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ON DECISION THEORY MODELS INCORPORATING THE ELLSBERG PARADOX

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Figure 1: A confused monkey

1 Abstract

The Ellsberg paradox is a paradox in decision theory under uncertainty in which decision makers, when faced with multiple lotteries shy away from lotteries in which probabilities are uncertain, contradicting the hypothesis of Von-Neumann and Morgenstern Expected Utility theorem (1963). The so called 'Ellsberg preferences' not only seem to suggest that Von-Neumann and Morgenstern theories do not accurately represent the preferences of decision makers in certain situations, but violate many other axioms of decision theory as well. In this paper, I show which axioms the Ellsberg paradox contradicts and then introduce a couple of models that have incorporated the Ellsberg paradox. I compare/contrast the models and create functions for decision makers in which the model would be consistent with the Ellsberg paradox. For example, in Segal's process of evaluating ambiguous lotteries, the function $f(p) = \frac{e^p - 1}{e - 1}$ incorporates both versions of the Ellsberg paradox.

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3 Literature Review

The Ellsberg paradox has been a thorn to the Von-Neumann-Morgenstern Expected Utility Hypothesis [1947] (which we refer to as VNM from now on) ever since the theory was first published. The paradox, in essence, states that a non-ignorable number of decision makers display aversion to uncertain lotteries even though the expected value of the lottery is no less than the lottery they end up choosing. The Ellsberg paradox has insinuated that at best, the axioms chosen by VNM are too restrictive, or at worst, that the axioms do not accurately model real behaviour.

One point to note is that some subjects display a strong aversion to ambiguity. Binmore, Stewart and Voorhoeve [2000] have showed that a non-neglectable proportion of subject would prefer a lottery with the known probability of winning at $\frac{2}{9} = 0.\overline{22}$ over the lottery with the unknown but the expected chance of winning of $\frac{1}{3} = 0.\overline{33}$. This implies that there the so called 'Ellsberg preferences' is a real phenomenon to some decision makers, and not the by-product of off-chance.

Furthermore, the Ellsberg paradox not only violates Expected Utility Theory, but also many other axioms in decision making theory, including First Order Stochastic Dominance and Sure Thing Principle.

In the paper, I am going to introduce two decision models that resolve the Ellsberg paradox, one formulated by David Schmeidler [1986] and another formulated by Uzi Segal. [1986] I chose these models because both models provide an interesting axiomatic approach to handling the Ellsberg preferences and are thus worth studying. I build up the model starting from the axioms. I will explain the motivation behind each theory and address the strengths and weaknesses of each. I will then create my own functions in the Uzi Segal model and show how it is compatible with the Ellsberg preferences.

The paper is organized as follows. In the first section, I will introduce Expected Utility Theory and its axioms. I will then introduce the Ellsberg paradox and show how it violates pretty much every axiom in decision theory. Then, I will introduce the Choquet Integral and show how Schmeidler used the Choquet Integral as his decision model. I will the integrate the Choquet Integral with maxmin expected utility and introduce the Gilboa-Schmeidler [1989] model. In the fourth section, I will introduce Anticipated Utility and show how to calculate the value of an ambiguous lottery using Anticipated Utility. I will then introduce Segal models and create functions in the model to represent the preferences. Then, I will compare and contrast both theories, and conclude by providing a synopsis of where the research could go from here.

3.1 Expected Utility Theory

In 1947, John Von-Neumann and Oskar Morgenstern demonstrated that under certain axioms and when a decision maker is faced with several options, which we call lotteries, there exists an utility function that represents the decision maker's preferences. A utility function is in essence a real numbered ranking of the decision maker's lotteries. Thus, if the decision maker is faced with two lotteries, A and B and u is the utility function that represents the decision maker's preferences, A is (weakly) preferred to B if and only if u(A)$\geq$u(B). If A is (weakly) preferred to B, we write in notation A$\succeq$B. If A is strictly preferred to B, we write A$\succ$ B. If A is indifferent to B, we write A$\sim$B. The axioms that were assumed in the Von-Neumann Morgenstern utility theorem were as follows.

Completeness: For any lotteries A and B, either A$\succeq$B or $B \succeq$A.

Either A is preferred to B or B is preferred to A or there is no preference.

Transitivity: For lotteries A, B and C such that A$\succeq$B and B$\succeq$C, A$\succeq$C.

This means that for any three lotteries, preferences will remain consistent.

Continuity: If there exists lotteries A, B, C such that A$\succeq B \succeq C$, there exists a number p $\in [0, 1]$ such that $pA + (1-p)C \sim B$.

This assumes that there is a certain point where the decision maker is better or worse off than a given middle option.

Independence: If $A \succ B$, then for any N and p $\in$ (0,1],

$$pA + (1-p)N \succ pB + (1-p)N.$$

This axiom states that preferences are invariant over the possibility of a third lottery occurring. The theorem then goes on to say that for any decision maker satisfying the four axioms above, there exists a utility function u that assigns to each outcome A a real number u(A) such that for any two lotteries A and B,

$$A \succeq B \Leftrightarrow Eu(A) \geq Eu(B)$$

where $Eu(L)$ is the expected value of u in L and u(L) is defined to be the utility of the lottery L

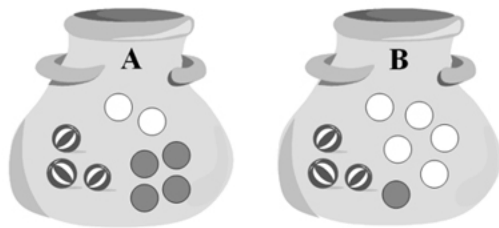

Figure 2: The Ellsberg Paradox

if L occurs as a consequence with probability 1.

Thus, if lottery L results in the prize A_1 with probability p_1,..., A_n with probability p_n then,

$$Eu(L) = p_1 u(A_1) + ... + p_n u(A_n)$$

The Ellsberg paradox is one (of quite a few) example of a paradox when people's choices seem to contradict the expected utility hypothesis. In the paradox, the decision maker has to pick between playing two urns with two different coloured balls in each urn. In one urn, the decision maker knows that the two coloured balls are evenly composed in number. In the other urn, the decision maker does not know the exact compositions of the colors of the balls, but know that the coloured-balls in the urn are randomly distributed. The decision maker is given several bets and is told that should he pick the correct coloured ball from the urn, he will win a certain amount of money. The Ellsberg paradox says that the decision maker will shy away from bets whose probability are unknown, even though there is no reason to believe that the probabilities will be more or less than another.

