INVESTIGATION OF ATTITUDE AND TRANSLATIONAL RATE COMMAND CONTROL LAWS FOR A SHIP-BASED HELICOPTER

REBECCA RIPLEY
FALL 2013

A thesis
submitted in partial fulfillment
of the requirements
for a baccalaureate degree
in Electrical Engineering
with honors in Electrical Engineering

Reviewed and approved* by the following:

Joseph F. Horn
Associate Professor of Aerospace Engineering
Thesis Supervisor

John Mitchell
Professor of Electrical Engineering
Honors Adviser

Constantino Lagoa
Professor of Electrical Engineering
Additional Reader

* Signatures are on file in the Schreyer Honors College.
ABSTRACT

One of the most difficult tasks for any rotorcraft pilot to perform is landing the aircraft on a ship deck. There are numerous effects that the pilot needs to be aware of and quickly correct for. Those effects include air wake turbulence, a moving ship deck, and landing in such a small and potentially cluttered area. Much research has been done to identify ways to widen the envelope of conditions under which pilots can safely perform ship deck landings and other ship-based operations. This paper looks at Translation Rate Command (TRC) control techniques to alleviate some of the pilot workload associated with ship-based operations. TRC time and frequency responses are compared to those found in the Attitude Command, for a controller with inner-loop Attitude Command and Outer-loop TRC. Results show that there is a significant trade-off between disturbance rejection and inner-loop stability margins; however, adding the TRC outer-loop does not degrade these margins. The TRC controller provides high levels of disturbance rejection while still maintaining adequate stability margins, and provides tighter responses than Attitude Command.
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ACKNOWLEDGEMENTS

I would like to thank my advisor, Joe Horn, for his help and guidance throughout this project and for allowing me the opportunity to explore this interest. I would like to thank Constantino Lagoa for his willingness to help as second advisor. Lastly, I would like to thank my Honors Advisor, Jack Mitchell, for his continued support throughout my entire career at Penn State. His open door has contributed greatly to my successes.
Chapter 1

Introduction

1.1 Problem Statement

Landing a rotorcraft on a ship deck is one of the most dangerous operations for any pilot to perform. Pilots must be aware of and account for the deck movements in high sea states, air wake turbulence, poor weather, high winds over the deck, and reduced visibility. These factors create a heavy workload in order for the pilot to maintain the aircraft's stability [1]. This means that tasks performed routinely such as take off, landing, and near Earth hover are much more difficult when performed over a ship deck. However, the Navy is interested in performing ship-based operations in even more severe winds and higher sea states, as they do not want to be held back by weather conditions in a mission critical situation [1]. It is imperative for pilots to perform ship-based tasks safely and reliably, as helicopters are one of the most essential weapons and assets the U.S. Navy uses in maritime operations [2].

Currently, one of the main limiting factor for ship deck operations is the pilot workload required to accurately compensate for all the aforementioned environmental factors [3]. To expand the ability to perform these operations a controller with a lower pilot workload (more control stabilization) and higher disturbance rejection is desired. Significant research efforts in previous years have been dedicated to investigating and trying to solve this problem.
1.2 Control Background

The main difference between TRC and other forms of control, are the parameters being commanded and the types of feedback. In Attitude Command, Attitude Hold (ACAH) for the roll and pitch axes the pilot inputs "yield a proportional pitch or roll attitude, respectively", and "the system must regulate the pitch and roll attitudes to their trim values when there are no command inputs".[4] ACAH is shown for the roll axis in Figure 1-1. The roll attitude is commanded by the lateral stick. It is apparent that the Attitude of the rotorcraft closely follows the commanded input. In ACAH there is attitude feedback, but no velocity feedback. Alternatively, in TRC mode the forward and side velocities are controlled with the pilot stick.[5] as shown in Figure 1-2. In this case the lateral velocity, not the roll angle, closely follows the lateral stick input. TRC is achieved in the outer-loop with an inner-loop attitude controller, thus there is velocity and velocity feedback. In Rate Command the rate of change is controlled. Rate Command, Heading Hold (RCHH) is shown in Figure 1-3. In RCHH for the yaw and vertical axes the pilot pedals and collective yield a proportional yaw rate or vertical velocity. In Figure 1-3 the yaw rate is commanded from the pedals and the yaw rate of the rotorcraft closely follows the input. RCHH is explained in terms of yaw and heave, since it will be used in both of these axes for both controllers used later in this investigation. RCHH is more representative of the natural response of the rotorcraft, and is the least complex method introduced. TRC is the most complex system introduced.
Figure 1-1. Roll Attitude and Lateral Velocity Response to Lateral Stick Input in Attitude Mode
Figure 1-2. Roll Attitude and Lateral Velocity Response to Lateral Stick Input in TRC Mode

Figure 1-3. Yaw Rate Commanded Input a Response in Rate Command
1.3 Literature Review

One possible solution to ship-based operation difficulties is Translation Rate Command (TRC). TRC is appealing for its "inherent gust proofing" and "relief of pilot inner loop stabilization" \cite{1}. Previous research and pilot analysis has shown that hover and tasks performed at low speed near the Earth using visual cues require increased control stabilization as visual cues become degraded \cite{6}. TRC introduces an outer-loop in the controller for this increased stabilization.

Research at the NASA Ames Research Center has shown the applicability of TRC for hovering and low-speed near-Earth maneuvers in conditions with limited visibility \cite{6}. In this investigation researchers used the NASA Ames Vertical Motion Simulator and the Singer-Link Dig 1 Computer Image Generator to simulate a helicopter flying in a degraded visual environment. A series of maneuvers were completed by pilots using three types of control: Rate Command, Attitude Command/Attitude Hold, and Translation Rate Command. Maneuvers included hover, a vertical landing, a pirouette, an acceleration/deceleration, and a sidestep. Pilots gave handling qualities ratings for each of the tasks \cite{7}. The rating system used the Cooper-Harper quality rating scale (HQR). This scale has ratings 1-10 where 1 is the best possible rating. The scale is broken into three levels. Ratings 1-3 are Level 1, ratings 4-6 are Level 2, rating 7-9 are Level 3, and a rating of 10 is unsafe for flight. Level 1 qualities are desired, Level 2 qualities are adequate, and Level 3 qualities are poor. In vertical landing and hover tasks near Earth using visual cues with TRC control achieved Level 1 handling qualities. For the remaining three tasks, TRC achieved a combination of Level 1 and Level 2 ratings. Rate Command resulted in "poor" Level 2 handling qualities for all maneuvers, and Attitude Command/Attitude Hold resulted in "good" Level 2 handling qualities \cite{6}.
One difficulty with research for this application, is that there are no clearly defined or quantitative requirements for optimal responses or disturbance rejection. In research conducted with the U.S. Navy X 22A variable stability (V/STOL) aircraft with attitude type TRC mode, researchers set out to identify "flying qualities and flight control system design criteria for hover and low speed flight" [1]. Through pilot simulations this research showed that TRC could provide adequate handling qualities for ship-based operations. The controller used the basic transfer function were V is the velocity, \( \delta \) is the stick deflection, K is the gain, and \( \tau \) is the time constant.

\[
\frac{V}{\delta} = \frac{K}{s\tau + 1}
\]

In pilot evaluations it was suggested that the ideal path mode time constant, \( \tau \), was between 1.5 seconds and 2.3 seconds for the longitudinal axis. Results were reported for this axis as it was identified as the more difficult to control in these simulations.

In research to develop a pilot assisted landing system (PALS) to land a helicopter on a ship deck, TRC with position hold control referenced to a lock grid on the ship deck was identified through pilot ratings as the favored PALS [2]. The PALS was tested with Attitude Command/Attitude Hold, TRC referenced to Earth with position hold, and TRC referenced to the ship with relative position hold. All three control types were tested in dusk and night conditions in sea state 6 and 30 knot winds at 30 degrees. TRC referenced to the ship exhibited the best handling qualities under both conditions. The TRC system reliably exhibited ratings showing desired performance in conditions with poor visibility. It even showed adequate performance ratings when the landing would have been almost unthinkable with attitude command attitude hold mode. It was also found that the TRC modes did in fact reduce the pilot workload [2].

A factor in aircraft design that has recently been of interest is the trade-off between stability margins, which ensure robustness, and disturbance rejection, which minimizes the effects
of atmospheric conditions. Stability margin requirements are well defined in MIL-F-9490D and more recently, SAE AS94900. Gain margins are required to be 6db or greater and Phase Margins 45 degrees or greater. However, there are situations when smaller margins are acceptable. Allowing for smaller stability margins provides more disturbance rejection. In a study conducted by the U.S. Army Aeroflightdynamics Directorate, two controllers were designed for the JUH-60, one to meet the traditional stability margin requirements and one with "relaxed margins" (4 dB, 35 deg). The two designs were flight tested and HQR ratings indicated pilot preference for the traditional stability margin requirements.

1.4 Purpose

The purpose of this paper is to investigate the properties of the baseline Dynamic Inversion controller with inner-loop Attitude Command and outer-loop TRC. The controller is used in conjunction with the linear model of the Penn State modified GENHEL software, GENHEL-PSU. This study will investigate the effects of feedback gains and time constants on stability margins, disturbance rejection bandwidths, and various time domain outputs such as attitude and velocities. Characterization of the controller is necessary for future studies which will aim to design a TRC controller for use in ship-based operations.
Chapter 2

Modeling and Simulation Environment

2.1 GENHEL Model

The linear model of the UH-60A Blackhawk was extracted from the non-linear GENHEL (GENeral HELicopter) model, a "non-linear, blade element flight dynamics model" [13]. The GENHEL model was originally created by Sikorsky Aircraft Corporation, then modified and published by U.S Army AFDD [13]. Penn State researchers have developed a modified version of the simulation model, GENHEL-PSU [11], [12]. The modified version includes a MatLab/Simulink interface, allowing researchers to input test controller configurations in place of the existing control systems on the UH-60A [13]. It also allows extraction of high order linear models, including that used in this research [13].

