THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

DEPARTMENT OF AEROSPACE ENGINEERING

SENSITIVITY ANALYSIS OF ONE-DIMENSIONAL DESCENT PROFILES FOR LOW ALTITUDE LUNAR LANDINGS

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Abstract

In order to find the optimal fuel-efficient way to descend from a low altitude orbit around the moon to the surface, a straight vertical drop to the surface will be modeled. This will set up a one-dimensional, two point boundary value problem that will be analyzed using optimization methods. Using basic propulsion and thrust equations, the thrust will be calculated at different times of the descent. This paper will analyze a descent with no thrusting as well as one with continuous and constant thrust. Using applied optimal control theory and inputting the differential equations into programming software, the optimal descent profile for the given thrust can then be analyzed. The variables for this feasible solution for the continuous burn will then be perturbed and the sensitivities of each will be analyzed. This analysis will be modeled similarly to the spacecraft that is being designed for the Penn State Lunar Lion project. The results from this analysis will then be used to determine the Lunar Lion's optimal descent profile to the surface of the moon. In the future, the case of throttle-able or step-function thrust can be analyzed using similar methods. In addition, the problem will be expanded to include a horizontal velocity component to make it a two-dimensional descent trajectory.

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Definitions of Variables

U_e = Exit Velocity at the End of the Thruster Nozzle [m/s]

P_e = Exit Pressure at the End of the Thruster Nozzle [Pa]

P_a = Ambient Pressure [Pa]

A_e = Exit Surface Area of the Nozzle [m^2]

I_{sp} = Specific Impulse of the Engine [s]

g = Gravitational Acceleration Near the Moon's Surface [m/s^2]

m_i = Initial Mass of the Spacecraft [kg]

m_f = Final Mass of the Spacecraft [kg]

r = Radial Distance of Spacecraft from the Moon's Center [m]

r_m = Radius of the Moon [m]

u = Radial Component of Velocity [m/s]

v = Tangential Component of Velocity [m/s]

H = Hamiltonian

τ = Thrust [N]

μ = Standard Gravitational Parameter [km^3/s^2]

$\dot{m}$ = Mass Flow Rate Through the Thruster [kg/s]

y = Altitude of spacecraft (vertical distance from Moon's surface) [m]

υ_1 = adjoint variable 1

υ_2 = adjoint variable 2

r_{tf} = Radius at the Time of Touchdown to the Moon's Surface [m]

1 Introduction

The Penn State Lunar Lion team is designing and building a spacecraft that will land on the moon. As part of the Google Lunar X Prize Competition, the Lunar Lion team has a need for analysis on the descent phase of their mission. A first step in analyzing this problem is to treat it as a one-dimensional (up/down) problem. In analyzing this descent as one-dimensional, some assumptions are also made and are discussed in this section.

A diagram of the spacecraft's motion relative to the surface of the moon during its descent phase can be seen in figure 1.

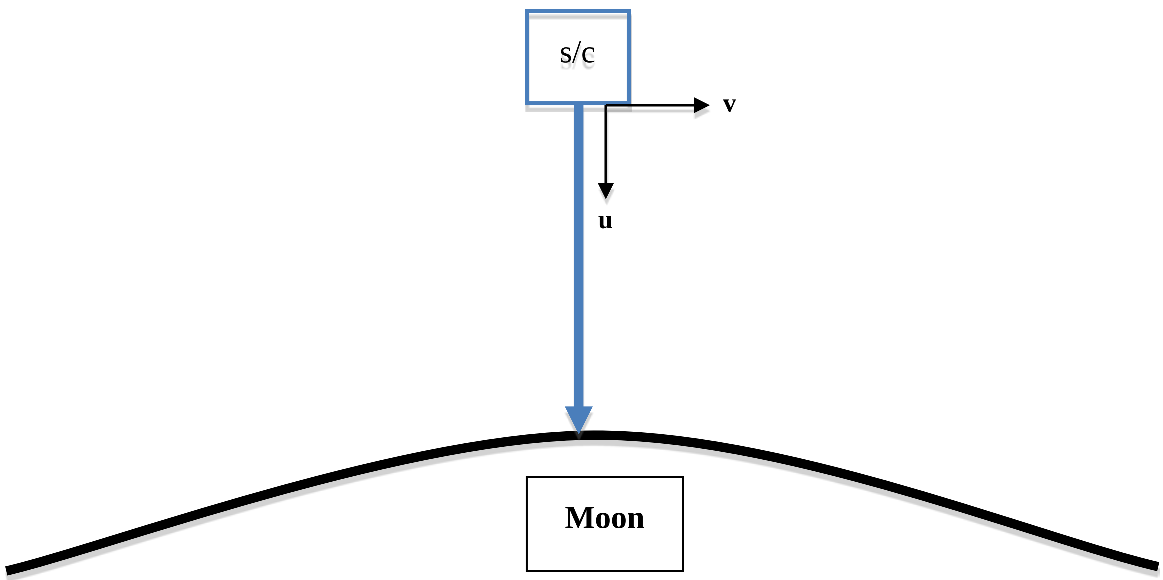


Figure 1- Mission Scenario Drawing

As Figure 1 shows, the spacecraft will drop straight down towards the moon after its orbital motion is stopped. For this problem, the radial velocity is changing with time but the tangential velocity remains constant at zero for the duration of the descent since there is no horizontal motion. This allows for a simple two-point boundary value problem to solve for the optimal descent path of the spacecraft.

1.1 Problem Statement

In order to reach the moon's surface, the spacecraft will have to de-orbit and start a descent towards the surface. In order to do this, the spacecraft must fire thrusters to slow its tangential velocity. In this thesis, the tangential velocity will actually be slowed to zero, so that the only force acting on the spacecraft is moon's gravity. The spacecraft will be orbiting at a low enough altitude that it will be outside the sphere of influence of any other gravitational source so the moon can be modeled as the only source of gravity. Once the thrusters on the spacecraft are fired at an altitude of 3 km and the orbital velocity of the spacecraft is brought to zero relative to the surface of the moon, the spacecraft begins its free-fall down towards the surface. The problem, however, is that if no other forces act on the spacecraft, it will continue to accelerate towards the moon's surface and impact the surface with significant velocity. This high velocity impact would not be survivable for the spacecraft's structural and instrumental components, which are very crucial for the mission's success.

1.2 Thesis Objective

In order to stop the spacecraft from being destroyed upon impacting the moon's surface, its vertical velocity will have to be significantly reduced. Ideally, the spacecraft's downward velocity will be slowed to a value very close to zero but it cannot be exactly zero. This is because if the velocity were slowed to exactly zero, then the spacecraft would just hover at that point and never reach the surface. The NASA Glenn Research Center even states that when landing a

Lunar Lander on the moon, the propulsion system must leave “the descent at a near zero, but not exactly zero, terminal velocity” [2]. There are many examples of this including a spacecraft docking with the International Space Station. The spacecraft needs to slow its velocity down as it approaches the docking station but it still has to have some residual velocity so that it can coast into the port. Similarly, an aircraft landing on a runway still needs to have some downward vertical velocity at the point of landing. It just needs to be small enough so that the wheels and the frame of the aircraft are not damaged when then plane lands. This is the same for the case of the lunar landing in that the spacecraft must be slowed to an acceptable landing velocity before touching down with the surface. This will be the main objective of this paper. Through applied optimal control methods and using MATLAB’s Runga-Kutta numerical integrator (ODE45), the optimal thrust characteristics to attain this acceptable landing velocity are found. This thesis analyzes only the continuous burn condition and analyzes how perturbing different values, such as the thrust magnitude, the specific impulse of the engines, and the mass of the spacecraft will affect the final conditions.

In order to complete these optimization tasks, certain assumptions about the mission are made. They are:

1. The spacecraft is orbiting close enough to the moon’s surface that the moon is the only gravitational source acting on it.
2. The acceleration of the spacecraft due to the moon is constant at 1.6 m/s^2.
3. The spacecraft does not have any tangential velocity during its descent phase.
4. The thrust applied by the thrusters is constant.
5. The spacecraft can survive a landing with a terminal velocity less than 5 m/s.

With these assumptions being made, the problem at hand becomes much simpler and the optimization is reduced to one-dimensional.

2 Analysis

In this chapter, the equations of motion and initial conditions are developed. Using basic propulsion equations, an expression for the constant mass flow rate through the thrusters is also found. The equations that are calculated in this section are then numerically integrated to find the optimal descent profile.

2.1 Thrust Determination

In order to determine the most fuel-efficient descent, the mass flow rate through the thruster must first be found. The thrust equation is:

$$\tau = \dot{m}U_e + (P_e - P_a) * A_e \tag{1}$$

Since it is assumed that the pressure at the exit of the nozzle is equal to the ambient pressure, this equation reduces to:

$$\tau = \dot{m}U_e \tag{2}$$

Since a constant thrust is being assumed for this thesis, the only unknown is the velocity at the exit of the nozzle. The characteristics of a Redmond hydrazine monopropellant rocket engine are used to model the descent thrusters. For the Lunar Lion spacecraft, three of these thrusters will be available, each with a specific impulse of about 228 seconds and a maximum thrust of 296 Newton [3].

To find the mass flow rate through the nozzle of the thruster, the specific impulse is:

$$I_{sp} = \frac{I}{mg} = \frac{U_e}{g} = \frac{\tau}{\dot{m}g} \tag{3}$$

As stated previously, this thrust is assumed to be constant and continuous for this case, and the thrust level will be at full power. Given the time of each burn, which will be found after integrating the differential equations of motion, the amount of propellant used during the descent is found as:

$$propellant\_used = \dot{m} * (\sum t_{burn}) \tag{4}$$

The value found using (Eq. 4) is then minimized so that the fuel optimal path to the moon's surface is found. This solution will include information on when to turn the thrusters on, how long to keep the thrusters on, and at what percent power to operate them. This analysis is found in the following sections of this thesis.

2.2 No Thrust Case

For this case, the thrusters will never be turned on, so the spacecraft will essentially be free-falling 3 kilometers towards the moon's surface. Because of the earlier assumption of constant acceleration, the classic constant acceleration equations can be used to find the functions of the altitude and velocity of the spacecraft. The final altitude of the spacecraft is 0 km, so the time of the descent will be found in order to find the final velocity. The time of descent can be found using the following equation:

$$t = \sqrt{\frac{2y_0}{g}} \tag{5}$$

The initial altitude (y_0) is 3000 meters. The radial velocity can also be calculated using:

$$u = u_O + gt \tag{6}$$

The initial radial velocity (u0) is 0 m/s. Note that since there is no atmosphere surrounding the moon, there is no terminal velocity that the spacecraft can reach, like on Earth. The crash velocity, however, can be found as a function of the altitude combing (Eq. 5) and (Eq. 6).

$$u = g\sqrt{\frac{2y_0}{g}} = \sqrt{2gy_0} \tag{7}$$

All three of these equations are evaluated for the duration of the freefall, which are found in the results and discussion section.