3.2 The Ellsberg Paradox

Game 1 Suppose that we have two urns; urn 1 contains 50 black and 50 red balls and urn 2 contains 100 balls of yellow and blue balls (but unspecified numbers of each). The decision maker chooses one ball from each urn. He wins $100 for each event ball he chooses correctly. Denote the following events as

A. A red ball is chosen from urn 1
B. A black ball is chosen from urn 1
C. A yellow ball is chosen from urn 2
D. A blue ball is chosen from urn 2

Formally, the decision maker needs to choose the bets from the product set (A,B) X (C,D). Then, the game is played again, except now, the decision maker must choose which urn to pick one ball from. If he wins, he receives $100. This time, the decision maker must choose a bet from the following events (A, B, C, D).

The decision makers who played the following game were indifferent between picking A and B and they were also indifferent between picking C and D. [Ellsberg] The majority also were indifferent between playing A (or B) to C (or D). However, a non-negligible portion of the decision makers preferred every bet from urn 1 (A or B) to every bet from urn 2 (C or D). In essence, they preferred bets in which probabilities were known to bets whose probabilities were unknown, but had no reason to believe were more or less than the known probabilities. Becker and Brownson [1964] and Mac Crimmon and Larsson [1979] also recorded evidence of such preferences in their studies.

Example 1 *There are four events in the first version of the above game, ((A,C),(A,D),(B,C),(B,D)). Let us assume that the deterministic outcomes are sums of dollars and assume that there are 2 deterministic outcomes, $0 and $100. An act assigns to each state a probability distribution over the outcomes. The bet "$100 if A, $0 otherwise in urn 1" is an act which assigns the degenerate lottery $100 to every state in event A with probability 1 and $0 to every state in event B with probability 1.*

These preferences seem to contradict themselves, since the probability of getting A and B is $\frac{1}{2}$ but strict preference of A and B to C and D seems to imply that the probability of getting C and D is $\frac{1}{2}$.

In my view, there is one possible criticism to the Ellsberg Paradox. If the interviewer does not tell the subjects the exact probabilities of each lottery, the subject may fail to calculate the correct expected value of each lottery. Specifically, if the decision maker fails to correctly calculate the expected value of the unknown lottery, he might perceive that the unknown lottery will result in a lower expected value than the lottery with the known probabilities. Thus, preferring the lottery with the known probabilities would then be consistent with VNM. Even if a small percentage of subjects were to make this mistake, it would create the illusion of a paradox. The best way to avoid this mistake is if the experimenter explicitly tells the subjects beforehand that the expected value of the unknown and the known lotteries are the same. Even then, some subjects may distrust the experimenter, as if the type of game triggers a deceit aversion mechanism. Again, the decision maker would believe that the expected value of the known lottery is higher than that of the unknown lottery.

Game 2 Suppose that we have a urn containing twenty red balls and forty balls of either green or blue (with the exact quantities unknown). Consider the following bets.

A. A red ball is chosen.
B. A green ball is chosen.
C. A red or blue ball is chosen.

D. A green or blue ball is chosen.

The subject has four choices. He has to pick either A or B and then either C or D. Formally, his strategy set is $\{(A,C),(A,D),(B,C),(B,D)\}$ If the subject's bet comes true, he wins \$100; otherwise, he gets nothing.

Empirically, studies have shown that the subject prefers A to B but D to C. Thus, the subject prefers bets in which the probabilities are exactly known and shies away from bets in which the chances of winning are unknown, even if it were to mean that the subject was less likely to win. In other words, this subject displays ambiguity aversion in his decision making.

Even though there may be valid reasons for such references, they run contradictory to subjective standard expected utility theory. For example, for any Borel Probability measure P that represents the individual's outcome of events, A$\succ$B implies that grabbing a red is more likely than grabbing a green out of the urn so we have $P(R) > P(Gr)$ (1) where R stands for the red ball, Bl stands for the blue ball and Gr stands for the green ball. However, D$\succ$C implies that grabbing a green or blue ball from the urn is more likely than grabbing a red or blue ball. Thus, we have $P(Gr) + P(Bl) > P(R) + P(Bl)$. Thus, $P(Gr) > P(R)$ which contradicts (1).

Perhaps the most ambitious attempt to represent the beliefs of a decision maker of uncertain events through additive prior probability was by Savage. Motivated by authors such as Ramsey and VNM, Savage suggested maximization of expected utility, carried out with respect to a prior probability from the decision maker's preferences over acts to be the criterion for decision makers. The act here assigns to each state an objective lottery with probability 1. The axiom that I will show to be violated by the decision maker in the above example is called the Savage's Sure Thing Principle.

The following two axioms, First Order Stochastic Dominance and Savage's Sure Thing Principle, are widely used in analyzing decision makers behaviour under uncertainty. First Order Stochastic Dominance is more widely used than Savage's Sure Thing Principle. For example Machina (1982), Quiggin (1982), Chew (1983), Yaari (1984) and Segal (1984) do not accept the Sure Thing Principle, but all accept the First Order Stochastic Dominance. Thus, the Ellsberg paradox not only challenges Expected Utility Theory, but every other theory of rational behavior under uncertainty in which probabilities are additive.

Definition 1 (Savage's Sure Thing Principle) *Let f, f', g, g' be lotteries and let S be an event. If on S, f=g and f'=g', and on not S, f=f' and g=g', then f is preferred to f' if and only if g is preferred to g'.*

Theorem 1 (Preferences violate Sure Thing Principle) *Let A, B, C, D be lotteries as above. Let $S = \{R \cup G\}$ Let f = A, f' = B, g'= D and g = C. Then on S, f = g, f'= g' and on not S, f*

= *f' and g = g'. However, we have f preferred to f' and g' preferred to g. This is a contradiction to Savage's Sure Thing Principle.*

Definition 2 (First Order Stochastic Dominance Axiom) *Let $F_a(x)$ be the probability of winning not more than x in lottery A. If for every x, $F_a(x) \leq F_b(x)$, then lottery A is weakly preferred to lottery B.*

Definition 3 (not S) *Given all possible events Ω and an event $S \in \Omega$, not S is defined to be $\Omega \backslash S$.*

Theorem 2 (Preferences violate First Order Stochastic Dominance Axiom) *Lemma 1: By this axiom, the lottery (x, S, 0, not S) is preferred to the lottery (x, T, 0, not T) $\Leftrightarrow$ the subject probability of S is greater than that of T.*

Proof: Since $A \succ B$, by Lemma 1, $\frac{1}{3} = P(R) \succ P(G)$. Since $P(R \cup B) + P(G) = 1$, $P(R \cup B)$ is bounded by $\frac{2}{3}$. However, this implies that $C \succ D$ This is a contradiction to the preference $D \succ C$.

