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COMPARISON OF VOLATILITY MODELS OF THE S&P 500 INDEX  
LAM NGUYEN  
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Reviewed and approved\* by the following:

James Miles  
Professor of Finance  
Joseph F. Bradley Fellow of Finance  
Thesis Supervisor and Honors Adviser

Jingzhi Huang  
Associate Professor of Finance  
David H. McKinley Professor of Business  
Faculty Reader

\* Signatures are on file in the Schreyer Honors College.

## Abstract

An accurate forecast of financial volatility is very crucial in many applications such as portfolio management when we need to figure out the Efficient Frontier, or risk management when we need to compute the Value-At-Risk, or hedging when we need to calculate the Hedge Ratio for the portfolio, etc. This thesis compares the forecasting power of different ARCH-type models during the recent financial crisis. Those models include ARCH, GARCH, EGARCH, TARCH, GJR-GARCH, SA-ARCH, P-ARCH, NA-ARCH, NA-ARCHK, APARCH, and NPARCH. Using six different loss functions and the Reality Check of White for data snooping, this study found that the results largely depend on the loss functions that are chosen. EGARCH models generally have the best performances during the financial crisis. After the test for data-snooping, we found that ARCH (1) is outperformed by other models for three out of six loss functions but perform just as well for the other three. The exact reverse applies to the GARCH (1, 1).

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# Chapter 1

## Introduction

Volatility in financial market has always been received a great amount of interest from both academics and practitioners at Wall Street. Volatility is a key input in many important financial models, including models in risk management, option pricing and hedging strategies. Granger and Spoon (2003) list many practical applications of volatility forecasting in financial markets. First, volatility is the key input to many models of investment and portfolio managements. Each individual has a certain level of risk-aversion, and a good forecast of portfolio volatility is keystone for measuring a portfolios risk. Second, since the first Basle Accord in 1996, volatility forecasting has become a compulsory exercise for many financial institutions. For instance, a risk manager of a financial institution needs a good forecast of volatility in order to compute the Value at Risk (VaR), which is defined as the threshold value such that the probability of a loss on a portfolio over a given time horizon exceeds this value is the given probability level. The risk manager then needs to set aside enough capital to sustain a loss of at least three times that VaR. Moreover, option traders need volatility estimates to put in the Black-Scholes model as well as other models of option pricing. Due to an importance of volatility to derivative contracts, people now can also buy contracts written on volatility itself, meaning that volatility becomes an underlying asset. In order to price such contracts, traders need an accurate forecast of not only volatility but also its second moment. And finally, hedge funds managers need to know future volatility in order hedge their investment in different financial assets. Officers at the Federal Reserve System are also known to pay close attention to implied volatility from the option markets as a way of measuring market sentiment. In his comprehensive study of the relationship between stock market volatility and macroeconomic time series, Schwert (1989) found that stock market volatility helps predict other macroeconomic volatilities but not vice versa, and he also found that stock market volatilities and macroeconomics volatilities are significantly high dur-



ing period of economic recessions. Nelson (1991) notes the study of change in volatility is important to our understanding of other essential problems in macroeconomics and finance, including the term structure of interest rates, irreversible investment, option pricings, and dynamic capital asset pricing theory.

A financial crisis in late 2007 reminds everybody the importance of having a good model of volatility forecasting and how severe the consequences would be by having wrong models. Brownlees, Engle and Kelly (2011) note a statement from former Federal Reserve Chairman Alan Greenspan to the Committee of Government Oversight and Reform (Jan. 2, 2009), saying that collapsed in the summer of last year because the data inputted into the risk management models generally covered only the past two decades, a period of euphoria. Had instead the models been fitted more appropriately to historic period of stress, capital requirements would have been much higher and the financial world would be in a far better shape today. In fact, academics have produced a great amount of literature over decades about modeling volatility. In academic literature, volatility of a financial asset is often measured by the standard deviation of that asset over a certain period of time. The standard deviation is a square root of the variance computed by the following formula

$$\sigma^2 = \frac{1}{N-1} \sum_{t=1}^n (R_t - \bar{R})^2$$

where  $\sigma^2$  is the sample variance,  $N$  is the number of observations,  $R_t$  is return observed at time  $t$ , and  $\bar{R}$  is the sample mean.

In other words, this is the measure of how much returns of that asset deviate from its expected return over time. As noted by Granger and Spoon (2003), when using standard deviation as a measure of risk, people often implicitly assume that an assets returns follow the normal distribution. However, it is important to remember that a sample standard deviation is a distribution-free parameter, meaning that it can come from any type of distribution. Thus, unless we specify a distribution function such as a normal distribution or Student t-distribution, it will be misleading to use  $\sigma$  as a risk measure. In addition, practitioners usually want to define risk as a probability of having a negative return, but a standard deviation is indifferent about signs of returns. People who only care about deviations below an expected return may want to look at the semi-variance suggested by Markowitz (1991) instead.

Although returns of financial assets are widely believed to be unpredictable according to the Efficient Market Hypotheses (EMH), it is still possible to provide a range in which returns will fall in between in the future. However, modeling volatility is difficult because a good model needs to be able to explain the two stylized facts about volatility: volatility clustering and mean-reversion. Volatility clustering refers to the fact that when volatility is high

(or low), it tends to remain high (or low) for a while. A possible explanation for volatility clustering is the news theory, which states that financial markets are moved by new information and information arrives in cluster, hence we have volatility clustering. Many researchers have found volatility clustering from different types of financial data, including stock returns, bond returns or exchange rates. For example, see Mandelbrot (1963), Fama (1965), Chou (1988), Schwert (1989) and Baillie et al. (1996). Mean-reversion, on the other hand, says that when volatility is high (or low), it will tend to come down (or go up) to its average level in the long run. People may differ from their assessments about what the normal level of volatility is and whether that level varies over time. Engle and Patton (2001) noted that implied volatility from options is consistent with the mean-reversion phenomena. Implied volatilities of long maturity options are less volatile than those of short maturity options, because they are closer to the normal level of volatility of an asset.

Besides the two stylized facts listed above, volatility still has many other important features, and many researchers have come up with different models to capture those features. Perhaps the most important feature is the leverage effect first documented by the empirical work of Black (1976), then by Christie (1982), and Nelson (1991). The leverage effect refers to the fact that a negative shock to returns will have a larger effect on volatility than a positive shock. The phenomenon is named the leverage effect because people believed that higher volatility is caused by a decrease in a firm value after a negative shock, leading to higher leverage; however, recent empirical researches suggest that leverage only has a minor effect on volatility. Indeed, many researchers have tried to include this leverage effect in their models of which the EGARCH of Nelson (1991), and the GJR-GARCH of Glosten, Jagannathan and Runkle (1993) are perhaps the most popular ones. Other important features of financial volatility were discovered by Philip Perron (1982) and Adrian Pagan and G. William Schwert (1990) who found that volatility time series have a unit root.

The major contributor for the literature on modeling volatility of financial market is Robert Engle (1982) who proposed the first ARCH model to measure the variance of U.K. inflation. Since then, he has written extensively on ARCH models and its applications in the financial market, such as the use of GARCH to capture the risk-return trade-off (Engle, Lilien and Robins 1987), how to use SAARCH to capture the asymmetric effect in stock returns (Engle 1990), and how to infer the impact of news on volatility from a GARCH model (Engle and Ng 1993). Besides providing practical guides to practitioners (Engle 2001), Engle (2002) also identified five new frontiers for academic researches in GARCH: (1) high-frequency volatility models, (2)

large-scale multivariate ARCH models, (3) derivative pricing models, (4) application of ARCH to non-negative process, and (5) the use of Least Squares Monte Carlo to examine simulated models. Following the spirit of the first ARCH model, researchers have come up with hundreds of models for financial volatility in the literature by now. For a comprehensive survey about GARCH-models, readers can refer to Bollerslev (2009).