The linear models used for this paper are 28th order state space systems extracted from the non-linear simulation at various operating points using a perturbation method. "The state vector includes 8 rigid body fuselage states (3 velocities, 3 angular rates, pitch and roll Euler angles), 12 rotor states (flapping and lagging dynamics in multi-blade coordinates), 3 inflow states (Pitt-Peters model), and 5 engine states (rotor speed and turbine engine states)" [13]. The input to the plant are the lateral, longitudinal, collective, and yaw inputs to the control mixer. These inputs are the outputs of the actuator. Three linear models were extracted for the flight conditions to be studied. The flight conditions chosen are typical environments in which basic ship-based operations are performed. These models were hover with zero mean wind, 30 knot mean head wind, and 30 knot mean wind at 30 degrees to the right of the aircraft.
2.2 Turbulence Models

2.1.1 Airwake Turbulence Model

A key factor to be aware of when testing and designing for ship deck operations, is the ability to accurately model the environment. Wind experienced over a ship deck can be markedly different from that present over an open area of land. The wind on a ship is deflected and redirected around the large ship structures, and all structures are in the immediate proximity of the landing deck \[14\]. This can create unusual gusts. In a Master's Thesis, written by Sade Sparbanie, an air wake turbulence model of a DDG81 ship was created, by identifying and modeling spectral properties of the gusts \[14\]. For this investigation, an air wake turbulence model for an LHA ship was used \[15\], where the turbulence filters were previously derived using a method similar to those in Ref. 14. The disturbance vector contains the average gust velocity on the body of the rotorcraft and the spatial variations modeled as equivalent angular rates. $u_g$, $v_g$, and $w_g$ are the average gust velocities in the lateral, longitudinal, and vertical axes respectively. $p_g$, $q_g$, and $r_g$ are the equivalent angular rates resultant from the wind gust. This disturbance enters the system as described in equation 2.1.

$$
\begin{bmatrix}
  u_g \\
  v_g \\
  w_g \\
  p_g \\
  q_g \\
  r_g
\end{bmatrix}
$$

(2.1)

$$
\dot{x} = Ax + Bu + Gw
$$

(2.2)

In the model used here, this turbulence can be turned on and off via a flag variable in MatLab.
2.1.2 CETI Turbulence Model

A second type of turbulence was also used in this study. The Control Equivalent Turbulence Input (CETI) models turbulence as pilot inputs \cite{16}. Researchers collected flight test data from a UH-60 hovering in air wake turbulence behind a building. From this data, pilot inputs that would achieve the same aircraft responses, assuming no turbulence, were identified. Thus the control equivalent method models turbulence as control inputs to the aircraft plant which are added to the output of the actuators, as opposed to Airwake Turbulence which is multiplexed. The CETI model consists of "Dryden-type," "white-noise driven filters" that can be scaled to simulate various levels of turbulence \cite{16}.
Chapter 3

Control Law Design

This section describes the basic control principles used to design the Attitude and TRC controllers. Attitude Command Attitude Hold (ACAH) response is achieved in the roll and pitch axes, and Rate Command Heading Hold (RCHH) in the yaw and collective axes in the inner-loop. Translational Rate Command (TRC) for forward and lateral velocities is achieved through the outer-loop of the controller.

To design the controller, the 28th order model described in section 2.1 was reduced further to a simple 4th order system\(^{[13]}\). Initially, the rotor RPM and engine states were removed by assuming constant rotor speed (this is a reasonable assumption if the aircraft has an effective rotor RPM governor, which is the case for the UH-60 model). The remaining 23 states are then divided into "fast" and "slow" states. "Fast" states are assumed to be in a quasi-steady state (their state derivatives are always approximately zero), and thus these states can be 'residualized'. This eliminates rotor blade states and leaves an 8 state model of the slower rigid body states of the rotorcraft. These states are the body velocities, angular rates, and roll and pitch attitudes. To reduce this 8th order model to the desired 4th order system, the rows and columns of the state space matrices corresponding to the attitudes and lateral and longitudinal body velocities ("very slow" states) are simply eliminated. This leaves a 4th order system with states roll rate, pitch rate, yaw rate, and vertical velocity (equation 3.2). This 4th order system was only used for the control design, the 23rd order model (representing the full aircraft dynamics with constant rotor RPM) is used for testing in Chapter 5. The inputs to the controller are the pilot inputs: lateral stick, longitudinal stick, pedal, and collective.
Dynamic Inversion was used to design both inner-loop and outer-loop control. It has become a popular control method in recent years \cite{18,19,20,21,22,23}. It has been adapted for rotorcraft control in various studies \cite{14,24,25}. Dynamic Inversion provides many benefits. The inversion decouples the axes from each other, which allows us to investigate each axis separately, as done in Chapter 5. Additionally the problems of Disturbance Rejection and Command Following are decoupled. In Dynamic Inversion, the A and B matrices (introduced in this chapter) need to be gain scheduled, but the PID or PI controllers can generally be held constant. This is a huge advantage as one controller can be designed to work over a wide envelope of operating conditions.

3.1 Attitude Controller Design

Pilot inputs are not directly fed to the controller. They are first transformed by a command filter to translate physical pilot inputs into reference or command inputs for the controller. The command filter for the roll and pitch axes are classic second order transfer
functions and the filter for the yaw and heave axes are simple first order filters.

\[
\begin{align*}
\phi_{cmd} & \rightarrow \frac{K_{roll}\omega_{nroll}}{s^2 + 2\xi_{roll}\omega_{nroll}s + \omega_{nroll}^2} \rightarrow s^2 \rightarrow \phi_c \\
\theta_{cmd} & \rightarrow \frac{K_{pitch}\omega_{npitch}}{s^2 + 2\zeta_{pitch}\omega_{npitch}s + \omega_{npitch}^2} \rightarrow s^2 \rightarrow \theta_c \\
r_{cmd} & \rightarrow \frac{K_{yaw}}{sT_{yaw} + 1} \rightarrow s \rightarrow r_c \\
w_{cmd} & \rightarrow \frac{K_{heave}}{sT_{heave} + 1} \rightarrow s \rightarrow \omega_c
\end{align*}
\]

Figure 3-1. Attitude Command Filters

To begin the controller design the basic state-space model is assumed

\[
\dot{x} = Ax + Bu \tag{3.1}
\]

where \(x(t)\) is the state vector, and \(u(t)\) is the control input vector (the input to the rotorcraft model). For the controller used in this investigation, the A and B matrices are defined by the linearized GENHEL-PSU model. An important condition to note for Dynamic Inversion control is a square system, meaning \(u(t)\) and \(x(t)\) vectors have the same number of rows (or equivalently the number of control inputs equals the number of system states) \([18]\). This is true of the system used in this investigation.

For this system,
where $p$ is the roll rate and $q$ is the pitch rate. Equivalently, the roll and pitch rates in the linear model can be expressed as time derivatives of the Euler angles,

$$ x = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\tau} \\ \dot{w} \end{bmatrix} $$

The goal of the controller is to have the output track some commanded value. In this case the output is identical to the states. The commanded values are the command filter response to the pilot inputs. The commanded state vector is referred to as $x_c$

$$ x_c = \begin{bmatrix} \dot{\phi}_c \\ \dot{\theta}_c \\ \dot{\tau}_c \\ \dot{w}_c \end{bmatrix} $$

The commanded states are compared to measured states to yield a tracking error $e(t)$

$$ e(t) = \begin{bmatrix} \int (\dot{\phi}_c - \dot{\phi}) \, dt \\ \int (\dot{\theta}_c - \dot{\theta}) \, dt \\ \int (\dot{\tau}_c - \dot{\tau}) \, dt \\ \int (\dot{w}_c - \dot{w}) \, dt \end{bmatrix} = \begin{bmatrix} \phi_c - \phi \\ \theta_c - \theta \\ \tau_c - \tau \\ w_c - w \end{bmatrix} $$

A pseudo-command, $v(t)$, is then defined by

$$ v(t) = Ax + Bu $$

Solving for $u(t)$ yields
\[ u(t) = B^{-1}(v - Ax) \] (3.7)

Then substitute \( u(t) \) into \( \dot{x}(t) \)

\[ \dot{x}(t) = Ax + Bu \] (3.8)

\[ \dot{x}(t) = Ax + BB^{-1}(v - Ax) \] (3.9)

\[ \dot{x}(t) = v(t) \] (3.10)

Since in the roll and pitch axes the integral of the commanded input is being controlled, the pseudo command in the roll and pitch axes will be looked at separately from the yaw and heave axes. The roll and pitch axes will be examined first. \( v(t) \) is chosen to command roll and pitch acceleration and to provide PID compensation in the roll and pitch axes. This is denoted by \( v_{\phi, \theta}(t) \)

\[ v_{\phi, \theta}(t) = \dot{x}_c + K_P e + K_D \dot{e} + K_I \int e \, dt \] (3.11)

Substitute \( \dot{x}(t) = v_{\phi, \theta}(t) \) and rearrange

\[ 0 = \dot{x}_c - \dot{x} + K_P e + K_D \dot{e} + K_I \int e \, dt \] (3.12)

From equation 3.5, the twice differentiated error signal in the roll and pitch axes is

\[ \ddot{e}(t) = \dot{x}_c - \dot{x} \] (3.13)

Substituting equation 3.13 into 3.12, and differentiating yields, equation 3.14, which describes the error dynamics of the roll and pitch axes.
\[
\ddot{e} + K_D \dot{e} + K_P \dot{e} + K_I e = 0
\]  \hspace{1cm} (3.14)

This equation defines the error dynamics of the system. The constants \(K_D\), \(K_P\), and \(K_I\) are chosen to reject external disturbances and so that error dynamics are stable. If the linear model of the rotorcraft was an exact model and there were no disturbances to the system there would be no error in the system. This means the system output would follow the commanded values exactly. Put another way, the closed loop transfer function of the output to pilot commanded input would simply be the command filter transfer function, shown in Figure 3-1. The PID compensation compensates "for discrepancies between the simplified inversion model and the actual higher order dynamics (which can be viewed as a type of disturbance)." \[13\]

To model this error to external disturbance, an external signal is summed with the output of the aircraft. This is modeled with the block diagram in Figure 3-2. In this representation the dynamic inversion control is modeled as a double integrator plant. \(Y(s)\) represents the attitude output which is \(\theta\) or \(\phi\).