4 Choquet Integral

The Ellsberg paradox (along with the Allais paradox) have called into question classical models of Expected Utility. The crux of the problem, as demonstrated above, lies in the hedging effects that are used under the independence axiom. Thus, slightly altering the independence axiom so that the axiom only holds when hedging effects are not present, Schmeidler [1986], Gilboa [1985], Quiggin [1982] and Yaari [1984] have built models to order the preferences in the absence of hedging effects. The functional they used in the model was the Choquet Integral and the models that they have created explain the paradoxes and allow more diversified patterns of behavior under uncertainy and risk and allow to separate the perception of risk and uncertainty from the valuation of the outcomes.

The basic idea of the Choquet integral is to assign a weight to every possible outcome and then to compute a weighted average of the values of the subsets. In order to build up the Choquet Integral, we must introduce some definitions.

4.1 Capacity

Let T be the sets of nature, that is in Game 1, A, B, C or D. Let $\epsilon \subset 2^T$ be a sigma-algebra of subsets of T, F $\in \epsilon$ be an event and Y the set of lotteries on a given consequence set. We define the acts to be the horse lotteries (as in Anscombe-Aumann) of mappings from F to Y. The set of all acts will be denoted by A.

Let S be an algebra of subsets of some non-empty set S_0.

Definition 4 (Capacity) *A capacity is a function*

$$v : S \to R$$

which satisfies
(i) Monotonicity: A, $B \in S$ and $A \subset S$ imply

$$v(A) \leq v(B)$$

(ii) Normalisation:

$$v(\emptyset) = 0, v(S) = 1$$

Definition 5 (Choquet Integral) *Let S be a set and $\mathcal{P}(\mathcal{S})$ be the set of subsets such that $\emptyset \in \mathcal{P}(\mathcal{S})$. Let v: $\mathcal{P}(\mathcal{S}) \to [0,1]$ be a capacity on (S, $\mathcal{P}(\mathcal{S})$). Let f: $S \to [0,\infty]$. Furthermore, suppose that S is a finite set. Then, the Choquet Integral of f w.r.t v is defined by:*

$$\int_S f \, \mathrm{d}u = \int_{-\infty}^{0} v(\{x : f(x) > t\} - 1)\mathrm{d}t + \int_0^\infty v(\{x : f(x) > t\}\mathrm{d}t$$

provided $\{x : f(x) > t\} \in \mathcal{P}(\mathcal{S})$

If S is finite, then the Choquet Integral of f can be written as follows
$\int_S f \, \mathrm{d}v = \sum_{i=1}^{n} (f(w_i) - f(w_{i-1}))u(A_i)$
where the subscript (.) indictes that the indices have been permuted in order to have $f(w_1) < ... < f(w_n)$, $A_i = \{w_1, ..., w_n\}$ *and* $f(w_0) = 0$, *by convention.*

Remark 1 *If v coincides with a probability measure P, then* $\int_S f \, \mathrm{d}v$ *coincides with the mathematical expectation of f(x) with respect to P,* $E_P(f(x))$.

Remark 2 *Let X be an act from the set of acts, A. Then, we can decompose X into the sum of the product of its sets of nature and the consequences of the lottery; that is,* $X = \sum_{i=1}^{n} y_i A_i^*$ *where* y_i *is the set of lotteries on a given consequence set with* $y_1 \leq y_2 \leq ... \leq y_n$ *and* A_i^* *is a partition of the states of nature T and a characteristic function of t; that is* $A_i^*(t) = 1$ *if* $t \in A_i, 0$ *otherwise. Then, one obtains:* $u(X) = \sum_{i=1}^{n} a_i A_i^*$ *with* $a_1 = f(y_1) \leq ... \leq a_i = f(y_i) \leq ... \leq a_n = f(y_n)$ *and the Choquet Integral of X is then given by*

$$\int_S f(X) \, \mathrm{d}v = a_1 + (a_2 - a_1)v[f(x) \geq a_2] + ... + (a_n - a_{n-1})v[f(x) \geq a_n]$$

Thus, the Choquet Integral, $\int_S f(X) \, \mathrm{d}v$ has a meaningful interpretation. The decision maker computes the value of X by taking for sure the minimum expected payoff a_1 and then adds to this the successive larger payoff $a_i - a_{i-1}$ $2 \leq i \leq n$ that is scaled to how likely the decision maker thinks $v[f(x) > a_i]$ will happen. Thus, the formula is an expectation operation with respect to not necessarily additive probabilities.

Example 2 *For instance, if* $u(A_3) \leq u(A_2) \leq u(A_1)$, *we have*

$$\int_S f \, \mathrm{d}u = u(A_3)[f(3, 2, 1) - f(2, 1)] + u(A_2)([f(2, 1) - f(1)] + u(A_1)f(1)$$

Weak Order (a) For all acts f and g: f $\succeq g$ or $g \succeq f$. (b) For all acts f, g and h in L; If $f \succeq g$ and $g \succeq h$, then $f \succeq h$.

Two acts f, g: S $\rightarrow Y$ are said to be **comonotonic** if for no s and t in S, $f(s) \succ f(t)$ and $g(t) \succ g(s)$.

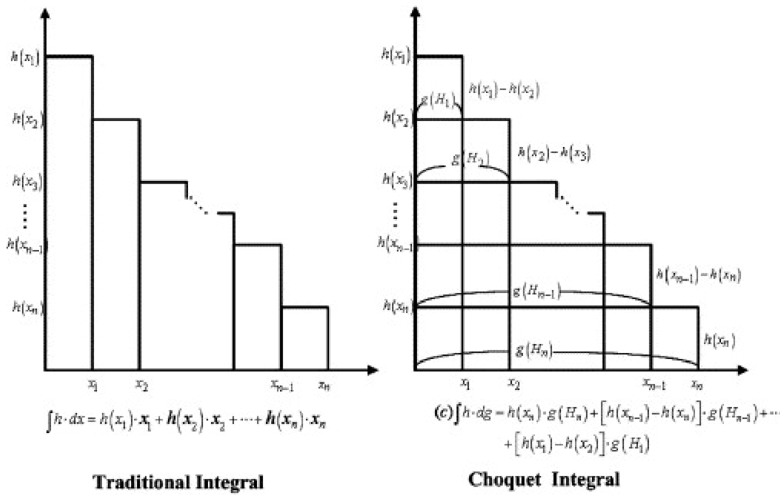

Figure 3: Choquet vs Riemann Integral

If X is a set of numbers and preferences respect the usual order on numbers, then any two X-valued functions f and g are comonotonic iff $(f(s) - f(t))(g(s) - g(t)) \geq 0$ for all s and t in S.

