount at the end of the investing period. Then, the rest of the investment can participate in the market indices by purchasing call options on the S&P 500 index.

Z. Bodie’s paper focuses on an individual and determines a certain amount that the individual needs when he or she retires. Then, Z. Bodie examines whether the different investment strategies, including the combination of treasury bills and call options, meet the target that has been
previously established. Moreover, Z. Bodie extends by proposing how this new way of investing can play an important role as an investment alternative in defined contribution plans.

In my paper, I aim to approach Z. Bodie’s thesis in a different way. Rather than defining an individual and his or her needs, I assume a certain amount for the initial investment- $1000. I utilize different and more up-to-date dataset in my analysis, such as, randomly generated returns of the S&P 500 index and historical average of risk-free returns. Moreover, I divide my analysis into three time periods: 10-year, 20-year and 30-year. Since individuals have their own definition of long-term investment, it is important to analyze in various time periods. And, I emphasize on direct comparison between the different investing strategies through tables and graphs. As the paper continues, the main analysis is on the risk of different investments. For the aging populations worldwide, this paper can be a guide for safe retirement investing.
A significant portion of this paper involves understanding of a useful financial instrument - the option. There are two kinds of options: the European option and the American option. The difference between the two is that in a European option, the option can only be exercised at the maturity date and in an American option, it can be exercised at any time within the maturity date. For the purpose of the paper, I use European options to value investment. An option is also divided into call options and put options. A call option provides the buyer the right to exercise the option and buy the underlying commodity at the strike price on the expiration date. On the other hand, a put option gives the buyer the right to sell the underlying commodity at the strike price on the expiration date.

In valuing options, understanding the concept of put-call parity is important. It basically shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and exercise date, and vice versa. To prove this statement, consider the following portfolios:

Portfolio A: one European call option plus a zero-coupon bond providing payoff of K at time T
Portfolio B: one European put option plus one share of the stock

The assumptions are that the stock pays no dividends, and both call and put options have the same strike price K and the same time to maturity T.

**Portfolio A**
If the stock price $S_t$ at time T is above K, the call option will be exercised. Then, the value of Portfolio A becomes $(S_T - K) + K$, which is simply $S_T$. If $S_t$ at time T is below K, the call option will expire worthless, leaving the portfolio value of K.
Portfolio B

If the stock price \( S_t \) at time \( T \) is below \( K \), the put option will be exercised. Then, the value of Portfolio B becomes \( (K-S_t) + S_t \), which is simply \( K \). If \( S_t \) at time \( T \) is above \( K \), the put option will expire worthless, leaving the portfolio value of \( S_t \).

Summarizing the results above, as \( S_t > K \), both portfolios are worth \( S_t \) at time \( T \), and as \( S_t < K \), both portfolios are worth \( K \) at time \( T \). At time \( T \), the portfolios have the same values; therefore, they must have the same values in the beginning. This put-call parity can be explained by the following equation:

\[
c + Ke^{-rT} = p + S_0
\]

If this relationship does not hold, there are arbitrage opportunities in which investors can short-sell in order to make riskless profits. The put-call parity is useful in understanding Black-Scholes Model in Appendix A.

In relation to my thesis, the put-call parity theory proves that buying shares of stock and put protection is identical to buying treasury bills and a call option. Therefore, the results of this thesis are applicable to both kinds of strategies even though the paper consistently refers only to buying treasury bills and a call option.
Chapter 3

Simulation and Analysis

Simulation

I simulated 1000 trials of hypothetical S&P 500 index returns, with each trial arranged in 30-year period. These random numbers are generated by the following characteristics of the S&P 500 index: the mean return of 11% and the standard deviation of 20%. The mean is derived from the historical average of the S&P 500 index and the standard deviation represents a common market volatility of 20%. Using this simulation of returns, I am also able to simulate the hypothetical S&P 500 index prices for each trial. Based on these hypothetical prices, I can measure the options’ payoff. The starting price of the S&P 500 index is $100 per share, and the calculation follows:

Hypothetical S&P 500 index price in n years= $100 \times (1 + r_1) \times (1 + r_2) \times \cdots (1 + r_n)

n= number of years
r= S&P 500 returns

For all the calculations, I use the average treasury bills return of 3.73% and the initial investment amount of $1000.