![Figure 3-2. Disturbance Model for Roll and Pitch Axes](image)

From Figure 3-2, the transfer function of the external disturbance to the system output is found to be
This is a third order system, meaning constants $K_D$, $K_P$, and $K_I$ are derived from the basic second order transfer function multiplied with an additional pole. The second order system, is the classical system with parameters $\omega_n$ and $\zeta$. For the roll axis these are designated $\omega_{roll_d}$ and $\zeta_{roll_d}$. The pole for the roll axis is designated as $p_{roll}$. The subscript "d" on the $\omega_n$ and $\zeta$ indicate that this is the parameter used in error dynamics of the closed loop system subject to disturbances, which can be different than the same parameters used in the command filter. This distinction was made for investigative purposes and in practice the error dynamics parameters are often selected to have similar frequency and damping as the command filter. This convention is used in the MatLab design tool and throughout the experiments reported in Chapter 5. The relations between $K_D$, $K_P$, and $K_I$ and $\omega_n$, $\zeta$, and $p$ for the roll axis are described by equations 3.16 through 3.19. The derivation of $K_D$, $K_P$, and $K_I$ is the same for the pitch axis.

\[
\frac{Y(s)}{\Delta(s)} = \frac{s^3}{s^3 + K_D s^2 + K_P s + K_I}
\] (3.15)

Finally, we can show the pseudo-commands for the roll and pitch axes.
Figure 3-3. Roll and Pitch Pseudo-Commands

Now the yaw and heave axes control is considered. This derivation is a little more straightforward as the error is defined without integrals as in equation 3.20

\[ e(t) = x_c - x \]  

(3.20)

\( v(t) \) is chosen to provide the commanded yaw and vertical acceleration and PI compensation in the yaw and heave axes. This is denoted by \( v_{\nu,yo}(t) \)
Substitute and rearrange (3.22)

Substituting equation 3.20 into 3.22, and differentiating yields, equation 3.23, which describes the 2nd order error dynamics of the yaw and heave axes.

\[ \ddot{e} + K_P \dot{e} + K_I e = 0 \] (3.23)

The constants \( K_P \) and \( K_I \) are chosen to reject external disturbances and so that error dynamics are stable. Again, if the linear model of the rotorcraft was an exact model and there were no disturbances there would be no error in the system. The PI compensation compensates "for discrepancies between the simplified inversion model and the actual higher order dynamics (which can be viewed as a type of disturbance)" \cite{13}. This is the same as in the case of the roll and pitch axes, but in this case PI compensation is used.

To model this error to external disturbance, an external signal is summed with the output of the aircraft. This is modeled with the block diagram in Figure 3-4. In this representation the dynamic inversion control is modeled as an integrator plant and \( Y(s) \) represents the output \( r \) or \( w \).
From Figure 3-4, the transfer function of the external disturbance to the system output is found to be

\[
\frac{Y(s)}{\Delta(s)} = \frac{s^2}{s^2 + K_P s + K_I} \tag{3.24}
\]

This is a second order system, meaning constants $K_P$ and $K_I$ are derived from the basic second order transfer function with parameters $\omega_n$ and $\zeta$. For the yaw axis these are designated $\omega_{\text{yaw}_d}$ and $\zeta_{\text{yaw}_d}$. Again, the subscript "d" on the $\omega_n$ and $\zeta$ indicate that this is the parameter used in the closed-loop system. The relationships between $K_P$ and $K_I$ and $\omega_n$ and $\zeta$ for the yaw axis are described by equations 3.25 through 3.27. The derivations of $K_P$ and $K_I$ are the same for the heave axis.

\[
s^2 + 2\zeta_{\text{yaw}_d} \omega_{\text{yaw}_d}s + \omega_{\text{yaw}_d}^2s = s^2 + K_P s + K_I \tag{3.25}
\]

\[
K_P = 2\zeta_{\text{yaw}_d} \omega_{\text{yaw}_d} \tag{3.26}
\]

\[
K_I = \omega_{\text{yaw}_d}^2 \tag{3.27}
\]

Finally, we can show the pseudo-commands for the yaw and heave axes.
The entire Attitude controller is shown in Figure 3-6, where $e(t)$ is defined as in equation 3.5. Also, note that $K_D$ is zero for the yaw and heave axes.

Figure 3-5. Yaw and Heave Pseudo-Commands

Figure 3-6. Attitude Controller
3.2 TRC Design

In this mode the pilot longitudinal and lateral stick inputs are used to control the forward and lateral velocity ($V_x$ and $V_y$) of the rotorcraft. Similar to the attitude controller, the pilot inputs are passed through command filters. The command filters for both longitudinal and lateral velocities are simple first order filters, shown in Figure 3-7.

![Figure 3-7. TRC Command Filters](image)

The state vector in this case is the velocity vector $V$. The state-space model in this case transforms the command inputs for forward and lateral velocities into $\phi_{cmd}$ and $\theta_{cmd}$ inputs for the Attitude controller according to (3.29) \[ x = V = \begin{bmatrix} V_y \\ V_x \end{bmatrix} \]  
(3.28)  
\[
\dot{x} = \begin{bmatrix} \dot{V}_y \\ \dot{V}_x \end{bmatrix} = \begin{bmatrix} Y_{Vy} & Y_{Vx} \\ X_{Vy} & X_{Vx} \end{bmatrix} \begin{bmatrix} V_y \\ V_x \end{bmatrix} + \begin{bmatrix} g \\ 0 \end{bmatrix} [\phi_{cmd}] + \begin{bmatrix} 0 \\ -g \end{bmatrix} [\theta_{cmd}] 
\]  
(3.29)
The B matrix was determined based on the forces experienced by the rotorcraft when rolling and pitching. For this model the inner-loop attitude controller and GENHEL-PSU linear model combined are considered the plant being controlled by the outer-loop TRC. Equation 3.29 can be solved for the inner-loop input\textsuperscript{[26]}.\n\[
\begin{bmatrix}
\phi_{cmd} \\
\theta_{cmd}
\end{bmatrix} = \begin{bmatrix}
g & 0 \\
0 & -g
\end{bmatrix}^{-1} \left( \begin{bmatrix}
V_y \\
\dot{V}_x
\end{bmatrix} - A \begin{bmatrix}
V_y \\
V_x
\end{bmatrix} \right)
\] (3.30)

Again, a Dynamic Inversion controller is used and a pseudo-command must be defined.\n\[
v(t) = Ax + Bu
\] (3.31)
Solving for \(u(t)\) yields\n\[
u(t) = B^{-1}(v - Ax)
\] (3.32)
Then substitute \(u(t)\) into \(\dot{y}(t)\)\n\[
\dot{x}(t) = Ax + Bu
\] (3.33)\n\[
\dot{x}(t) = Ax + BB^{-1}(v - Ax)
\] (3.34)\n\[
\dot{x}(t) = v(t) = \dot{V}(t)
\] (3.35)
Again, a tracking error must be defined\n\[
e(t) = V_c - V = \begin{bmatrix}
V_{yc} \\
V_{xc}
\end{bmatrix} - \begin{bmatrix}
V_y \\
V_x
\end{bmatrix}
\] (3.36)
PI compensation is used to regulate tracking error and provide the pseudo-command.\n\[
v(t) = \dot{V}_c + K_pe + K_i \int e \, dt = \dot{V}(t)
\] (3.37)
where \( v(t) \) is the pseudo-command, \( \dot{V}_c \) is the acceleration command derived from the command filter.

Substituting equation 3.36 into 3.37, and differentiating yields, equation 3.38, which describes the 2\(^{nd}\) order error dynamics of the lateral and longitudinal velocity axes.

\[
\ddot{e} + K_p \dot{e} + K_i e = 0 \tag{3.38}
\]

The constants \( K_D \) and \( K_I \) are chosen to reject external disturbances and so that error dynamics are stable, just as they were chosen for the yaw and heave axes in Attitude command.

To model this error to external disturbance, an external signal is summed with the output of the aircraft. This is modeled with the block diagram in Figure 3-8. In this representation the dynamic inversion controller is modeled as an integrator plant and \( Y(s) \) represents the output \( V_x \) or \( V_y \).

Figure 3-8. Disturbance Model for Lateral and Longitudinal Velocity Axes

From Figure 3-8, the transfer function of the external disturbance to the system output is found to be

\[
\frac{Y(s)}{\Delta(s)} = \frac{s^2}{s^2 + K_p s + K_i} \tag{3.39}
\]
K_p and K_I of this second order system are derived from $\tau_{Vx,d}$ and $\tau_{Vy,d}$, time constants of a first order filter. Again, the "d" is used to distinguish between the command filter parameters and the closed-loop parameters. The derivation of $K_p$ and $K_I$ for the longitudinal axis are shown in equations 3.40 - 3.43. The derivation for the lateral axis is the same.

\[
s^2 + K_{px}s + K_{Ix} = \left( s + \frac{1}{\tau_{Vx,d}} \right) \left( s + \frac{1}{\tau_{Vx,d}} \right)
\]  

(3.40)

\[
s^2 + K_{px}s + K_{Ix} = s^2 + \frac{2}{\tau_{Vx,d}} s + \frac{1}{\tau_{Vx,d}^2}
\]  

(3.41)

\[
K_{px} = \frac{2}{\tau_{Vx,d}}
\]  

(3.42)

\[
K_{Ix} = \frac{1}{\tau_{Vy,d}^2}
\]  

(3.43)

Now that the gains have been defined, Figure 3-9 shows the TRC pseudo-commands.
This leads the Figure 3-10, which shows the closed-loop TRC Dynamic Inversion Controller where the Plant block refers to the complete inner-loop attitude controller and the UH-60 model.
Chapter 4

UH-60 Attitude/TRC Mode Design Tool

As stated, the purpose of this paper is to investigate the functionality and any benefits or limitations of the TRC and Attitude Controllers designed for the linearized models of the UH-60. Previous work developed a Simulink model of the controller\textsuperscript{[27],[28]}. In order to investigate the controller, it became important to have a quick and easy way to configure the controller, simulate responses, evaluate various stability metrics, and compare the different results. To accomplish this task, a design tool with a Graphical User Interface (GUI) was created. The GUI was designed in MatLab using the "GUIDE" (Graphical User Interface Design Environment) toolbox. The GUI consists of two main screens: Time History Analysis and Frequency Analysis. A basic description of the GUI is provided in this section. A more detailed description and software guide is provided in Appendix A.

4.1 Controller Configuration in the GUI

When the GUI is opened the user is prompted to choose the default configuration or to modify the controller. All defaults are listed in Table 4-1. The default values were selected based on previous work \textsuperscript{[27],[28]}. The command filter default values were selected to meet ADS-33 standards for small amplitude response to pilot inputs (bandwidth and phase delay requirements) and response types for TRC \textsuperscript{[29]}. The disturbance rejection parameters were chosen to be similar to the command filter values, with some tuning to get reasonable stability margins and disturbance rejection requirements. However, the selections of these parameters was not rigorously tested. One of the motivations of this thesis is to provide a design tool and analysis to
better understand the best selection of these parameters. Once the controller is set-up, the user is prompted to select time domain or frequency response analysis.