The bet \$100 if II_A is an act that assigns the degenerative lottery of receiving "\$100 with probability one" to each state in II_A and "\$0 with probability one" to each state in II_B. Consider the bets II_A and II_R. Let A be the state $[I_B, II_R]$, e.g. a black ball is chosen in urn 1 and a red ball is chosen from urn 2 and let B be the state $[I_R, II_B]$. Then we know that $[II_R(A) - II_R(D)] \geq 0$ and $[II_B(A) - II_B(D)] \leq 0$. Thus $[II_R(A) - II_R(D)][II_B(A) - II_B(D)] \leq 0$ and so II_A and II_R are not comonotonic.

Monotonicity For all acts f and g, and states, if f(s) $\succeq$ g(s) on S, then f $\succeq$ g.

Comonotonic Independence
For all pairwise comonotonic acts f, g and h and for all α in (0,1): $f \succ g$ implies $\alpha f + (1 - \alpha)h \succ \alpha g + (1 - \alpha)h$

Nondegeneracy: Not for all f and g in L, f$\succeq$ g.

4.2 Main Theorem of Choquet Integral

In this theorem, the independence axiom is replaced by the stronger comonotonic independence axiom. A good case why the independence axiom cannot be assumed is as follows. Suppose that S, the set of all states, is $[0, \pi]$ and that $f(s) = |(sin(s))|$ and $g(s) = |cos(s)|$. Suppose that the decision maker prefers the act g(s) because he believes that the interval $[\frac{\pi}{4}, \frac{3\pi}{4}]$ is more probable than its complement. Now suppose that we have an extremely complex function $h(s) = cos(64s)sin(32s)$. It is immediately not clear whether $f' = \frac{1}{2}f(s) + \frac{1}{2}h(s) \succ g' = \frac{1}{2}g(s) + \frac{1}{2}h(s)$. This is because when coupled with another act, the combination of acts may define a much larger set of lotteries, and the decision maker might do a careful retrospection and comparison of the new acts f' and g' and determine that g' is preferred to f'. However, if f, g and h are pairwise comonotonic (that is any two of f, g and h are comonotonic) then the choice between f and g as well as f' and g' are not much at all different. Thus, the comonotonic independence gets rid of the contradiction that we encounter in the Ellsberg preferences.

Schmeidler (1984), (1986) proved that if a preference relation satisfies weak order, comonotonic independence, continuity, monotonicity and nondegeneracy, then there exists a unique nonadditive probability v on S and an affine real valued capacity u on S such that for all acts $f, g \in A$, we have

$$f \succeq g \leftrightarrow \int_S f\,\mathrm{d}u \succeq \int_S g\,\mathrm{d}u$$

Suppose that

$$I(t) = \int_{-\infty}^{0} v(\{x : f(x) > t\} - 1)\mathrm{d}t + \int_{0}^{\infty} v(\{x : f(x) > t\}\mathrm{d}t.$$

Dellarchie [1970] proved that for $a \geq b$ I(a) $\geq I(b)$ and $I(\lambda a) = \lambda I(a)$ for $\lambda > 0$. He also proved that $I(a + b) = I(a) + I(b)$. Since from above, we know that the decision maker can use the Choquet Integral to represent preferences, we now want to investigate the opposite direction, when a continuous and monotonic functional I (a map from a vector space to its underlying scalar field), that is additive on comonotonic functions can be representable through an integrable operation as above with respect to a monotonic set function v. Schmiedler [1986] result is stated below.

Schmeidler's Integral Representation without Additivity

Theorem: Let $I : B \to \Re$ satisfy $I(S^*) = 1$ as given. Suppose also that the functional I satisfies
(i) Comonotonic additivity: a and b comonotonic implies $I(a + b) = I(a) + I(b)$
(ii) Monotonicity: $a \geq b$ on s implies $I(a) \geq I(b)$.
Then, defining $v(E) = I(E^*)$ on Σ we have for all t in B,

$$I(t) = \int_{-\infty}^{0} v(\{x : f(x) > t\} - 1)\mathrm{d}t + \int_{0}^{\infty} v(\{x : f(x) > t\}\mathrm{d}t.$$

A capacity is called **convex** if for all A and B $\in S$

$$v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$$

A preference relation satisfies **uncertainty aversion** if for all f, g $\in A$ and for $\alpha \in (0, 1)$, f $\sim g$ implies $\alpha f + (1 - \alpha)g \succeq f$

An interesting and useful corollary that Schmeidler proved is the equivalence of the following two properties. v is convex if and only if for all a in b, I(a)=min $\int a\mathrm{d}p | p \in core(v)$.

Afterwards, Gilboa and Schmeidler proved in 1989 if a preference relation satisfies completeness, transitivity, certainty independence, continuity, and monotonicity on the set measurable finite step functions from T to Y, there exists an affine utility function u: Y $\rightarrow R$ and a non-empty, closed and convex set C of finitely additive probability measures on S such that

$$f \succeq g \Leftrightarrow \min_{P \in C} \int u(f(\cdot))\mathrm{d}P \geq \min_{P \in C} \int u(g(\cdot))\mathrm{d}P$$

4.3 Application to the Ellsberg Paradox

In Game 2, we can let the capacities equal
$v(R) = \frac{1}{3}$ $v(G) = v(Bl) = \frac{1}{6}$
$v(G; Bl) = \frac{2}{3}$ $v(R; G) = v(R; Bl) = \frac{1}{2}$
Then
$v(A) = [u(100) - u(0)]v(R) + u(0)$
$v(B) = [u(100) - u(0)]v(G) + u(0)$
$v(C) = [u(100) - u(0)]v(R; Bl) + u(0)$
$v(D) = [u(100) - u(0)]v(G; Bl) + u(0)$

With the Choquet Integral, it is our job to assign a value to every possible outcome. Therefore, it is important and imperative to assign an appropriate value to our capacity. For example, if we mis-assign the value of our capacity, this can throw off our value quite a bit. The Ellsberg paradox will still be 'rectified' as long as the assumptions in the theorem are met, but the value of the Choquet Integral will not accurately represent the utility of the decision maker.

One thing to note is that the Choquet Integral is not an additive, but rather a sub-additive integral. This allows the Ellsberg preferences to be consistent when used with the Choquet Integral.