Utilizing the data above, I test the outcomes of three different investment strategies.

Portfolio 1 consists of 100% S&P 500 index. I calculate the payoff by multiplying the initial investment amount by the product of (1+ simulated returns).

Portfolio 2 consists of 60% S&P 500 index and 40% treasury bills. Same calculation applies here.

Portfolio 3 consists of 90% treasury bills and 10% invested in call options. This third strategy contains Black-Scholes Model for option pricing, which is explained in Appendix A. The combination of Black-Scholes Options Pricing Model and the hypothetical S&P 500 index prices provide the payoff information from the options.

◊ For simplicity, the portfolios are named: Portfolio 1, Portfolio 2 and Portfolio 3.
Portfolio 3
Portfolio 3 represents an investment strategy that combines treasury bills and call options. First, I invest $900 in treasury bills growing at 3.73% for 10 years. Then, I invest $100 in call options on a share that is currently trading at a price equal to the exercise price of the option with standard deviation equal to the typical market standard deviation of 20%. I assume that the option expires in 10 years, and I repeat the steps for different time periods: 20-year and 30-year. The simulation of hypothetical S&P 500 index prices provides the information about the index price at the end of each maturity period. The difference between this simulated price and the exercise price of $100 is multiplied by the number of option purchased to calculate the payoff. The number of option purchased is simply $100 divided by the call price determined by Black-Scholes Model.

Analysis

<table>
<thead>
<tr>
<th>Table 1-1</th>
<th>100% S&amp;P500</th>
<th>60% S&amp;P/40% T-bills</th>
<th>90% T-bills/10% call options</th>
</tr>
</thead>
<tbody>
<tr>
<td>10yr</td>
<td>2603</td>
<td>2248</td>
<td>1622</td>
</tr>
<tr>
<td>20yr</td>
<td>6696</td>
<td>4989</td>
<td>6107</td>
</tr>
<tr>
<td>30yr</td>
<td>17203</td>
<td>10351</td>
<td>21504</td>
</tr>
<tr>
<td>Average amount ($)</td>
<td>2139</td>
<td>973</td>
<td>123</td>
</tr>
<tr>
<td>Median amount ($)</td>
<td>4850</td>
<td>3826</td>
<td>3664</td>
</tr>
<tr>
<td>Standard deviation ($)</td>
<td>1523</td>
<td>7417</td>
<td>12902</td>
</tr>
<tr>
<td>% below target (100% T-bills)</td>
<td>22.5</td>
<td>22.5</td>
<td>26.5</td>
</tr>
</tbody>
</table>

Table 1-1 above summarizes the outcomes of three different investment strategies. For each strategy, the results are divided into three time periods: 10-year, 20-year and 30-year.

For all three time periods, it is clear that Portfolio 1 provides the highest average and median amounts, followed by Portfolio 2 and Portfolio 3. As the time period lengthens, the difference between the portfolios significantly increases. In a 30-year period, the average amounts of the portfolios 1, 2 and 3 are $17203, $11523 and $4958 respectively. Therefore, it is true that investing heavily on market indices provides superior returns. However, there is always a trade-off associated with a high return- the risk. The standard deviation shown in the table represents how much variation exists from the average investment value. The standard deviation of 1622 in Portfolio 1 is much higher, especially when compared to the standard deviation of 398 in Portfolio 3. As Portfolio 2 possesses a significant portion in the S&P 500 index, it also shows relatively high standard deviation of 973. It is also interesting to notice that the volatility difference between the portfolios increases as the time period lengthens. For Portfolio 1, the
standard deviation significantly increases as the investing period is longer, thereby rejecting investor’s common thinking that buying and holding stocks for a long period of time greatly reduces the risk.

