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ANALYSIS OF AT RISK MIDDLE SCHOOL STUDENTS' ERRORS WHEN SOLVING FRACTIONS

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ABSTRACT

As a country, we are falling behind in teaching our children mathematics. Mathematics is a clear predictor of future academic success and is vital for a student's education. Students' understanding of fractions is the strongest predictor of future knowledge of algebra and overall mathematics achievement, even after controlling for parents' education, and income. (NMAP, 2008). Consequently, there is an immediate need to improve teaching and learning of fractions according the National Mathematics Advisory Panel. This thesis aims to address the above issues by conducting an error analysis to study the error patterns made by struggling students on fraction problems involving addition, subtraction, multiplication, and division. Discussion includes patterns of errors that occurred most frequently and recommendations for future instruction.

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Chapter 1

Introduction

Demands of Mathematics Knowledge and Skills Increasing

Overall, the more education a person receives, the higher his or her pay will be will be. Figure A adopted from *Why Study Math* (Kromer) is a chart titled Earnings vs. Educational Attainment that shows this trend. Among those in many professions, engineers, Sociologists, and bank tellers, for example, use mathematics daily in their jobs (Kromer). Examinations of jobs that require daily mathematics, as shown in green circles in Figure A, illustrate an even greater correlation between educational attainment and earnings. Further, among jobs that do not require a college education, a strong mathematical sense is still required. Ninety percent (90%) or more of carpenters, electricians, surveyors, and bank tellers hold a high school diploma or GED, yet these occupations require number sense, geometry, large data sets, and complication analysis of formulas during their daily operations.

Outside of employment, mathematical situations can be found anywhere you look in daily life. Calculating a tip, choosing a cell-phone plan, using a receipt, betting on poker, and investing for retirements are a few of the abundant examples of daily tasks that require proficiency with arithmetic skills, geometry, and estimation skills.

Common Core State Standards Overview

The Common Core State Standards (CCSS), or the Common Core, provides a consistent, clear description of what students are expected to learn in mathematics and language arts. A team of the nation’s governors and education commissioners, worked to establish the common core in order to set high standards and clear expectations that are aligned to the expectations in college are careers. These standards promote equity by ensuring that all students are well prepared with the skills and knowledge necessary to thrive in today's world. (CCSSI)

The CCSS was established to assure that when met the skills and competencies will help to develop students who are able to compete and collaborate with their peers in college and beyond. (CCSSI) Through the core an intense focus is placed on mathematics skills that begins in kindergarten and continues through completion of 12^{th} grade. This focus is not by coincidence. The highly skilled committee spent years researching what our students need to succeed and focused on those competencies necessary for future success. (CCSSI)

Previous Findings in Error Analyses of Problems Involving Fractions

To fully comprehend fraction related mathematical topics, six areas of instruction need to be taught as separate entities and then bridged together. Kieren (1980) suggests that the five concepts that need to be taught independently are:

1. Part-whole relationships
2. Rations,
3. quotients
4. Measures
5. Operators

Kieren continues to stress the importance of focusing on each area independently, because students need to learn the why behind what they are doing before learning the how. Far too often students' simply "plug and chug" numbers into the prescribed formulas without knowing *why* they are doing it. Later on, the leaner may try to apply a known protocol when it does not make sense to. For example, a student could try to use a natural, number protocol and add the numerators and denominators of a fraction addition problem. (Kieren, 1980) If the algorithm is taught before the student is cognitively aware enough, they will not have the conceptual ability to know when to apply the concept.

Fractions are Recognized as Important

In 2006 Brown and Quinn conducted an analyses that shed a disturbing light on the state of mathematics learning in the United States. They reported that the majority of high school students struggle with and have deficiencies in decimals, percentages, fractions, and simple algebra. These are all areas that encompass skills necessary for future endeavors and are vital for academic success.

Brown and Quinn declared that the lack of experience with fraction concepts and fraction computation in our current educational system to be inexcusable. Before starting ninth grade, students need to have had at least two years of informal exposure of fraction concepts and three years for the development of formal concepts and computational fluency. Being able to comprehend whole number concepts can help to building fraction related competency, which can then be extended to form algebraic concepts. Generalizations occur through this process, which are beneficial to the student.

The National Mathematics Advisory Panel (NMAP, 2008) asserts that a foundational knowledge of fractions is vital for students' success in algebra. Additionally, results suggest that fraction knowledge is linked to algebra readiness, more so than number magnitude knowledge in general. Specifically, it appears that students' magnitude knowledge of unit fractions (those with a numerator of 1), appears very important. (Booth, 2012). If we want our students to succeed in their futures, a strong skill set of fractions is essential.

Additionally, the "Nations Report Card" displayed low achievement in fractions and algebraic concepts for the age seventeen student. (Booth, 2010). As discussed, mathematical skills are used far beyond high school, and college for that matter. Occupations now seek out those with a higher-level understanding of mathematics, yet we are sending seventeen year olds out into the work force despite their low lack of skills in such vital areas essential to future mathematical related success.

When Fractions are Introduced

According to the Common Core Standards (CCS) (CCWS, 2012) fractions are one of the three critical areas that instruction should focus on in fourth grade. It states that it is vital for students to " develop an understanding of fraction equivalence, addition, and subtraction with like denominators, and multiplication of fractions by whole numbers." Problems that were included in the practice set included multiplication of fractions by whole numbers and addition and subtraction with like denominators. This was done to be able to hone in on *what types of errors were presented* and *when* in the sequence of instruction that students begin making errors.