Table 4-1. Controller Set-up Default Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{n\text{roll}}$</td>
<td>2.5 rad/sec</td>
<td>Roll Command Filter frequency</td>
</tr>
<tr>
<td>$\omega_{n\text{pitch}}$</td>
<td>2.5 rad/sec</td>
<td>Pitch Command Filter frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{roll}}$</td>
<td>0.8</td>
<td>Roll Command Filter damping ratio</td>
</tr>
<tr>
<td>$\zeta_{\text{pitch}}$</td>
<td>0.8</td>
<td>Pitch Command Filter damping ratio</td>
</tr>
<tr>
<td>$\tau_{\text{yaw}}$</td>
<td>0.4 sec</td>
<td>Yaw Axis time constant</td>
</tr>
<tr>
<td>$\tau_{VZ}$</td>
<td>2 sec</td>
<td>Heave Axis time constant</td>
</tr>
<tr>
<td>$\omega_{n\text{roll}_d}$</td>
<td>3.5 rad/sec</td>
<td>Roll Attitude Feedback gain frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{roll}_d}$</td>
<td>0.9</td>
<td>Roll Attitude Feedback damping ratio</td>
</tr>
<tr>
<td>$\omega_{n\text{pitch}_d}$</td>
<td>1.5 rad/sec</td>
<td>Pitch Attitude Feedback gain frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{pitch}_d}$</td>
<td>0.9</td>
<td>Pitch Attitude Feedback damping ratio</td>
</tr>
<tr>
<td>$\omega_{n\text{heave}_d}$</td>
<td>1 rad/sec</td>
<td>Heave Attitude Feedback gain frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{heave}_d}$</td>
<td>0.9</td>
<td>Heave Attitude Feedback damping ratio</td>
</tr>
<tr>
<td>$\omega_{n\text{yaw}_d}$</td>
<td>2 rad/sec</td>
<td>Yaw Attitude Feedback gain frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{yaw}_d}$</td>
<td>0.9</td>
<td>Yaw Attitude Feedback damping ratio</td>
</tr>
<tr>
<td>$p_{\text{roll}}$</td>
<td>0.5</td>
<td>Additional pole for integrator on attitude error, roll axis</td>
</tr>
<tr>
<td>$p_{\text{pitch}}$</td>
<td>0.5</td>
<td>Additional pole for integrator on attitude error, pitch axis</td>
</tr>
<tr>
<td>$p_{\text{heave}}$</td>
<td>0</td>
<td>Additional pole for integrator on attitude error, heave axis</td>
</tr>
<tr>
<td>$p_{\text{yaw}}$</td>
<td>0</td>
<td>Additional pole for integrator on attitude error, yaw axis</td>
</tr>
<tr>
<td>$K_{Vx\text{cmd}}$</td>
<td>-50</td>
<td>X Direction Velocity gain for TRC Commands</td>
</tr>
<tr>
<td>$K_{Vy\text{cmd}}$</td>
<td>50</td>
<td>Y Direction Velocity gain for TRC Commands</td>
</tr>
<tr>
<td>$\tau_{Vx}$</td>
<td>2 sec</td>
<td>X Direction Time Constant of Command Filter</td>
</tr>
<tr>
<td>$\tau_{Vy}$</td>
<td>2 sec</td>
<td>Y Direction Time Constant of Command Filter</td>
</tr>
<tr>
<td>$\omega_{n\text{Act}}$</td>
<td>40 rad/sec</td>
<td>Actuator Frequency</td>
</tr>
<tr>
<td>$\zeta_{\text{act}}$</td>
<td>0.8</td>
<td>Actuator Damping Ratio</td>
</tr>
<tr>
<td>$\tau_{Vx_d}$</td>
<td>2 sec</td>
<td>X Direction Time Constant of Feedback</td>
</tr>
<tr>
<td>$\tau_{Vy_d}$</td>
<td>2 sec</td>
<td>Y Direction Time Constant of Feedback</td>
</tr>
</tbody>
</table>
4.2 Time History GUI

The Time History GUI can simulate inputs on any of the four pilot input axes: lateral, longitudinal, pedal, or collective pilot inputs. These can be configured directly in the GUI or, for more complex inputs, the input can be configured in a MatLab script which is then opened and run from the GUI. All time domain inputs used in this investigation were configured from the GUI.

To configure an input in from the GUI select "Define Input Below" from the dropdown menu labeled "Input". The user can simulate an input on any of the four input axes: lateral, longitudinal, pedal, or collective. For each axis, the user has a choice of no input or three input types: step, pulse, or doublet. The user has control over various aspects of these inputs and simulations. Users chose a Start Time (a time in seconds greater than zero), a simulation time, a duration time (i.e. the duration of the input, can be ignored for step inputs), the maximum rate of change for the input, and the amplitude of the input. The amplitude of "±1" corresponds to the maximum pilot input. For the ACAH mode this commands a roll attitude of 60 degrees in the roll axis, pitch attitude of 30 degrees in the pitch axis, a yaw rate of 60 degrees/second in the yaw axis, and 50 ft/sec in the vertical axis. For TRC mode the amplitude of "±1" commands 50 ft/sec in the lateral axis and 50 ft/sec in the longitudinal axis. The "TRC" checkbox indicates if outer-loop TRC command should be turned on. No check in this box indicates Attitude Command.

Then the user can select which of the three linear models, discussed Chapter 2, is desired. Last the user selects the type of turbulence mode: CETI, Ship Airwake, Both, or neither.

Once the input has been configured and the desired control type, linear model, and turbulence have been selected, the user selects to run the defined simulation. If the model contains an unstable pole the user will be warned and asked if they want to attempt the simulation anyway. When the simulation completes, a success message will appear. Once this message
appears the user can then select which output plots to view. The outputs options are described in Table 4-2.

Table 4-2. Output Options of Time History GUI

<table>
<thead>
<tr>
<th>Output Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Velocities</td>
<td>Plots of Forward, Lateral, and Vertical Body Velocities</td>
</tr>
<tr>
<td>Angular Rates</td>
<td>Plots of Roll, Pitch, and Yaw Rates</td>
</tr>
<tr>
<td>Attitude</td>
<td>Plots of $\phi$, $\theta$, and $\psi$ attitude angles</td>
</tr>
<tr>
<td>Inertial Velocities</td>
<td>Plots of Northern, Eastern, and Downward Velocities</td>
</tr>
<tr>
<td>Position</td>
<td>Plots of North, East, and Downward Position</td>
</tr>
<tr>
<td>Actuator Output</td>
<td>Plots of Lateral, Longitudinal, Pedal, and Collective Actuator Outputs</td>
</tr>
<tr>
<td>Eigen Values</td>
<td>Pole-Zero map of overall system poles and zeros</td>
</tr>
<tr>
<td>Simulated v. Ideal</td>
<td>$\phi$, $\psi$, Yaw Rate, and Vertical Body Velocity simulated responses plotted with ideal their responses (For use in Attitude Command)</td>
</tr>
<tr>
<td>TRC Sim v. Ideal</td>
<td>Forward and Lateral Body Velocity simulated responses plotted with their ideal responses (For use in TRC mode)</td>
</tr>
<tr>
<td>Position Map</td>
<td>Northern Position plotted against Eastern Position</td>
</tr>
<tr>
<td>Actuator Input</td>
<td>Plots of Lateral, Longitudinal, Pedal, and Collective Actuator Inputs against time</td>
</tr>
</tbody>
</table>

**Tracking Errors**

<table>
<thead>
<tr>
<th>Rate</th>
<th>Roll, Pitch, and Yaw Rate Tracking Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Velocity</td>
<td>Vertical Velocity Tracking Error</td>
</tr>
<tr>
<td>Attitude</td>
<td>Roll and Pitch ($\phi$ and $\theta$) Attitude Tracking Error</td>
</tr>
<tr>
<td>Integral Vertical Speed</td>
<td>Vertical Speed Error Integrated</td>
</tr>
<tr>
<td>Integral Yaw Rate</td>
<td>Yaw Rate Error integrated</td>
</tr>
</tbody>
</table>
4.3 Frequency Response GUI

In the Frequency Response GUI, bode diagrams showing the input and output relationships for any input to any output can be generated. It also gives the option to obtain the stability margins for each of the lateral, longitudinal, pedal, and collective axes. These stability margins are obtained by breaking the loop at the actuator and acquiring the open loop transfer function frequency response.

When the Frequency Response GUI is opened, the user must select a linear model from the dropdown menu. Like the Time History GUI, the user can select TRC control by checking the box labeled "TRC". No check in this box indicates Attitude Command. Once these two parameters are set, the user can select to look at the lateral, longitudinal, pedal, or collective stability margins (i.e. gain margin and phase margin for each of these axes) by clicking the labeled pushbuttons. To generate a Bode plot of the closed loop transfer function relationship between a chosen input to output the user must select the input and output from the appropriate dropdown menus and then click "Generate Response".

Disturbance rejection properties can also be evaluated from the GUI. Disturbance Rejection Bandwidth (DRB) is a measure of how well a system can reject an external disturbance\[^{[8]}\]. It will be fully explained in section 5.3. If a DRB input to output is selected in the dropdown, menus the DRB can be calculated with the "DRB" pushbutton.

The input and output options available to view in a Bode diagram are listed in Table 4-3.
Table 4-3. Input and Output Options in Frequency Response GUI

<table>
<thead>
<tr>
<th>Input Options</th>
<th>Output Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Lateral Input</td>
<td>• Forward Body Velocity</td>
</tr>
<tr>
<td>• Longitudinal Input</td>
<td>• Lateral Body Velocity</td>
</tr>
<tr>
<td>• Collective Input</td>
<td>• Vertical Body Velocity</td>
</tr>
<tr>
<td>• Pedal Input</td>
<td>• Roll Rate</td>
</tr>
<tr>
<td>• Roll DRB Input</td>
<td>• Pitch Rate</td>
</tr>
<tr>
<td>• Pitch DRB Input</td>
<td>• Yaw Rate</td>
</tr>
<tr>
<td>• Yaw DRB Input</td>
<td>• Roll Attitude</td>
</tr>
<tr>
<td>• North Velocity DRB Input</td>
<td>• Pitch Attitude</td>
</tr>
<tr>
<td>• East Velocity DRB Input</td>
<td>• Yaw Attitude</td>
</tr>
<tr>
<td></td>
<td>• North Velocity</td>
</tr>
<tr>
<td></td>
<td>• East Velocity</td>
</tr>
<tr>
<td></td>
<td>• Downward Velocity</td>
</tr>
<tr>
<td></td>
<td>• North Position</td>
</tr>
<tr>
<td></td>
<td>• East Position</td>
</tr>
<tr>
<td></td>
<td>• Downward Position</td>
</tr>
<tr>
<td></td>
<td>• Roll Rate Tracking Error</td>
</tr>
<tr>
<td></td>
<td>• Pitch Rate Tracking Error</td>
</tr>
<tr>
<td></td>
<td>• Vertical Velocity Tracking Error</td>
</tr>
<tr>
<td></td>
<td>• Yaw Rate Tracking Error</td>
</tr>
<tr>
<td></td>
<td>• Roll Attitude Tracking Error</td>
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<td>• Pitch Attitude Tracking Error</td>
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<td></td>
<td>• Integrated Vertical Speed Error</td>
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<td>• Integrated Yaw Rate Error</td>
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<td>• Roll DRB Output</td>
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<td>• Pitch DRB Output</td>
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<td>• Yaw DRB Output</td>
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<td>• Lateral Act. Output</td>
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<td>• Collective Act. Output</td>
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<td>• Pedal Act. Output</td>
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<td></td>
<td>• Roll Attitude Ideal</td>
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<td>• Pitch Attitude Ideal</td>
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<td>• Yaw Rate Ideal</td>
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<td>• Vertical Body Velocity Ideal</td>
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<td>• Forward Velocity Ideal</td>
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<td>• Lateral Velocity Ideal</td>
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<td></td>
<td>• North Velocity DRB Output</td>
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<td>• East Velocity DRB Output</td>
</tr>
</tbody>
</table>
An important aspect of the Bode plot generation option is that you can view the relationship between any of the available inputs to outputs. While you can view any input/output combination, generally "on-axis responses" are the most valuable to examine. While not all combinations make sense, this input and output method is the easiest way to give the user the option to view any of the relationships of interest.
Chapter 5

Results

All tests performed in this investigation were done for all three linearized models (hover with zero mean wind, 30 knot mean head wind at 0 degrees, and 30 knot mean wind at 30 degrees to the right of the rotorcraft). It was found that results for the three states were very similar and thus this paper will only present the results for hover with zero mean wind. In each test, the default value, listed in Table 4-1, for each parameter not under investigation was used. Additionally, it is important to note the tests were done on the 28th order system described in section 2.1.