The last column of the Table 1-1 shows how the portfolios perform compared to a base target, which is a portfolio consisted of 100% treasury bills. This base portfolio can be considered as the safest investment strategy, with all of its allocation on securities backed by the government. Surprisingly, the percentage of the outcomes below the target is lower for Portfolio 1 and 2 than for Portfolio 3. As the time period lengthens, the difference becomes smaller. However, the result still shows that Portfolio 1 and 2 perform better when the base target is established as a portfolio of 100% treasury bills.

The analysis above shows that investing in market indices results in higher returns but higher risk. On the other hand, investing in government bills provides lower risk but lower returns. Portfolio 3 takes an advantage of treasury bill’s low risk, and benefits in times of bullish market by participating in options. Now, I directly compare the values of portfolios to each other, which is shown in Table 1-2.

**Table 1-2**

<table>
<thead>
<tr>
<th>% Insurance kicked in</th>
<th>90% T-bills/10% call options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10yr</td>
</tr>
<tr>
<td>vs. 100% S&amp;P500</td>
<td>20.4</td>
</tr>
<tr>
<td>vs. 60% S&amp;P500/40% T-bills</td>
<td>19.0</td>
</tr>
</tbody>
</table>

I characterize Portfolio 3 as an insurance against inferior market conditions. Therefore, I analyze in what percentage Portfolio 3 outperforms Portfolio 2 and 3, respectively. This also means in what percentage the insurance kicks in. In a 20-year period, 16.5% of Portfolio 3 outcomes beat Portfolio 1 outcomes. The effect of insurance decreases when compared to a portfolio of lower allocation on market indices- Portfolio 2. This is obvious since higher the allocation on market indices, higher the chance of insurance. As the time period lengthens, the percentage of insurance kicked in reduces.
The analysis above shows the effect of Portfolio 3 during times of inferior market conditions. It provides insurance when the stock market is bearish. It is important to focus on how Portfolio 3 shines against other portfolios when market indices, such as the S&P 500 index provide low or negative returns. Based on Table 1-2, it is hard to visualize the portfolios’ performance. Therefore, I arrange the outcomes into graphs with Value of Investment on Y-axis and Stock Price on X-axis.

(I excluded three data points from each graph which showed negative stock prices. Since negative stock prices are unrealistic, I removed those three outliers.)
Figure 1-1

10-Year comparison

20-year comparison

30-year comparison
The graphs in Figure 1-1 summarize the outcomes from three investment strategies based on certain stock prices. For all three portfolios, positive correlation is shown between the strategies and the stock price. Since both stock prices and the S&P 500 index returns are derived from the same randomly generated numbers, it is obvious that Portfolio 1 and 2 have a direct, positive relationship with the stock price. For Portfolio 3, the correlation does not seem as strong because only 10% of initial investment amount is allocated to call options participating in market indices. By analyzing the graphs, it is clear that Portfolio 1 provides the highest possible returns, followed by Portfolio 2 and Portfolio 3.

Now, to see the effect of Portfolio 3, I focus on the left-hand-side of the graph where the stock prices are low. This area represents times of inferior market performance and is enlarged in the following graphs.
Figure 1-2

10-Year comparison

20-year comparison

30-year comparison
In the graphs of Figure 1-2, it is obvious that Portfolio 3 outperforms Portfolio 1 and Portfolio 2 when the stock prices are low. This area of low stock prices is significant in the analysis since it reflects the poor performance by the S&P 500 index. Therefore, it is observed that Portfolio 3 provides stable returns even in times of bearish market. In all time periods, Portfolio 3’s effect of insurance is clearly portrayed. One interesting observation is that as time period lengthens, the range of stock prices in which Portfolio 3 provides insurance increases. In 10-year, 20-year and 30-year, Portfolio 3 outperforms the others up to approximate amounts of $130 per share, $195 per share and $290 per share respectively. Therefore, I assume that in longer time periods, the mix of treasury bills and call options provides insurance for larger range of hypothetical stock prices. However, in this longer time periods, the investor has to be willing to give up the higher potential of Portfolio 1, 100% S&P 500 index.