Given the fact that there are too many models around, an important question is what model specifications is the best model for financial volatility. This question is hard to answer since research has shown that different asset classes will favor different models. In this paper, I attempt to select the best GARCH-type model for predicting the volatility of the S&P 500 Index by comparing the forecasting powers of eleven GARCH-type models, namely the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982), the General-ARCH (GARCH) model of Bollerslev (1986), the Exponential-GARCH (EGARCH) model of Nelson (1991), the Threshold ARCH (TARCH) of Zakoian(1994), the GJR-GARCH model of Glosten, Jagannathan and Runkle (1993), the Simple Asymmetric ARCH (SAARCH) suggested by Engle (1990), the Power ARCH (PARCH) by Higgins and Bera (1992), the non-linear ARCH (NARCH), the non-linear ARCH with one shift (NARCHK), the Asymmetric Power ARCH (APARCH) of Ding, Granger and Engle (1993), and the non-linear PARCH (NPARCH). Note that these models are just general specifications for the conditional variances. In order to estimate a GARCH-type model, we will need to determine a certain number of lags for each ARCH and GARCH terms in the model we want to estimate. Also, before we can estimate the conditional variance model, we will first need to specify a model for the conditional mean of stock returns, which can be either a constant since stock returns follows a random walk or GARCH-in-mean as suggested by Engle (1987). Thus, different lags and different conditional mean models correspond to different volatility models. Because the number of models available are too large, in this paper, I will restrict the maximum number of lags to 2 for ARCH and GARCH terms, and the conditional means will be either Random Walk model or GARCH-in-mean model. Specifically, I divide my sample into two sub-samples. I use the first sub-sample to estimate models and use the other for testing. After estimating a model, I measure its out-of-sample forecasting powers by different loss functions and then rank those models accordingly. The raw ranking generally suggests that EGARCH model is the best model specifications for forecasting volatility. However, after implementing the Reality Check for data-snooping of White (Sullivan, Timmerman and White 1999), none of the model outperforms the simple GARCH(1,1), regardless of the inability of GARCH(1,1) to capture the leverage effect of stock returns. The finding is

consistent with the paper of Hansen and Lunde (2005). In fact, they found that the GARCH (1, 1) is the best model for forecasting volatility when they compare 330 different volatility models using daily exchange rate data (DM/USD) and IBM stock prices. Also, Granger and Spoon (2003) noted that GARCH (1, 1) is empirically proven to be the most popular structure for many financial time series.

To better understand volatility, we can look at Figure 1, which is the graph of the S&P 500 Index closing prices from 1990 to 2012. Clearly, we can see that there is an upward trend from 1990 to 2012 regardless of the two market crash in 2000 and late 2007. The key observation from this picture is that stock price is not a stationary process, and it explains why we need to model stock returns instead of the prices. Indeed, the non-parametric Breitung test suggests that the natural log of daily prices is a unit root process. The null hypothesis in this test is that the natural log of daily prices is a unit root process, and the alternative hypothesis is that it is trend stationary. Note that we test for a unit root hypotheses using natural log of daily prices because the price level is always positive and cannot be a unit root process. The unit root tests confirm the validity of the Efficient Market Hypotheses (EMH), which says that the best forecast for next period stocks price is current price. The results of the test can be seen from Appendix A. The results also imply that we would need to specify an ARMA model for the return (the first difference of natural log of daily prices) instead of the daily prices themselves, before we are able to model volatility with a GARCH-type model.

Furthermore, Figure 2 is the graph of the returns of the S&P 500 Index over the same time period. Comparing Figure 1 and Figure 2, we can see the three important properties of volatility that are volatility clustering, mean-reversion and leverage effect. Specifically, volatility was low during 1990 to 1998 when the stock market experienced many good years, and then it became higher due to the stock market crash in 2000 and stayed at that level until 2003. After that, volatility reversed to its low level from 2004 to 2007, and it went up again during the recent financial crisis. All those observations suggest that volatility will tend to stay high (or low) for a while before it reverses to its normal level, and a negative shock to returns will lead to a higher level of volatility. Additionally, from this time serie of stock returns, we can understand why a Random Walk model for the conditional mean is an appropriate model for stock return. As noted above, we should specify an ARMA model for the Indexs returns, and the task is simply to choose the best order P for Autoregressive and Q for Moving Average of the general ARMA(P,Q) model. In order to select the best order for the ARMA (P, Q) model for the Indexs return, I use all three Akaike, Hannan-

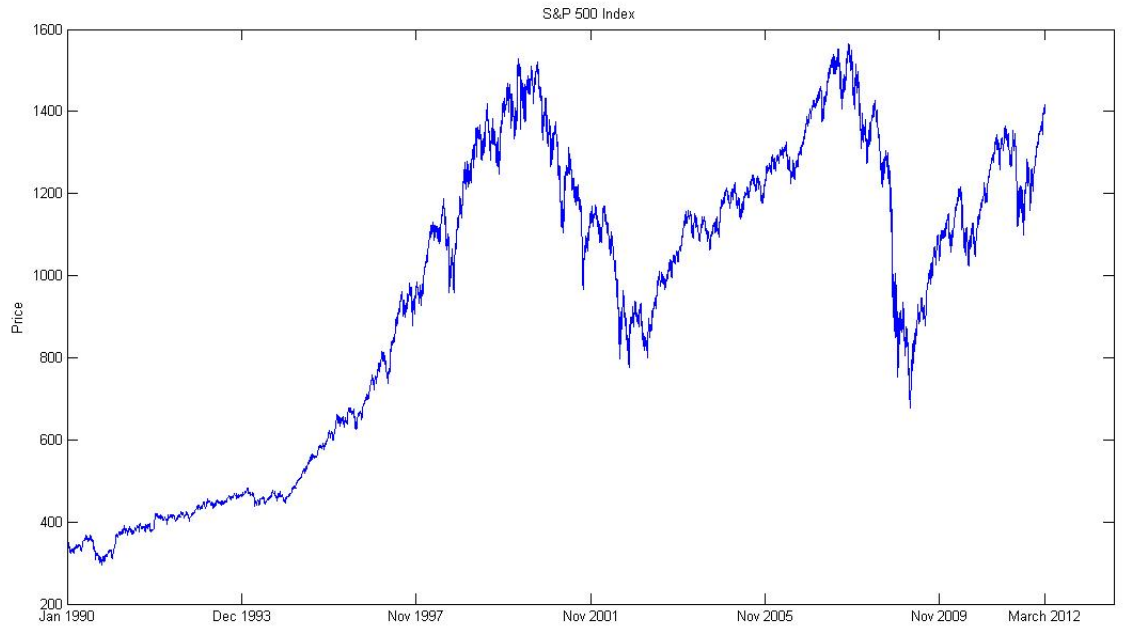


Figure 1: Price of the S&P 500 Index

Quinn, and Bayesian Information Criteria. The procedure is to estimate each model with different values of  $P$  and  $Q$ , and then take the model that has the lowest values from those criteria. In this case, most of the Information Criteria, except for Akaike, indicate that the Index's return is just a white noise process  $ARMA(0,0)$ , which is consistent with the EMH. The results for this selection procedure can be seen from Appendix B.

This paper follows the framework from that of Hansen and Lunde (2005); however, my study differs from theirs since I used returns of the entire stock market (i.e. S&P 500) whereas they only used the return of IBM stock. The behavior of a portfolio of stocks might be different from the stock of one individual blue-chip company like IBM. Also, I measured the out-of-sample forecasting powers of those models during the financial crisis of 2008 (i.e. from 12/03/2007 to 08/13/2009), when market experienced very high degree of volatility. Since the only purpose of volatility model is to give accurate forecasts of volatility in time of turmoil, their performance during the chosen time period will provide more insights to their practical implications than their performances when the market is calm. The paper is organized as follows: Section II will describe the general forms of different types of GARCH-type models. Section III will describe the data and my methodology for estimating

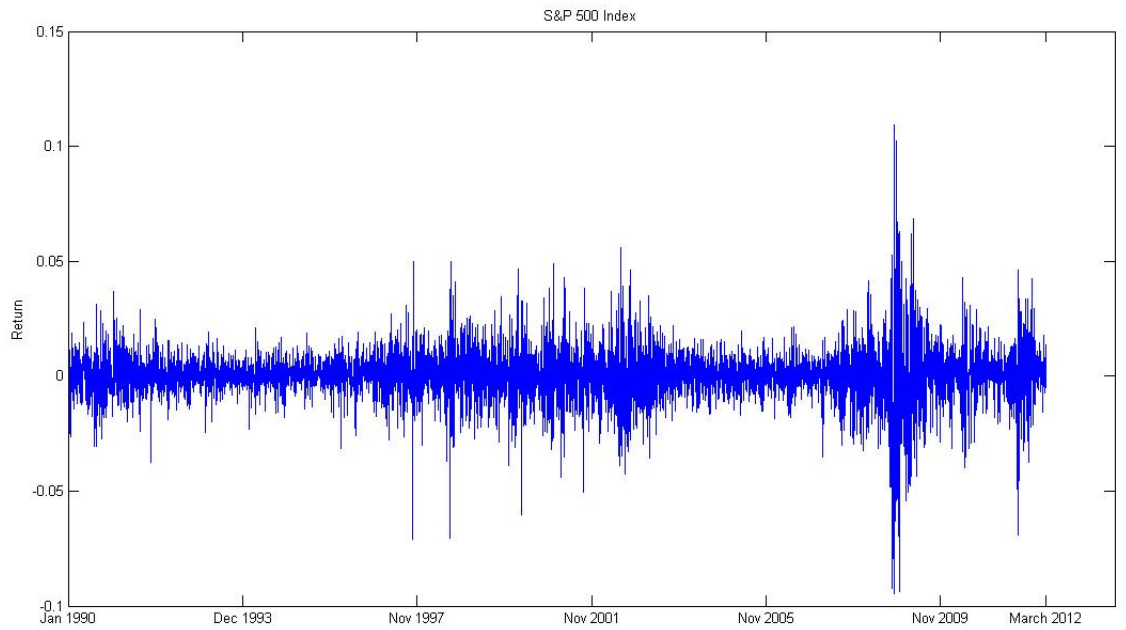


Figure 2: Return of the S&P 500 Index

and evaluating. Section IV will provide the results. And Section V is the conclusion.