2.3 Continuous Thrust Case

The continuous thrust case is when the thrusters will be turned on for the duration of the descent phase. As stated earlier, the thrusters will cause a constant thrust and can either be turned on to one quarter, one half or full power to produce upwards thrust. For this case, simple constant acceleration equations cannot be used because the thrust introduces a force in the opposite direction. Using the formulation found in Ref. 1, the two initial equations for the velocity of the spacecraft are:

$$\dot{r} = u \tag{8}$$

$$\dot{u} = g - \frac{\tau}{m_o - \left|\dot{m}\right| t} \tag{9}$$

In addition to these equations, some initial conditions need to be specified in order to perform the integration. Those boundary conditions are as follows: [1]

$$r(0) = r_o \tag{10}$$

$$u(0) = 0 \tag{11}$$

$$v(0) = 0 \tag{12}$$

The Hamiltonian can then be set up as:

$$H = \lambda_r u + \lambda_u \left(g - \frac{T}{m_o - \left|\dot{m}\right| t} \right) \tag{13}$$

where λ_r and λ_v are the Lagrange multipliers for this problem. Therefore, the conditions that need to be met are: [1]

$$\dot{\lambda}_r = 0 \tag{14}$$

$$\dot{\lambda}_u = -\lambda_r \tag{15}$$

Two more boundary conditions are then needed in order to complete the integration and they are as follows: [1]

$$\lambda_r(t_f) = 1 + \frac{\upsilon_2 \sqrt{\mu}}{2\left[r(t_f)\right]^{3/2}} \tag{16}$$

$$\lambda_u(t_f) = \upsilon_1 \tag{17}$$

$$\lambda_v(t_f) = \upsilon_2 \tag{18}$$

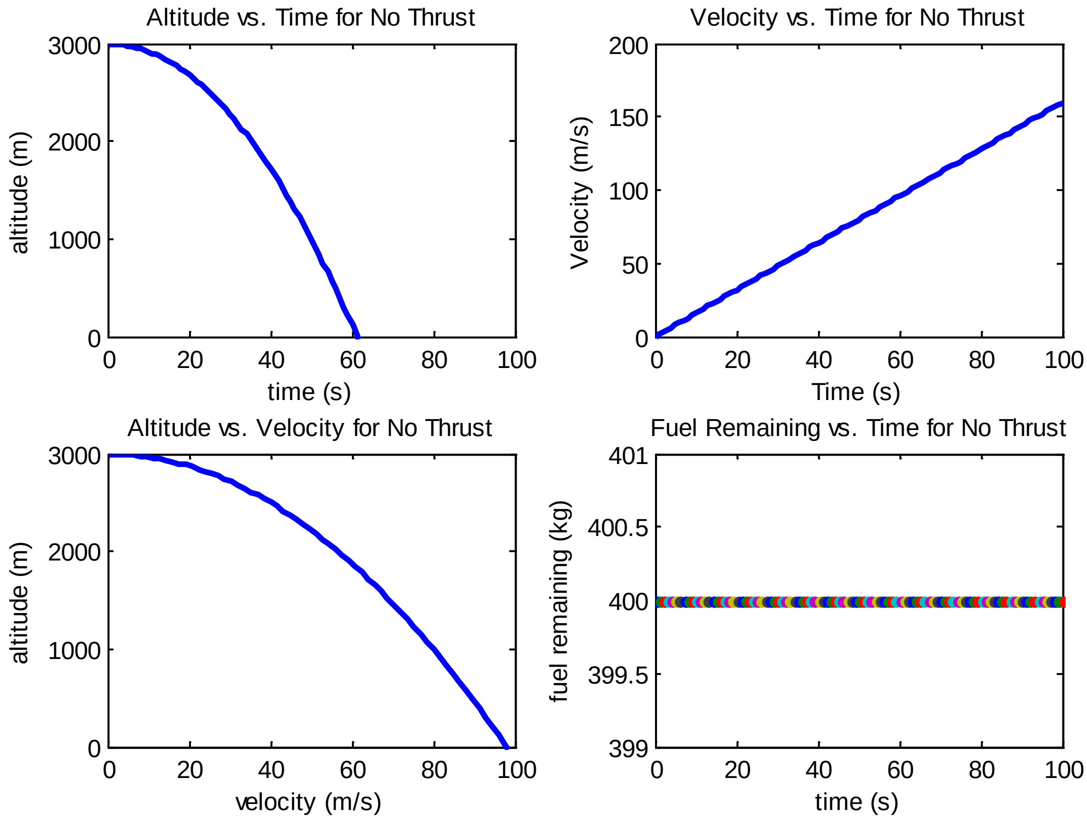
The four differential equations, (Eqs. 8-9, 14-15) can then be solved subject to the six boundary conditions (Eqs. 10-12, 16-18). The adjoint variables υ_1 and υ_2 can be changed accordingly so that the integration yields the proper results [1]. These differential equations are analyzed using MATLAB’s ODE45 numerical integrator to compute the optimal descent path.

3 Results and Discussion

In this chapter, the results of several simulations are displayed. These results are analyzed for various combinations of thrusting and the appropriate equations are used to calculate the amount of propellant used for each case. Then the optimal thrust descent profile can be found for this mission (subject to the assumptions that are made).

3.1 No Thrust Case

After applying the necessary equations from the calculations section into MATLAB, the plots seen in figure 3 were plotted. In the first graph in the top left, it is clear that the altitude gradually decreases as time goes on and as more time elapses, the altitude decreases at a faster rate. The velocity versus time graph shows that the velocity is increasing at a constant rate as time increases, which makes sense due to the constant acceleration of the spacecraft. In the Altitude versus Velocity graph, it is clear that the velocity increases very rapidly as the altitude of the spacecraft decreases. This is because since there is no thrust, there is no force to counter the moon's gravitational attraction and so it keeps pulling the spacecraft towards it's surface, increasing it's velocity. In the last graph, it is pretty clear that no propellant is expelled during this descent which makes sense because the thrusters are never turned on.



<u>Figures 2</u>- No Thrust Case Plots

As Figures 2 shows, if the spacecraft is allowed to free fall towards the moon's surface, it will impact the surface with a velocity of almost 100 m/s. This velocity is much too high for any of the structural or technical components of the spacecraft to survive. Also, it can be seen that the time of flight for this case is around 60 seconds, which will be used as a baseline time of flight for the cases with thrust added in. Also, due to the constant acceleration of the spacecraft due to the moon's gravity, the velocity increases constantly with time which will also change when thrust is added in.

3.2 Continuous Thrust Case

In the previous case, the spacecraft will impact the surface of the moon with a very high final velocity if it is allowed to free fall to the surface. It is for this reason that thrusters need to be fired in the direction opposing the gravitational attraction of the moon. For this case, these thrusters will fire at a constant rate and the final conditions at the surface of the moon will then be analyzed. A preliminary plot showing the goal of adding thrust to the descent can be seen in Figure 3:

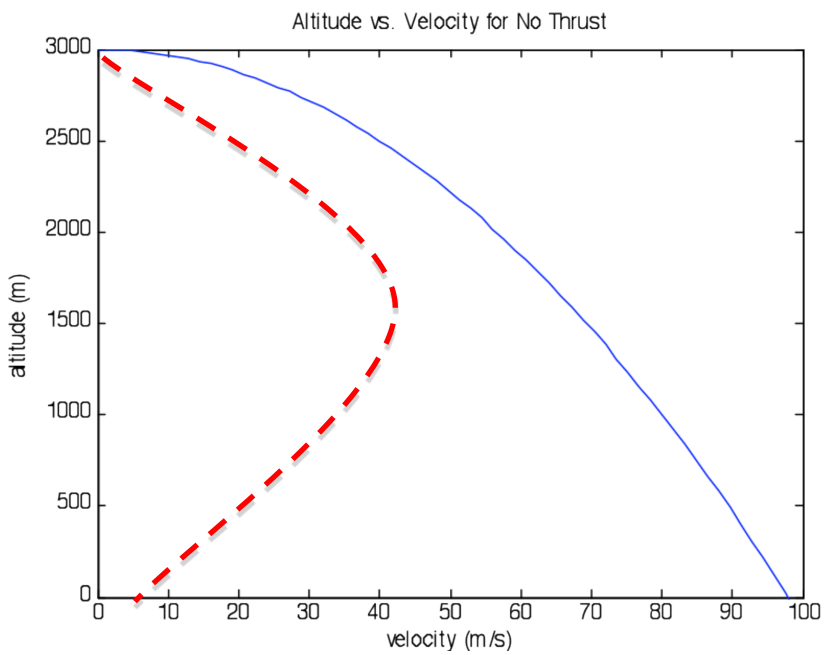
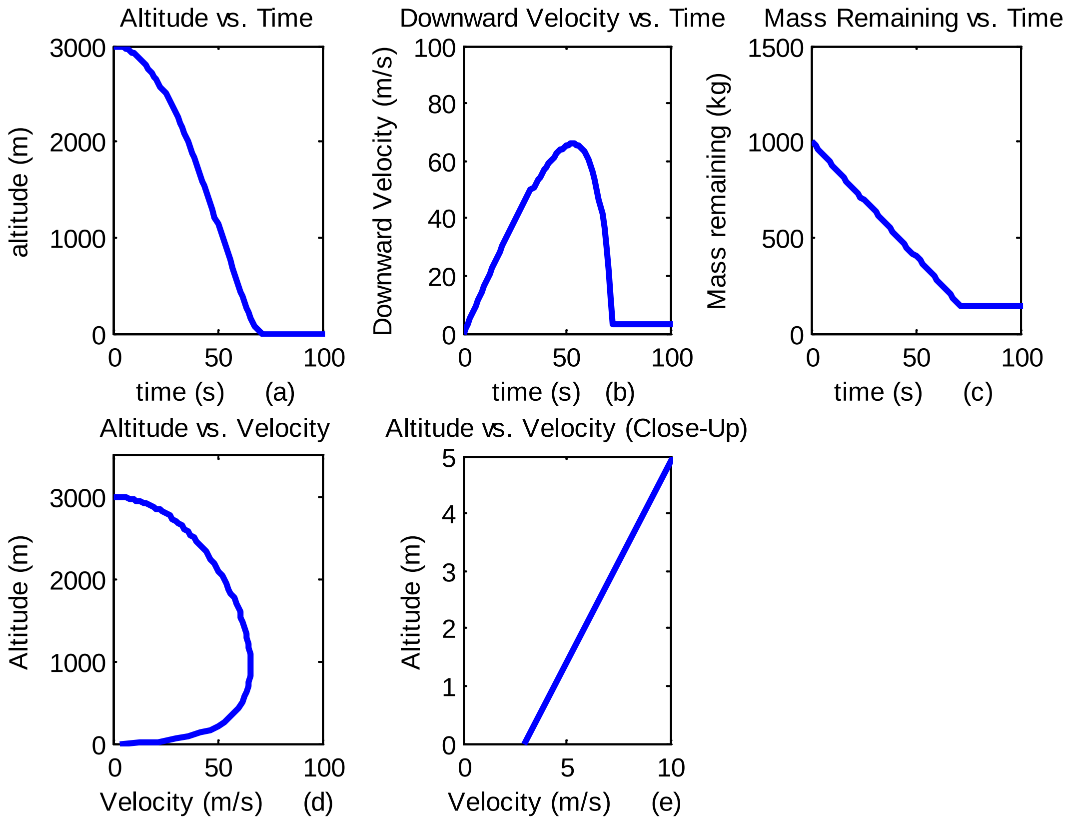


Figure 3- Ideal Thrust Outcome Example

In Figure 3, the solid line is the velocity of the spacecraft in the no thrust case. The dashed line represents an approximate curve for what the altitude versus velocity graph should look like when thrust is added. Ideally, the thrusters will reduce the final vertical velocity to a survivable velocity of around 5 m/s. For a given initial mass of the spacecraft and an I_{sp} of the thrusters, an ideal thrust magnitude can be determined to attain the desired final conditions. For an initial mass of 1,000 kilograms and an I_{sp} of 228 seconds, the integration was run for a variety of different thrust values and the ending conditions were analyzed. It is determined that the feasible solution for the thrust for this case is 4,450 Newton.

In Figures 4, the results from the analysis of the continuous thrust case are shown for a constant value of thrust of 4450 N. As Figure 4(a) shows, the spacecraft will make contact with the moon at approximately 70 seconds after beginning its descent. Figure 4(b) shows that the spacecraft's velocity will increase to a certain point, but then begin to decrease very rapidly after that point is reached. This point where the velocity begins to decrease is the point at which the upwards acceleration due to the thrust begins to be greater than the downward acceleration due to the moon's gravity. Since the thrusters are being fired continuously and at a constant rate, the mass of the spacecraft will decrease linearly with time as can be seen in Figure 4(c). Figure 4(d) shows the altitude of the spacecraft as a function of its velocity and this shape is exactly the desired one that is shown in Figure 3 with the dashed line. Since the origin is hard to see, however, a zoomed in portion of the altitude verses velocity graph is shown in Figure 4(e). The final velocity of the spacecraft when it makes contact with the moon's surface can be seen very clearly in this figure. For this specific case, the spacecraft touches down with a final downward velocity of about 3 m/s, which is below the 5 m/s that was recommended earlier in order for the spacecraft to survive the impact.