5.1 Effects of \( \omega_{n_d} \) in Attitude and TRC Control

The first test performed observed the effect of varying the parameter \( \omega_{n_d} \) for each of the roll, pitch, and yaw axes (\( \omega_{nroll_d}, \omega_{npitch_d}, \omega_{nyaw_d} \)). The proportional, derivative, and integral gains are directly related to \( w_{n_d} \) by equations 3.17 - 3.19 for the roll and pitch axes and by 3.26 and 3.27 in the yaw axis. Thus, increasing \( w_{n_d} \) increases all inner-loop gain. For each axis \( \omega_{n_d} \) was varied and measurements of the gain and phase margins associated with that axis were recorded. Additionally, to find the root mean square (RMS) value of the attitude output (\( \theta_{RMS}, \phi_{RMS}, \psi_{RMS} \)) a 20 second simulation was run with no pilot inputs and CETI turbulence. The RMS value is a measure of the average magnitude of each attitude angle over the 20 second simulation. In these simulations, the RMS value represents the average effect of the turbulence on the rotorcraft. A lower RMS implies good disturbance rejection. After all simulations had been run, the 20 second simulation was questioned as having not been long enough. To investigate this, 60 second
simulations were run for a few cases. It became apparent that the same trends were visible in the 20 second simulations as in the 60 second simulations (see Appendix B). Thus, the data presented in this section and the following are all based on the 20 second simulation. The results are shown in Figure 5.1 - 5.3.

Figure 5-1. Effects of ω$_{\text{roll}}$ in Attitude Command
Figure 5-2. Effects of $\omega_{npitch_d}$ in Attitude Command
Before discussing these results, we must take a look at the sharp drop in yaw axis gain margin plot. The gain margin becomes negative at $\omega_{nyaw_d} = 2$ radians/second indicating that the closed-loop system is seemingly unstable. However, when this simulation was run the unstable warning, as discussed in Chapter 4, did not appear. This means there was no pole with real part
great than $10^6$. This is confirmed in the pole zero plot shown in Figure 5-4. A zoomed in version is shown in Figure 5-5 to show that the poles are all in the left half plane. To take a closer look at why MatLab reported a negative gain margin, via the "margin" command, the open-loop bode plot showing the stability margins for the yaw axis when $w_{nyaw,d}=1.5$ radians/second and $w_{nyaw,d}=2$ radians/second were generated and are shown in Figure 5-6 and Figure 5-7.

Figure 5-4. Pole Zero Map for $\omega_{nyaw,d} = 2$ radians/second, Attitude Command
Figure 5-5. Zoomed in Pole Zero Map for $\omega_{\text{yaw},d} = 2$ radians/second, Attitude Command
Figure 5-6. Bode Diagram of Gain and Phase Margin for $\omega_{\text{yaw}_{\text{d}}}$ = 1.5 radians/second, Attitude Command
In the case of $\omega_{\text{yaw}} = 2$ radians/second stability margin analysis indicates that the system contains a phase crossover at 0 rad/sec. This is now shown in Figure 5-7, generated from the "margin" command in MatLab; however, it is clear that there is a second gain margin. The phase response crosses -180 degrees at about 13 radians/second and the magnitude response at this point is about -23 dB. This means the second gain margin is about 23 dB at 13 radians/second. This discrepancy arises from the fact that stability margin analysis is typically reserved for Single Input/Single Output (SISO) systems. When performing this analysis for this system, it is viewed as SISO, where the input is the yaw command and the output is the yaw attitude. However, this system is really a Multi-Input/Multi-Output (MIMO) system with four control channels for the roll, pitch, yaw, and heave axes. Since the channels are not perfectly
decoupled, stability margin analysis is not perfect for this system. However, it is still a fairly good representation of the system in most cases. The plot of the effects of $\omega_{yaw,n}$ in Attitude Command, Figure 5-3, is re-plotted in Figure 5-8 with the second gain margin instead of that from the low frequency mode. It is clear the second gain margin fits the trend of the rest of the data much better. This low frequency mode appeared in other measurements for this system; however, only the higher frequency margins will be reported in this Chapter. It is interesting to note that in some cases even using the MatLab command "allmargin", which should report all stability margins, the margins seen are not reported. This is believed to be a bug with MatLab. When this occurred the desired gain margin was found with a "by hand" method, using lines and cursor on the MatLab plots to identify the margin. Plots showing margins as a result of the low frequency modes found from the "margin" command are shown in Appendix C.
From Figure 5-1, 5-2, and 5-8 it is observed that, generally, increasing $\omega_{n_d}$ decreases stability margins. However, in the roll axis gain margin and phase margin both increase until $\omega_{roll_d}=3.5$ radians/second and then decrease until the system is barely stable at 11
This small rise in stability margins is interesting, but does not significantly change the behavior of \( \omega_{n\theta, d} \). The gain and phase margins for the pitch and yaw axes only decrease. The pitch axis remains stable until about \( \omega_{npitch, d} = 12 \) radians/second, and the yaw axis remains stable until about \( \omega_{nyaw, d} = 17 \) radians/second. As it would be dangerous to fly with margins that are just barely stable, the practical limitations of these parameters are much less.

The attitude RMS values \((\phi_{RMS}, \theta_{RMS}, \psi_{RMS})\) always decrease for all three axes. The values drop fast when \( \omega_{n, d} \) is small. As \( \omega_{n, d} \) increases the RMS values drop to almost zero, meaning the turbulence is almost completely rejected for high values of \( \omega_{n, d} \).

This same test was then performed with TRC mode on turned on. The results are very similar to those for Attitude Command and are reported in Figure 5-9 through 5-11 on the same plot as the Attitude results for each axis.
Figure 5-9. Effects of $\omega_{\text{roll}}$ in Attitude and TRC Control
Figure 5-10. Effects of $\omega_{\text{npitch}_d}$ in Attitude and TRC Control
Figure 5-11. Effects of $\omega_{\text{yaw}_d}$ in Attitude and TRC Control

The similarity of these results implies that adding out-loop TRC mode does not significantly affect the inner-loop attitude stability margins. This means using TRC control does not affect the ability of the inner-loop to stabilize the attitude of the rotorcraft.
5.2 Effects of $\omega_{n,d}$ and $\tau_{V,d}$ in TRC mode

To further identify the effects of the TRC mode a similar test was conducted. Instead of only varying $\omega_{n,d}$, $\omega_{n,d}$ was varied proportionally with $\tau_{V,d}$ for both the longitudinal and lateral axes. For each axis, $\omega_{n,d}$ and $\tau_{V,d}$ were varied to keep the same proportionality as in the default settings. The roll axis parameter $\omega_{n\text{roll},d}$ is varied with the y axis parameter $\tau_{V_{y,d}}$ because in the rotorcraft model the y axis velocity is controlled by the lateral cyclic. Similarly, the x axis velocity is controlled by the longitudinal cyclic so pitch axis parameter $\omega_{n\text{pitch},d}$ is varied with the x axis parameter $\tau_{V_{x,d}}$. These relationships are described by equations 5.1 and 5.2.

\begin{align}
\omega_{n\text{roll},d} &= \frac{7}{\tau_{V_{y,d}}} \quad (5.1) \\
\omega_{n\text{pitch},d} &= \frac{3}{\tau_{V_{x,d}}} \quad (5.2)
\end{align}

For each axis as $\tau_{V,d}$ and $\omega_{n,d}$ were varied the respective inner-loop gain margin, inner-loop phase margin, attitude output RMS, and velocity output RMS were recorded. The results are shown in Figure 5-12 and Figure 5-13.
Figure 5-12. Effects of $\tau_{Vyd}$ in TRC Control
According to equations 5.1 and 5.2 increasing $1/\tau_{V_d}$ is the same as increasing $\omega_{a_d}$. This means that the results in Figures 5-12 and Figure 5-13 agree with those reported in section 5.1. Increasing $1/\tau_{V_d}$ decreases Phase and gain margins and decreases the RMS value of the velocity. It also decreases the RMS value of the roll angle, $\phi$. It may seem the lateral axis acts differently because when only $\omega_{roll_d}$ was increased, the lateral phase margin and lateral gain margin increased to a maximum and then decreased, but in this experiment they only decrease. However, in Figure 5-12 the first measurement shown is from $1/\tau_{Vy_d}=.5$ rad/sec ($\tau_{Vy_d} = 2$ seconds), which by equation 5.1 means $\omega_{roll_d}=3.5$ radians/second, the maximum from Figure 5-1. Thus Figure 5-12 starts at the maximum. In this test $\phi_{RMS}$ decreased to about .25 degrees, which fairly small but still larger than the value in Figure 5-1. However, the RMS of the x
velocity decreased to .05 ft/second, which is negligible and shows the disturbance rejection capabilities of TRC.

The pitch angle RMS, $\theta_{\text{RMS}}$, decreases to a minimum and then increases. This a trend that was not apparent when only $\omega_{\text{pitch}_d}$ was varied. The minimum, $\theta_{\text{RMS}}=.25$ degrees, occurs at about $\tau_{Vx_d} = 1$ second, which from equation 5.2 means $\omega_{\text{pitch}_d}=3$ radians/second. Like in the lateral axis, this minimum is higher than that achieved when only $\omega_{\text{pitch}_d}$ was varied. This minimum implies that this is the optimal configuration for the pitch axis in TRC to maximize disturbance rejection. $\tau_{Vx_d} = 1$ second is also provides maximal disturbance rejection in the X velocity as the RMS value of X velocity output levels off at its minimum, .1 ft/second, at about this point.