# Chapter 2

## General Specifications of GARCH-type models

To model a time series with a GARCH-type model, we will need to specify an equation for the conditional mean, an equation for the conditional variance and the distribution for the error terms. Particularly, for the conditional mean of stock returns, we can specify the conditional mean as a Random Walk as suggested by the Efficient Market Hypotheses, or we can use the GARCH-in-mean model as suggested by Engle, Liliens and Robins (1987) who showed that this model could be used to capture the time-varying risk premiums. The Random Walk model for the conditional mean implies that returns is a stationary process around a constant, while the GARCH-in-mean model allows for returns to be higher in period of high volatility by including the conditional variances in the conditional mean equation, hence capturing the trade-off between risk and return. However, our interest here is not on the conditional mean but on the conditional standard deviation of the returns, which is usually interpreted as the volatility. Since the seminal paper of Engle(1982), researchers have developed many different GARCH-type models aimed to capture different features of volatility, including volatility clustering, mean-reversion, and asymmetric response to news. The formulas for these models are specified below. The GARCH model is a natural extension of the ARCH model of Engle(1982). The only difference is that conditional variance is modeled as an ARMA process in GARCH. A superior of GARCH over ARCH is that GARCH can capture volatility clustering and mean-reversion by modeling volatility as a function of past volatilities and placing restriction on the parameter estimates to allow for mean-reversion. Beside ARCH and GARCH, all the other models are designed to successfully capture the asymmetric response of stock returns to news in various ways. For instances, one of the limitations of GARCH model is that it completely ignores the size of a shock to returns. As noted above, it is an empirical fact that a negative shock

will lead to higher volatility than a positive one. Also, estimating a GARCH model might be difficult due to those restrictions placed on the model parameters. Thus, Nelson (1991) introduces a new type of GARCH-model namely the EGARCH in which he modeled the natural log of volatility instead of volatility itself. By modeling the natural log of the conditional variance, there is no need to place restrictions on the parameters to ensure that the conditional variance will be positive, and the EGARCH will also be able to capture the leverage effect. Another example is the GJR-GARCH model, which is another approach to capture the leverage effect in financial time series. GJR-GARCH model of conditional variance includes a dummy variable that equal 1 when the disturbance is negative. It implies that if there was a negative shock to returns, conditional variance in next period would be higher than normal. Moreover, power ARCH, such as APARCH, can be used to capture the long memory property of stock market returns, which refers to the fact that the absolute value of stock market returns have significant correlations over very long lags. Interestingly, Ding, Granger and Engle (1993) also shown that APARCH nests seven other GARCH-type models, including popular models such as ARCH, GARCH and GJR-GARCH. Although each specification can provide very interesting information about volatility, for the purpose of this study, I will only pay attention to the model's out-of-sample performance as measured by different loss functions. Finally, we will need to specify the distribution for the error terms which can be either normal distribution or t-distribution. Fama (1965) found that stock market returns have fat-tail distributions, and the t-distribution for the error terms will enable the model to efficiently capture that property.

1/ Conditional Mean Equations:

Random Walk Model:  $y_t = \beta_0 + \epsilon_t$

GARCH-in-mean model:  $y_t = \beta_0 + \sum_{i=0}^p \beta_i \sigma_{t-i}^2 + \epsilon_t$

2/ Conditional Variances Equations:

1/ Autoregressive Conditional Heteroscedasticity (ARCH):

$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2$

2/ General ARCH (GARCH):

$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2$

3/ Exponential GARCH (EGARCH):

$\ln Var(\epsilon_t) = \ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i z_{t-i} + \sum_{i=1}^p \delta_i (|z_{t-i}| - \sqrt{\frac{2}{\pi}}) + \sum_{j=1}^q \gamma_j \ln \sigma_{t-j}^2$

where  $z_t = \frac{\epsilon_t}{\sigma_t}$

4/ Threshold ARCH (TARCH):

$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i |\epsilon_{t-i}| + \sum_{i=1}^p \delta_i (|\epsilon_{t-i}|)(\epsilon_{t-i} > 0) + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2$

5 /Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH):

$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2 + \sum_{i=1}^p \delta_i (\epsilon_{t-i}^2)(\epsilon_{t-i} > 0)$



6/ Simple Asymmetric ARCH (SAARCH):

$$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2 + \sum_{i=1}^p \delta_i \epsilon_{t-i}$$

7/ Power ARCH (PARCH):

$$Var(\epsilon_t)^{\frac{\varphi}{2}} = \sigma_t^\varphi = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^\varphi + \sum_{j=1}^q \gamma_j \sigma_{t-j}^\varphi$$

8/ Non-linear ARCH (NARCH):

$$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i (\epsilon_{t-i} - \kappa_i)^2 + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2$$

9/ Non-linear ARCH with one shifts (NARCH-K):

$$Var(\epsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i (\epsilon_{t-i} - \kappa)^2 + \sum_{j=1}^q \gamma_j \sigma_{t-j}^2$$

10/ Asymmetric Power ARCH (A-PARCH):

$$Var(\epsilon_t)^{\frac{\varphi}{2}} = \sigma_t^\varphi = \alpha_0 + \sum_{i=1}^p \alpha_i (|\epsilon_{t-i}| + \kappa_i \epsilon_{t-i})^\varphi + \sum_{j=1}^q \gamma_j \sigma_{t-j}^\varphi$$

11/ Non-linear Power ARCH (NPARCH):

$$Var(\epsilon_t)^{\frac{\varphi}{2}} = \sigma_t^\varphi = \alpha_0 + \sum_{i=1}^p \alpha_i (|\epsilon_{t-i}| - \kappa_i)^\varphi + \sum_{j=1}^q \gamma_j \sigma_{t-j}^\varphi$$

# Chapter 3

## Data and Methodology

I download the daily closing prices adjusted for dividends and stock splits of the S&P 500 Index from Yahoo!Finance, then I compute 3,000 daily returns in the period between 11/09/1997 and 08/13/2009. I divide my sample into two sub-samples: one is used for estimation and the other for evaluation. The first sub-sample is from 11/09/97 to 11/30/2007, and the second sub-sample is from 12/03/2007 to 08/13/2009, which is the period of the financial crisis of 2008. I estimate the coefficients for each model using the first sub-sample and use those estimations to continuously make one-step-ahead forecasts of volatility over the second sub-sample. Those forecasts will be put into different loss functions to produce a measure for the model's performance. Because different loss functions will tend to favor a particular type of model, I use all six loss functions specified below to determine which forecasting model is the best one. Since this is one-step-ahead forecast, the benchmark of our forecasts here is the daily return of the next trading day. Hansen and Lunde (2005) noted that daily return is a very noisy benchmark of volatility, and they used intra-day returns in their research instead. However, in this paper, I do not make any attempt to say anything about the forecasting power of any individual model but rather to compare them to each other. Thus, by using the same benchmark to evaluate all models, I expect the comparison will be accurate.

Loss Functions:

$$\mathbf{MSE}_1 = n^{-1} \sum_{t=1}^n (\sigma_t - h_t)^2$$

$$\mathbf{MSE}_2 = n^{-1} \sum_{t=1}^n (\sigma_t^2 - h_t^2)^2$$

$$\mathbf{QLIKE} = n^{-1} \sum_{t=1}^n (\log(h_t^2) + \sigma_t^2 h_t^{-2})$$

$$\mathbf{R}^2\mathbf{LOG} = n^{-1} \sum_{t=1}^n [\log(\sigma_t^2 h_t^{-2})]^2$$

$$\mathbf{MAD}_1 = n^{-1} \sum_{t=1}^n |\sigma_t - h_t|$$

$$\mathbf{MAD}_2 = n^{-1} \sum_{t=1}^n |\sigma_t^2 - h_t^2|$$

where

$\sigma_t$  is realized volatility at time  $t$

$h_t$  is forecasted volatility at time t

Note that the criteria  $\mathbf{MSE}_2$  is similar to the  $R^2$  of the regression  $r_t^2 = a + bh_t^2 + u_t$ , and the criteria  $\mathbf{R}^2\mathbf{LOG}$  is similar to the  $R^2$  of the regression  $\log(r_t^2) = a + b\log(h_t^2) + u_t$ . The  $\mathbf{MAD}_1$  and  $\mathbf{MAD}_2$  are less sensitive to outliers than  $\mathbf{MSE}_2$ .