First Grade

Students are using fraction language to talk about shapes. (CCWS, 2012). For example, the words two, three, or four equal shares are used. In addition to this language, students are expected to describe shares using vocabulary such as *halves, thirds, a third of, etc.*

Second Grade

Second graders work on increasing the vocabulary introduced in first grade. They also work on using this language in visuals, as well. (CCWS, 2012).

Third Grade

In the third grade, students begin to learn with unit fractions. Unit fractions are fractions with a numerator of 1. (CCSW, 2012). For the sake of increasing the number of opportunities for the mathematical practice of focusing on precision, students are asked to specify the whole and elaborate on what the phrase “equal parts” means. A focus is placed on the number line, and using unit fractions with the number line. Equivalent fractions are used at an introductory level during grade three to better prepare them for the work to come in the following academic year. Another area of instruction in grade three that will appear over and over again is the ability to compare fractions. Students need to be able to translate a fraction onto a point on a number line in order to start conceptualizing negative numbers, which will be introduced in the sixth grade. (CCWS, 2012).

Fourth Grade

In grade four, students begin to multiply the numerator and denominator of a fraction by the same number. This exercise will extend to “comparison, addition, and subtraction of fractions and the introduction of finite decimals.” (CCWS 2012). This is also the year where composition and decomposition of fractions with the same denominator occurs. Building on what was learned in grade 3, they are able to add and subtract fractions with like denominators. Another vital skill in this year is the process of multiplying a fraction by a whole number. (CCWS, 2012).

Fifth Grade

In fifth grade, students work on adding and subtracting fractions with different denominators where one denominator is a factor of the other one. This allows for only one fraction to be changed and is a terrific method of scaffolding the process. Multiplication and division of fractions is also a topic students are exposed to. In grade five, students work on understanding that multiplication can be used as a means of scaling. This is noted as a way to work on students being able to reason abstractly. (CCWS, 2012).

Previous Research Shows at Risk Kids struggle with Learning Fractions

Many children struggle with learning about fractions. Siegler and Pyke (2012) found that low achieving students got less from instruction than typically or high achieving children. In this study, the students were all from the same classroom getting the same instruction, but had a different learning experience. The most alarming finding from this study is that as high achieving students progress from sixth to eight grade, their fraction arithmetic accuracy becomes higher. Sadly, low achieving students' accuracy was low in both grades and did not have the expected growth in the two-year span.

Additionally, Garnett (1998) wrote that students with learning disabilities have difficulty grasping the written symbol system and concrete materials, which are used to teach about fractions. Visuals, such as a number line or drawing a picture to represent a fraction problem, are not as accessible to students with a learning disability.

Knowing these facts, this study chose to focus on students who were low achieving in mathematics. The hope is that this study will provide a meaningful contribution to the field and spark discussions on how we can better educate these students.

We Need Sound Means to Teach Fractions

The Institute of Educational Sciences (IES) is a government run organization that does research on behalf of the United States Department of Education. In September 2010, they released a document titled *Developing Effective Fractions Instruction for Kindergarten Through*

8^{th} Grade. This guide was created to show the five recommendations intended to help improve students understanding of fractions. The five recommendations are:

> "1.Build on students informal understanding of sharing an proportionality to develop initial fraction concepts
>
> 2.Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
>
> 3. Help students understand why procedures for computations with fractions make sense.
>
> 4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems
>
> 5. Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them." (Siegel et al, 2010)

IES guide instructional recommends that equal sharing activities be used as a tool to introduce fractions. This can be done by dividing set of objects equally as well as single whole objects. Once this is completed, students should extend this activity by ordering fractions in order from smallest to largest and learning about equivalent fractions. Building from this activity, students understanding can be taken to a more advanced level by working with proportions. Activities should start with similar proportions and as comfort levels increase, so should the number of different proportions introduced in an activity. (Siegel et al, 2010)

Once students are able to understand proportions, they are ready to learn that fractions are numbers that extend the number line beyond whole numbers. At this stage, number lines should be used as the central representation tool in teaching. Activities should allow the student to locate and compare fractions on number lines. This will improve their understanding of fraction equivalence, fraction density (the concept that are an infinite number of fractions in-between any two fractions, and recognizing the existence of negative fractions.) Students are expected to understand that fractions can be represented as fractions, decimals, and percentages. Students should be able to translate among these forms. (Siegel et al, 2010)

To ensure long-term understanding of fractions, the next step is to help students understand that procedures for computations make sense. Again, number lines, along with models and other visual representations should be used when modeling computational procedures. A key part of this stage is to give students opportunities to estimate solutions to problems and judge the reasonableness of problems involving computations. This will help them to learn that they are doing things for a reason, instead of just applying the algorithms because they were taught to do so in a certain situation.