5.3 Disturbance Rejection Bandwidth

To find the Disturbance Rejection Bandwidth (DRB), a disturbance input is added to the output state, as shown in Figure 5-14. It is then possible to determine the closed loop transfer function of the system output to this disturbance. An example of this is shown in the bode plot in Figure 5-15. The DRB is defined as the point where the magnitude response hits -3db, shown in Figure 5-15. The DRB is a measure of how well the "hold functionalities of the system" reject atmospheric disturbances [8], where a higher DRB frequency indicates better disturbance rejection performance. Strong disturbance rejection capabilities are extremely beneficial in conditions of degraded weather and visibility.
Figure 5-14. Block Diagram of Disturbance Input and Output

Figure 5-15. Bode Diagram of Disturbance Output to Disturbance Input
5.3.1 Attitude DRB

For each axis as $\omega_{n,d}$ was varied and the respective DRB and the maximum value of the magnitude response for the disturbance input to disturbance rejection output was recorded. The results are shown in Figure 5-16.

![Figure 5-16. Attitude Disturbance Rejection Bandwidth](image)

The DRB of each axis is very similar and increases from about 1 radian/second to 15 radians/second with $\omega_{n,d}$. This means a larger natural frequency, $\omega_{n,d}$, makes the system less susceptible to disturbances. In application this means the aircraft is more resistant to wind gusts and turbulence when a larger $\omega_{n,d}$ is used to design the controller. This trend matches that seen section 5.1 which recorded the RMS of the attitude output to a simulation with CETI turbulence and no pilot inputs.
5.3.2 TRC DRB

To look at the DRB in TRC mode, $\omega_{n,d}$ was varied proportionally with $\tau_{V_d}$ for both the longitudinal and lateral axes. For each axis, $\omega_{n,d}$, and $\tau_{V_d}$ were varied to keep the same proportionality as in the default settings, described by equations 5.1 and 5.2. For each $\omega_{n,d}$, and $\tau_{V_d}$ configuration the respective DRB (Eastern Velocity DRB for $V_y$ and Northern Velocity DRB for $V_x$) was recorded. The results are shown in Figure 5-17. For this simulation the rotorcraft is assumed to be facing north, so the northern velocity is equivalent to the longitudinal velocity, and the eastern velocity is equivalent to the lateral velocity. The GUI uses the "north" and "east" terminology; however, for the sake of clarity the rest of the paper will refer to the velocities as lateral and longitudinal.

Figure 5-17. TRC Disturbance Rejection Bandwidth
Similar to the results in Attitude Command, increasing $1/\tau_{V\_d}$, which by equations 5.1 and 5.2 is equivalent to increasing $\omega_{n\_d}$, increases the DRB. This matches the results in section 5.2 where the RMS values of the velocities decrease with increasing $1/\tau_{V\_d}$. This means selecting a small time constant in the feedback filter, increases the DRB of the system and makes it more robust to wind gusts and turbulence. An interesting note is that the DRBs for the velocities in TRC mode are less than those in Attitude Command, both increasing from about 0.5 radians/second to 1.5 radians/second. This is logical as controlling variables in the outer-loop naturally takes longer than the inner-loop commands.

### 5.4 Rotorcraft Trajectory to Turbulence

To visualize the effects of turbulence on the rotorcraft, 20 second simulations were run with no pilot inputs and CETI Turbulence. The simulations were run in Attitude and TRC mode with various values for, $\omega_{n\_d}$ and $\tau_{V\_d}$. Again, they were varied to maintain the same proportionality. Each axis was varied separately, and the values for the other axis were kept at their default values. The smallest value of $\tau_{V\_d}$ was chosen as the smallest shown in Figures 5-12 and 5-13. The results are in Figure 5-18 through Figure 5-34.
Figure 5-18. Trajectory of $\omega_{\text{pitch},d} = 5$ radians/second and $\tau_{Vx,d} = 0.6$ seconds, Attitude Command

Figure 5-19. Trajectory of $\omega_{\text{pitch},d} = 5$ radians/second and $\tau_{Vx,d} = 0.6$ seconds, TRC Command
Figure 5-20. Trajectory of $\omega_{\text{pitch},d}$ = 3 radians/second and $\tau_{V_x,d}$ = 1 seconds, Attitude Command

Figure 5-21. Trajectory of $\omega_{\text{pitch},d}$ = 3 radians/second and $\tau_{V_x,d}$ = 1 seconds, TRC Command
Figure 5-22. Trajectory of $\omega_{\text{pitch}_d} = 2$ radians/second and $\tau_{Vx_d} = 1.5$ seconds, Attitude Command

Figure 5-23. Trajectory of $\omega_{\text{pitch}_d} = 2$ radians/second and $\tau_{Vx_d} = 1.5$ seconds, TRC Command
Figure 5-24. Trajectory of $\omega_{\text{pitch},d} = 1.5$ radians/second and $\tau_{Vx,d} = 2$ seconds, Attitude Command

Figure 5-25. Trajectory of $\omega_{\text{pitch},d} = 1.5$ radians/second and $\tau_{Vx,d} = 2$ seconds, TRC Command
Figure 5-26. Trajectory of $\omega_{\text{roll},d} = 10$ radians/second and $\tau_{V_y,d} = .7$ seconds, Attitude Command

Figure 5-27. Trajectory of $\omega_{\text{roll},d} = 10$ radians/second and $\tau_{V_y,d} = .7$ seconds, TRC Command
Figure 5-28. Trajectory of $\omega_{\text{roll}_d} = 7$ radians/second and $\tau_{V_y,d} = 1$ seconds, Attitude Command

Figure 5-29. Trajectory of $\omega_{\text{roll}_d} = 7$ radians/second and $\tau_{V_y,d} = 1$ seconds, TRC Command
Figure 5-30. Trajectory of $\omega_{\text{roll}_d} = 4.667$ radians/second and $\tau_{V_y,d}=1.5$ seconds, Attitude Command

Figure 5-31. Trajectory of $\omega_{\text{roll}_d} = 4.667$ radians/second and $\tau_{V_y,d}=1.5$ seconds, TRC Command
Figure 5-32. Trajectory of $\omega_{\text{roll},d} = 3.5$ radians/second and $\tau_{V_y,d} = 2$ seconds, Attitude Command

Figure 5-33. Trajectory of $\omega_{\text{roll},d} = 3.5$ radians/second and $\tau_{V_y,d} = 2$ seconds, TRC Command
The plots show that for small values of $\tau_{Vx,d}$ and large values of $w_{npitch,d}$ the longitudinal position response is tighter than the lateral response. This is because the $w_{npitch,d}$ controls the longitudinal velocity and the high gain results in good disturbance rejection, as shown in Figure 5-17. The responses for the TRC mode are generally tighter than those from Attitude Command in all cases. The TRC responses for small values of $\tau_{Vx,d}$ and large values of $w_{npitch,d}$ are especially tighter for the longitudinal position. Additionally, it is interesting to note that these responses in TRC mode are centered around zero in both directions, but the in Attitude Command they are only centered in the lateral direction, as there is less tendency to drift in TRC mode since the velocity is being regulated by the controller.

Results are similar in the roll axis. The plots show that for small values of $\tau_{Vy,d}$ and large values of $\omega_{nroll,d}$ the lateral position response is tighter than the longitudinal response. This is because the $w_{nroll,d}$ controls the lateral velocity and the high gain results in good disturbance rejection, as shown in Figure 5-26. The responses for the TRC mode are generally tighter than those from Attitude Command. The TRC responses for small values of $\tau_{Vy,d}$ and large values of $\omega_{nroll,d}$ are especially tighter for the lateral position. Additionally, it is interesting to note that these responses in TRC mode are centered around zero in both directions, but the in Attitude Command they are only centered in the lateral direction, the same as they were when the pitch axis was under investigation.

To examine the effects of varying the gains in both axes, Figure 5-34 and Figure 5-35, show the trajectory for $\omega_{npitch,d}=2$ rad/s, $\tau_{Vx,d}=1.5$ seconds, $\omega_{nroll,d}=4.667$ rad/s, and $\tau_{Vy,d}=1.5$ seconds. Figure 5-34 and Figure 5-35 show that increasing $\omega_{n,d}$ and $\tau_{V,d}$ in both axes tightens the response in both axes. Additionally, TRC mode again provides a tighter response than Attitude.
Figure 5-34. Trajectory for $\omega_{\text{pitch},d}=2$ rad/s, $\tau_{Vx,d}=1.5$ seconds, $\omega_{\text{roll},d}=4.667$ rad/s, and $\tau_{Vy,d}=1.5$ seconds, Attitude Command

Figure 5-35. Trajectory for $\omega_{\text{pitch},d}=2$ rad/s, $\tau_{Vx,d}=1.5$ seconds, $\omega_{\text{roll},d}=4.667$ rad/s, and $\tau_{Vy,d}=1.5$ seconds, TRC
5.5 Actuator Input

To study the effects of turbulence on the actuator inputs, 20 second simulations were run with no pilot inputs and CETI Turbulence. The simulations were run in Attitude and TRC mode with varying values for $\omega_{n,d}$, and $\tau_{V_d}$. Again, they were varied to maintain the same proportionality. For each test configuration the RMS value of the actuator input for each axis was recorded. The results are in Figure 5-37.

The RMS value of the actuator inputs for Attitude and TRC control are almost identical in each axis and never increase above .3 in either axis. This means the values of $\omega_{n,d}$ and $\tau_{V_d}$ are realizable in the real system. The actuator is a physical device that is limited in motion to $\pm 5$
inches of equivalent pilot stick input (this corresponds to the full range of control motion). Since the RMS values in Figure 5-37 are so small compared to this limit, it is reasonable to assume that in these configurations the actuator will not be saturated by the turbulence.
Chapter 6

Conclusions

The results in sections 5.1 - 5.3 show a clear tradeoff between stability margins and disturbance rejection. As the natural frequency of each axis increases, the inner-loop stability margins decrease while both inner-loop and outer-loop DRBs increase.

6.1 Attitude Conclusions

Table 6-1 and Table 6-2 show the recommended settings for Attitude Command and the stability margins and DRBs this configuration achieves.