5 Two Part Decision Making Process

In 1986, Uzi Segal of the University Toronto proposed another way to resolve the so called Ellsberg paradox. Suppose that a decision maker is faced with the following lottery. A decision maker is at a race track betting on a horse race with two horses named Hoof and Heart. Hoof will win the race p percent of the time. Suppose that the decision maker is faced with the lottery $1,000,000 if Hoof wins and 0 otherwise. The decision maker has some rough beliefs on p, but obviously cannot assign a pinpointed value to it. Thus, he assigns a probability density function to his belief. The first stage is over the random variable p and its outcome is denoted as $\overline{p}$. In the second stage, we play the lottery ($1,000,000, $\overline{p}$, 0, not $\overline{p}$).

Uzi Segal proved that under certain conditions, the lottery ($1,000,000, $\overline{p}$, 0, not $\overline{p}$) is preferred to the lottery ($1,000,000, Hoof wins, 0, otherwise); in essence, he proved that when comparing two lotteries, the lottery with the probability of winning being the mean of the cumulative's distribution function is preferred to the lottery played with the chance of winning being all the possible values of the lottery.

5.1 Anticipated Utility

Allais paradox, MacCrimmon [1968], and Kunreuther et al. [1978] which examined behaviors of home owners in flood and earthquake-prone areas are all examples of the belief that people in real life situations do not conform to the VNM axioms. As a response, Quiggin [1980] generalized expected utility by using a set of weaker axioms than VNM in which he provided an axiomatic approach to his model, anticipated utility.

Anticipated Utility is a more general version of Expected Utility. We measure the value of the lottery by Anticipated Utility in Segal's theorem. Since it does not satisfy the additive property, the Ellsberg paradox, we can manoeuvre around the additivity paradox and make it compatible with the Ellsberg paradox.

Let L_1 be the set of bounded random variables over $\mathbb{R}$. For A$\in F_a$, define the cumulative distribution function F_a by
$F_a(x) = Pr(A \leq x)$ and $A^+ = inf\{x : F_a(x) = 1\}$ and $A^- = sup\{x : F_a(x)\} = 0$.

Let L_1^* be the set of elements of L_1 which have a finite range. The elements of L_1^* are called prospects and they are denoted by vectors of the form $(x_1, p_1; x_2, p_2; ...; x_n, p_n)$ with $x_1 \leq x_2 \leq ... \leq x_n$ and $\sum p_i = 1$. (x_i, p_i) is interpreted as the lottery yielding x_i dollars with probability p_i. Thus, by the definition of cumulative density function, we have

$$F_A(x) = \begin{cases} 0 & \text{if } x \leq x_i \\ \sum_{j=1}^{i} p_j & \text{if } x_i \leq x < x_{i+1} \\ x & \text{if } \geq x_n \end{cases}$$

On L_1, we assume the existence of a complete and transitive binary relation. We say that a function $U : L_1 \to \mathbb{R}$ represents a relation $\succeq$ if for all A, B $\in L_1$, A $\succeq$ B iff U(A) $\geq$ U(B). For example, the expected utility functional, a utility function widely used to measure the value of a lottery, is one such representation, and it is defined by

$$\int_{-\infty}^{\infty} u(x)\, \mathrm{d}F_A(x)$$

On L_1^*,

$$V(x_1, p_1; ...; x_n, p_n) = \sum p_i u(x_i).$$

Over the last two decades, several alternatives have been proposed to Expected Utility Theory. Quiggin (1982) suggested the following generalization of the above, which he called Anticipated Utility.

$$V(A) = u(A^-) + \int_{A^-}^{A^+} u'(x) f(1 - F_A(x))\, \mathrm{d}x = \int_{A^-}^{A^+} u(x)\, \mathrm{d}f(1 - F_A(x))$$

where f:$[0, 1] \to [0, 1]$ are decision weight functions satisfying $f(0) = 0$ and $f(1) = 1$.

On L_1^* this takes one of the following equivalent forms.

$$V(x_1, p_1; ...; x_n, p_n) = u(x_n) f(p_n) + \sum_{i=1}^{n-1} u(x_i)[f(\sum_{j=i}^{n} p_j) - f(\sum_{j=i+1}^{n} p_j)] \quad (1)$$

$$= u(x_1) f(p_1 + ... + p_n) + \sum_{i=2}^{n} [u(x_i) - u(x_{i-1})] f(\sum_{j=i}^{n} p_j) \quad (2)$$

5.2 Anticipated Utility verses Expected Utility

Suppose that we have a lottery A= $(x_1, p_1; x_2, p_2; ...; x_n, p_n)$. Now, we want to measure the value of this lottery according to expected utility and anticipated utility. If we draw this on our figure with the outcomes on the x-axis and the probabilities on the y-axis, we would have the following figure.

To calculate the value of the lottery A using expected utility, we measure the set A_0. We employ the usual Riemann definition to calculate the area under the curve.

To calculate the value of the lottery A using anticipated utility, we also have to measure the set A_0. However, instead of being confined to the usual Riemann definition of the area under the curve, we may employ more general measures to measure the set. Thus, expected utility is a special case of anticipated utility.

5.3 Introducing two-stage lotteries

In this subsection, we show how we use anticipated utility to calculate the value of a two-stage lottery.

Let $L_2^* = \{(A_1, p_1; ...; A_m, p_m) : \sum p_i = 1 p_1, ..., p_m > 0; A_1, ..., A_m \in L_1^*\}$. Elements of L_2^* are called two stage lotteries. A lottery X$\in$ L_2 yields a ticket to the lottery A_i with probability $p_i.i = 1, ..., m$. Thus, at time t_1, the decision maker faces the lottery $(1, p_1; 2, p_2, ...; m, p_m)$. After he wins some number i, he faces the lottery A_i at time $t_2 > t_1$. We assume that the decision maker discount factor is 1. Thus, once he knows he will win a certain amount of money, his decision making is independent of the time he actually receives the money.

Let $\succeq_2$ be a preference relation on L_2. Since the decision maker's discount factor is 1, we can create a natural isomorphism between L_1^* and L_2^*, namely, a lottery

$$(x_1, p_1; ...; x_n, p_n) \sim ((x_1, 1), p_1; (x_2, 1), p_2; ...; (x_n, 1), p_n)$$

Thus, we may omit the subscript 2 in the preference relation and the preference relation over one stage and two stage lotteries is denoted by $\succeq$.

Now, we have to have some way to compare lotteries in L_2^* with lotteries in L_1. The following two axioms deal with the subject.