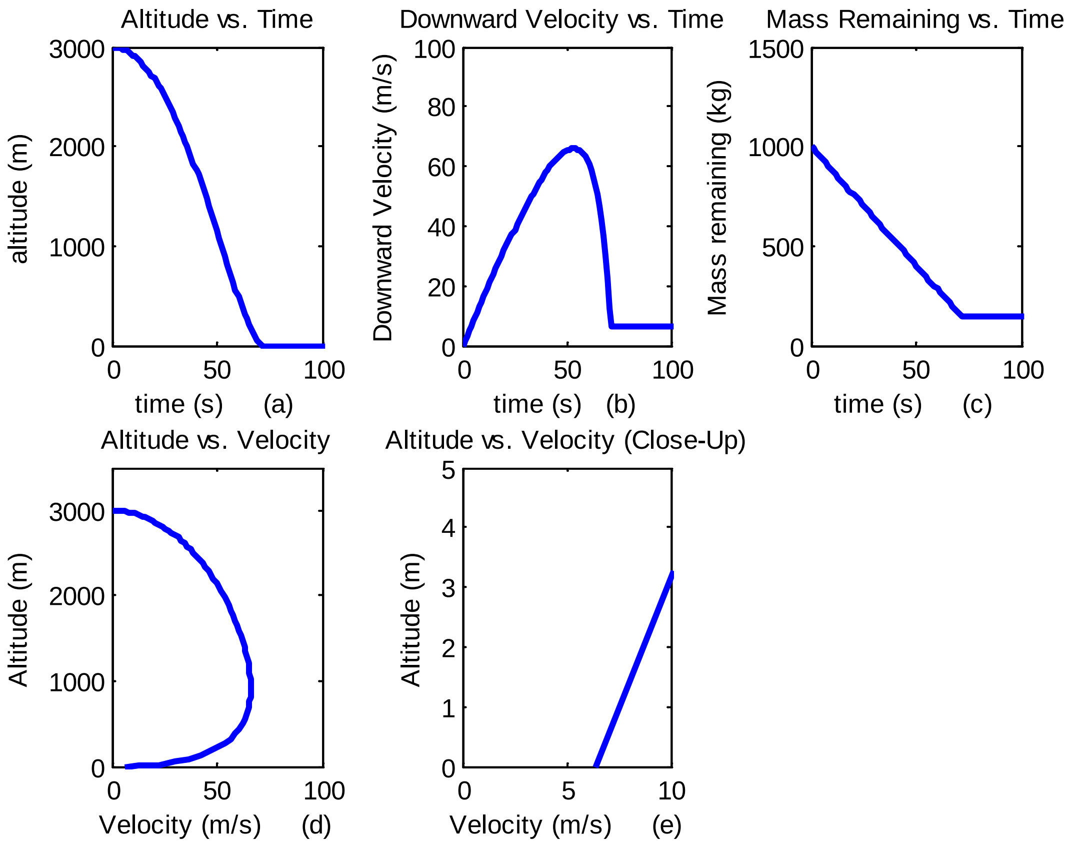


Figures 4- Plots for Thrust = 4450 N, M = 1000 kg, I_{sp} = 228 s

Now that a feasible solution for this case has been determined, the variables that are input into the differentiation will now be perturbed so that the sensitivities of each of these variables can be analyzed. The three variables that can be adjusted for this integration that have an effect on the design of the spacecraft are the initial mass, the I_{sp} of the thrusters and the magnitude of the thrust applied.

3.3 Sensitivity of Thrust Magnitude

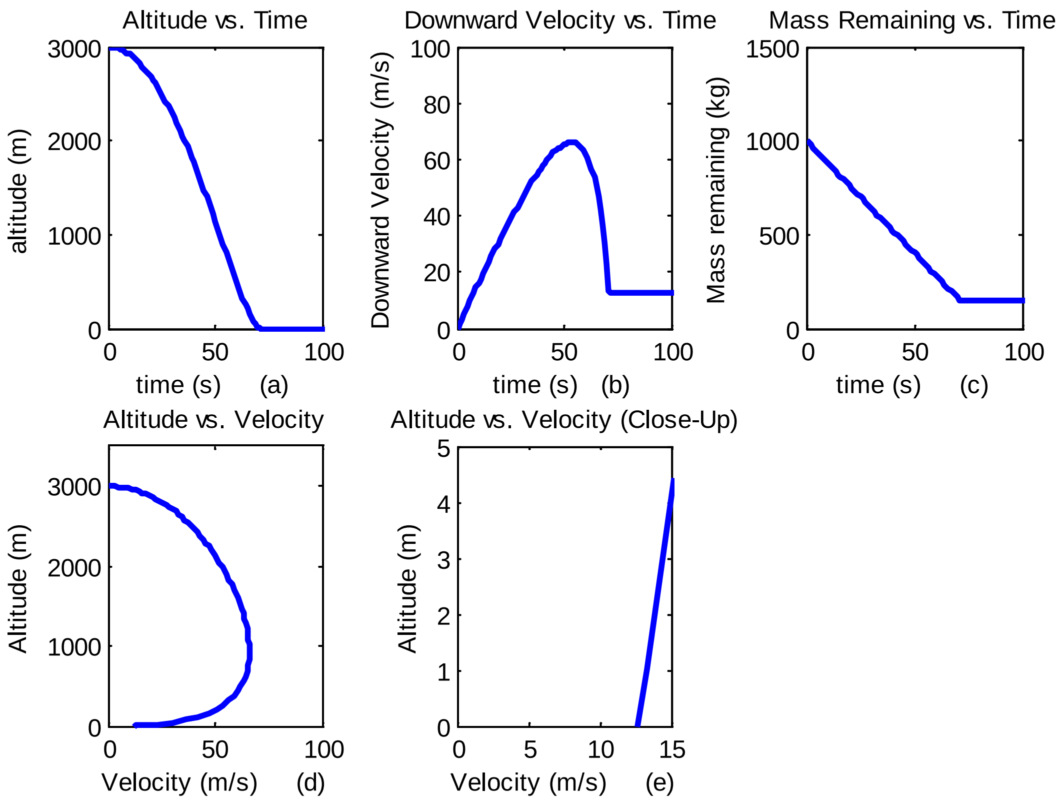
The first variable that is analyzed is the magnitude of the thrust. In this section, the integration will be run again, holding everything constant except for the thrust. From the results of this, the sensitivity of the thrust on the final conditions can be analyzed. First, the thrust will be decreased slightly to see if the spacecraft will still be able to survive the impact with the moon at the final velocity. A lower thrust magnitude is optimal because the lower the thrust during the descent, the lower the amount of propellant used. For the first perturbation, the thrust will be reduced by 1 N and the results can be seen in Figures 5:



Figures 5- Plots for Thrust = 4449 N, M = 1000 kg, I_{sp} = 228 s

At first glance, these plots may appear exactly the same as the ones in figure 4, but when looking closer at Figure 5(e), it is clear that the final velocity value has changed. Now, the spacecraft will impact the surface of the moon with a downward velocity of around 6 m/s, which is just above the survivable velocity. The spacecraft will most likely be able to survive this impact, but some of the main sensors, tools, or computers may be damaged during landing.

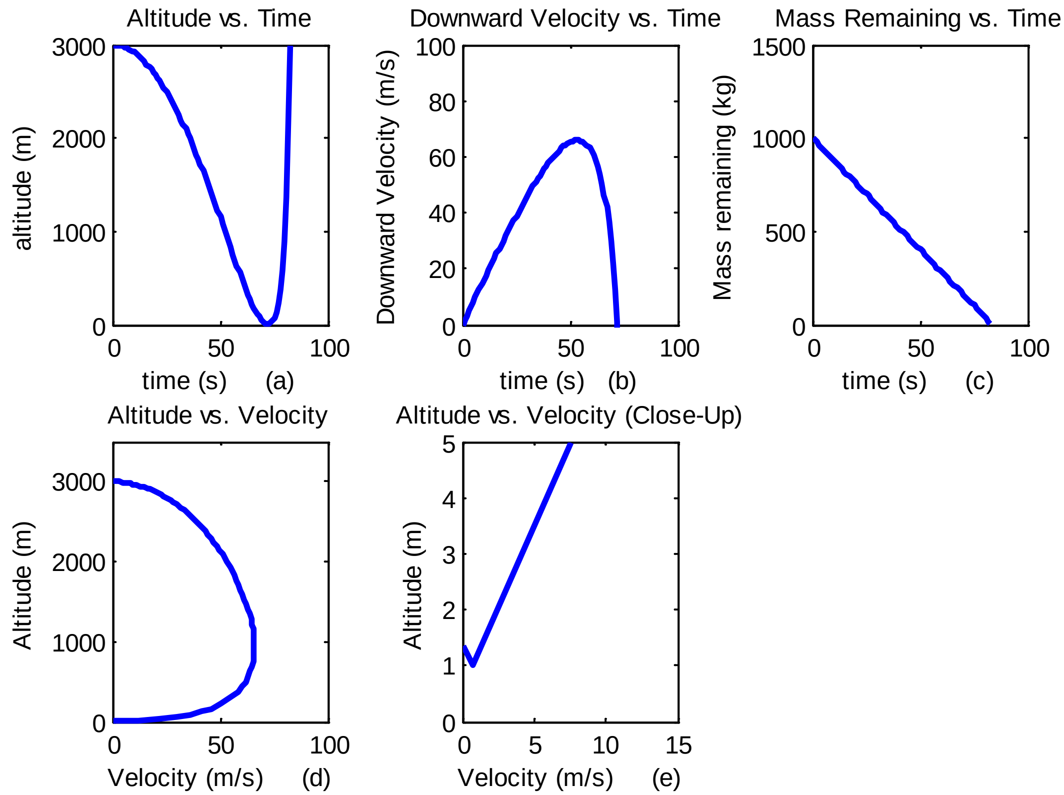
Next, the continuous thrust will be decreased by 5 N less than its original value as can be seen in Figures 6:



Figures 6- Plots for Thrust = 4445 N, M = 1000 kg, I_{sp} = 228 s

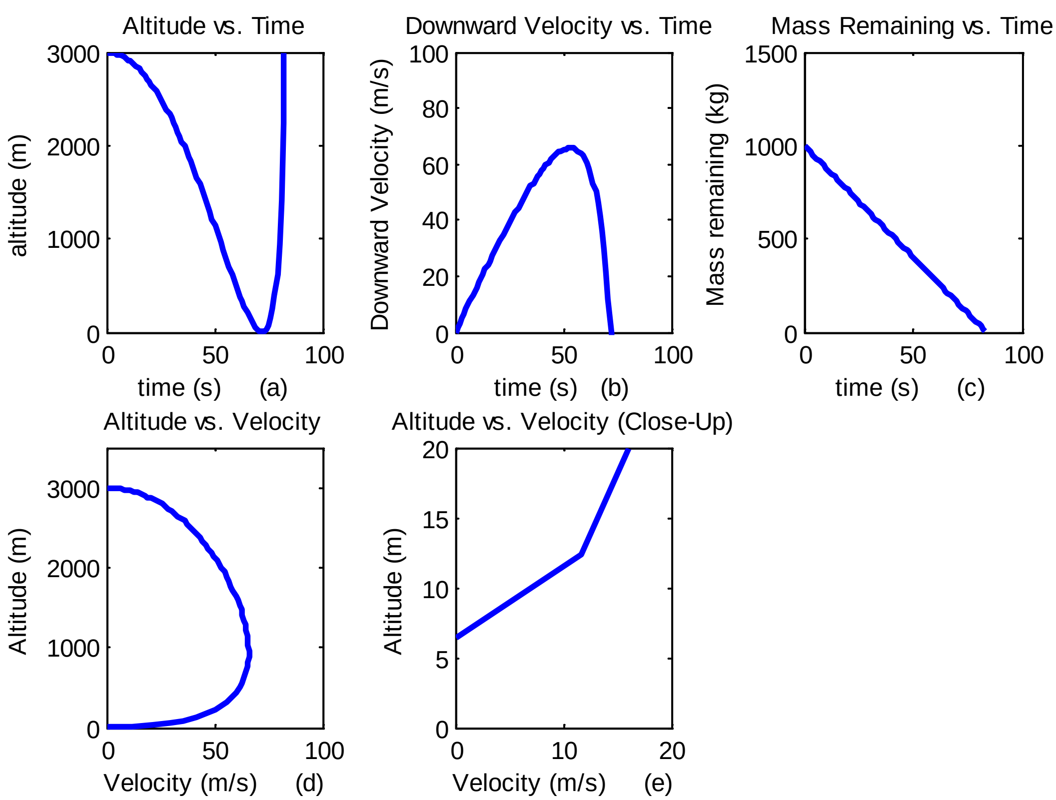
Again, figure 6(e) shows that this change in thrust increases the final velocity of the spacecraft when it touches down with the moon. For this change, however, the velocity increases all the way to around 13 m/s. This velocity is too high in order for the spacecraft to survive and maintain basic functionality. So, based off these results, it can be concluded that decreasing the thrust magnitude by as little as 5 Newtons will result in a final downward velocity that is not survivable for the spacecraft.