The final recommendation is that professional development programs should place a “strong emphasis on improving teachers’ understanding of fractions and how to teach them”. (Siegel et al, 2010) Teachers need to spend more time learning how to use varied concrete and pictorial representations of fractions and operations involving fractions. Additionally, teachers need to be able to assess students understanding and misunderstandings of fractions so that they can correctly instruct the learner when errors do occur. (Siegel et al, 2010)

When Designing Instruction it is Important to Understand Where Students Typically Make Mistakes

When students make errors and formulate mathematical misconceptions, teachers should recognize the errors, prescribe an appropriate instructional focus, and implement an effective and efficient reteaching plan. The first step in this process, recognizing the errors, in completed through a systematic examination of students mathematics work. (Ashlock, 2002). An error analysis is an essential step to help teachers guide their instruction. By finding the kind and quantity of error that occurs, instruction can be guided based off of the pinpointed errors.

In a study completed to assess if teachers could identify errors in student work and alter instruction based off of the errors made, it was found that over 50% of the teachers were able to find and explain the error pattern presented. (Riccomini, 2005). This is a promising statistic because it assures us that most educators do have the skills necessary to identify the errors that their students are making. From there, they need to be able to take that knowledge and transform their instruction to offer the most effective instruction possible.

Description of an Error Analysis and its Applications

The purpose of an error analysis is to identify the patterns of errors students are making so that instruction can be targeted to correct misconceptions that are causing the errors. When an error analysis is conducted and the only material available is the permanent product (in this study the permanent product is the students' problem sets), the first step is to ensure that an error code key is created to cover every possible type of error. This is done by a thorough examination of the problem set to ensure that every possible error type is included. These errors are categorized by type (in this study the four main error categories were the four operations included in the problem set: addition, subtraction, multiplication, and division). Once the key is created, each individual problem set will be assessed problem by problem. Looking at the first problem, the coder will examine the students work to decode if an error was made. If an error was made, they will learn establish what type of error was made, and then mark that error type in the corresponding category of the error analysis key. The same process is done for the second problem, and so on, until the entire problem set is completed. This process is done for every problem set in the sample. Once all of the data has been collected, the analysis can serve as a tool to identify the error patterns that are occurring most often in order to tailor future instruction.

Previous Studies Find Consistent Error Patterns in Problems Involving Fractions

Brown and Quinn's conducted a study in 2010 that found five distinct categories that students' fraction errors fell into. They were:

1. Algorithmic applications
2. Applications of basic fraction concepts in word problems
3. Elementary algebraic concepts
4. Specific arithmetic skills that are perquisite to algebra
5.comprehension of the structure of rational numbers
6. Computational fluency

There were numerous errors cited as a result of the misapplication of an algorithm. For example, when asked to find the sum of two fractions with different denominators, almost 48% of the sample was unable to correctly solve the problem. The authors assert that the "algorithmic application of finding the lowest common denominator" was a frequently occurring error made on this problem. (Driscoll, 1982). Under the same umbrella as the misuse of algorithmic applications, when asked to multiply two fractions with unlike denominators, 58% of students answered incorrectly. The three main errors, misapplying the standard multiplication algorithm, adding the denominators and multiplying the numerators, and finding the least common denominator before multiplying, were all a result of the misapplication of an algorithm.

The second category of errors, misapplication of basic fraction concepts in word problems, was also a cause for concern. Not only was there a disturbing rate of incorrect answers found, but also many solutions did not make sense in regards to what the question was asking. By encouraging students to use a visual representation of the problem, they may have a better starting off point to solve the problem. (Brown, Quinn, 2002). For example, if the question is asking how many people will arrive in the first bus, a pictorial representation will hopefully point out to the student that an answer with a fraction remaining will not make sense, because people cannot be broken down into fractions. Students need to be able to make sense of problems related to fractions in order to succeed in future employment placements. For example, when a contractor needs to find out how many squares of tile to order, they need to have the prowess to know how to turn their real world problem into a math problem and come up with a solution that makes sense.

Elementary algebraic concepts are the third category Brown and Quinn established. The examples presented again stressed the need for students to be able to utilize algorithms and visual representations when solving a problem. Once algebra was added to problems that required the same processes as number only problems, students were unable to apply the algorithms needed.

The fourth category was specific arithmetic skills that are prerequisite to algebra. The understanding of arithmetic skills that are used in algebra is the key to success for understanding basic algebra (Brown, Quinn, 2006). For example, one problem asked the students' to find 18/0. Although students' are taught that division by zero is undefined, they are taught that as an independent concept. (Brown, Quinn, 2006). Division by zero, and other specific arithmetic skills, need to be taught by connecting it to rational numbers and different situations to fully ensure comprehension. By showing the arithmetic concept in different types of problems, students will truly understand the skills and be able to apply them as they are exposed to more mathematical contexts.

Comprehension of the structure of rational numbers was found to be the fifth category of concern. For example, one problem asked the students to find ½ of 2/3. None of the students surveyed were able to show the basic knowledge of the concepts that one-half of two-thirds is one- third. (Brown, Quinn, 2006). Once again, the multiplication algorithm was overextended as a result of not understanding the structure of the numbers presented.

The sixth and final category addressed was computational fluency. Wu (2001) asserts that fluent computation with numbers is the building blocks for being able to “symbolically manipulate numbers”, which will lead to success in algebra. He asserts that students cannot simply memorize algorithms to be successful, they need to be able to compute with speed and accuracy. Increasing the opportunities students have to solve basic algebraic questions and explicit, immediate feedback will help to increase computational fluency.