By creating a relationship between the value of investment from three different investment strategies and the hypothetical stock prices, it is clear that Portfolio 3 is able to provide safe and stable returns throughout any kind of market conditions. This analysis shows that Portfolio 3 shines the brightest when the volatility of market indices is high.

Now, I arrange the data in a different way. I divide 1000 final values of investment into percentile graphs and compare the three investment strategies.
Figure 1-3

Percentile Histogram (10-year)

Percentiles

Value of Investment ($)

Percentile Histogram (20-year)

Percentiles

Value of Investment ($)

Percentile Histogram (30-year)

Percentiles

Value of Investment ($)
The percentile graphs in Figure 1-3 show a similar pattern of observation. In the 10-year graph, Portfolio 1 has the highest values for all percentiles except 0.2, 0.1 and 0.05. In the 20-year and 30-year graphs, Portfolio 1 outperforms other portfolios for all percentiles except 0.1 and 0.05. Because the paper focuses on times of inferior market performance, I emphasize my analysis on 0.05 percentile.

<table>
<thead>
<tr>
<th>Table 1-3</th>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-year</td>
<td>784</td>
<td>1047</td>
<td>1298</td>
</tr>
<tr>
<td>20-year</td>
<td>1114</td>
<td>1501</td>
<td>1892</td>
</tr>
<tr>
<td>30-year</td>
<td>1902</td>
<td>2342</td>
<td>2828</td>
</tr>
</tbody>
</table>

According to the values in Table 1-3, Portfolio 3 has the highest values for all time periods. This shows that the downside of Portfolio 3 is well protected by allocating 90% of the initial investment into treasury bills. Portfolio 2 has 40% of its investment into treasury bills, so it has better protection than all-equity Portfolio 1.
Chapter 4

Conclusion

Based on various arrangements of the outcomes of three different investment strategies, Portfolio 3, the combination of treasury bills and call options, is the safest compared to other portfolios that invest in equities. Portfolio 3 especially shines against the others in times of inferior market returns. On the other hand, Portfolio 1, all-equity portfolio, has the potential of superior returns; however, during bearish market, the portfolio simply fails. The purpose of retirement investing focuses on how stable and safe the portfolio’s returns are over any time period. Whether the market moves up or down, Portfolio 3 continues to provide stable performance because it ensures downside protection at the expense of slightly reduced upside potential. Unlike a portfolio of 100% treasury bills, Portfolio 3 does allow participation in market indices through call options and is able to reach a higher amount. Therefore, call options can add excitement and extra income to the investor in times of bullish market. An individual can never predict the stock movements in the market, and a tremendous economic downturn like that in 2008 is bound to happen in the future. For the aging populations worldwide, their retirement portfolio should invest in combination of treasury bills and call options to possess the right amount of money that they need in the future. Investing in equities may increase the possibility of higher outcomes; however, no protection against inferior market returns exists. And, failure in achieving the target for retirement can seriously hinder a person’s life and goals. Therefore, I advise the investors to divert from equities and invest in treasury bills and call options.

One of the extensions to the paper includes finding the optimal allocation amounts in Portfolio 3. Depending on how an individual perceives risk, he or she can change the allocation amounts to fit his or her own investing style. Furthermore, as individual ages, he or she can allocate more on treasury bills and less on call options; this adjustment can be done yearly or every five years, etc. Creating a portfolio that includes the adjustments of allocation amounts can produce a different result. Moreover, a possible extension to the paper is strategic investment in options. Purchasing call options of different maturities affects the options’ payoff and may increase the value of final investment.
**Appendix A**

**Black-Scholes Model**

Black-Scholes Model suggests that the value of an option can be calculated through the following inputs: stock price, exercise price, risk-free rate, time and volatility of the market.

**Assumptions**

1. Stock price behavior corresponds to the lognormal model with \( \mu \) and \( \sigma \) constant.
2. There are no transactions costs or taxes. All securities are perfectly divisible.
3. There are no dividends on the stock during the life of the option.
4. There are no riskless arbitrage opportunities.
5. Security trading is continuous.
6. Investors can borrow or lend at the same risk-free rate of interest.
7. The short-term risk-free rate of interest, \( r \), is constant.