In order to get a full understanding of the sensitivity of the thrust magnitude, however, the magnitude will need to be changed in the opposite direction as well. For the next case, the thrust will now be increased by one Newton to a total of 4451 N and the results are seen in Figures 7:



<u>Figures 7</u>- Plots for Thrust = 4451 N, M = 1000 kg, I_{sp} = 228 s

With this small increase in the thrust, not only does the (e) plot change, but plot (a) changes very drastically as well. In figure 7(a), the altitude of the spacecraft decreases towards zero but then quickly jumps almost straight back up to the initial height almost instantaneously. Using the graphic in figure 7(e), the reason for this sharp jump can be seen. The spacecraft's velocity will actually slow to zero m/s before it ever even comes into contact with the moon's surface. The upwards force from the thrusters will overcome the downwards force of the moon's gravity at about 1 m and the spacecraft will hover at one meter instantaneously, before starting to accelerate back up towards orbit. The spacecraft will continue to accelerate and move upwards until it runs out of propellant, at which point it will enter into a free fall and impact the moon's surface at a very high velocity. This is not an acceptable outcome for this descent because the spacecraft never even touches down on the moon's surface. In order to see how sensitive the thrust is, it will also be integrated for a thrust 5 N higher than the feasible solution, which can be seen in Figures 8:

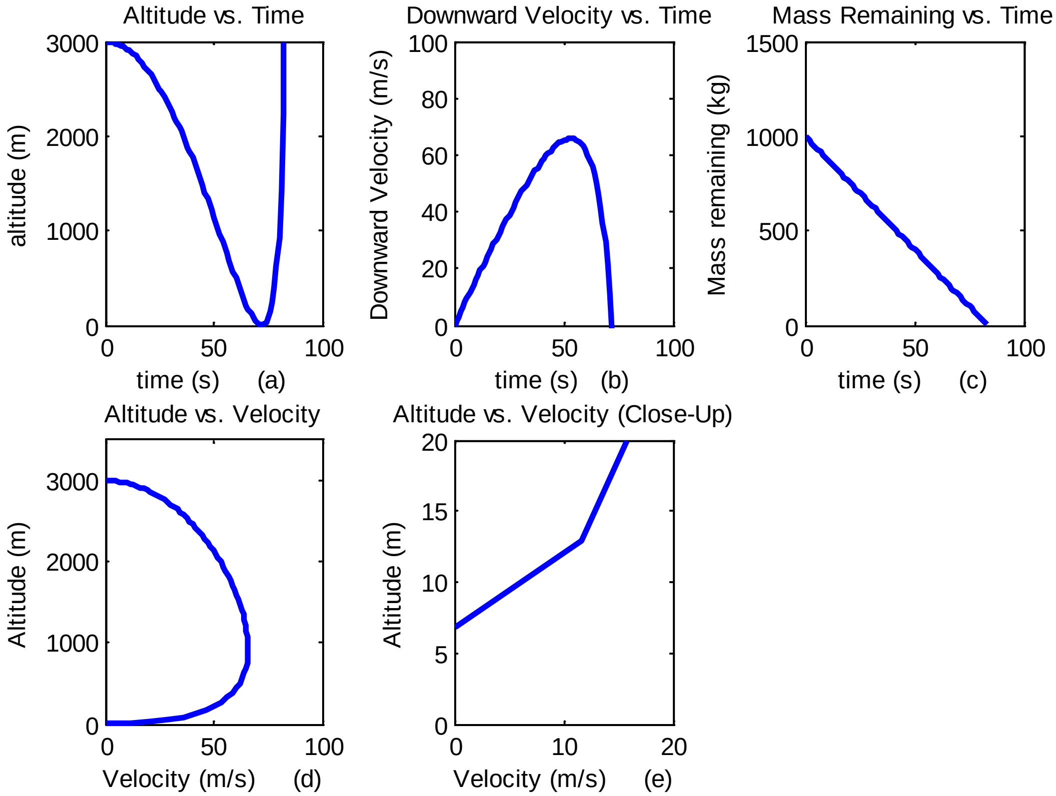


Figures 8- Plots for Thrust = 4455 N, M = 1000 kg, I_{sp} = 228 s

As figure 8(a) shows, the spacecraft will turn come near the surface and then accelerate back upwards, just as in the previous case. Figure 8(e) shows that the spacecraft's downward velocity will reach zero m/s when it is approximately 6 m above the surface of the moon. This case is no longer feasible because the spacecraft never touches down on the moon's surface. After analyzing these four perturbations of the thrust magnitude, it can be concluded that the thrust is a very sensitive variable. The thrust absolutely cannot be increased from the ideal 4450 N because it will never reach the surface of the moon and the descent cannot be properly completed. If the thrust is reduced, however, the descent may be still be completed but not without risking some damage to the spacecraft or internal instruments. If the thrust is reduced by only 5 N, then the final downward velocity of the spacecraft is too high and the spacecraft will be destroyed.

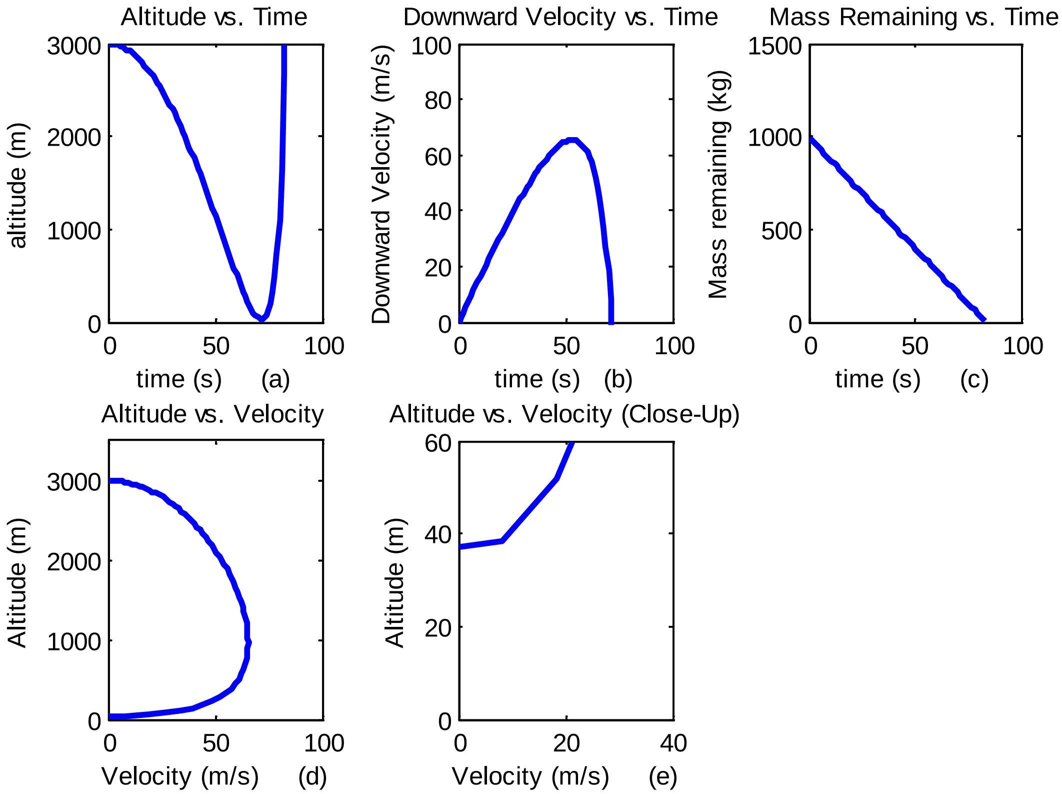
3.4 Sensitivity of Initial Mass

The same type of analysis will now be conducted on the variable of initial mass. For this case, the thrust magnitude and specific impulse will be help constant at 4450 N and 228 s respectively. The initial mass of the spacecraft will then be perturbed and the change in the results of the integration will be analyzed. To start, the mass of the spacecraft is decreased by only one kilogram to 999 kg. The results can be seen in Figures 9:



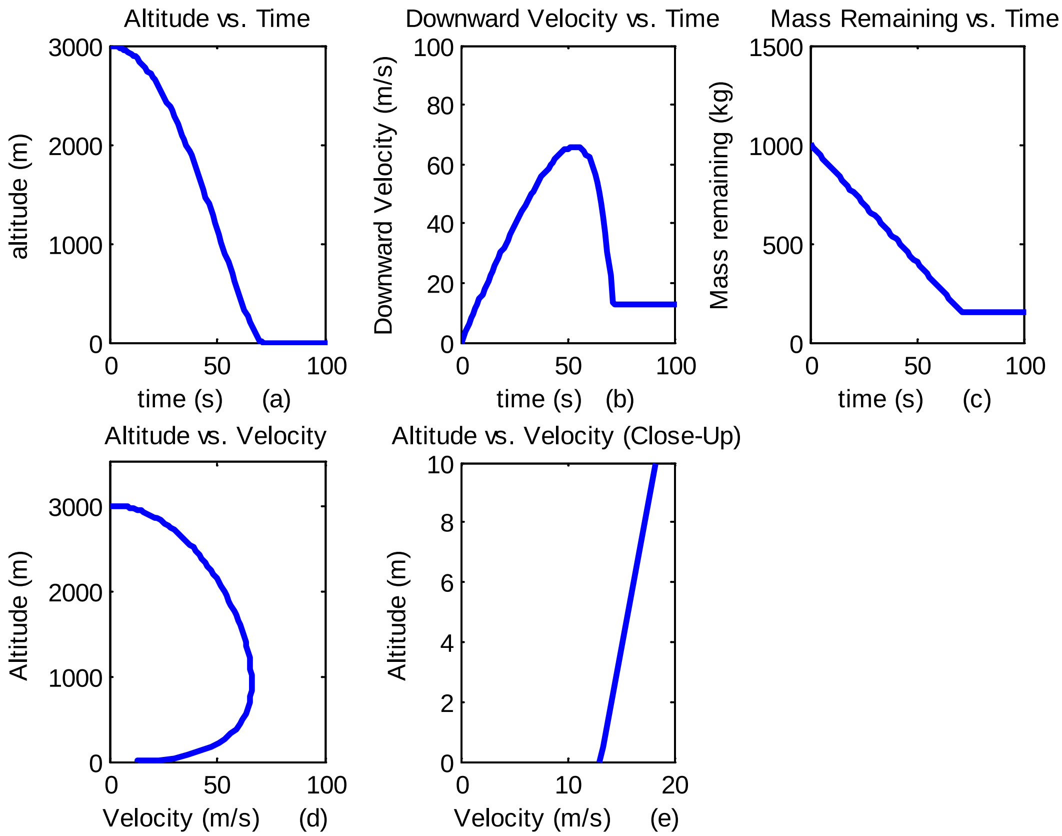
<u>Figures 9</u>- Plots for Thrust = 4450 N, M = 999 kg, I_{sp} = 228 s