Research Questions

1. Can the types of errors at-risk students make when solving fractions problems be categorized?
    a. If so, what are the types of errors made and their relative frequency?
2. Do the types of errors vary by the type of problem?
    a. If so, which types of problems are the most challenging for students?
3. Are there trends in the types of errors made across specific problem types?
    a. If so, can we identify and describe these trends?

Chapter 2

Methods

Design

Data from the practice packets of 31 students in a suburban seventh grade classroom participated in the study. Students were enrolled in a class of struggling mathematics students. Data about each student was provided by the teacher without student identifiers.

The practice packets included 30 items. The items included each of the following: 6 addition problems (fraction added to fraction, whole number added to fraction, fraction added to mixed number, mixed number added to improper fraction, whole number added to mixed number, and mixed number added to mixed number), 6 subtraction problems (whole number minus fraction, fraction minus fraction, improper fractions minus mixed number, whole number minus improper fraction, and mixed fraction minus mixed fraction), 12 multiplication problems (fraction times whole number, fraction times fraction, whole number times mixed number, mixed times whole number, mixed fraction times mixed fraction, mixed fraction times improper fraction, whole number times mixed fraction, and fraction times improper fraction), and 6 division problems (fraction divided by fraction, fraction divided by whole number, mixed number divided by improper fraction, mixed fraction divided by fraction, whole number divided by mixed number, and mixed number divided by mixed number.) The breakdown of the problem set can also be found in a table form in Figure B.

Items from the packet were derived from Pearson Prentice Hall *Connected Mathematics 2*, the textbook used in the samples classroom. Additionally, *Math Connects* and *Everyday*

Mathematics, Grade 6 were used for gathering problems for the problem set. .In addition, original problems were also created for the practice packets. To ensure the packets were consistent with what students were exposed to in the classroom, teacher created worksheets were collected and analyzed for comparison Discussion with the teacher assured pedagogical methods she used and sequence of instruction regarding fractions as presented in class. Analyses of teacher created materials included an examination of both the types of fractions the students were exposed to and their relative frequency. The practice packets were developed to be parallel. For example, multiplication problems were included twice as often as addition, subtraction, or division problems in the problem set because they were seen twice as often in classroom related activities. As a result, 12 of 30 total problems were multiplication problems in the practice packets

After generating approximately 500 problems as a team using *Connected Mathematics 2*, *Math Connects* and *Everyday Mathematics Grade 6*, the problems were shared with the classroom teacher. She provided feedback about the level of item difficulty, and consulted with the team to ensure that all problems were consistent with what students would encounter in their daily classroom problems. Once teacher input was received, items were modified to fit with the classroom curriculum. The final packets were compiled to be distributed to students. Students completed the packets during regular class time over two days. As the student handed in their packet, their classroom teacher recorded the time it took them to complete their problem set.

Coding Rules and Decisions

The purpose of this study was to examine the types of errors that were made by struggling students while completing fraction problems in order to offer suggestions for improving future fraction instruction. Students work was systematically analyzed according to the created codes. In the first round of coding individual problems were determined to be correct, incorrect, or not fully reduced problems were coded as correct with a “1” if they had the correct answer in the correct format as per the classroom teacher. Correct format for an answer required the fraction to be a fully reduced number that is a fraction, whole number, or a mixed number. Improper fractions, and fractions that were not fully reduced were coded with a ".5" as a result of not being in the required format. Incorrect answers were coded with a "0". This code can be found in Appendix A.

The coding scheme was created with advice from Karen Fries, a doctoral candidate working in Special Education under Dr. Paul Riccomini. Karen shared a coding scheme she created for a different project under Dr. Riccomini, which was used as a preliminary guide for establishing this error code.

Challenges in Coding

Coding provided to be challenging for a number of reasons. For the code to be valid and reliability, the terms had to be clear and descriptive enough that any reader could understand what each code meant independently.

Multiple Errors in One Problem

Additionally, another problem that presented itself was when a problems error seemed to cover more than one of the codes available. After getting advice from Dr. Sperling and Dr. Riccomini, it was decided that a problem could have multiple error codes. The errors made would be coded in ordered of importance, most to least. For example, if in a multiplication problem if the student multiplied the numerator, left the same denominator, and did not fully reduce it could be categorized as Ag or Ac. This solution allowed the code to read as Agc, because the solution did have both errors, but error "Ag", multiplied numerator, left the same denominator, was more at fault for the error.

Reliability Checks

Reliability checks were conducted at each stage of the coding process. Shawn Gardiner, a graduate student in Special Education, completed the same coding process independently using the key.

Initial Reliability Check

The first reliability check was doing using the correct/partially correct/incorrect scheme described above. Shawn completed this independently and when the coding results were compared, a 100% inter-rater reliability rate between the two coders was found.

Final Reliability Checks

A second and third reliability checks were completed to ensure inter-rater reliability once the final code, Appendix B, was established. In the second reliability check, an inter-rater reliability rate of 83.3% was achieved. At this point, there was a discussion of the errors made and both coders made adjustments to their coding analysis. Through discussion, the coders were able to reach agreement on 87 of the 90 problems. The third and final inter-rater reliability rate achieved was a 96.7%.