Also, I compute the Information Criterias to see which model best fits the data, and whether the one that has the best in-sample performance will also be the one that has the best out-of-sample performance. The formulas for those Information Criterias are specified below.

**Information Criterias:**

Akaike Information Criteria:  $AIC = -2 \times \ln(\text{likelihood}) + 2 \times k$

Bayesian Information Criteria:  $BIC = -2 \times \ln(\text{likelihood}) + \ln(N) \times k$   
where

k= number of parameters estimated

N=number of observations

**The Test for Data-Snooping Bias:**

Due to data-snooping bias, the fact that one model outperforms others in a particular data set does not necessarily mean that the model is superior to others. The higher performance may simply due to chance. Thus, in order to accurately compare the forecasting power among models, we need to have a test for data-snooping. In this paper, I will use the Reality Check of White (Sullivan, Timmerman and White 1999). As being said above, each model will generate a series of one-step-ahead forecasts, which will be compared to a series of realized squared-returns by a particular loss function. Let say that the model k yield the forecast  $h_{k,1}, h_{k,2}, h_{k,3}, \dots, h_{k,n}$ . Then, we can compute the value of the loss function using that forecast and the realized return as inputs. For example, if we use MSE1 as the loss functions, we will have a new serie of performance measure that is  $(h_{k,1}^2 - r_1^2)^2, (h_{k,2}^2 - r_2^2)^2, (h_{k,3}^2 - r_3^2)^2, \dots, (h_{k,n}^2 - r_n^2)^2$ . Note that the average of this serie is the value for the particular loss function specified above. Suppose that our benchmark model is model numbered 0. We will define the relative performance of model k to model 0 at time t as

$$X_{kt} \equiv u_{0,t} - u_{k,t}$$

Then, by the law of large numbers, the expected performance of model k relative to the benchmark model can be calculated as the sample averare of  $X_{kt}$

$$\lambda_k \equiv E[X_{kt}] = \bar{X}_{n,k} = n^{-1} \sum_{t=1}^n X_{kt}$$

If model k outperforms the benchmark model, we will have a positive value of  $\lambda_k$ . Thus, the null hypotheses that no model performs better than the benchmark model is equivalent to

$$H_0 : \lambda_{max} \equiv \max \lambda_k \leq 0 \text{ where the maximum is taken over all the models}$$

If the null hypotheses  $H_0$  is rejected, then we can say that there is at least one model that significantly outperforms the benchmark model. Data-snooping bias refers to the fact that we can get a positive value for  $\lambda_{max}$  just by chance; however, we can correct for that problems by estimating the distribution of  $\lambda_{max}$  under the null hypotheses by stationary bootstrap, and then from that distribution, we will be able to derive the p-value and conclude whether  $\lambda_{max}$  is significantly positive. The procedure of generating the distribution of  $\lambda_{max}$  is described in details in Hansen and Lunde (2005).

# Chapter 4

## Results

Since all the models are estimated using Maximum Likelihood, failure to converge is a very common problem. And thus, I omitted all GARCH-type models that encounter difficulties in convergence in the estimation process; however, the number of models that are omitted is small. Because GARCH (1, 1) is the most popular model used by practitioners, it is interesting to see how models in the GARCH family perform relative to other more sophisticated models. There are two key observations from the final results: first, ARCH and GARCH models underperform other sophisticated models, and second, the evidences seem to suggest that models in the EGARCH family have the best out-of-sample forecasting power. Those observations can be explained by the fact that ARCH and GARCH models do not capture the leverage effect in stock returns, which has been well-documented in many previous research, and thus they do not perform as well in out-of-sample forecasting.

Based on different loss functions and two Information Criteria, Table 1 is the ranking of 42 models estimated with normal distribution for the error terms and Random-Walk conditional mean model. The model that is ranked number one is the model that performs best according to that criterion. One thing is clear from the table is that different functions do tend to favor different models. For example, the EGARCH family dominates the top 4 models based on MSE1, MAD1 and MAD2, but other loss functions suggest different winners, such as TARCH(2,1) by MSE2, GJRGARCH(1,1) by QLIKE, and NPARCH(1,2) for R2LOG. Although the best models seem to be different according to different loss functions, we can see that the ARCH and GARCH specifications are clearly inferior relative to other models that can capture the leverage effect in stock returns, such as EGARCH, TARCH or SAARCH. Particularly, ARCH and GARCH models are near the bottom when it comes to in-sample performance ranked by AIC and BIC, and for out-of-sample performance, all loss functions but R2LOG and

MAD2 suggest the same story. For instance, ARCH(1) is ranked 42 by 5 out of 8 criteria, and GARCH(1,1) is ranked 41 by 2 out of 8 criteria. In term of in-sample performance, the Information Criteria suggest that the EGARCH, TARCH, and SAARCH models fit the sample the best. Also, the EGARCH, TARCH, NPARCH and GJR-GARCH models have strong out-of-sample performance since they all have been placed at the top by some loss functions, especially EGARCH models, which have been placed at the top by 3 out of 6 loss functions. Furthermore, Table 2 consists of the ranking of 39 models estimated with the student t-distribution for the error terms and Random-Walk conditional mean model. Except for R2LOG and MAD2, all the other loss functions and Information Criteria place ARCH and GARCH model at the bottom of the list. EGARCH, TARCH and SAARCH are shown to have the best in-sample and out-of-sample performances once again. Thus, the two tables clearly indicate that models that allow for asymmetric effect of stock returns outperform those that do not.

Table 3 consists of models that have GARCH-in-mean as a conditional mean model and normal distribution for the error terms. In term of in-sample performance, both AIC and BIC say that EGARCH(2,2) is the best model, following by SAARCH(2,2), NPARCH(4), and NARCHK(2,2). About out-of-sample performance, MSE1, MAD1 and MAD2 are again in favor of models in the EGARCH family, shown by the fact that those models are consistently ranked on top of the list. Moreover, this time, MSE2 and R2LOG also suggest that EGARCH models are the best, but QLIKE is in favor of GJR-GARCH models. ARCH and GARCH models continue to underperform based on almost all of the loss functions and Information Criteria, except for R2LOG that places ARCH models in the top 10. Moreover, table 4, which includes models estimated with GARCH-in-mean and t-distribution for the error terms, also confirms the underperformance of ARCH and GARCH models. Almost all loss functions in Table 4, except for R2LOG and MAD2 place ARCH and GARCH models at the bottom of the list. And once again, the evidence confirms the superiority of EGARCH models because all criteria consistently place those models in the top ten, especially MSE1, MAD1 and MAD2 placed them in the top 3. Other models that perform well are the TARCH as suggested by MSE2, AIC and BIC, and GJR-GARCH as suggested by QLIKE. Therefore, just by looking at the performance of these models in this sample, we can say that the more sophisticated models clearly outperform the simple models such as ARCH and GARCH in every specifications, and in this contest, models in the EGARCH family are the winners.

Nevertheless, as noted above, the performances of those models in this particular sample may suffer from data-snooping bias. For that reason, it is necessary to perform a data-snooping test to see whether one model spec-

ification is actually better than another. In order to do the Reality Check of White, we would need to select a benchmark model. In this study, my benchmark models will be ARCH(1) and GARCH(1,1). The former is the most simple form of GARCH-type model, and the latter is the most common used by practitioners. I divide my model spaces into four sub-categories: (1) Models with Random Walk conditional mean and normal distribution for the error terms, (2) Models with Random Walk conditional mean and student t-distribution for the error terms, (3) Models with GARCH-in-mean and normal distribution for the error terms, and (4) Models with GARCH-in-mean and student t-distribution for the error terms. Diving the model spaces into each sub-categories will make it easier for us to say whether one model specification is better than another. Then, I compute the p-value for each benchmark model and each criterion by using the Reality Check of White. The final results are presented in Table 5, Table 6, Table 7 and Table 8. A significant p-value (i.e p-value less than 0.05) means that there is at least one model that outperforms our benchmark model, while an insignificant p-value says that the benchmark model is doing just as well as other more sophisticated models. Surprisingly, although those tables consist of statistics of different model specifications for the conditional mean and the distribution of the error terms, the central message from all those tables is the same. By observing the p-values, we can see that MSE1, MSE2 and QLIKE generally suggest that ARCH(1) underperforms, while R2LOG, MAD1 and MAD2 say that ARCH(1) performs just as well as other models. The reverse applies to GARCH(1,1) when MSE1, MSE2 and QLIKE indicates that GARCH(1,1) 's forecasting power is as well as other models, but R2LOG, MAD1 and MAD2 suggests that there are other models that outperform GARCH(1,1). Thus, we can see clearly that different loss functions will favor different models and lead to different conclusions. If we measure forecasting power by MSE1, MSE2 and QLIKE, we will conclude that GARCH(1,1) is an adequate model to forecast volatility. However, if we use R2LOG, MAD1 and MAD2, then we will conclude that ARCH(1) is a good forecasting model.