All of the plots in figure 9 look very similar to the ideal case except for figures 9(a) and 9(e). Decreasing the mass results in the same phenomenon that happens when the thrust is increased, that is the slight hover near the surface followed by an immediate and fast ascent back towards orbit. For this case, however, the spacecraft stops around 7 m above the surface, which is much higher than the 1 m that it stopped at when the thrust was only increased by 1 N. This leads to the hypothesis that the variable initial mass is much more sensitive than the thrust magnitude. In order to prove this hypothesis, the integration will now be run for an initial mass that is 5 kg less than the optimal mass. The results can be seen in Figures 10:



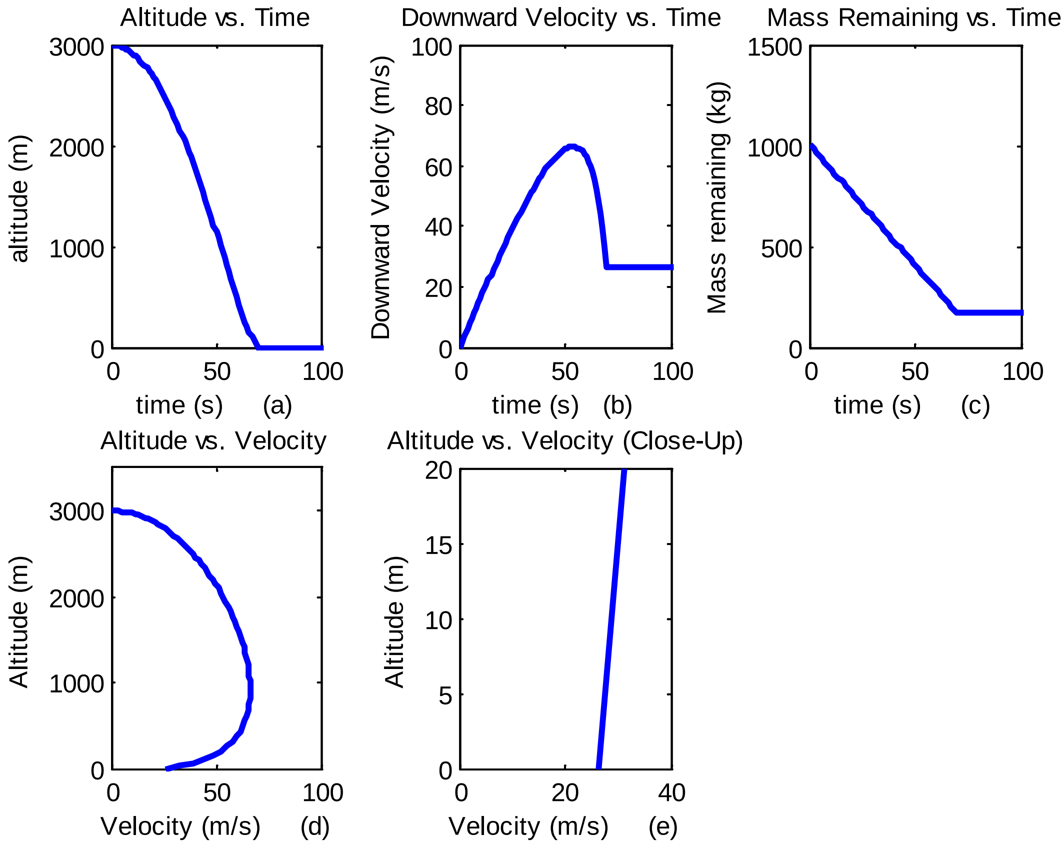
Figures 10- Plots for Thrust = 4450 N, M = 995 kg, I_{sp} = 228 s

The results seen in figure 10(e) verify the previously stated hypothesis; because now the spacecraft loses its downwards velocity around 38 m about the surface. This height is much greater than the height where the spacecraft turns around if the thrust is increased by only 5 N. If the same extreme changes in the final conditions happen for an increased initial mass as well, then the initial mass will in fact be a more sensitive variable to the final conditions. The results for an initial mass only one kilogram greater than the ideal mass can be seen in Figures 11:



<u>Figures 11</u>- Plots for Thrust = 4450 N, M = 1001 kg, I_{sp} = 228 s

As figure 11(e) shows, the final downward velocity of the spacecraft when it makes contact with the moon for this case is around 13 m/s. This is a very extreme jump from the 3 m/s that it landed with when the spacecraft was only one kg lighter. This final velocity for this case is not survivable and therefore increasing the mass will not yield a feasible solution to the problem. To verify that this is the case, the analysis is conducted for a spacecraft that is 5 kg heavier and the results from this analysis can be seen in Figures 12:

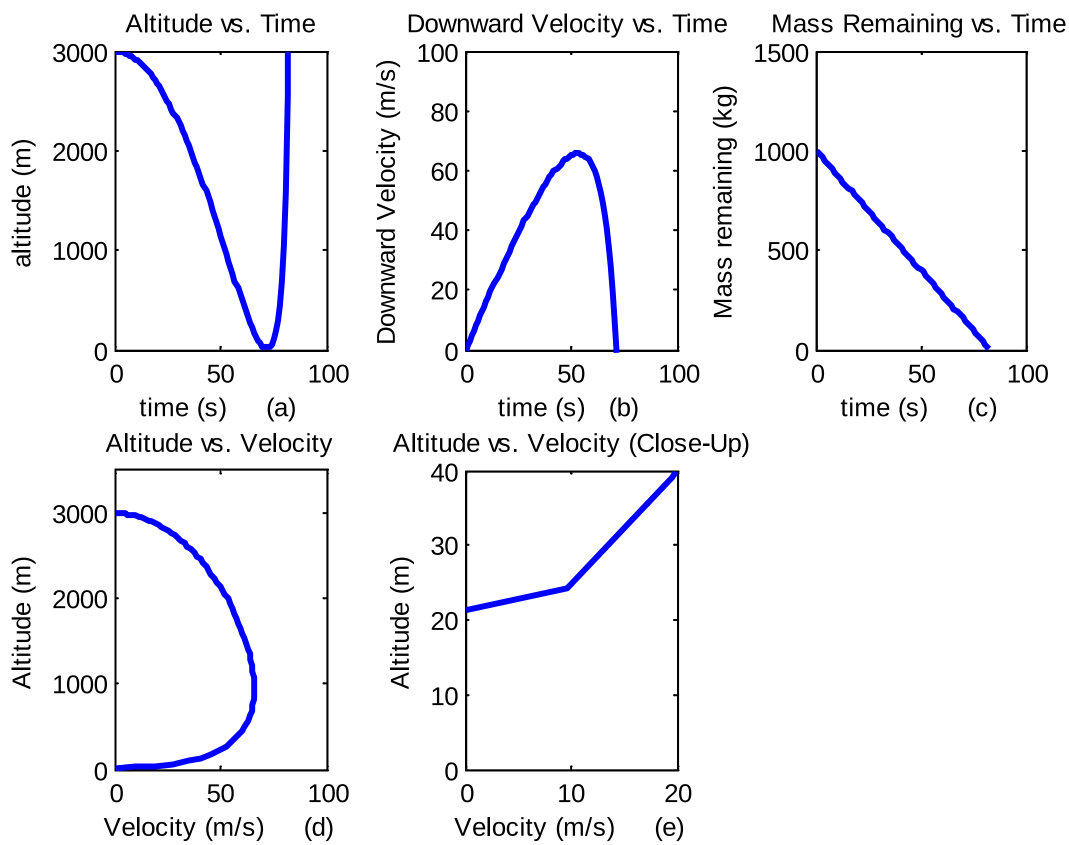


Figures 12- Plots for Thrust = 4450 N, M = 1005 kg, I_{sp} = 228 s

Figure 12(e) verifies that increasing the mass will result in a mission failure because the spacecraft would impact the moon's surface with a downward velocity of around 25 m/s, which is not survivable at all. The spacecraft would be totally destroyed and useless if it contacted the moon at this speed. Therefore, it can be concluded that the initial mass is indeed a much more sensitive variable with regards to the final conditions near the surface of the moon. A change in the initial mass of just one kilogram in either direction will result in the failure of the mission of landing the spacecraft safely on the moon's surface. This means that the initial mass has extremely high sensitivity to perturbations and needs to be very precise in the design of the spacecraft.

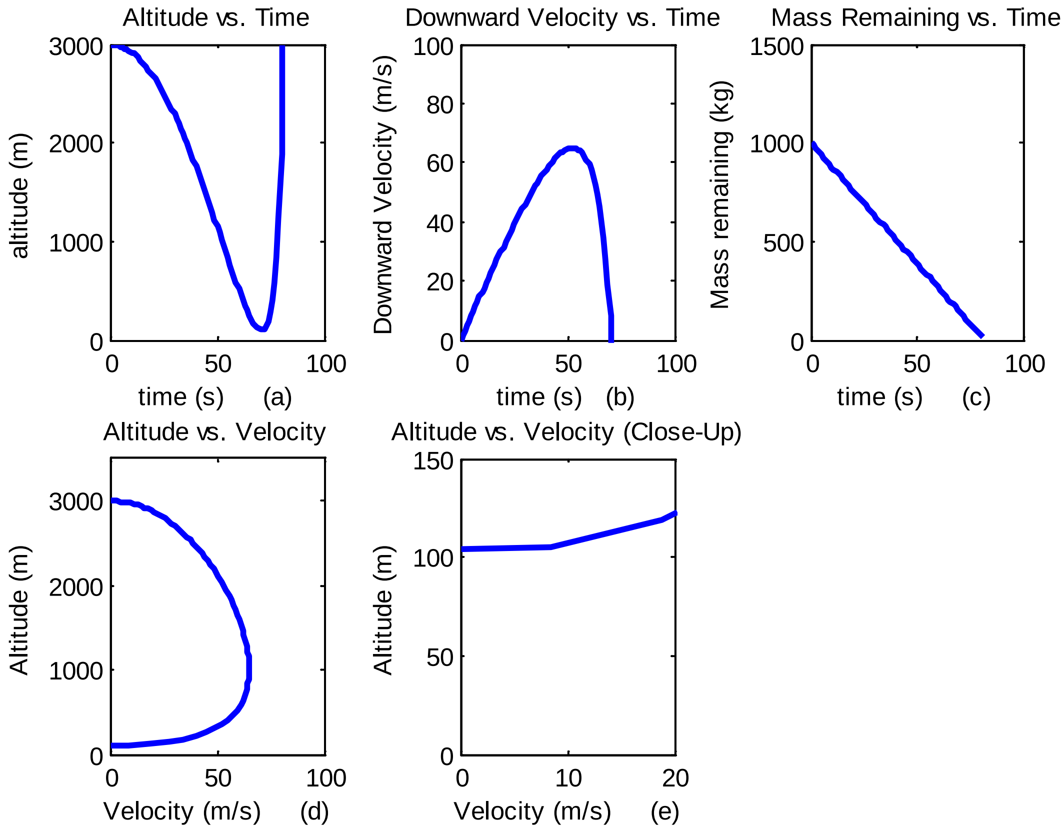
3.5 Sensitivity of Specific Impulse

The third and last variable that will be tested for sensitivity is the specific impulse of the thrusters. This is a constant value that is dependent on the type and size of the thruster. First, the specific impulse will be decreased by one second from its ideal value of 228 seconds. The results for this change can be seen in Figures 13:



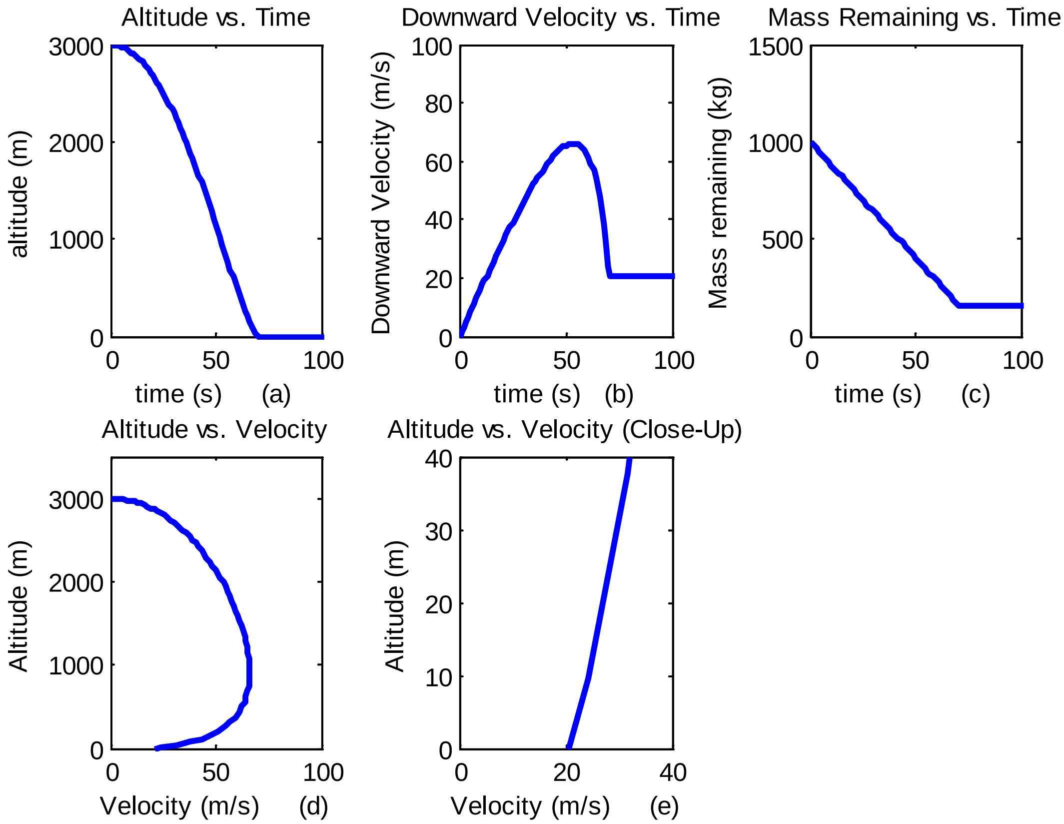
<u>Figures 13</u>- Plots for Thrust = 4450 N, M = 1000 kg, I_{sp} = 227 s

Figure 13(a) shows, once again, that the spacecraft will descend to an altitude near the surface of the moon but then quickly ascend until all of the propellant is expelled. As figure 13(e) shows, the spacecraft will perform this maneuver when it is still approximately 22 m away from the surface of the moon. That is the furthest distance away from the surface for a change of only 1 unit away from the ideal condition so far. The problem will now be analyzed and integrated for a specific impulse that is 5 seconds less than the optimal condition and the results are shown in Figures 14:



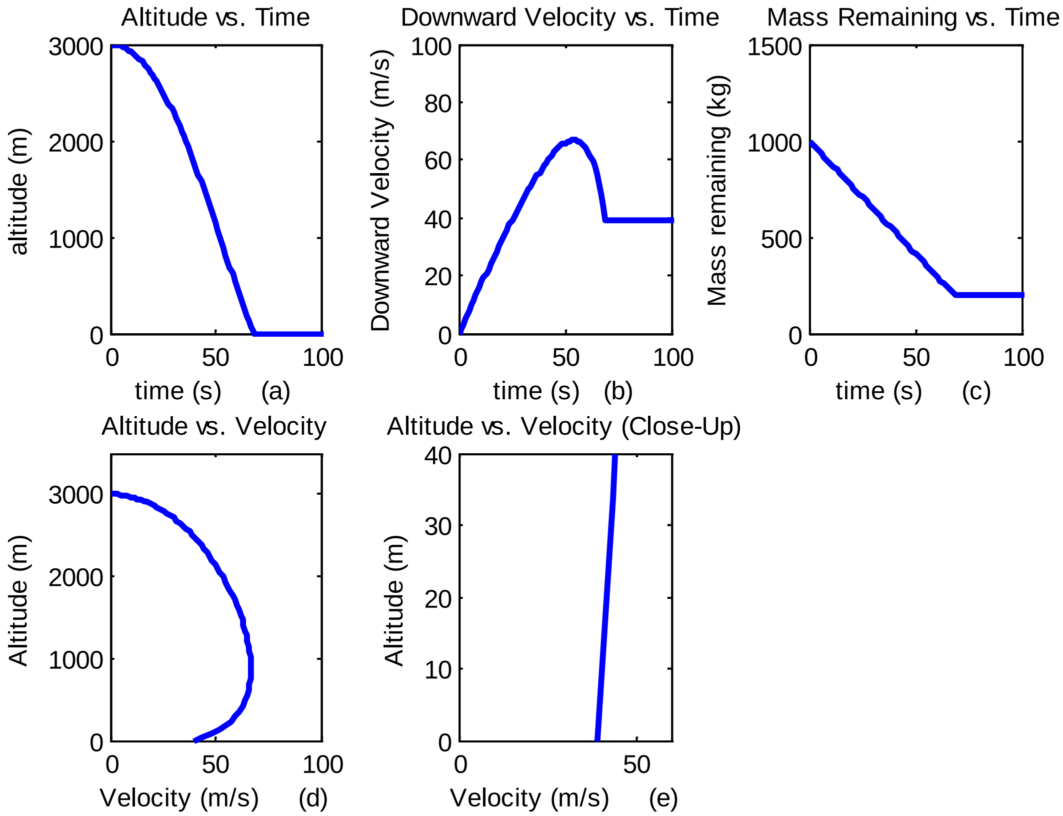
Figures 14- Plots for Thrust = 4450 N, M = 1000 kg, I_{sp} = 223 s

As figure 14(e) shows, the spacecraft will not come anywhere close to the surface of the moon before it turns around and heads back into orbit. The spacecraft will not even get within 100 m of the moon's surface for this case, which is a whole order of magnitude greater than the case where the thrust magnitude was only 5 N less. This data shows that the specific impulse is by far the most sensitive variable in this problem. However, in order to be completely sure and stay consistent, the same kind of analysis will be done for if the specific impulse is increased instead of decreased. For a specific impulse that is only one second greater than the optimal specific impulse, the results can be seen in Figures 15:



<u>Figures 15</u>- Plots for Thrust = 4450 N, M = 1000 kg, I_{sp} = 229 s

Figure 15(a) shows that at least the spacecraft is now coming into contact with the surface of the moon. However, in order to determine what velocity it will impact the surface with, figure 15(d) and more specifically 15(e) need to be analyzed. Figure 15(e) clearly shows that the final downward velocity of the spacecraft for this case will be very close to 20 m/s. This velocity is extremely high and is 17 m/s greater than the velocity that the spacecraft would impact with if the specific impulse was only one second less. This extreme difference further proves that the specific impulse is the most sensitive variable and needs to be calculated very carefully in the mission design process. In order to double check to make sure that a much higher specific impulse will not have different effects, the integration is run for a specific impulse of 233, which is 5 seconds greater than the ideal specific impulse. The results can be seen in Figures 16:

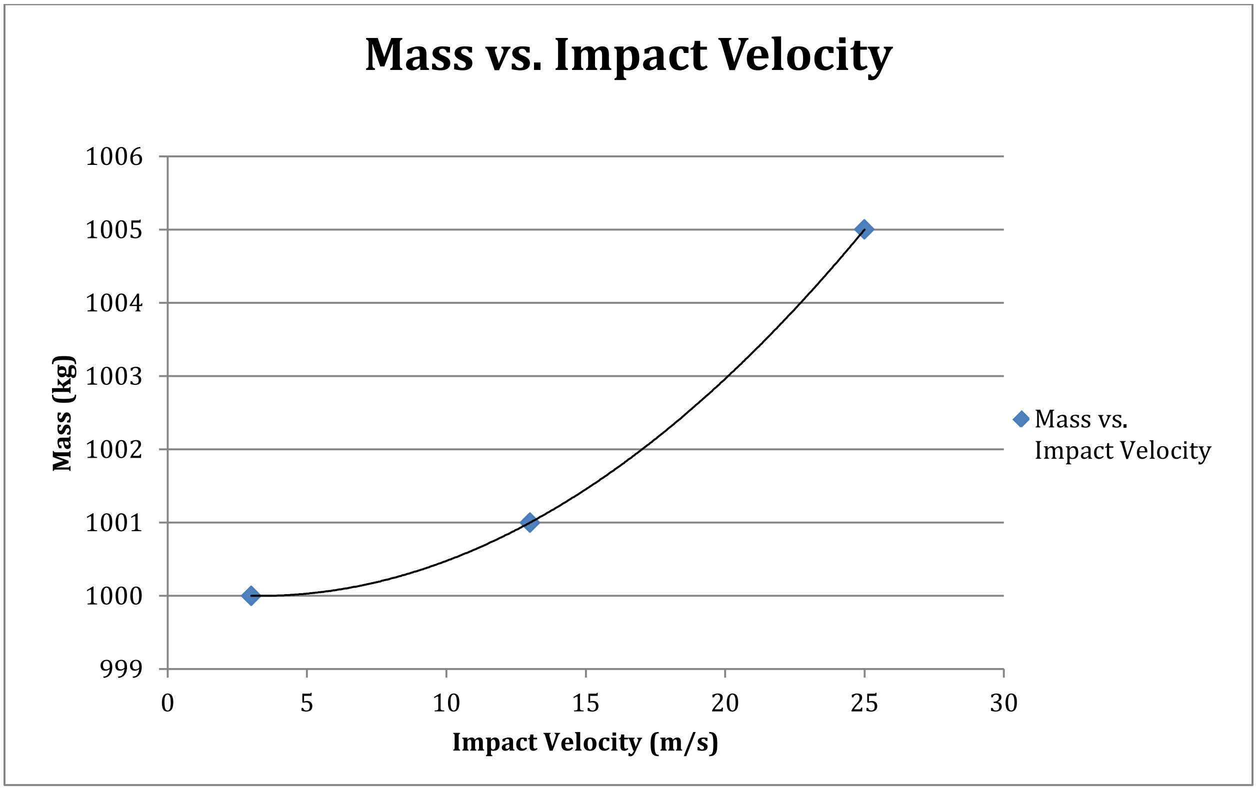


<u>Figures 16</u>- Plots for Thrust = 4450 N, M = 1000 kg, I_{sp} = 233 s

As figure 16(e) shows, the final downward velocity of the spacecraft is now around 45 m/s, which is almost double the final downward velocity for a spacecraft that is 5 kg heavier. This 45 m/s is absolutely a non-survivable impact speed and so this case is not a feasible solution to the problem. Since both the impact speed and the altitude at which the spacecraft turns around are extremely different from the ideal conditions, it can be concluded that the specific impulse is the most sensitive variable. A small change in this in any direction will yield results that are catastrophic to the mission's success.

3.6 Summary Plots of Perturbations

In addition to the plots generated from running the integration, three summary plots were also plotted using the data collected from Figures 4-16. The results from the variations in the initial mass of the spacecraft can be seen in Figure 17:



<u>Figure 17-</u> Summary Plot of Mass Perturbations

As Figure 17 shows, the impact velocity increases as the initial mass of the spacecraft also increases. As previously stated, an increase of only one kilogram will result in an unsafe impact velocity of the spacecraft. This graph, however, shows that an increase in only a fraction of a kilogram will result in an unsafe landing velocity for the spacecraft. A summary plot of the thrust values was also created and can be seen in Figure 18.