Table 6-1. Recommended Settings for Attitude Command

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Recommended Value</th>
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<tbody>
<tr>
<td>$\omega_{roll_d}$</td>
<td>4.5 rads/s</td>
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<tr>
<td>$\omega_{pitch_d}$</td>
<td>2 rads/s</td>
</tr>
<tr>
<td>$\omega_{yaw_d}$</td>
<td>6.5 rads/s</td>
</tr>
</tbody>
</table>
Table 6-2. Stability Margins and DRBs Achieved by Recommended Settings, Attitude Command

<table>
<thead>
<tr>
<th>Metric</th>
<th>Achieved Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll Phase Margin</td>
<td>45 deg</td>
</tr>
<tr>
<td>Roll Gain Margin</td>
<td>15 dB</td>
</tr>
<tr>
<td>Roll Attitude DRB</td>
<td>2 rads/s</td>
</tr>
<tr>
<td>Pitch Phase Margin</td>
<td>45 deg</td>
</tr>
<tr>
<td>Pitch Gain Margin</td>
<td>16 dB</td>
</tr>
<tr>
<td>Pitch Attitude DRB</td>
<td>1 rads/s</td>
</tr>
<tr>
<td>Yaw Phase Margin</td>
<td>45 deg</td>
</tr>
<tr>
<td>Yaw Gain Margin</td>
<td>12 dB</td>
</tr>
<tr>
<td>Yaw Attitude DRB</td>
<td>2.8 rads/s</td>
</tr>
</tbody>
</table>

Table 6-2 shows that this controller can achieve gain margins much greater than the minimum 6 dB specified by MIL-F-9490 \(^9\) while maintaining the minimum phase margin. Even at this minimum, it provides very strong disturbance rejection capabilities. A typical DRB is on the order of 1 radian/second and all those reported in Table 6-2 are equal to or greater than this value \(^8\).

The trajectory response of the UH-60 in Attitude Command with parameters in Table 6-1 to a 20 second simulation with CETI turbulence is shown in Figure 6-1.
Figure 6-1. Trajectory of Rotorcraft in Attitude Command with Recommended Parameters

6.2 TRC Conclusions

One of the most surprising and interesting results found in TRC mode is that adding the outer-loop does not significantly degrade the stability margins. This important to note because stability margins are a measure of the robustness \cite{8}. Essentially, we can add TRC mode without affecting the robustness of the rotorcraft control system.

The recommended settings to achieve traditional stability margin requirements (6 dB, 45 deg) in TRC mode are shown in Table 6-3. The stability margins and DRBs achieved from these gains and time constants are listed in Table 6-4.
Table 6-3. Recommended Settings for TRC Mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Recommended Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\tau_{V_y,d}$</td>
<td>0.65 rads/s</td>
</tr>
<tr>
<td>$\omega_{nroll_d}$</td>
<td>4.55 rads/s/s</td>
</tr>
<tr>
<td>$1/\tau_{V_x,d}$</td>
<td>0.8 rads/s</td>
</tr>
<tr>
<td>$\omega_{npitch_d}$</td>
<td>2.4 rads/s</td>
</tr>
</tbody>
</table>

Table 6-4. Stability Margins and DRBs Achieved by Recommended Settings, TRC Control

<table>
<thead>
<tr>
<th>Metric</th>
<th>Achieved Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll Phase Margin</td>
<td>45 deg</td>
</tr>
<tr>
<td>Roll Gain Margin</td>
<td>12 dB</td>
</tr>
<tr>
<td>Roll Attitude DRB</td>
<td>2 rads/s</td>
</tr>
<tr>
<td>Lateral Velocity DRB</td>
<td>0.7 rads/s</td>
</tr>
<tr>
<td>Pitch Phase Margin</td>
<td>45 deg</td>
</tr>
<tr>
<td>Pitch Gain Margin</td>
<td>9 dB</td>
</tr>
<tr>
<td>Pitch Attitude DRB</td>
<td>1.1 rads/s</td>
</tr>
<tr>
<td>Longitudinal Velocity DRB</td>
<td>0.75 rads/s</td>
</tr>
</tbody>
</table>

The results in the pitch axis are of particular interest when compared to the results from the study done by the Army Aeroflightdynamics Directorate (AFDD) mentioned in the introduction. In the controllers used in that study to control the same rotorcraft used here, the DRBs resulting from choosing a phase margin of 45 degrees were Pitch DRB = 0.6 radians/second and Longitudinal Velocity DRB = 0.45 radians/second \([8]\). The values reported in Table 6-3 result in bandwidths of almost twice these values.

However, in the roll axis this was not true. The roll DRB reported is about twice that reported by the AFDD, but the Lateral Velocity DRB was about the same.

The gain margins resultant from the recommend settings are both much larger than the minimum 6dB, while still satisfying a 45 degree phase margin and exhibiting strong disturbance...
rejection capabilities. This is the same result seen with Attitude Command. The attitude DRBs are about the same in Attitude Command and TRC. TRC mode only lowers the gain margins slightly, which is not significant as they are still well about 6dB. This indicates that the TRC controller here may be applicable to ship-based operation, as it does not degrade the abilities of the inner-loop Attitude Command. Additionally, the tighter responses from TRC mode are ideal for ship-based operations. While there is still much more to study about this controller to determine if it truly will be adequate for ship-based operations, this preliminary study indicates it has a lot of potential in this area.

The high gain margins are also interesting to note because it means it is possible to increase disturbance rejection capabilities by relaxing only the phase margin. For instance, in the pitch axis, if the values are chosen to satisfy only gain margin requirements, the pitch margin would be 6 dB, with phase margin of about 35 degrees, pitch attitude DRB of 1.25 rad/s, and Longitudinal Velocity DRB of .9 rad/s. As previous studies have looked at the effects of relaxing stability margins\[8\], these results may present an opportunity to study the effects of relaxing only the phase margin in return for even higher disturbance rejection.

The trajectory response of the UH-60 with parameters in Table 6-3 to a 20 second simulation with CETI turbulence is shown in Figure 6-2.
When Figure 6-2 is compared to Figure 6-1, it is clear that the recommended values in TRC provide a much tighter response than the Attitude Command recommendations. This is another benefit TRC mode provides to ship-based operations. The tighter response is ideal for maneuvering in the limited space on a ship deck.
Appendix A

Design Tool Guide

Introduction

This program allows the user to configure the TRC/Attitude controller and simulate time history responses to selected inputs as well as generate frequency response characteristics.

Model: The aircraft model used in this program is a linearized model of the UH-60, GENHEL-PSU. The model has three different linearized states: Hover with no wind, 30 Knot Wind at 0 degrees, and 30 Knot Wind at 30 degrees. Selection of one of these states from the GUI window will load the selected model as the rotorcraft plant in the controller.

Controller: The controller used can be configured for TRC or Attitude command.

Controller Inputs: pilot inputs (Lateral stick, Longitudinal stick, Collective, and Pedal), TRC flag (to turn on/off TRC control), Loop Break Inputs for Stability Margin Analysis (Lateral, Longitudinal, Collective, and Pedal loops breaks), and Disturbance Rejection Bandwidth Inputs for DRB Analysis (Roll, Pitch, Yaw, North Velocity and East Velocity DRB Input). The user has direct control over pilot inputs and the TRC flag, the others will be set internally by the program depending on the selected task. Additionally the controller can be configured to input air wake or CETI turbulence.

Controller Outputs: Forward, Lateral, and Vertical Body Velocities; Roll, Pitch, and Yaw Rates; Roll, Pitch, and Yaw Attitude; North, East, and Downward Velocities; North, East, and Downward Position; Roll, Pitch, and Yaw Rate Tracking Error; Vertical Velocity Tracking Error;
Roll and Pitch Attitude Tracking Error; Integrated Vertical Speed Error; Integrated Yaw Rate Error; Lateral, Longitudinal, Collective, and Pedal Loop Break Output; Roll, Pitch, and Yaw Disturbance Rejection Outputs; Roll and Pitch Attitude Ideal Response; Yaw Rate Ideal Response; Vertical Body Velocity Ideal Response; Ideal Forward and Lateral Velocities for when TRC Command is used; Northern and Eastern Velocity DRB output; Lateral, Longitudinal, Collective, and Pedal Actuator inputs. These outputs are called in the various output options available in the GUI.

**Open Program and Configure Controller**

A. To start the program, open MatLab and make sure the program files are in the opened directory. The necessary files are: TimeGUI.fig, TimeGUI.m, simulate.m, FreqGUI.fig, FreqGUI.m, gust_filters.mat, OpenGUI.m, inputdlgcol.m, lat_pulse.m, long_pulse.m, UH60_TRC_eval.mdl, Lin-models-gusts (folder), Default_Control.m, Init_GUI_param.m, User_defined_control.m. The function of each of these files can be found in section 4.

B. **Run OpenGUI.m**

C. The following pop-up message will appear. Select the desired action.

![Image of pop-up message](image_url)

Figure A-1. Edit Controller Prompt
D. If "Yes" from Figure A-1 is selected, the prompt in Figure A-2 will appear. Edit desired values and click "ok".

![Student Version > Filter Parameters](image)

Figure A-2: Pop-up to Edit Control Parameters

If "No" is selected, all control parameters will be set to the default values Figure A-2 and listed in Table 4-1. The screen above will not appear at all.

**Note:** When editing values, you do not need to change every value, any unedited value will be set to the value shown in Figure A-2. All of these values are set in the base workspace so they will be accessible to other parts of the program (i.e. UH60_TRC_eval.mdl). Additionally, other controller parameters will be derived from these values.
E. Now you will be prompted to chose a GUI: Time History Analysis or Frequency Response Analysis. Select the desired GUI. Once a selection is made, the appropriate GUI will appear. Each GUI can be opened from the other.

![GUI Selection Prompt](image)

**Figure A-3. GUI Selection Prompt**

**Time History GUI**

A. Select an input method. Select option "Define Input Below" from the drop down to use the GUI to configure an input. Or select another input to load a specific file for the inputs. This is a good option for non-standard inputs. If an input file is selected the settings below are ignored and the user can skip to step H below.

B. Select the from the drop down menu of the axis on wish you wish to simulate the input, chose the input type "step", "pulse", or "doublet".

*Note:* It is possible to enter input on two different axes. However since both inputs are synchronized to the same time vector (based on the same start time, simulation time, duration, and maximum rate) the inputs will occur at the same time. They will also have the same amplitude. For this reason any unsynchronized inputs should be defined from an input file.

C. Type in a "Start Time" which is the time at which the input action occurs after zero, and must be greater than zero.
D. Type in a "Simulation Time" which is the time for which the simulation will run. It will run from 0 seconds to the time entered.

E. Type in a "Duration" which is the time a pulse or doublet lasts. This must be smaller than the "Simulation Time".

Note: When entering step inputs, this parameter is ignored by the program so this value does not need to be changed. When entering doublet inputs, the duration is the time for BOTH pulses.