# Chapter 5

## Conclusion

In my thesis, I have compared the performances of a large number of different forecasting models for stock market volatility. There are almost 160 forecasting models that are combinations of two different conditional mean equations, eleven conditional variances equations, and two distributions for the error terms. The method of comparison is to compute the out-of-sample forecasts for each mode during the period of the 2008 financial crisis, and then use those forecasts to calculate the value for a particular loss functions. Since different loss functions tend to favor different model specifications, I use all six different loss functions to compare the out-of-sample performance, and I also use AIC and BIC to measure the in-sample performances. In addition, I perform the Reality Check of White to test for data-snooping bias with two benchmark models, which are ARCH(1) and GARCH(1,1). The final result is that EGARCH models have the best performance during the financial crisis of 2008, while ARCH and GARCH models fall behind. It can be understood by the fact that EGARCH models are able to capture asymmetric response to news of stock market volatility, while ARCH and GARCH models cannot. Nevertheless, although the evidence during the financial crisis is in favor of EGARCH models, the Reality Check of White suggests that the result might be due to data-snooping bias. When I use ARCH(1) as the benchmark model, the test indicates that our benchmark will be outperformed if we measure performances by MSE1, MSE2 and QLIKE; however, there will be no difference in performance if we use R2LOG, MAD1 and MAD2 as loss functions. The exact opposite applies when I use GARCH(1,1) as the benchmark model. Thus, in conclusion, one needs to be cautious about choosing an appropriate forecasting model for volatility because different way of measuring forecasting power will indeed yield different results.



Table 1: The overall ranking of models that have Random Walk conditional mean and normal distribution for the error terms. Model that is rank #1 in each column is the best model based on that criterion. Whenever there is a tie, the ranking will be divided between the two models. Note that the first number next to a model name indicates the number of lags for the ARCH term, and the second number indicates the number of lags for the GARCH term in that model. For example, GARCH11 means that the model specification is GARCH, and there are one lag for ARCH and one lag for GARCH.

Model	MSE1	MSE2	QLIKE	R2LOG	MAD1	MAD2	AIC	BIC
ARCH1	42	42	42	10	27	12	42	42
ARCH2	39	37	38	9	11	7	40	40
ARCH3	36	38	37	8	14	15	38	38
ARCH4	38	36	36	16	30	31	36	36
GARCH11	34	31	28	41	41	38	32	28
GARCH12	32	29	27	39	39	37	33	31
GARCH21	26	19	22	32	32	34	30	30
GARCH22	28	13	26	36	38	35	28	29
EGARCH11	2	12	23	4	2	2	10	9
EGARCH12	4	11	24	7	4	3	12	11
EGARCH21	1	3	4	5	1	1	3	3
EGARCH22	3	10	20	6	3	4	13	24
TARCH11	6	5	13	12	6	9	9	6
TARCH12	7	6	21	13	8	11	11	10
TARCH21	5	1	5	11	7	10	4	4
TARCH22	20	2	25	27	28	29	1	1
GJRGARCH11	29	7	1	37	33	39	24	19
GJRGARCH12	31	8	2	40	37	41	25	25
GJRGARCH21	24	4	3	35	34	40	8	12
GJRGARCH22	35	9	8	42	42	42	5	5
SAARCH11	15	24	12	22	15	16	16	15
SAARCH12	19	28	17	23	21	20	17	20
SAARCH21	8	15	18	14	9	13	6	7
SAARCH22	23	32	31	30	29	28	2	2
PARCH11	33	33	35	38	40	36	34	32
PARCH12	30	30	34	34	36	32	35	35
PARCH21	25	20	33	31	31	30	31	34
PARCH22	27	16	32	33	35	33	29	33
NARCH11	13	22	10	20	17	18	19	17
NARCH12	17	26	15	25	23	22	22	22
NARCH21	9	14	19	15	10	14	7	8
NARCHK11	13	22	18	10	20	17	18	17
NARCHK12	17	26	15	25	23	22	22	22
NARCHK21	21.5	34.5	29.5	28.5	25.5	26.5	14.5	13.5
NARCHK22	10.5	17.5	6.5	17.5	19.5	24.5	26.5	26.5
APARCH1	13	22	10	20	17	18	19	17
APARCH2	17	26	15	25	23	22	22	22
APARCH3	21.5	34.5	29.5	28.5	25.5	26.5	14.5	13.5

Table 2: The overall ranking of models that have Random Walk conditional mean and student t-distribution for the error terms. Model that is rank #1 in each column is the best model based on that criterion. Whenever there is a tie, the ranking will be divided between the two models. Note that the first number next to a model name indicates the number of lags for the ARCH term, and the second number indicates the number of lags for the GARCH term in that model. For example, GARCH11 means that the model specification is GARCH, and there are one lag for ARCH and one lag for GARCH.

Model	MSE1	MSE2	QLIKE	R2LOG	MAD1	MAD2	AIC	BIC
ARCH1	39	39	39	2	10	6	39	39
ARCH2	37	36	36	6	8	7	37	37
ARCH3	35	37	35	4	11	25	36	36
ARCH4	36	35	34	12	27	27	35	35
GARCH11	33	33	29	38	38	38	31	26
GARCH12	31	29	25	35	36	36	30	28
GARCH21	26	16	23	31	33	33	27	27
GARCH22	27	14	24	33	35	31	26	29
EGARCH11	2	8	7	5	2	2	8	5
EGARCH12	3	9	8	7	3	3	9	10
EGARCH21	1	2	3	3	1	1	4	3
EGARCH22	4	4	5	8	4	5	2	2
TARCH11	6	6	9	9	7	9	10	7
TARCH12	7	7	14	11	9	10	11	11
TARCH21	22	1	37	27	26	26	34	34
TARCH22	14	3	22	13	23	24	1	1
GJRGARCH11	28	10	1	34	30	34	12	12
GJRGARCH12	30	11	2	36	31	35	13	17
GJRGARCH21	23	5	4	29	28	30	7	9
GJRGARCH22	34	12	6	39	39	39	5	6
SAARCH11	10	18	13	16	14	15	16	13
SAARCH12	15	24	18	20	18	14	20	20
SAARCH21	5	13	17	10	5	8	6	8
SAARCH22	19	28	28	24	22	13	3	4
PARCH11	32	34	32	37	37	37	33	30
PARCH12	29	30	33	32	34	32	32	32
PARCH21	24	17	30	28	29	28	29	31
PARCH22	25	15	31	30	32	29	28	33
NARCH11	12	20	11	18	16	20	18	15
NARCH12	17	26	20	22	20	17	22	22
NARCHK11	12	20	11	18	16	20	18	15
NARCHK12	17	26	19 20	22	20	17	22	22
NARCHK21	20.5	31.5	26.5	25.5	24.5	22.5	14.5	18.5
NARCHK22	8.5	22.5	15.5	14.5	12.5	11.5	24.5	24.5
APARCH1	12	20	11	18	16	20	18	15
APARCH2	17	26	20	22	20	17	22	22
APARCH4	20.5	31.5	26.5	25.5	24.5	22.5	14.5	18.5
NPARCH11	8.5	22.5	15.5	14.5	12.5	11.5	24.5	24.5

Table 3: The overall ranking of models that have GARCH-in-mean and normal distribution for the error terms. Model that is rank #1 in each column is the best model based on that criterion. Whenever there is a tie, the ranking will be divided between the two models. Note that the first number next to a model name indicates the number of lags for the ARCH term, and the second number indicates the number of lags for the GARCH term in that model. For example, GARCH11 means that the model specification is GARCH, and there are one lag for ARCH and one lag for GARCH.