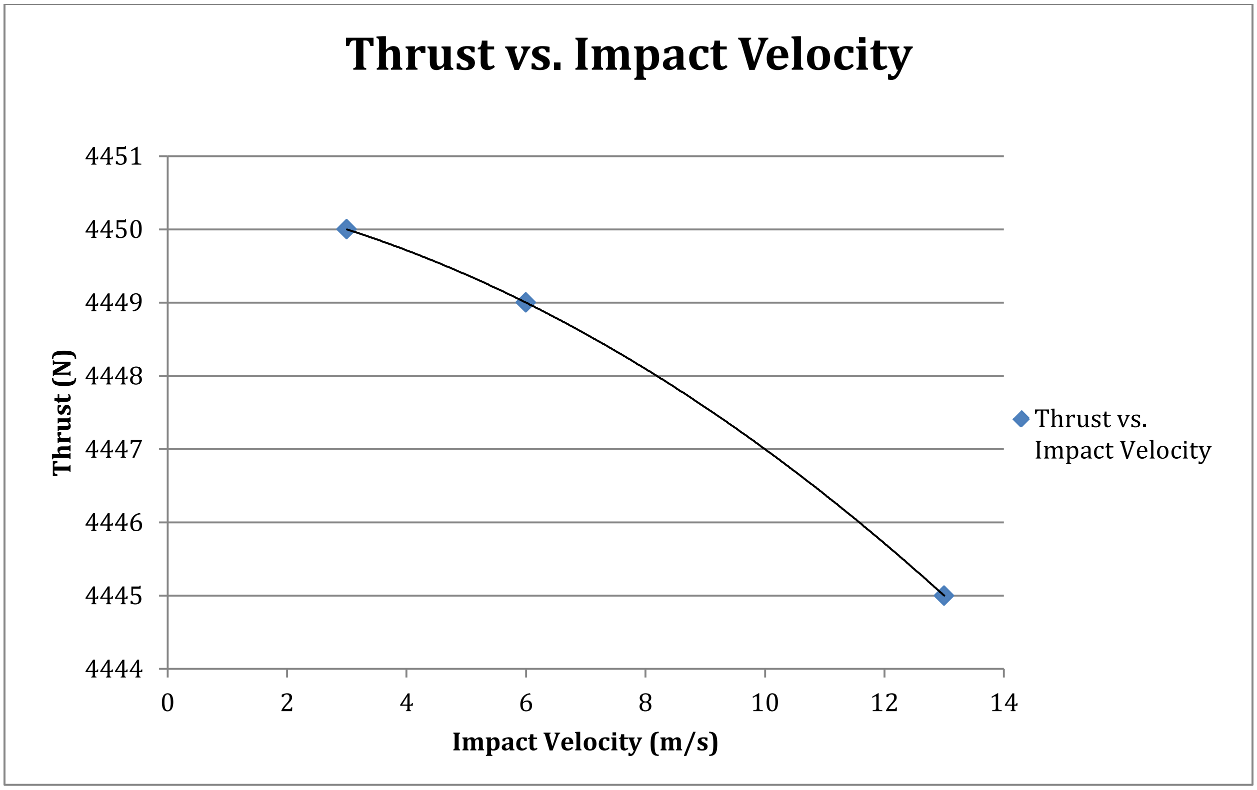


Figure 18- Summary Plot of Thrust Perturbations

The results from Figure 18 are very similar to those found in Figure 17. Any small decrease in thrust will result in an increase in the impact velocity. The sensitivity of this variable is not as severe, as the final impact velocity only reaches 13 m/s, as opposed to the 25 m/s that the spacecraft reaches at the final condition of Figure 17. A summary plot of the specific impulse's effect on the spacecraft's impact velocity was then made and can be seen in Figure 19:

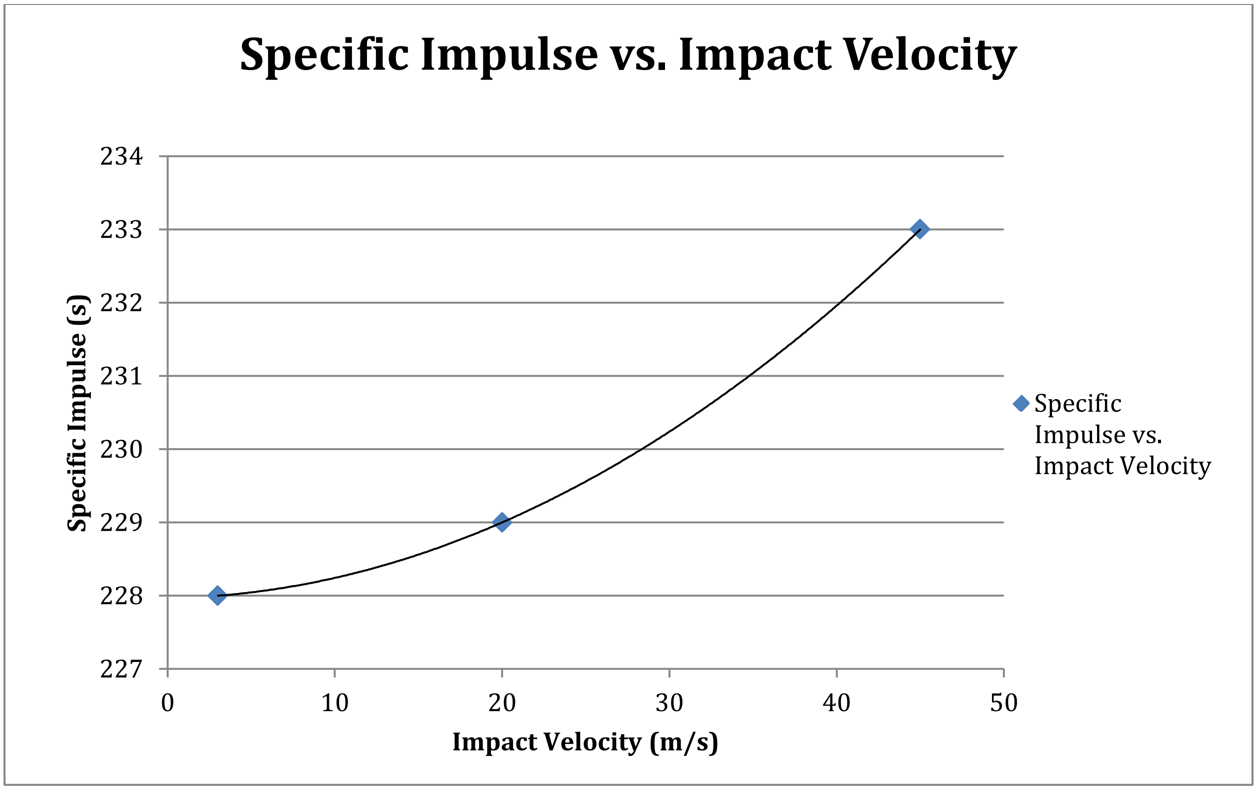


Figure 19- Summary Plot of Specific Impulse Perturbations

As Figure 19 shows, the specific impulse is the most sensitive variable in this problem. The impact speed of the spacecraft increases the most rapidly with the changes in specific impulse. A change of only five seconds yields an impact velocity that is eight times greater than the acceptable velocity for a safe landing (5 m/s). It is for this reason that the specific impulse is the most sensitive variable to perturbations.

4 Conclusions and Future Work

Based on the results, a descent trajectory with zero thrust (freefall) will result in an impact with the moon's surface that is not survivable. It is for this reason that the thrusters on the spacecraft must be fired during the descent to slow the spacecraft's downward velocity to one that is low enough at impact to survive.

4.1 Conclusions

The differential equations of the spacecraft were numerically integrated to produce graphs that fully show the descent characteristics. For a given initial mass of 1,000 kg and specific impulse of 228 seconds, the minimum thrust to satisfy the final boundary conditions was found to be 4,450 N. This solution led to a final downward velocity of 3 m/s at the surface of the moon, which is survivable for the spacecraft. This successful case for the no-coast solution was then explored further to determine the sensitivities of three of the variables in this problem.

Those three variables are the thrust that the spacecraft can produce, the initial mass of the spacecraft, and the specific impulse of the thrusters. Holding two of these variables constant and changing the third, the final boundary conditions were calculated and analyzed for each case. It is concluded that the specific impulse of the thrusters is the most sensitive variable in this problem. Any change in this value results in the final boundary conditions that are furthest from those for the optimal case. The variable with the second highest sensitivity is the initial mass of the spacecraft. All of the perturbations in this value result in boundary conditions that produce a failed mission as well, but they are not as drastic as in the specific impulse case. And the variable with the lowest sensitivity is the thrust magnitude. The perturbations in this still yield a plausible scenario as long as the thrust does not change by very much. Since this problem is extremely sensitive to the parameters, it is recommended that a variable thrust propulsion system be used for this mission. In addition to this system, some type of feedback control algorithm will need to be implemented that incorporates the thrust as a function of altitude and/or velocity.

4.2 Future Work

In the future, there will still be more possible work on this type of problem. A second or third feasible solution can be found using the same integration method presented in this thesis. Changing the initial conditions and the values for the thrust and initial mass could yield more cases where the velocity of the spacecraft is within the survivable range of values. These cases could then be analyzed and a fuel-efficient optimal descent trajectory could then be found. Also, in the future, the thrust could be modeled as throttle-able instead of just constant and continuous. It could be turned on for a certain period of time and then switched off for another time interval. This may end up resulting in a more fuel-efficient descent than just using a continuous thrust level. If this were the case, each level of thrust would require the calculation of a different mass flow rate through the thrusters. In the current design, there are three thrusters on this spacecraft, so the thrust values must each be multiplied by three to get the values for the entire spacecraft. Using (Eq. 3), the mass flow rate is then determined for different stages of the descent, depending on the fraction of the total power that is being supplied to the thrusters, as can be seen in Table 1:

Table 1- Mass Flow Rate Calculations

Thruster Power	Maximum Thrust (N)	Specific Impulse (s)	Mass Flow Rate (m/s)
25% Power	222	228	0.09936
50% Power	444	228	0.19871
100% Power	888	228	0.39742

Note that the specific impulse stays constant for all of the cases. This is because the specific impulse is a property of the thruster engines and is independent of how much power is being applied to the thrusters. Given the time of each burn, which will be found after integrating the differential equations of motion, the amount of propellant used during the descent can be found. This table will prove useful for when the variable thrust condition case is analyzed in the future. This type of analysis will also yield an optimal thrust profile.

In order to identify the times when the thrusters are on and when they are off, a switching function could be implemented. This switching function will be implemented so that the time that the thrusters are on can be easily output for each case. This function will be: [4]

$$K(t) = \frac{\lambda_v(t)}{r(t)} + \frac{\lambda_r(t)}{I_{sp} g} \tag{19}$$

This function will provide a plot that looks very similar to Figure 20.

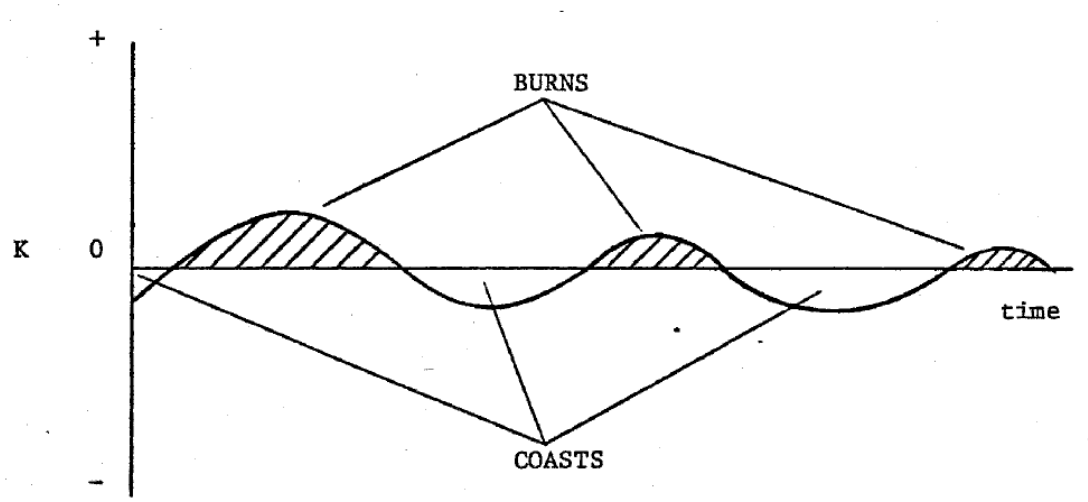


Figure 20- Switching Function Graph Sample [4]

The portions of the graph where K>0 show the times when the thrusters are on, thus providing information on total burn time, which ultimately yields the total mass expelled. From this information on the burn profile, an optimal thrust profile for a set of initial conditions can then be found.