Participants

The worksheets were derived from classes of at-risk students enrolled in seventh grade math classes tracked for at-risk students in a school district in the Mid-Atlantic region of the United States. The District serves 6,900 students. Of these students, 14% are a minority, and 27% are economically disadvantaged. The tracked classes represented students performing below grade level in mathematics.

Procedures

On September 28th, 2012 I met with Dr. Paul Riccomini, a professor of Special Education at The Pennsylvania State University, and Karen Fries, a doctoral student from the Special Education department at The Pennsylvania State University, to discuss error analysis methods. Dr. Riccomini has done extensive research on the most common procedural errors made by students while evaluating fractions; Karen has also conducted significant amounts of research related to error analysis, specifically with respect to procedural errors while evaluating fractions. Under their direction I was given a coding system Karen had created for a similar error analysis to use as an example. Her error analysis was based off of the article *Algebra Students' Difficulty with Fractions, An Error Analysis,* Brown and Quinn. They also recommend that I read the IES Practice Guide, *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.* Using the resources given to me by Dr. Riccomini and Karen, I gained a fundamental understanding of the separation process; I grouped the error key into the four operations - addition, subtraction, multiplication, and division. These four sets would then contain sub-groups that would describe and distinguish between the specific error types. The rationale behind grouping errors by operation was in hopes of making trends in the data more readily available, an idea that I noticed in both Karen's work as well as the aforementioned articles.

Although advised to start by creating a code to score three student packets at random, I wanted to gauge the frequency and severity of error types by doing an initial analysis of the collected data. I was familiar with error types in fractions from the previously mentioned readings, however I was interested in seeing possible differences in the types of errors as the material assessed in this data differed from that of the readings. While reviewing the data, I would mark a frequency tally next to repeated error types to begin to establish initial trends.

Before continuing, I reviewed my notes from the discussion we had with the classroom teacher to ensure that my scoring was consistent with the school's standards and the expectations taught in the students' textbooks with respect to the correct process for writing a fraction solution.

According to the classroom standards, and the classroom textbook, for a solution to be considered correct, it must be simplified to a mixed number. As a result, I realized that every operation group would include a "did not simplify" sub-code.

Once a code was created, three packets were scored at random. This step was done to ensure that no error types showed up that the code did not account for. I began by scoring three packets for correct and incorrect solutions. After scoring all three, my peer, Shawn Gardiner scored them independently for correct and incorrect. We scored a 100% reliability rate in this phase.

Shawn Gardiner is a student pursuing her masters' degree in special education. She is currently enrolled in a course with Dr. Riccomini and is familiar with error analysis. Once the code was developed, Shawn was given the three packets to score. Independently, she scored three packets containing 30 problems each without my presence. Once she was finished scoring using my key, we met to check our reliability rate. We scored 75 out of 90 problems with the same coding. This gave us an initial reliability rate of 83.3%. From there, Shawn and I met to see if any agreement could be reached on problems that were coded differently. Through discussion, we were able to reach agreement on 87 of the 90 problems. This gave us a reliability rate of 96.7%. The reliability check was conducted in order to ensure that the variation in the code's interpretation was as insignificant as possible. The reliability rating of 96.7% suggests, with a high degree of confidence, that an error analysis using my error code key could be independently replicated in order to produce an accurate error analysis.

On October 30th, I presented my code to Dr. Rayne Sperling, a professor and researcher in the educational psychology department at The Pennsylvania State University who has conducted countless research studies related to educational psychology, and Dr. Riccomini. I systemically explained the process detailed above so that they could understand the rationale behind the code. In addition to some minor formatting and rewording changes (e.g. simplify not reduce,) Dr. Sperling and Dr. Riccomini suggested improvements be made in order to make each operational set more parallel. Dr. Sperling suggested making the addition and the subtraction subset codes parallel, because the operations performed in these steps are almost identical. Similarly, multiplication and division sets were made to have almost parallel sub-codes. Division would have one additional sub code, as a result of an additional step being completed while evaluating a division problem.

A key discussion point in this meeting was the three problems in which a coding error agreement could not be reached between the scorers. They each saw a different issue as the "error" in the problem, and both scorers had valid points. As a result, Dr. Sperling and Dr. Riccomini recommended that the problem should be coded with more than one sub code. The rationale behind this is that sometimes more than one error can contribute to an incorrect answer. Additionally, having the ability to note more than one subset of error type will provide valuable information once all the data is collected. Continued analysis will strive to isolate trends within the subsets in order to document errors that frequently occur. If there are a significant number of subsets of errors occurring together, there could be strong implications for instruction.

After this meeting, the codes were reworked to be parallel from operation to operation. Additionally, the language was reworded to ensure consistency throughout the coding scheme. After checking all of these areas over, it was time to conduct another error analysis. Again, each packet was individually assessed problem by problem. Each answer was coded as a single or multiple error code, which was stored in an excel spreadsheet. Once all of the packets had been coded, they were given to Shawn Gardiner for a reliability check. After she completed coding the students' packets, a reliability check was done between the two coders. An inter-reliability rate of 76% was reached at this point. At this point, the two scorers went through each question together and made adjustments to the codes they assigned to each problem. After the discussion, the inter-rater reliability rate reached 95%.

Once the inter-rater reliability reached an acceptable level, analytics began on the error code and its results. Data was put into a summary chart to help with further analysis. See Figure C.