**Table 1-4**

<table>
<thead>
<tr>
<th>S</th>
<th>E</th>
<th>r</th>
<th>T</th>
<th>s</th>
<th>D1</th>
<th>D2</th>
<th>N(D1)</th>
<th>N(D2)</th>
<th>Call Price</th>
<th>Options investment</th>
<th># of options bought</th>
</tr>
</thead>
<tbody>
<tr>
<td>10yr</td>
<td>100</td>
<td>100</td>
<td>0.037331</td>
<td>10</td>
<td>0.2</td>
<td>0.906476</td>
<td>0.174021</td>
<td>0.817658</td>
<td>0.607966</td>
<td>39.91014</td>
<td>100</td>
</tr>
<tr>
<td>20yr</td>
<td>100</td>
<td>100</td>
<td>0.037331</td>
<td>20</td>
<td>0.2</td>
<td>1.281951</td>
<td>0.387524</td>
<td>0.900070</td>
<td>0.650816</td>
<td>59.16032</td>
<td>100</td>
</tr>
<tr>
<td>30yr</td>
<td>100</td>
<td>100</td>
<td>0.037331</td>
<td>30</td>
<td>0.2</td>
<td>1.570063</td>
<td>0.474618</td>
<td>0.9418</td>
<td>0.68247</td>
<td>71.91052</td>
<td>100</td>
</tr>
</tbody>
</table>

The figure above represents my Black-Scholes Model for valuing the option’s payoff. Five inputs - stock price, exercise price, risk-free rate, time and volatility of the market - are utilized to calculate \( D_1 \) and \( D_2 \).
The equations for D1, D2 and Call Price are as follows:

\[
D1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(\frac{r+\sigma^2}{2}\right)t}{\sigma \sqrt{t}}
\]

\[
D2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(\frac{r-\sigma^2}{2}\right)t}{\sigma \sqrt{t}} = D1 - \sigma \sqrt{t}
\]

Call price = \( S_0 N(D1) - Ke^{-rT}N(D2) \)

Put price = \( Ke^{-rT}N(-D2) - S_0 N(-D1) \)

I manually choose the current stock price (S or S0) to be $100 per share. And, the exercise price (E) is also $100 because the paper’s purpose is to exercise the option only when the market is bullish. Therefore, having an exercise price lower than $100 in order to bring more profits is not considered in the paper. The risk-free rate, r, represents the historical average of Treasury bills returns over 1926 to 2009. A risk-free rate of 3.7% is appropriate to use in the calculations. Time, T, shows the duration of the options, how long they last. I assume volatility of the stock price, s, to be the volatility of the S&P 500 index prices, which is 20%.

Through the computations, call prices for each time period are found. This determines how many options the investor can buy when I divide the options investment amount by the call price. Then, number of options bought is multiplied by the difference between the hypothetical S&P 500 index price at the maturity date and the beginning stock price, which is $100.

It is important to notice that when the hypothetical price is below $100, the call option will not be exercised at the maturity date. Therefore, no payoff comes from the options.

In the real market of options, there is no way of purchasing a fraction of an option. Investors must buy a whole amount. However, in the analysis, I do not round up the number of options bought and use the number presented in the table.
Appendix B

Data

The only historical data used in the paper is the Treasury bills returns from 1926 to 2009. My thesis advisor provided the data.

Another data is the randomly generated numbers by 30x1000. This is simply done by clicking on “data analysis” and then, “Random generation” on Excel. Then, input “Normal” for distribution and provide the mean and the standard deviation. In my case, the mean is 11% and the standard deviation is 20%. The mean is derived from historical average of the S&P 500 index returns and the standard deviation represents a common market volatility of 20%. I assume that hypothetical returns of the S&P 500 index are normal distributed.
References


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Minor in Economics

University Park, PA
Graduating May 2012

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Dean’s List for 7/7 semesters

ACTIVITIES

Penn State Investment Association
Member of Energy Sector

Korean Student For Christ
Member of Alumni Team

SKILLS

Fluency in Korean