F. Type in a "Max Rate" which is the maximum rate at which the input can change.

G. Type in an "Amplitude" which is the amplitude of the pulse. The amplitude of "1" corresponds to a command of approximately 60 degrees in the roll axis, 30 degrees in the pitch axis, and a yaw rate of 60 degrees/second.

H. If TRC Command is desired check the box labeled "TRC". If attitude command is desired, make sure the box is unchecked.

I. Select the linear model of the aircraft to be used.

J. If turbulence is desired, select the type of turbulence from the "Turbulence" drop down menu. The options are "air wake", "CETI", "BOTH" (Both air wake and CETI turbulence), or "none".

K. Once all of these parameters have been set as desired, click "RUN" on the GUI. This opens the simulate.m file which runs a simulation of the model (UH60_TRC_eval.mdl) with the selected settings and previously entered controller parameters. Before running the simulation a check for stability is conducted. If there is a pole with real part greater than $10^{-6}$ in the Right Half Plane (RHP) an error will appear. The user will be asked whether or not to run the simulation. If the user selects "Yes" the simulation will attempt to run regardless of the potential error, otherwise it will not.
Note: All parameters do not need to be set in the above order. ALL MUST be set to desired values before selecting "RUN". Different configurations and inputs can be simulated sequentially simply by changing the settings in the GUI and selecting "RUN" again.

L. Once a simulation has been run by clicking "RUN", a success message will appear. Then different output figures can be generated. To view a figure, simply click on the push button for the desired figure.

M. Selecting "Frequency Response" in the bottom right corner will open the Frequency Response GUI. Anything run from this GUI will run with the same control gains that were initially entered.

NOTE: As linear model and TRC mode can be changed from both GUIs always make sure to reselect the desired setting before running simulations or obtaining frequency responses.
Changing the setting in one GUI will NOT change it in the other. The program is always set to the most recent selection for a parameter, whether it was in the current GUI or the previous.

**Frequency Response GUI**

A. Select the linear model of the aircraft to be used.

B. If a frequency response under TRC command is desired check the box labeled "TRC". If attitude command is desired, make sure the box is un-checked.

C. To look at the stability margins for each of the axes, select the push button under "Stability Margins" for the desired axis. This will display a Bode Diagram showing the phase and gain margins.

D. To look at various frequency responses select an "Input" and "Output" under the "Frequency Responses" heading.

E. Click "Generate Response". This will generate the bode plot of the output variable to the input variable. This will also perform a check for stability. If there is a pole with real part greater than $10^{-6}$ in the Right Half Plane (RHP) an error will appear (The same as from the Time History GUI). The program will still attempt to determine a frequency response, unless it errors out.

F. If a DRB input to DRB output relationship is selected, the DRB can will be calculated by clicking "DRB". As with the "Generate Response" pushbutton, it will perform a check for stability. If there is a pole with real part greater than $10^{-6}$ in the Right Half Plane (RHP) an error will appear (The same as from the Time History GUI). The program will still attempt to determine a frequency response, unless it errors out.

G. The last pushbutton is "Max" which finds the maximum value of the bode plot and the frequency at which it occurs. Again, the same stability check is preformed.
F. Selecting "Time History" in the bottom right corner will open the Time History GUI. Anything run from this GUI will run with the same control gains that were initially entered.

**NOTE**: As linear model and TRC mode can be changed from both GUIs always make sure to reselect the desired setting before running simulations or obtaining frequency responses. Changing the setting in one GUI will NOT change it in the other. The program is always set to the most recent selection for a parameter, whether it was in the current GUI or the previous.

**Software Guide**

**OpenGUI.m**

This is the initialize file, the file where all control parameters are set, and the file that will open the selected GUI. Then the user is prompted to change controller parameters or chose the default settings. Their response prompts `User_defined_control.m` or `Default_Control.m` to open to set the control parameters. Next the file calls the `Init_GUI_param.m` file which sets all variables that the user will later select from the GUIs to initial values (This prevents the user from having to go through and re-enter values that already appear as desired in the GUI). These are the values only used in the GUI, that were not initialized in the `Default_Control.m` or the `User_defined_control.m` file. Then the script prompts the user to chose a GUI which will be opened from the initialize file. All values set and loaded from the initialize file and scripts called within it are made available in the workspace.

**TimeGUI.fig**

This file contains the layout for the Time History GUI.
Figure A-6. Time History GUI

TimeGUI.m

This file contains the code controlling the Time History GUI. In this GUI all parameters set are sent to the workspace so they can be used in other functions and files apart from the GUI file that set them. The "Input" Dropdown sets an InputMethod variable to tell the simulation to use an input from a file or an input defined in the GUI. If a file is selected the rest of the inputs under the drop down will be ignored, and the simulation will be run by sending the input file to the simulation (for a simulation time of 10 seconds). If an input is configured in the GUI, all of the variables set will be output to the workspace, to be configured into an input matrix before simulation (this takes place in the simulate.m file).

The TRC Checkbox turns TRC mode on and off by setting iTRC_Force_On to "1" or "0", turning on TRC mode if the checkbox is checked. The "Linear Model" and "Turbulence" drop down menus work by selecting the appropriate matrices to be loaded to the workspace.
Selecting "RUN" opens the file simulate.m and runs simulation with the selected inputs and control parameters. The results of the simulation are output to the workspace. If the input was configured in a GUI an input matrix is configured as the input. To add an input file: add the file to the project directory, add the file name to the drop down menu options (done by opening the .fig in GUIDE), and add a switch statement setting "R" to the file name in the input function in TimeGUI.m.

Selecting any of the output options loads the output from the simulation in the workspace into the function called and plots the appropriate outputs.

FreqGUI.fig

This file contains the layout for the Frequency Response GUI.

Figure A-7. Frequency Response GUI
FreqGUI.m

This file contains the code controlling the Frequency Response GUI. In this GUI all parameters set are sent to the workspace so they can be used in other functions and files apart from the GUI file that set them. The TRC Checkbox turns TRC mode on and off by setting \textit{iTRC\_Force\_On} to "1" or "0", turning on TRC mode if the checkbox is checked. The "Linear Model" drop down menu works by selecting the appropriate matrices of those loaded in the initialize file.

\textbf{gust\_filters.mat}

This file contains the gust filters for LHA airwake model

\textbf{inputdlgcol.m}

This is a function exactly like the MatLab function \textit{inputdlg()}; however, this function allows the desired input prompts to be displayed in columns. This function was downloaded from mathworks.com.

\textbf{lat\_pulse.m, long\_pulse.m}

These are two file inputs and are selectable from the Time History GUI as file inputs.

\textbf{UH60\_TRC\_eval.mdl}

This is the Simulink model of the controller.

\textbf{Lin-models-gusts (folder)}

This file contains the three linearized models of the UH-60.
**simulate.m**

The simulate script will configure the input matrix if the input was specified in the GUI. The first column in the matrix is the time vector which is determined from the start time, duration time, maximum rate, and simulation time. The rest of the columns are the pilot inputs. The only inputs being used in these simulations are lateral axis, longitudinal axis, collective, and pedal. (The unused inputs are TRC mode flag (the TRC force on variable is always used to turn on TRC mode instead), Loop Break Inputs (lateral, longitudinal, collective, and pedal) and Disturbance Rejection Bandwidth Inputs for DRB Analysis (roll, pitch, yaw, northern velocity, and eastern velocity). The file then runs the simulation and outputs the results to the workspace. If a file is selected as an input, then the simulation is run with the file as the input, and results are similarly output to the workspace. This file will also check for the unstable poles and warn the user before the simulation is run.

**Init_GUI_param.m**

This script initializes all the parameters that will be used in the GUI to the values that are shown initially upon opening the GUI.

**User_defined_control.m**

The file first sets the step size for gust simulation which is .01 seconds and loads the *gust_filters.mat* file for LHA airwake model. Then linear dynamic models for the aircraft in hover, in 30 knots wind at 0 degrees, and in 30 knots wind at 30 degrees are loaded. It selects hover as the default mode. It sets all the loop breaks to off, TRC mode to off, and turbulence to off. This file opens the screen where the user can set the controller parameters. It will also take these values to compute other parameters.
**Defualt_Control.m**

The file first sets the step size for gust simulation which is .01 seconds and loads the `gust_filters.mat` file for LHA airwake model. Then linear dynamic models for the aircraft in hover, in 30 knots wind at 0 degrees, and in 30 knots wind at 30 degrees are loaded. It selects hover as the default mode. It sets all the loop breaks to off, TRC mode to off, and turbulence to off. This will initialize the controller model to the default settings. These are the settings shown initially when the user is prompted to enter controller values.

**Note:** Running either `Default_Control.m` or `User_defined_control.m` on their own will set up all Simulink model parameters so the model can be run on its own.
Appendix B

Results of 20 Second Simulations and 60 Second Simulations Compared

Figure B-1. Comparison of Effects of $\omega_{\text{roll,d}}$ on $\phi_{\text{RMS}}$ in Attitude Command for 20 and 60 Second Simulations
Figure B-2. Comparison of Effect of $\tau_{\text{Vx,d}}$ on $\theta_{\text{RMS}}$ and X Velocity RMS in TRC Mode for 20 and 60 Second Simulations
Appendix C

Plots With Negative Gain Margins

Figure C-1. Effects of $\omega_{nroll_d}$ in Attitude and TRC Control, Unadjusted Gain Margins
Figure C-2. Effects of $\omega_{nyaw_d}$ in Attitude and TRC Control, Unadjusted Gain Margins
Figure C-3. Effects of $\tau_{vy_d}$ in TRC Control, Unadjusted Gain Margins
Figure C-4. Effects of $\tau_{Vxd}$ in TRC Control, Unadjusted Gain Margins
REFERENCES


ACADEMIC VITA

Rebecca Ripley
Rebecca.H.Ripley@gmail.com

Education

B.S., Electrical Engineering, 2013, Pennsylvania State University, University Park, PA

Honors and Awards

- Penn State Evan Pugh Senior Award, 2013
- Clifford B. Holt Jr. Memorial Scholarship 2012
- Penn State Evan Pugh Junior Award, 2012
- Lockheed Martin Scholar, 2011
- Penn State Sophomore Award, 2011
- Penn State Freshman Award, 2010

Association Memberships/Activities

- Institute of Electronic and Electrical Engineers
- Eta Kappa Nu, Electrical Engineering Honor Society
- Phi Sigma Rho, National Sorority for Women in Engineering

Professional Experience

The Boeing Company, Electrical Engineering Intern, 2010

- Engaged in Osprey (V-22) production engineering wiring and design
- Edited and revised group engineering in preparation for official release
- Responsible for wire harness design and form board creation
- Compiled engineering packages for release to suppliers