Chapter 3

Data Analysis and Results

Frequency of Errors by Error Type

Most Frequently Occurring Errors

Looking at Figure D*,* there are some error types that are strong outliers. Focusing on those errors that were made the most frequently, the highliers, there are seven types that stood out for having over fifty occurrences throughout the sample populations' problem sets. This can be found in D. These were Aa –multiplication, no work shown (81), Ab- multiplication- misapplied standard multiplication algorithm (108), Af- performed other operation (55), Bf – subtraction- misapplied standard subtraction algorithm (52), Cf -addition- misapplied standard addition algorithm (50), and Da – division- no work shown (116).

Least Frequently Occurring Errors

There were ten error types that occurred less than ten times throughout the samples' problem sets. These can be found in Figure E. These lowliers were Bb – subtraction- incorrectly reduced number (5), Bc – subtraction - did not fully reduce (8), Bd subtraction- performed addition operation incorrectly (0), Ca --addition- no work shown (8), Cb – addition- incorrectly reduced number (4), Cc -addition- did not fully reduce (9), Cd -addition- performed addition operation incorrectly (5), Ce addition- performed subtraction operation incorrectly (0), Dc – division- did not fully reduce (3), De – division- incorrectly reduced (2).

Frequency of Errors by Problem Type

Of the four operations included in the problem set (addition, subtraction, multiplication, and division), students had the highest counts of errors in problems that required division. Problems that required multiplication were the second highest category for frequency of errors. Fraction problems that required subtraction had the third highest error count by problem type. Lastly, addition had the lowest count of errors by problem type. Figure F offers a comprehensive look at error frequency by individual problem.

Evidence of Comprehension

There were some specific problems in which the overall success rate was marginally higher when compared to the overall success rate. Of the 30 presented problems, there were four individual problems in which 20 or more participants found the correct answer. The first problem, in which 22 people answered it correctly, was a subtraction problem of a whole number minus a proper fraction. The second problem with 20 people completing it correctly was a subtraction problem involving two proper fractions. The third problem, with 24 correct counts, was the highest count of the entire problem set. This problem was an addition problem involving a whole number and a mixed number. The fourth problem with 20 people getting the correct answer was an addition problem between a whole number and a proper fraction. All of these problems involved addition and subtraction. Figure F shows a comprehensive look at the percentage of solutions that were incorrect by problem type.

Chapter 4

Discussion

Conclusion

Can the types of errors at-risk students make when solving fractions problems be categorized?

Yes they can.

If so, what are the types of errors made and their relative frequency?

Misapplied standard algorithm- 233 counts/ 648 total errors

36% of errors in this sample came from the misapplication of a standard algorithm.

No work shown- 226 counts/ 648 total errors

34.9% of errors in this sample came from no work shown.

Did not fully reduce- 37 counts/ 648 total errors

6% of the errors in this sample came from not fully reducing.

Found least common denominator before multiplying-

66 counts/ 373 applicable errors

17.7% of the errors in applicable problems in this sample came from finding the least common denominator before multiplying.

Performed other operation- 16 counts/ 173 applicable errors

9.2% of the errors in applicable problems in this sample came from performing another operation.

Do the types of errors vary by the type of problem?

Yes, the types of errors vary by the type of problem.

If so, which types of problems are the most challenging for students?

Division appears to be the most challenging type. Of the 288 correct problems counted for, only 13 of them were from problems where division was required. Although problems requiring division accounts for 20% of problem set, the number of correct answers counted for problems that require division was merely 4.5% of the total number of correct answers.

Are there trends in the types of errors made across specific problem types?

Yes, there are trends in the types of errors made across specific problem types.

If so, can we identify and describe these trends?

The most concerning trend lies in the division problems that were presented. Of the 173 errors accounted for in problems requiring division, 116, or 67% of them, were found to not have any work present. This alarming percentage comes from a lack of procedural knowledge. When the population sampled did show work in a problem requiring division, they got it correct 59% of the time. When compared to the overall division success rate of 4.5%, this is a true outlier. This shows that the majority of students lack the procedural knowledge necessary to solve a problem that requires the multistep process of dividing fractions.

Recommendations

Require Students to Show Their Work

The highest occurring error in this study proved to be from students not showing any of their work. In addition to this being a cause for concern in regards to error correction and analysis, showing no work allows for careless errors to be made that otherwise may not have been if the students had written out their work. Taking the time to write out each step will allow for the student to be able to check their work more effectively as well. By showing their work, students will have more opportunities to practice commonly used operations, which will also aid their fluency.

Error Correction Instruction

There needs to be instructional time dedicated to correcting errors as a class. The teacher should present relevant problems that are already completed incorrectly to the class. As a class, the problem should be presented so that students are made aware of frequently occurring errors, before they are given the opportunity to make these mistakes themselves. Students should be asked to mark off the step in which the error occurred, then complete the problem correctly individually. By seeing the errors firsthand, they will be able to recognize the common misconceptions and be made hyperaware of them before making the same errors firsthand. These errors can serve as a source of learning for the student. By highlight the common errors, and careless mistakes, students hopefully will take note of them and be cognizant not to make the same mistakes on their own work.