Reduction of Compounds Lotteries Axiom (RCLA) If the decision maker is indifferent to the resolution timing of the uncertainty, then he may assume both stages to be conducted at time t_1. Thus, a two-stage lottery can be reduced to a simple one-stage lottery. Formally, let $A_i = (x_1^i, p_1^i; ...; x_n^i, p_n^i)$ $i = 1, ..., m$. Then,

$$(A_1, p_1; ...; A_m, p_m) \sim (x_1^1, p_1 p_1^1; ...; x_{n_1}, p_1 p_{n_1}^1; ...; x_1^m, p_m p_1^m; ...; x_{n_m}^m, p_m p_{n_m}^m)$$

Independence Axiom (IA) The relation $\succeq$ on L_2^* induces several relations on L_1^*. The Independence Axiom assumes that these relations coincide and are equal to $\succeq$ on L_1^*. Formally,

$$(A_1, p_1; ...; B, p_i; ...; A_m, p_m) \succeq (A_1, p_1; ...; C, p_i; ...; A_m, p_m) \Leftrightarrow B \succeq C$$

RCLA axiom paradox

Anticipated Utility is compatible with both the RCLA and Independence Axiom. However, we can create an example where in fact RCLA some decision makers might not accept RCLA. We take from Kahneman and Tversky [1979]. (a) Choose between
$A_1 = (3000, 1)$ and $A_2 = (0, 0.2; 4000, 0.8)$
(b) Choose between
$B_1 = (0, 0.75; 3000, 0.25)$ and $B_2 = (0, 0.8; 4000, 0.2)$
(c) Choose between
$X_1 = (0, 0.75; A_1, 0.25)$ and $X_2 = (0, 0.75; A_2, 0.25)$

By IA, $X_1 \succeq X_2$ iff $A_1 \succeq A_2$, while by RCLA, we have that $X_1 \succeq X_2$ iff $B_1 \succeq B_2$. However, Kahneman and Tversky [1979] found in their studies that most people prefer A_1 to A_2, B_2 to B_1 and X_1 to X_2.

Define the certainty equivalence to be the certain value that is equally attractive to the risky asset. Thus, if A is the risky asset, the certainty equivalence of A, CE(A) satisfies (CE(A),1) $\sim$ A. If $\succeq$ satisfies IA, then

$$(A_1, p_1; ...; A_m, p_m) \sim (CE(A_1), p_1, ..., CE(A_m), p_m)$$

If $\succeq$ can be represented by the Anticipated Utility Function, then CE(A)=$u^{-1}(V(A))$. Let $(A_1, p_1; ...; A_m, p_m) \in L_2^*$ and assume without loss of generality that $CE(A_1) \leq ... \leq CE(A_m)$. Then the above implies that

$$(A_1, p_1; ...; A_m, p_m) \sim (u^{-1}(V(A_1)), p_1; ...; u^{-1}(V(A_m)), p_m)$$

By the anticipated utility function, this becomes

$$V(A_1, p_1; ...; A_m, p_m) = V(A_1) + \sum_{i=2}^{m} [V(A_i) - V(A_{i-1})] f(\sum_{j=1}^{m} p_j).$$

5.4 The value of an ambiguous lottery

We consider the lottery (X,S,0,not S). Here, we think of this as the lottery with outcome X if S occurs and 0 otherwise. If the chance of S occurring is p, the value of the said lottery is given by

$u(X)f(p)$. If the decision maker does not know the exact value of S, but has some rough beliefs on S, then we can assign a probability density functions to his beliefs. If his beliefs are discrete, then $P^*(p)$ will denote the probability that the probability of S is p. If his beliefs are non-discrete, then $F^*(p)$ is the probability that the probability of S is not greater than p. In this section, we assume that whenever the decision maker is confronted with an ambiguous lottery, that he/she consider it as a two-stage lottery. The first stage is over the random variable p and its outcome is denoted by $\bar{p}$. The distribution function is F^*. In the second stage, the decision maker participates in the lottery $(X, \bar{p}$,0, not $\bar{p})$.

Let x>0. For each p, let y(p)=CE(x,p,0,1-p)=$u^{-1}(u(x)f(p))$. That is, we define y(p) to be the certainty equivalence of a lottery X with probability p.

For every 0≤ $y \leq x$, let p(y) be defined implicitly by (x,p(y);0,1-p(y)) $\sim$ (y,1) and explicitly by p(y)=$f^{-1}(\frac{u(y)}{u(x)}$. Thus, p(y) is the probability that the decision maker would be indifferent in playing the lottery (x,p(y);0,1-p(y)) or accepting y with probability 1. Let F^* be a distribution function over the possible values of the probability of S in the lottery $(x, S;0$,not S). If the decision maker considers this lottery as an ambiguous two stage lottery, then the probability that the certainty equivalence of (X,S;0, not S) is not greater than y is given by $G(y) = F^*(f^{-1}(\frac{u(y)}{u(x)}))$. Let α^* and $\mathbb{B}^*$ be the minimal and maximal possible values of the probability of S, that is, $\alpha^* = sup\{p : F^*(p) = 0\}$ and $B^* = inf\{p : F*(p) = 1\}$. By above, the value of the ambiguous lottery (x,S,0,not S) equals

$$u(x)f(\alpha^*)+\int_{u^{-1}[u(x)f(\alpha^*)]}^{u^{-1}[u(x)f(\mathbb{B}^*)]} u'(y)f(1-G(y))\mathrm{d}y = u(x)f(\alpha^*)+\int_{u^{-1}[u(x)f(\alpha^*)]}^{u^{-1}[u(x)f(\mathbb{B}^*)]} u'(y)f(1-F^*(f^{-1}(\frac{u(y)}{u(x)})))\mathrm{d}y$$

Let $z = f^{-1}(\frac{u(y)}{u(x)})$. Then, $f(z)u(x) = u(y)$. Then $f'(z)u(x) = u'(y)$. Substituting this back into the equation, we have

$$u(x)f(\alpha^*) + u(x)\int_{\alpha^*}^{\mathbb{B}^*} f'(z)f(1 - F^*(z))\mathrm{d}z$$

We let $u = f(1 - F^*(z))$ and $\mathrm{d}v = f'(z)\mathrm{d}z$ and integrate by parts.

$$u(x)\int_{\alpha^*}^{\mathbb{B}^*} f(z)f'(1 - F^*(z))F^{*'}(z)\mathrm{d}z$$

Now suppose that the range of F^* is finite such that the probabilities of S are $\alpha^* = p_1 \leq ... \leq p_m = \mathbb{B}^*$. Now, the value of the ambiguous lottery is

$$u(x)f(p_1) + u(x)\sum_{i=2}^{m}[f(p_i) - f(p_{i-1})]f(\sum_{j=i}^{m} P^*(p_j))$$