Model	MSE1	MSE2	QLIKE	R2LOG	MAD1	MAD2	AIC	BIC
ARCH1	39	39	38	8	22	10	39	39
ARCH2	34	36	36	6	16	11	37	37
ARCH3	33	35	35	5	10	23	35	35
ARCH4	36	34	34	14	27	27	34	34
GARCH11	29	27	24	36	35	35	30	26
GARCH12	27	25	21	31	33	34	31	29
GARCH21	23	15	13	27	29	30	28	28
GARCH22	24	8	20	30	31	31	26	27
EGARCH11	2	14	22	3	2	2	12	10
EGARCH12	3	13	23	4	3	3	13	12
EGARCH21	1	4	4	2	1	1	5	5
EGARCH22	4	1	5	13	4	4	1	1
TARCH11	5	2	18	9	5	8	10	9
TARCH12	8	3	19	10	8	9	11	11
GJRGARCH11	31	6	1	37	36	36	24	20
GJRGARCH12	32	7	2	38	38	37	25	25
GJRGARCH21	30	5	3	33	37	38	9	13
GJRGARCH22	35	12	9	39	39	39	6	6
SAARCH11	9	20	10	18	11	13	16	16
SAARCH12	13	24	17	19	15	17	17	21
SAARCH21	6	11	11	11	6	6	8	8
SAARCH22	19	29	27	25	23	26	2	2
PARCH11	28	28	33	32	34	33	32	30
PARCH12	26	26	32	28	30	29	33	33
PARCH21	22	16	29	26	28	28	29	32
PARCH22	25	9	28	29	32	32	27	31
NARCH11	11	18	7	16	13	15	19	18
NARCH12	15	22	15	21	18	19	22	23
NARCH21	7	10	12	12	7	7	7	7
NARCHK11	11	18	7	16	13	15	19	18
NARCHK12	15	22	15	21	18	19	22	23
NARCHK21	17.5	30.5	25.5	23.5	20.5	21.5	14.5	14.5
NARCHK22	20.5	32.5	<sup>20</sup> 30.5	34.5	25.5	24.5	3.5	3.5
APARCH1	11	18	7	16	13	15	19	18
APARCH2	15	22	15	21	18	19	22	23
APARCH3	17.5	30.5	25.5	23.5	20.5	21.5	14.5	14.5
APARCH4	20.5	32.5	30.5	34.5	25.5	24.5	3.5	3.5
NPARCH11	38	38	39	7	24	12	38	38
NPARCH12	37	37	37	1	9	5	36	36

Table 4: The overall ranking of models that have GARCH-in-mean and student t-distribution for the error terms. Model that is rank #1 in each column is the best model based on that criterion. Whenever there is a tie, the ranking will be divided between the two models. Note that the first number next to a model name indicates the number of lags for the ARCH term, and the second number indicates the number of lags for the GARCH term in that model. For example, GARCH11 means that the model specification is GARCH, and there are one lag for ARCH and one lag for GARCH.

Model	MSE1	MSE2	QLIKE	R2LOG	MAD1	MAD2	AIC	BIC
ARCH1	38	38	38	2	7	4	38	38
ARCH2	37	36	37	5	20	7	37	37
ARCH3	35	37	36	3	19	24	36	36
ARCH4	36	35	35	11	26	26	35	35
GARCH11	31	33	29	37	37	34	32	27
GARCH12	29	29	24	33	33	32	31	29
GARCH21	24	16	21	28	28	31	28	28
GARCH22	26	14	25	32	32	30	27	30
EGARCH11	2	8	7	4	2	2	9	6
EGARCH21	1	3	4	1	1	1	4	3
EGARCH22	3	4	5	6	3	3	2	2
TARCH11	6	5	14	9	6	21	10	7
TARCH12	7	6	16	10	12	22	11	11
TARCH21	22	1	32	12	25	25	26	26
TARCH22	19	2	23	13	24	23	1	1
GJRGARCH11	32	9	1	34	34	36	12	12
GJRGARCH12	33	10	2	36	35	37	13	17
GJRGARCH21	28	7	3	31	30	35	8	10
GJRGARCH22	34	11	6	38	38	38	5	5
SAARCH11	10	20	13	14	8	9	16	13
SAARCH12	14	24	20	20	15	8	20	20
SAARCH21	4	12	15	7	4	5	6	8
SAARCH22	18	28	28	24	23	20	3	4
PARCH11	30	34	34	35	36	33	34	31
PARCH12	27	30	33	30	31	29	33	33
PARCH21	23	19	30	27	27	27	30	32
PARCH22	25	15	31	29	29	28	29	34
NARCH11	12	22	11	18	10	14	18	15
NARCH12	16	26	18	22	17	11	22	22
NARCH21	5	13	22	8	5	6	7	9
NARCHK11	12	22	11	18	10	14	18	15
NARCHK12	16	26	18	22	17	11	22	22
NARCHK21	20.5	31.5	<sup>21</sup> 26.5	25.5	21.5	16.5	14.5	18.5
NARCHK22	8.5	17.5	8.5	15.5	13.5	18.5	24.5	24.5
APARCH1	12	22	11	18	10	14	18	15
APARCH2	16	26	18	22	17	11	22	22
NPARCH11	20.5	31.5	26.5	25.5	21.5	16.5	14.5	18.5
NPARCH12	8.5	17.5	8.5	15.5	13.5	18.5	24.5	24.5

Table 5: Performances of models with Random Walk conditional mean and normal distribution for the error terms. The table shows the performance of the benchmark model along with the worst, median and best models. The p-values is computed from the the Reality Check of White. A significant small p-value (i.e  $<0.05$ ) indicates that there is at least one model outperforms the benchmark model.

Criterion	Benchmark: ARCH(1)				p-values
	Benchmark	Performance		Best	
		Worst	Median	Best	
MSE1	0.00029399	0.00029399	0.00023171	0.00020847	0.0202
MSE2	0.00000179	0.00000179	0.00000138	0.00000127	0.0328
QLIKE	-5.33696938	-6.95323896	-6.91889048	-5.33696938	0.0010
R2LOG	6.22201157	6.80367136	6.39519811	6.04070711	0.4232
MAD1	0.01157935	0.01209021	0.01151428	0.01101740	0.2660
MAD2	0.00052899	0.00056926	0.00053620	0.00050615	0.4404

Criterion	Benchmark: GARCH(1,1)				p-values
	Benchmark	Performance		Best	
		Worst	Median	Best	
MSE1	0.00024169	0.00029399	0.00023171	0.00020847	0.1568
MSE2	0.00000140	0.00000179	0.00000138	0.00000127	0.5282
QLIKE	-6.90375376	-6.95323896	-6.91889048	-5.33696938	0.8542
R2LOG	6.63398361	6.80367136	6.39519811	6.04070711	0.0044
MAD1	0.01195667	0.01209021	0.01151428	0.01101740	0.0334
MAD2	0.00056178	0.00056926	0.00053620	0.00050615	0.0274

Table 6: Performances of models with Random Walk conditional mean and student-t distribution for the error terms. The table shows the performance of the benchmark model along with the worst, median and best models. The p-values is computed from the the Reality Check of White. A significant small p-value (i.e  $<0.05$ ) indicates that there is at least one model outperforms the benchmark model.

Criterion	Benchmark: ARCH(1)			p-values	
	Benchmark	Worst	Median		Best
MSE1	0.00029159	0.00029159	0.00023346	0.00020913	0.0260
MSE2	0.00000179	0.00000179	0.00000138	0.00000119	0.0330
QLIKE	-5.40534544	-6.95323896	-6.92343283	-5.40534544	0.0098
R2LOG	6.19655561	6.84313774	6.46385479	6.15372419	0.6786
MAD1	0.01152191	0.01215111	0.01159706	0.01110169	0.3846
MAD2	0.00052574	0.00057328	0.00054015	0.00051070	0.5622

Criterion	Benchmark: GARCH(1,1)			p-values	
	Benchmark	Worst	Median		Best
MSE1	0.00024741	0.00029399	0.00023171	0.00020847	0.1106
MSE2	0.00000141	0.00000179	0.00000138	0.00000127	0.2006
QLIKE	-6.90241146	-6.95323896	-6.92343283	-5.40534544	0.9272
R2LOG	6.75206280	6.80367136	6.39519811	6.04070711	0.0036
MAD1	0.01214093	0.01209021	0.01151428	0.01101740	0.0188
MAD2	0.00057123	0.00056926	0.00053620	0.00050615	0.0186

Table 7: Performance of models with GARCH-in-mean and normal distribution for the error terms. The table shows the performance of the benchmark model along with the worst, median and best models. The p-values is computed from the the Reality Check of White. A significant small p-value (i.e  $<0.05$ ) indicates that there is at least one model outperforms the benchmark model.