Increase Focus on Procedural Knowledge and More Levels of Scaffolding for These Skills

Teachers can help their students by increasing the amount of time spent on introducing a new skill before allowing the student to practice it independently. As shown in the data above, students struggled with knowing how to do certain types of problems, especially division problems. By allowing for more time for the teacher to model problems involving the division of fractions, students should have a better grasp of it before incorrectly practicing it independently. In addition to increasing the time spent modeling, the teacher should spend a good portion of the allotted instruction time scaffolding the students by providing support and class wide examples before releasing them to practice independently. Independent practice should not be done until the student truly understands the skill and its appropriate application. During the modeling portion of instruction, students should be presented with every type of problem they will see where this procedural skill can be used. In addition, they need to see non-examples, or instances where the procedural knowledge skill being taught could not work. For example, if the class was focusing on the division of fractions, the teacher should present an array of problems that include mixed numbers, whole numbers, and improper fractions. Additionally, the teacher should present problems that show a different operation than division and highlight that the steps being taught would not work in that case. By practicing examples and non-examples, the student will become more aware of the context of the skill and will be less likely to overgeneralize the procedure, which was a very common error in this study.

Figure A

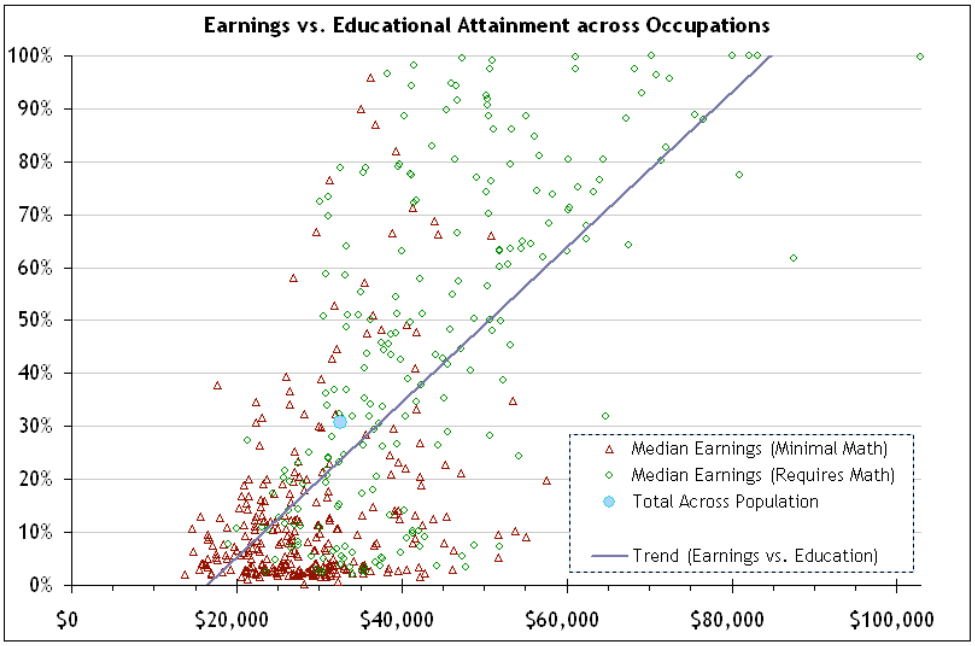
Earnings vs. Educational Attainment across Occupations
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
$0
$20,000
$40,000
$60,000
$80,000
$100,000
Median Earnings (Minimal Math)
Median Earnings (Requires Math)
Total Across Population
Trend (Earnings vs. Education)

Figure B

Problem Set Problem Type Breakdown

Problem Type / Operation	Addition	Subtraction	Multiplication	Division	Total
Fraction / Fraction	1	1	1	1	4
Fraction / Whole Number	1	1	2	1	5
Fraction / Mixed Number	1	1	1	1	4
Mixed Number / Improper Fraction	1	1	2	1	5
Whole Number / Improper Fraction	0	1	1	1	3

Whole Number / Mixed Fraction		1	0	2	1	4
Mixed Fraction / Mixed Fraction		1	1	3	0	5
Total		6	6	12	6	30

Figure C

Correct	Aa	Ab	Ac	Ad	Ae	Af	Ag	Ba	Bb	Bc	Bd	Be	Bf	Ca	Cb	Cc	Cd	Ce	Cf	Da	Db	Dc	Dd	De	Df
9	0	12	1	2	3	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	21	2	0	2	1	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	3	2	0	11	0	0	0	0	0	0
16	2	5	0	6	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	1	0	0	0	3	4	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	3	0	0	0	1	5	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	13	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	5	0	2	0	3
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	3	0	2	0	2
9	2	12	1	5	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	2	0	0	0	0	0
1	6	11	1	9	2	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	8	8	2	6	2	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	9	7	1	8	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	4	1	2	1	5
11	0	0	0	0	0	0	0	3	4	1	0	2	8	2	0	0	0	0	1	0	0	0	0	0	0
8	4	13	1	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	5	0	4	0	3	10	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	1	0	0	10	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	0	5	0	0	0	0	0	0
5	11	9	1	5	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	6	0	2	0	1	11	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	6	1	1	0	3
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	5	1	1	0	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	8	0	0	0	0	0	0
8	7	8	4	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	3	1	1	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0
10	6	7	1	1	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	12	9	2	2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	13	7	2	3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
288	81	108	17	53	11	11	22	21	5	8	0	10	50	8	4	9	5	0	50	116	25	3	10	2	16