Now consider the case when $x < 0$. Let $\bar{f}(p) = 1 - f(1-p)$. By the value of the lottery (x,p;0,1-p) is u(x)$\bar{f}(p)$. The value of an ambiguous lottery (x,S; 0,not S) equals

$$u(x)\bar{f}(\mathbb{B}^*) + \int_{u^{-1}[u(x)\bar{f}(\mathbb{B}^*)}^{u^{-1}[u(x)\bar{f}(\alpha^*)} u'(y)f(F^*(f^{-1}(\frac{u(y)}{u(x)})) = u(x)\bar{f}(\alpha^*) + u(x)\int_{\alpha^*}^{\mathbb{B}^*} \bar{f}'(z)\bar{f}(1-F^*(z))$$
$$= u(x)\int_{\alpha^*}^{\mathbb{B}^*} \bar{f}(z)\bar{f}'(1-F^*(z))F^{*'}(z)\mathrm{d}z$$

If $x < 0$ and F^* is discrete, then this reduces to

$$u(x)\bar{f}(p_1) + u(x)\sum_{i=2}^{m}[\bar{f}(p_i) - \bar{f}(p_{i-1})]\bar{f}(\sum_{j=i}^{m} P^*(p_j))$$

Ambiguous vs. Non-ambiguous lotteries

In this section, we talk about how in under certain conditions, a lottery in which its probability is known is preferred to a lottery in which its probability is unknown. Let F^* be a distribution function of S in the lottery (x,S;0, not S). First, consider when the decision maker's beliefs are symmetric around the mean of F^*, $\bar{p}$. That is, for every $\epsilon > 0$, we have that $F(\bar{p}-\epsilon) = 1 - F(\bar{p}+\epsilon)$ In game 1, the decision maker has no reason to think for any $0 \leq i \leq 100$ that the combination of i yellow and $100 - i$ blue balls is more likely than $100 - i$ yellow and i blue balls. Thus, the decision maker has a symmetric belief of yellow and blue balls about the mean. Similarly in Game 2, the decision maker has no reason to believe that the combination of i green and $40 - i$ blue balls is any more likely than $40 - i$ green and i blue balls.

In order to proof our upcoming theorem that known lotteries are preferable to ambiguous lotteries in certain conditions, we have to define elasticity.

The elasticity of a differentiable function f: $\mathbb{R} \to \mathbb{R}$ is given by $\frac{xg'(x)}{g(x)}$.

Example 3 *The elasticity of* $f(p) = 1 - (1-p)^{\frac{1}{n}}$ *is given by* $\frac{p(1-p)^{\frac{1}{n}-1}}{n(1-(1-p))^{\frac{1}{n}}}$.

Lemma 1 *: The elasticity of the anticipated utility decision-weights function f is nondecreasing iff* $f(p)f(q) \leq f(pq)$.

Proof: Segal [1986] pp 32

Theorem 3 *Let F^* be a distribution function over the possible values of the probability of S in the ambiguous lottery (x,S;0,not S). Let us assume that F^* is symmetric around $\overline{p}$. If f is convex, if its elasticity is nondecreasing and if the elasticity of $\overline{f}$ is nonincreasing,then (x,$\overline{p}$; 0,not$\overline{p}$) is preferred to (x,s;0,not S).*

Proof: Segal [1986] pp 32-33

Example 4 *This theorem does not hold for all convex functions f. For an example of a strictly convex function let f be*

$f(p) = 1 - cos(\frac{\pi * p}{2})$ *for* $0 \leq p \leq 1$.

Also assume that $P^*(0.999) = P^*(0.995) = \frac{1}{2}$.

Note that f is strictly convex for $0 \leq p < 1$. *Then*

$$V(1, S; 0, notS) = u(1)[1 - cos(\frac{\pi * 0.999}{2})] + u(1)[(1 - cos(\frac{\pi * 0.999}{2} - (1 - cos(\frac{\pi * 0.995}{2})f(\frac{1}{2})$$

$$= 0.995287u(1) > 0.995287627u(1) = f(0.997)u(1) = V(1, 0.997; 0, 0.003)$$

So in this case, we have that the decision maker prefers the ambiguous lottery over the known lottery.

Example 5 *The function* $f(p) = p^2$ *for* $0 \leq p \leq 1$ *satisfies all assumptions in the theorem.*

Since $f''(p) = 2$ *f is strictly convex in the interval.*

The elasticity of f, $\frac{pf'(p)}{f(p)} = \frac{2p^2}{p^2} = 2$ *is clearly non-decreasing.*

The elasticity of $\overline{f(p)}$ *is non-increasing. The result is messy (so we refrain from doing it here) but simple calculus shows that* $\overline{f(p)} = \frac{2p(1-p)}{1-(1-p)^2}$ *and* $\overline{f'(p)} = \frac{-2p^3}{(2p-p^2)^2}$. *The numerator is always negative and the denominator positive, thus* $\overline{f(p)}$ *is non-increasing.*

In the Ellsberg paradox in Game 1, we have that P(25Red)=P*(75Red)=0.5. Then* $\bar{p} = 0.5$. *Using,* $f(p) = p^2$ *for* $0 \leq p \leq 1$, *we have*

$$u(100)f(0.25) + u(100)[f(0.75 - 0.25)]f(P^*(0.5)) =$$

$$u(100)0.1875 < u(100)0.25 = u(100)f(0.5)$$

Thus, the decision-maker favors the sure-lottery over the ambiguous lottery in this example.

The above theorem only works whenever the distribution is symmetric about the mean. We need to ask the question, would the theorem still hold if F^* is not symmetric? The following result answers our question.

Theorem 4 *Let F^* be a distribution function over the possible values of the probability of S in the ambiguous lottery $(x, S; 0, notS)$. Denote its mean value by $\overline{p}$, and let f be convex and twice differentiable.*
(a) $x > 0$: If $\frac{f''}{f'}$ is nonincreasing and if the elasticity of f is nondecreasing, then $(x, \overline{p}; 0, 1 - \overline{p})$ is preferred to (x,S;0,not S).
(b) $x < 0$: If $\frac{\overline{f''}}{\overline{f'}}$ is nondecreasing and if the elasticity of $\overline{f}$ is nonincreasing, then (x,$\overline{p}$;0,1-$\overline{p}$) is preferred to (x,S;0,1-S).