Criterion	Benchmark: ARCH(1)				p-values
	Benchmark	Performance			
		Worst	Median	Best	
MSE1	0.00028732	0.00028732	0.00023513	0.00020748	0.0090
MSE2	0.00000176	0.00000176	0.00000137	0.00000131	0.0400
QLIKE	-5.58035707	-6.95273352	-6.91831827	-5.57978678	0.0422
R2LOG	6.19581413	6.88273478	6.36378717	6.05648422	0.4996
MAD1	0.01155215	0.01222746	0.01147998	0.01095879	0.1986
MAD2	0.00052834	0.00057840	0.00053132	0.00050310	0.3338

Criterion	Benchmark: GARCH(1,1)				p-values
	Benchmark	Performance			
		Worst	Median	Best	
MSE1	0.00024104	0.00028732	0.00023513	0.00020748	0.1258
MSE2	0.00000139	0.00000176	0.00000137	0.00000131	0.6364
QLIKE	-6.90955353	-6.95273352	-6.91831827	-5.57978678	0.6530
R2LOG	6.62794161	6.88273478	6.36378717	6.05648422	0.0074
MAD1	0.01195026	0.01222746	0.01147998	0.01095879	0.0186
MAD2	0.00056224	0.00057840	0.00053132	0.00050310	0.0144

Table 8: Performance of models with GARCH-in-mean and student t-distribution for the error terms. The table shows the performance of the benchmark model along with the worst, median and best models. The p-values is computed from the the Reality Check of White. A significant small p-value (i.e  $<0.05$ ) indicates that there is at least one model outperforms the benchmark model.

Criterion	Benchmark: ARCH(1)			p-values	
	Benchmark	Worst	Median		Best
MSE1	0.00028796	0.00028796	0.00022986	0.00020774	0.0246
MSE2	0.00000178	0.00000178	0.00000137	0.00000117	0.0328
QLIKE	-5.54923153	-6.95377254	-6.92548776	-5.54923153	0.0004
R2LOG	6.16648340	6.87736464	6.41188407	6.16546822	0.5410
MAD1	0.01148338	0.01223142	0.01151077	0.01103771	0.3436
MAD2	0.00052412	0.00057935	0.00053575	0.00050726	0.5152

Criterion	Benchmark: GARCH(1,1)			p-values	
	Benchmark	Worst	Median		Best
MSE1	0.00024672	0.00028796	0.00022986	0.00020774	0.0942
MSE2	0.00000140	0.00000178	0.00000137	0.00000117	0.1716
QLIKE	-6.90653086	-6.95377254	-6.92548776	-5.54923153	0.8934
R2LOG	6.74154806	6.87736464	6.41188407	6.16546822	0.0010
MAD1	0.01212970	0.01223142	0.01151077	0.01103771	0.0162
MAD2	0.00057124	0.00057935	0.00053575	0.00050726	0.0112



## Appendix A

This appendix provides the result for the non-parametric Breitung unit root test. The test indicates that the natural log of daily prices of the S&P 500 Index is a unit root process, thus we should estimate an ARMA(P,Q) for the differenced time series instead of the original one. The output below is taken directly from the output produced by EASYREG.

JrgBreitung's nonparametric unit root test is based on the following simple but very clever idea: Let  $y_t$ ,  $t=1,\dots,n$ , be a unit root process:  $y_t = y_{t-1} + u_t$ , where  $u_t$  is a zero-mean stationary process. Compute the partial sums  $Y_t = y_1 + y_2 + \dots + y_t$ , and then the ratio

$$B(n) = \frac{[Y_1^2/1^2 + Y_2^2/2^2 + \dots + Y_n^2]/n^2}{[y_1^2/1^2 + y_2^2/2^2 + \dots + y_n^2]/n}$$

Under the unit root hypothesis  $B(n)/n$  converges in distribution to a function of a standard Wiener process, which is free of nuisance parameters. On the other hand, if  $y_t$  is stationary then  $B(n)$  itself converges in distribution, hence  $B(n)/n$  converges in probability to zero. If the alternative hypothesis is that  $y_t$  is stationary with a non-zero mean, then  $y_t$  is first demeaned, and if the alternative is that  $y_t$  is trend stationary, then  $y_t$  is first detrended.

Null hypothesis:

H0:  $y_t$  is a unit root with drift process.

Alternative hypothesis:

H1:  $y_t$  is a trend stationary process.

Reference:

Breitung, J. (2002): "Nonparametric Tests for Unit Roots and Cointegration", Journal of Econometrics 108, 343-364.

Test result for  $y_t = \text{LN}[\text{Prices}]$

Test statistic:  $B(n)/n = 0.01808$  ( $n = 5608$ )

Significance level Critical value Conclusion

5% 0.00355 Accept H0

10% 0.00450 Accept H0

Note that JrgBreitung only reports the critical values for  $n = 100$ ,  $n = 250$  and  $n = 500$ . Therefore, the critical values used here are the ones for  $n = 500$ .

## Appendix B

This appendix provides the results for the selection procedure using Akaike, Hannan-Quinn and Schwarz (or Bayesian) Information Criteria. The sample of the return is the first sub-sample consisting of 4,000 observations of daily returns.  $a(1,i)$  is the  $i$ -th autoregressive lag, and  $a(2,j)$  is the  $j$ -th moving average lag in the ARMA model. The output is produced by EASYREG.

First available observation:  $t = 1$  (=1.01)

Last available observation:  $t = 5617$  (=469.01)

First chosen observation:  $t = 1$  (=1.01)

Last chosen observation:  $t = 4000$  (=334.04)

Time series:

$Y(t) = \text{Return}$

Model:  $Y(t) = b(1)x(1) + u(t)$ ,

where

$x(1) = 1$

Maximal model for  $u(t)$ :

$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$ ,

where  $e(t)$  is white noise and  $L$  is the lag operator.

The most parsimonious model for  $u(t)$  will be determined on the basis of the Akaike, Hannan-Quinn and Schwarz information criteria

Model 0:

$u(t) = e(t)$

Information criteria:

Akaike: -9.1773058E+000

Hannan-Quinn: -9.1767480E+000

Schwarz: -9.1757323E+000

Model 1:

$[1 - a(1,1)L]u(t) = e(t)$

Information criteria:

Akaike: -9.1768162E+000

Hannan-Quinn: -9.1757007E+000

Schwarz: -9.1736692E+000

Model 2:

$[1 - a(1,1)L - a(1,2)L^2]u(t) = e(t)$

Information criteria:

Akaike: -9.1768486E+000

Hannan-Quinn: -9.1751753E+000

Schwarz: -9.1721281E+000

Model 3:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) = e(t)$$

Information criteria:

Akaike: -9.1773512E+000

Hannan-Quinn: -9.1751201E+000

Schwarz: -9.1710572E+000

Model 4:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) = e(t)$$

Information criteria:

Akaike: -9.1768672E+000

Hannan-Quinn: -9.1740783E+000

Schwarz: -9.1689996E+000

Model 5:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) = e(t)$$

Information criteria:

Akaike: -9.1775490E+000

Hannan-Quinn: -9.1742024E+000

Schwarz: -9.1681080E+000

Model 6:

$$u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1768172E+000

Hannan-Quinn: -9.1757017E+000

Schwarz: -9.1736702E+000

Model 7:

$$[1 - a(1,1)L]u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1763177E+000

Hannan-Quinn: -9.1746444E+000

Schwarz: -9.1715971E+000

Model 8:

$$[1 - a(1,1)L - a(1,2)L^2]u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1763510E+000

Hannan-Quinn: -9.1741199E+000

Schwarz: -9.1700570E+000

Model 9:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1768517E+000

Hannan-Quinn: -9.1740629E+000

Schwarz: -9.1689842E+000

Model 10:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1763727E+000

Hannan-Quinn: -9.1730261E+000

Schwarz: -9.1669316E+000

Model 11:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) = [1 - a(2,1)L]e(t)$$

Information criteria:

Akaike: -9.1770613E+000

Hannan-Quinn: -9.1731569E+000

Schwarz: -9.1660467E+000

Model 12:

$$u(t) = [1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1768575E+000

Hannan-Quinn: -9.1751842E+000

Schwarz: -9.1721370E+000

Model 13:

$$[1 - a(1,1)L]u(t) = [1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1763925E+000

Hannan-Quinn: -9.1741614E+000

Schwarz: -9.1700984E+000

Model 14:

$$[1 - a(1,1)L - a(1,2)L^2]u(t) = [1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1758810E+000

Hannan-Quinn: -9.1730921E+000

Schwarz: -9.1680134E+000

Model 15:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) = [1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1764440E+000

Hannan-Quinn: -9.1730973E+000

Schwarz: -9.1670029E+000

Model 16:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) =$$

$$[1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1759764E+000

Hannan-Quinn: -9.1720720E+000

Schwarz: -9.1649618E+000

Model 17:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) =$$

$$[1 - a(2,1)L - a(2,2)L^2]e(t)$$

Information criteria:

Akaike: -9.1767083E+000

Hannan-Quinn: -9.1722461E+000

Schwarz: -9.1641202E+000

Model 18:

$$u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1775003E+000

Hannan-Quinn: -9.1752692E+000

Schwarz: -9.1712063E+000

Model 19:

$$[1 - a(1,1)L]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1769985E+000

Hannan-Quinn: -9.1742097E+000

Schwarz: -9.1691309E+000

Model 20:

$$[1 - a(1,1)L - a(1,2)L^2]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1765757E+000

Hannan-Quinn: -9.1732291E+000

Schwarz: -9.1671347E+000

Model 21:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) =$$

$$[1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1761840E+000

Hannan-Quinn: -9.1722797E+000

Schwarz: -9.1651695E+000

Model 22:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1755839E+000

Hannan-Quinn: -9.1711217E+000

Schwarz: -9.1629958E+000

Model 23:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3]e(t)$$

Information criteria:

Akaike: -9.1763925E+000

Hannan-Quinn: -9.1713725E+000

Schwarz: -9.1622309E+000

Model 24:

$$u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1770003E+000

Hannan-Quinn: -9.1742115E+000

Schwarz: -9.1691327E+000

Model 25:

$$[1 - a(1,1)L]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1764985E+000

Hannan-Quinn: -9.1731519E+000

Schwarz: -9.1670574E+000

Model 26:

$$[1 - a(1,1)L - a(1,2)L^2]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1759633E+000

Hannan-Quinn: -9.1720589E+000

Schwarz: -9.1649487E+000

Model 27:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1755822E+000

Hannan-Quinn: -9.1711201E+000

Schwarz: -9.1629941E+000

Model 28:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1756191E+000

Hannan-Quinn: -9.1705992E+000

Schwarz: -9.1614575E+000

Model 29:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4]e(t)$$

Information criteria:

Akaike: -9.1758568E+000

Hannan-Quinn: -9.1702791E+000

Schwarz: -9.1601217E+000

Model 30:

$$u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1777062E+000

Hannan-Quinn: -9.1743596E+000

Schwarz: -9.1682651E+000

Model 31:

$$[1 - a(1,1)L]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1772338E+000

Hannan-Quinn: -9.1733294E+000

Schwarz: -9.1662192E+000

Model 32:

$$[1 - a(1,1)L - a(1,2)L^2]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1767118E+000

Hannan-Quinn: -9.1722497E+000

Schwarz: -9.1641237E+000

Model 33:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3]u(t) = [1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1762180E+000

Hannan-Quinn: -9.1711981E+000

Schwarz: -9.1620564E+000

Model 34:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4]u(t) =$$

$$[1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1759668E+000

Hannan-Quinn: -9.1703891E+000

Schwarz: -9.1602316E+000

Model 35:

$$[1 - a(1,1)L - a(1,2)L^2 - a(1,3)L^3 - a(1,4)L^4 - a(1,5)L^5]u(t) =$$

$$[1 - a(2,1)L - a(2,2)L^2 - a(2,3)L^3 - a(2,4)L^4 - a(2,5)L^5]e(t)$$

Information criteria:

Akaike: -9.1753702E+000

Hannan-Quinn: -9.1692347E+000

Schwarz: -9.1580615E+000

Optimal model according to

Akaike: Model 30

Hannan-Quinn: Model 0

Schwarz: Model 0



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**ACADEMIC VITA**  
**LAM NGUYEN**  
2509 Thomas Street, Harrisburg, PA 17103  
(717) 608 - 2915  
lhn5017@psu.edu

**Education:**

2009-2012 *The Pennsylvania State University*, University Park, PA  
The Schreyer Honor College, Smeal College of Business  
Bachelor of Science in Finance  
Minor in Economics

**Academic Honors and Awards:**

2009-2012 Dean's List  
2011-2012 Wherry Honors Scholarship in Business  
2011-2012 Bunton Waller Scholarship  
2011-2012 Dipple Trustee Scholarship  
2011-2012 PA State Grant  
2012 American Economic Association Minority Scholarship  
2012 Lewis Scholarship Fund  
2012 Whitney Trustee Scholarship  
2012 Equal Opportunity General Scholarship  
2010 Academic Competitiveness Grant

**Research Experiences:**

**Fall 2012** Schreyer Honor Thesis titled "*Comparison of Volatility Models of the S&P 500 Index*"

**Abstract:** An accurate forecast of financial volatility is very crucial in many applications such as portfolio management when we need to figure out the Efficient Frontier, or risk management when we need to compute the Value-At-Risk, or hedging when we need to calculate the Hedge Ratio for the portfolio, etc. This thesis compares the forecasting power of different ARCH-type models during the recent financial crisis. Those models include ARCH, GARCH, EGARCH, TARCH, GJR-GARCH, SA-ARCH, P-ARCH, NA-ARCH, NA-ARCHK, APARCH, and NPARCH. Using six different loss functions and the Reality Check of White for data snooping, this study found that the results largely depend on the loss functions that are chosen. EGARCH models generally have the best performances during the financial crisis. After the test for data-snooping, we found that ARCH (??) is outperformed by other models for three out of six loss functions but perform just as well for the other three. The exact reverse applies to the GARCH (1, 1).

**Summer 2012** American Economic Association Summer Training at the University of New Mexico

Presenting a group poster titled:

*“How to Save a Life: Study of the Relationship between Healthcare Expenditure, New Drug Approvals and Life Expectancy.”*

**Abstract:** With rising health care expenditure, there is some concern about the return of investment on health care research and development. To determine whether the benefits of R&D outweigh the costs, this study examines the relationship between health care expenditure, medical innovations, and life expectancy. We use the Finite Distributed Lag (FDL) model and the Vector Autoregression (VAR) model as our empirical frameworks. The FDL model suggests that medical innovations have a positive and significant effect on life expectancy. However, the VAR model suggests that medical innovations do not Granger-cause life expectancy, and both life expectancy and health care expenditure Granger-cause medical innovations.

**Summer 2011 McNair Summer Research Internship**

Writing a research paper titled:

*“Return in Investing in Equity Mutual Funds from 1990 to 2009”*

**Abstract:** Mutual fund is a financial institution that pools money from many small investors to invest in securities such as stocks, bonds and money market instrument. Actively managed mutual funds are funds that try to outperform a particular benchmark index, such as the S&P 500. Using different financial models such as CAPM, Fama-French 3-Factors model and Carhart 4-Factors Model to evaluate the performance of actively managed mutual funds from 1990 to 2009, this study found that only a small number of funds can actually outperform their benchmarks. Moreover, by looking at simulated returns from strategy of purchasing top mutual funds based on performance, we can see there are evidences indicating those superior performances are due to luck and not skills. Finally, the study found that there is a significant negative relationship between fund returns and factor measured fund expenses, including expenses ratios and fund turnovers.

**Publications and Presentations:**

Nguyen, Lam, Mackenzie Alston, Rosa M. Daz Rivera and Robert Granados (2012). How to Save a Life: Study of the Relationship between Healthcare Expenditure, New Drug Approvals and Life Expectancy. *Poster Presentation at the AEA Summer Training Program at the University of New Mexico*

Nguyen, Lam (2011). Return in Investing in Equity Mutual Funds from 1990 to 2009. *Penn State McNair Scholar Journal. Vol.18*

Nguyen, Lam (2010). Return in Investing in Equity Mutual Funds from 1990 to 2009. *Presented at the Annual McNair Conference at Penn State.*

**Relevant Course Work:**

**Finance:** Financial Management of the Business Enterprise; Security Analysis and Portfolio Management; Financial Markets and Institutions; Derivatives; Multinational Financial Management.

**Graduate Courses in Economics:** Introduction to Mathematical Economics (ECON 500), Econometrics (ECON 501)

**Economics:** Introductory Microeconomics and Policy; Introductory Macroeconomics and Policy; Intermediate Macroeconomics Analysis (Honor); Intermediate Microeconomics Analysis (Honor); Statistical Foundations for Econometrics; Introduction to Econometrics (Honor), International Economics.

**Mathematics:** Elementary Statistics; Technique of Calculus; Honors Calculus with Analytic Geometry II; Honors Calculus and Vector Analysis; Matrix Algebra; Introduction to Real Analysis (Honor), Classical Analysis I (Honor)

**Research Methods:** McNair Scholar Research Methods

**Professional Activities:**

***Affiliations:***

2012-2013 Member of American Economic Association (AEA)

2010-2012 The Ronald E. McNair Post-Baccalaureate Achievement Program at Penn State

**Student Community and Leadership:**

Taiji Club, Treasurer and Safety Officer, University Park, PA since 2009  
*(Winning four gold medals in Taiji Competition at the ICMAC tournament at Pittsburg and Washington DC in 2012)*

Penn State Investment Association, University Park, PA since 2009

Business and Society House, University Park, PA since 2009

Penn State THON, Treasurer for Rules and Regulations Committee in 2010-2011