Figure D

Error Code Frequency Count – Highliers

Error Code	***Frequency***
Aa –multiplication, no work shown	81
Ab- multiplication- misapplied standard multiplication algorithm	108
Af- performed other operation	55
Bf – subtraction- misapplied standard subtraction algorithm	52
Cf - addition- misapplied standard addition algorithm	50
Da – division- no work shown	116

Figure E

Error Code Frequency Count – Lowliers

Error Code	***Frequency***
Bb – subtraction- incorrectly reduced number	5
Bc – subtraction - did not fully reduce	8
Bd subtraction- performed addition operation incorrectly	0
Ca --addition- no work shown	8
Cb – addition- incorrectly reduced number	4
Cc -addition- did not fully reduce	9
Cd -addition- performed addition operation incorrectly	5
Ce addition- performed subtraction operation incorrectly	0
Dc – division- did not fully reduce	3

Figure F

Frequency of Errors by Problem Type

Problem Number	***Problem Type***	***Frequency of Incorrect Answers***	***Percent Incorrect***
1	Multiplication	21	70%
2	Division	27	90%
3	Addition	16	53.3%
4	Multiplication	14	46.7%
5	Subtraction	8	26.7%
6	Subtraction	10	33.3%
7	Addition	17	56.7%
8	Division	28	93.3%
9	Division	27	90%
10	Multiplication	21	70%
11	Addition	6	20%
12	Multiplication	29	96.7%
13	Multiplication	27	90%
14	Multiplication	25	83.3%

15	Division	29	96.7%
16	Subtraction	19	63.3%
17	Multiplication	22	73.3%
18	Subtraction	20	66.7%
19	Addition	15	50%
20	Addition	10	33.3%
21	Multiplication	25	83.3%
22	Subtraction	20	66.7%
23	Division	28	93.3%
24	Division	28	93.3%
25	Addition	11	26.7%
26	Multiplication	22	73.3%
27	Subtraction	17	56.7%
28	Multiplication	20	66.7%
29	Multiplication	25	83.3%
30	Multiplication	25	83.3%

Appendix A

Error Analysis Code #1

1. Correct

.5 Correct, but not in proper form (not fully reduced, improper fraction)

0 Incorrect

Appendix B

Error Analysis Final Code

A. Multiplication
    a. No work shown
    b. Misapplied standard multiplication algorithm
    c. Did not fully reduce
    d. Found LCD before multiplying
    e. Incorrectly reduced number
    f. Performed other operation
    g. Multiplied numerator, left same denominator
B. Subtraction
    a. No work shown
    b. Incorrectly reduced number
    c. Did not fully reduce
    d. Performed addition operation incorrectly
    e. Performed subtraction operation incorrectly
    f. Misapplied standard subtraction algorithm
C. Addition
    a. No work shown
    b. Incorrectly reduced number
    c. Did not fully reduce
    d. Performed addition operation incorrectly
    e. Performed subtraction operation incorrectly
    f. Misapplied standard addition algorithm
D. Division
    a. No work shown
    b. Misapplied standard division algorithm
    c. Did not fully reduce
    d. Found LCD before multiplying
    e. Incorrectly reduced
    f. Performed other operation

X=correct

BIBLIOGRAPHY

Ashlock, Robert B. *Error Patterns in Computation: Using Error Patterns to Help Each Student Learn*. Boston: Allyn & Bacon, 2010. Print.

Booth, Julie L.; Newton, Kristie J." Fractions: Could they really be the gatekeeper's doorman?" *Contemporary Educational Psychology* vol. 37 issue 4 October, 2012. p. 247-25

Brown, Geroge, and Quinn J. Robert. "Algebra Students' Difficulty with Fractions: An Error Analysis." *Australian Mathematics Teacher* 62.4 (2006): 28-40. Print.

Carter, John A., Gilbert J. Cuevas, and Roger Day. *Math Connects.* Columbus, OH: Glencoe/McGraw-Hill, 2012. Print.

Driscoll, M. (1982). Research Within Reach: Secondary School Mathematics. A Research Guided Response to the Concerns of Educators. Missouri: (ERIC Document Reproduction Service No. ED 225 842).

Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. In T. E. Kieren (Ed.), Recent Research on Number Learning (pp. 125-149)

Kromer, Phillip. (2007) "Why Study Math?" *Why Study Math?* Web.

Lappan, Glenda. *Connected Mathematics 2.* Boston, MA: Pearson/Prentice Hall, 2006. Print.

"Mission Statement." *Common Core State Standards Initiative*. N.p., n.d. Web.

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education. Retrieved December 16, 2010, from http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Riccomini, Paul J. "Identification and Remediation of Systematic Error Patterns in Subtraction." *Learning Disability Quarterly* 28.3 (2005): 233-43. Print.

Siegler, Robert, Thomas Carpenter, Francis Fennell, David Geary, James Lewis, Yukari Okamoto, Laurie Thompson, and Jonathan Wray. "Developing Effective Fractions Instruction For Kindergarten Through 8th Grade." *What Works Clearinghouse* (2010).

University of Chicago. School Mathematics Project. N.p.: Wright Group/McGraw-Hill Education, C2012. Print.

Wu, H. (2001, Summer). How to prepare students for algebra. American Educator, 25 (2), 10-17.

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