In addition, this analysis assumes that the spacecraft's orbital velocity could be instantaneously slowed to zero, so that it can drop straight down to the surface. In some cases, however, this assumption might not hold, so this analysis would have to be redone. It would be a very similar type of problem except that the tangential velocity of the spacecraft would not be zero for the duration of the flight. Adding in a new dimension and modeling the spacecraft as falling to the surface on a curved path instead of a straight downwards one does make the problem more complex, but certainly one that can be analyzed effectively.

Appendix A

In order to produce all of the graphs in this report, MATLAB programming software needed to be utilized. The raw code that was used to calculate many of the values in the calculations section as well as produce the graphs and figures seen in the results and discussion section is provided in this appendix. Please note that there are two different MATLAB codes: The first is for the initial case of having no upwards thrust during the descent to the moons surface and the second is the continuous thrust case, where the thrusters are fired at a constant magnitude throughout the descent.

A.1 No Thrust Case Code

```
%Jason Harmon
%Honor's Thesis
%Schreyer Honors College
%Optimizing 1-D descent trajectories- No Thrust Case
%3/14/12

clear
clc

%initial input variables

g = 1.6; %in m/s^2. Acceleration due to moon's gravity
t = [0:1:100] ;  %Sets up a table of times from 0s to 100s in increments of 1s.

m_i =  500 ;   %in kg. Initial mass of the spacecraft
m_p_i = 400 ;  %in kg. Initial mass of propellant at the start of descent

y_0 = 3000 ;   %in m. The initial height of the spacecraft
y = y_0 - .5*g*t.^2 ;   %Calculates the height of the spacecraft as a funtion of time

y_dot_0 = 0 ;  %in km/s. The initial velocity of the spacecraft will be modeled as 0 for this
problem. This problem will begin after thrusters have instananeously
```

```
y_dot = y_dot_0+ g*t ;

thrust = 0 ;    %in Newtons. This is the no thrust case, so the thrust is 0.

m_p_f = 400 ;  %in kg. Final mass of propellant when spacecraft reaches moon
m_f = 500 ;   %in kg. Final mass of the spacecraft when it reaches the moon

fuel_remain = m_p_f ; %This is the amount of fuel remaining onboard the spacecraft.

subplot(2,2,1)
p = plot(t,y)
title('Altitude vs. Time for No Thrust');
xlabel('time (s)');
ylabel('altitude (m)');
AXIS([0 100 0 3000]);
set(p,'LineWidth',2.5);
hold on;

subplot(2,2,2)
q = plot(t,y_dot)
title('Velocity vs. Time for No Thrust');
xlabel('Time (s)');
ylabel('Velocity (m/s)');
AXIS([0 100 0 200]);
set(q,'LineWidth',2.5);
hold on;

subplot(2,2,3)
z = plot(y_dot,y)
title('Altitude vs. Velocity for No Thrust');
xlabel('velocity (m/s)');
ylabel('altitude (m)');
AXIS([0 100 0 3000]);
set(z,'LineWidth',2.5);
hold on;

subplot(2,2,4)
l = plot(t,fuel_remain)
title('Fuel Remaining vs. Time for No Thrust');
xlabel('time (s)');
ylabel('fuel remaining (kg)');
set(l,'LineWidth',2.5);
hold on;
```

A.2 Continuous Thrust Code

```
%Jason Harmon
%Honor's Thesis
%Schreyer Honors College
%Optimizing 1-D descent trajectories- Continuous Thrust Case
%3/14/12

clear
clc
%These are variables that can be changed for different scenarios:

thrust = 4455; %in Newtons. This is the continuous thrust of the hydrazine engines
m_p_i =900 ;  %in kg. Initial mass of propellant at the start of descent

%initial input variables

g_0 = 9.8; %in m/s^2. Acceleration due to Earth's gravity
g = g_0/6; %in m/s^2. Acceleration due to moons' gravity
mu = 4902777900000; %in m^3/s^2. Standard gravitational parameter of moon
r_moon = 1737400; %in m
Isp = 228; %in seconds. This is the value of the MRM-122 Redmond Hydrazine Rocket Engine
y_0 = 3000;  %in meters. The initial altitude of the spacecraft
y_f = 0;    %in meters. The final altitude of the spacecraft
y_dot_0 = 0;  %in m/s. The initial vertical velocity of the spacecraft
x_dot_0 = 0;  %in m/s. The intial horizontal velocity of the spacecraft

m_s =  100 ;   %in kg. Initial structural mass of the spacecraft
m_i = m_s + m_p_i ; %in kg. Initial total mass of the spacecraft

t = [0:1:200] ;  %Sets up a table of times from 0s to 200s in increments of 1s.

m_dot = thrust / (g*Isp); %in kg/s. This is the mass flow of the propellant through the thruster

m_tot = m_i-m_dot*t;
m = m_tot';

c = Isp*g_0;   %exit velocity of thrusters

%This is where the integration begins:

x0=[3000;0;m_i;-1;-1;-1];        %Our initial conditions
tspan = [0:1:200];       %Outputs the values calculated from t=0 to t=200 seconds. The time
increases by 1 every time.
```

```
options = odeset('RelTol',1.0e-8,'AbsTol',1.0e-6);  %Establishes relative and absolute tolerances
[t,x] = ode45('THESISEOMS',tspan,x0,options);   %generates a table of values

r = x(:,1);
u = x(:,2);
m_final = x(:,3);
lam_r = x(:,4);
lam_u = x(:,5);
lam_m = x(:,6);

%k_1 = lam_u/m;
%k_2 = lam_m/c;
%k_1_fin = k_1(:,1);   %We only need the first column of the k_1 matrix to add it to the k_2
%single column matrix

%k = k_1_fin + k_2;   %Calculates the switching funtion, k

alt = r;

t_final = t(r<10 & r>0)

if r<10 & r>0
   t_graph = t_final + 10;
else
   t_graph = 100;
end

z = [t,x];      %Creates a table with t in it as well

%An altitude vs. time graph will now be plotted

subplot(2,3,1)
q_1 = plot(t,alt)
title('Altitude vs. Time');
xlabel('time (s)     (a)');
ylabel('altitude (m)');
axis([0 t_graph 0 3000]);
set(q_1,'LineWidth',2.5);
hold on;

%A thrust vs. time graph will now be plotted

%subplot(2,3,2)
%plot(t,-k)
%title('K vs. Time');
%xlabel('time (s)     (b)');
```

```
%ylabel('K');
%axis([0 t_graph -.1 .1]);
%hold on;

%A relative velocity vs. time graph will now be plotted

subplot(2,3,3)
q_2 = plot(t,u)
title('Downward Velocity vs. Time ');
xlabel('time (s)   (c)')
ylabel('Downward Velocity (m/s)');
axis([0 t_graph 0 100]);
set(q_2,'LineWidth',2.5);
hold on;

%A Mass vs. time graph will now be plotted

subplot(2,3,4)
q_3 = plot(t,m_final)
title('Mass Remaining vs. Time');
xlabel('time (s)     (d)');
ylabel('Mass remaining (kg)');
axis([0 t_graph 0 1500]);
set(q_3,'LineWidth',2.5);
hold on;

subplot(2,3,5)
q_4 = plot(u,alt)
title('Altitude vs. Velocity');
xlabel('Velocity (m/s)     (e)');
ylabel('Altitude (m)');
axis([0 t_graph 0 3500]);
set(q_4,'LineWidth',2.5);
hold on;

subplot(2,3,6)
q_5 = plot(u,alt)
title('Altitude vs. Velocity (Close-Up)');
xlabel('Velocity (m/s)     (f)');
ylabel('Altitude (m)');
axis([0 20 0 20]);
set(q_5,'LineWidth',2.5);
hold on;
```

```
%This is the function that is called by ODE45.

function xdot=THESISEOMS(t,x)
xdot = zeros(6,1);

thrust = 4455; %in Newtons. This is the maximum thrust of the hydrazine engines

%initial input variables

g_0 = 9.8; %in m/s^2. Acceleration due to Earth's gravity
g = g_0/6; %in m/s^2. Acceleration due to moons's gravity
Isp = 228; %in seconds. This is the value of the MRM-122 Redmond Hydrazine Rocket Engine

m_dot = thrust / (g*Isp); %in kg/s. This is the mass flow of the propellant through the thruster

c = Isp*g_0;   %exit velocity of thrusters

%These are the differential equations that the ODE45 integrator will integrate

if x(1)>0
k = (x(5)/x(3))+(x(6)/c);
xdot(1)=-x(2);
xdot(2)=g-k*(thrust/x(3));
xdot(3)=-m_dot;
xdot(4)=0;
xdot(5)= -x(4);
xdot(6)= -x(5)*(thrust/(x(3)*x(3)));
else
    x(1) = 0;
end
return
```

References

[1] Bryson Jr., Arthur E., Ho, Yu-Chi. “Applied Optimal Control: Optimization, Estimation, and Control”. Hemisphere Publishing Corporation. Revised Printing. (1975). Print.

[2] McGuire, Melissa L., Oleson, Steven R. “COMPASS Final Report: Low Cost Robotic Lunar Lander”. NASA Glenn Research Center, Cleveland, Ohio. December, 2010. Web.

[3] "MR-107." *Encyclopedia Astronautica*. Web. 23 Mar. 2012. <http://www.astronautix.com/engines/mr107.htm>.

[4] Zondervan, K.P., “Optimal Low Thrust, Three Burn Orbit Transfers with Large Plane Changes”, Ph.D. Dissertation, California Institute of Technology, May, 1983. Web.

ACADEMIC VITA
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Education

The Pennsylvania State University – The Schreyer Honors College, August 2008 – May 2012
Candidate for B.S. in Aerospace Engineering with a minor in Engineering Leadership Development
Expected: May 2012

Ecole Mohammadia d'Ingenieurs (EMI) Spring 2012 Rabat, Morocco
Worked as a member of an international team consisting of students from a Moroccan engineering school (EMI) and Penn State University on addressing water resource problems in a small village in northern Africa. The team designed, constructed and presented a final prototype of a water chlorinator for the village to the President of ONEP (Office of National Potable Water) and the system will be tested and implemented in the village and potentially other villages very soon.

Corvinus University of Budapest Summer 2011 Budapest, Hungary
Worked as part of an international team consisting of students from Hungary and Gaza on a semester long project that combined engineering and business skills, culminating in a final project which received first place.

The National University of Singapore Summer 2009 Singapore, Singapore
Participated in a summer engineering course at the University through Penn State's Engineering Design Program. Had the opportunity to collaborate with international students and present a finalized design project to a diverse group of faculty.

Work Experience

ProGasket Aerospace and Automotive Haskell, NJ
Engineering Intern *Summer 2011*

- Designed various parts for the aerospace industry using the CadKey CAD program.
- Aided the director of engineering in the design and manufacturing of die sets for press machines.
- Completed material and cost quotes for various new contracts that were obtained throughout the summer

Awards

- Schreyer Honors College Academic Excellence Scholarship
- Diefenderfer Scholarship
- Sigma Gamma Tau – National Aerospace Engineering Honors Society
- Phi Eta Sigma – National Honors Fraternity
- Dean's List every semester

Activities

- Donor Relations Executive Chairperson of Atlas THON
- American Institute of Aeronautics and Astronautics