Proof: Segal [1986] pp 33-34

Example 6 *Let $f(p) = \frac{p+p^{12}}{2}$. First for $p, q \in [0, 1]$ $f(p)f(q) \leq f(pq)$. Thus, the elasticity of f is non-negative. Since f is twice differentiable, we have that $f'(p) = \frac{1}{2} + 6p^{11}$ and $f''(p) = 66p^{10}$. Thus, $g(p) = \frac{f''(p)}{f'(p)} = \frac{66p^{10}}{\frac{1}{2}+6p^{11}} = \frac{132p^{10}}{1+12p^{11}}$ Thus, $g'(p) = \frac{(1+12p^{11})(1320p^9)-(132p^{10})(132p^{10})}{(1+12p^{11})^2}$. Since the denominator is always non-negative for $p \in [0, 1]$, we only care about the sign of the numerator. The numerator simplifies to $h(p) = 1320p^9 - 1584p^{20}$. For sufficiently small p, h(p) is positive and for sufficiently large p, h(p) is negative. Thus, $\frac{f''(p)}{f'(p)}$ is first increasing, then decreasing.*

Now for our lottery, let f(p)=$\frac{p+p^{12}}{2}$. and let $P^*(\frac{45}{100}) = \frac{10}{11}$ and $P^*(1) = \frac{1}{11}$. Then, $\overline{p} = 0.5$. However, we have that
V(1,S,0,not S)=$u(1)f(0.45)+u(1)[f(1)-f(0.45)]f(\frac{1}{11}) = 0.295485u(1) > 0.2501u(1) = \frac{0.5+0.5^{12}}{2} = V(1, 0.5; 0, 0.5)$
We have shown that the $\frac{f''(p)}{f'(p)}$ is a necessary assumption in the theorem.

We now present an example that is consistent with both Game 1 and Game 2 above.

5.5 Game 1

Consider for $0 \leq p \leq 1, f(p) = \frac{e^p-1}{e-1}$. For Game 1, we have that $P^*(25yellow) = P^*(75yellow) = 0.5$. Thus, V(C)=V(1, S, 0, not S)= $u(1)f(0.25)+u(1)[f(0.75)-f(0.25)]f(P^*(0.75)) = 0.34831u(1) < 0.37754u(1) = V(1, 0.5, 0, 0.5) = V(A)$. Similar calculation yields $V(B) > V(D)$. Thus, we see that the bet with the certain probability is preferred by the decision maker in Game 1.

5.6 Game 2

For Game 2, the decision maker has the belief that the likelihood of 10 green balls or 30 green balls is the same. Thus, $P^*(10green) = P^*(30green) = 0.5$. Using our function, we get
$V(C) = u(1)f(\frac{30}{80}) + u(1)[f(\frac{50}{80} - f(\frac{30}{80})]f(0.5) < f(\frac{40}{60})u(1) = V(D)$.
Thus, the function is consistent with the Ellsberg paradox in Game 2.

5.7 Ambiguous vs Two-Stage Lotteries

Yates and Zakowski (1976), compared ambiguous lotteries and two-stage lotteries. In each of the following games, we have a decision-maker who chooses a color (red or black) of the ball before he plays.

Game 1: In this game, there is one urn and 5 red and 5 black balls. The decision maker chooses one ball and he wins a dollar if the ball he picks coincides with the ball he chose.

Game 2: In this game, there is an urn with 11 balls numbered from 0 to 10. The decision maker chooses a ball, say i and then places i red balls and 10-i black balls into a second urn. Then, the decision maker chooses a ball from the second urn and he wins a dollar if the ball he picks coincides with the ball he chose.

Game 3: There is an unknown quantity of red and black balls in one urn. The decision maker wins a dollar if the ball he picks coincides with the ball he chose.

After running these tests with subjects, 78% of the subjects preferred Game 1 to Game 3 and 68% of the subjects preferred Game 2 to Game 3. The first preference is supported by Theorem 3. We can also make the argument that decision makers will prefer Game 1 to Game 2.

We use our function $f(p) = \frac{e^p - 1}{e-1}$ as above. In game 2, for each i, the decision maker believes that P*(i red balls) is equally likely. Thus, $P^*(0) = P^*(\frac{1}{11}) = P^*(\frac{2}{11}) = ... = P^*(1) = \frac{1}{11}$. The value of the lottery of the 1st game is V(Game 1)=0.37754. By Theorem 3, the value of Game 2 will be lower than Game 1. We avoid the calculations because of the extreme messiness. Thus, we have that for the decision maker, Game 1 $\succeq$ Game 2.

6 Conclusion

Both models of choice under ambiguity aversion have represented preferences using some expected utility function. Schmeidler first characterized acts according to a capacity function, which measured the decision maker's belief of the likelihood of some state of nature and the choquet integral.

This allowed us to now represent preferences without the use of the additive identity. Schmeidler [1986] weakened the independence axiom to comonotonic independence and showed that if a functional satisfies comonotonic independence and monotonicty, it is equivalent to a Choquet Integral on the vector space B. One of the interesting corollaries in the article showed that if the capacity function is convex (that is for all A, B $\in S$ v(A $\cup$ B) + v(A $\cap B) \geq v(A) + v(B)$), then for all a in the vector space B, $I(a) = min \int a dp | p \in core(v)$. Afterwards, Schmeidler and Gilboa [1989] characterized preferences using the functional $U(h) = \min_{Q \in D} E_Q[u(h)]$ where D is a set of priors over the state space S. The two-part decision making process with anticipated utility is a different way to calculate the value of an ambiguous lottery, but lies within the same concept of the Choquet Integral. Anticipated Utility is a generalization of expected utility, and it calculates the value of an ambiguous lottery piecewise. In general, we take a look at the marginal difference between each successive probability of a decision maker generated belief function. Segal showed that under certain conditions using anticipated utility, the decision maker prefers the certain lottery to the ambiguous one. We go on to explain the Ellsberg paradox using Segal's theorems and our functions.

In many regards, the Choquet Integral and the two-part decision making process of evaluating an ambiguous lotteries are more alike than different. In order to evaluate their lotteries, the decision maker must create his own belief function. In the Choquet Integral, this is the capacity function and in the two-part model, this is the user-generated f. Both evaluations use the Riemann sum computed piecewise to come up with the figures.

One question that have not been addressed in this project is the degree of ambiguity of the lotteries. An interesting question is how does increasing ambiguity of the lotteries affect the preferences of the decision maker. If we assume that decision makers are more ambiguous as ambiguity of the lottery increases, we may study whether they are increasingly or decreasingly more ambiguous.

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Date: September 2009-May 2010
Title: Research Assistant
Institution: Penn State Department of Economics Supervisor's Name: Dr. Edward Coulson

Date: September 2008-December 2009
Title: Calculus Tutor
Institution: Penn State Math Learning Center

Awards and Scholarships

Duke Masters Scholar —Spring 2013
Liberal Arts Scholarship, Summer 2012
Schreyer International Thesis Grant —Fall 2011

Skills

Programming: C++, LaTeX
Statistical Software: Stata
MS Office: Word, PowerPoint, Excel
Languages: English (native), Korean (fluent)