QUANTIFIED RISK MODELS FOR PANAMANIAN MICRO-FINANCE

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ABSTRACT

Global Brigades is building a microfinance operation in Piriati Embera, Panama. The use of the traditional Grameen banking model’s savings groups leaves the community vulnerable to massive simultaneous default triggered by external events such as flooding. This paper examines the theory and application of actuarial concepts as they apply to microfinance operations in Piriati Embera. This paper will suggest an insurance-based solution to promote growth and sustainability in the community.
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Chapter 1

Introduction

During my junior year at Penn State I participated in a community service project through the recently founded Penn State chapter of Penn State Global Business Brigades. In March of 2012, we traveled to a rural community in Piriati Embera, Panama. Our purpose was two-fold: to teach personal finance concepts to rural households and businesses, and to develop a micro-finance savings and loan cooperative.

The first objective was met with great success. During this trip I witnessed the raw power of financial education transforming the local status quo. Concepts that we take for granted—record-keeping, inventory forecasting, and marketing, were completely foreign concepts to these local businesses. Simple student-hosted training workshops helped Panamanian business owners create more effective solutions for their internal processes. Through entrepreneurial spirit, it seems that the community members have the opportunity to pull themselves out of poverty. I am very proud of the work that our 18-student team was able to accomplish during our one-week trip to Piriati Embera.

The second objective—to establish savings and loan microfinance cooperatives, intrigued my developing actuarial background. At first glance, the microfinance project appeared to contain elements of risk, and as an actuarial student, my natural reaction was to want to quantify it!

The Global Business Brigade microfinance system is based on a Grameen banking model. Underneath this banking structure, the operation will utilize savings groups to tackle the difficulties of establishing collateral in a rural community with limited legal enforcement. Groups of five neighboring friends will collectively act as cosigners for a loan made to one of the savings
group members. The motivation behind this structure is to create a “social pressure” incentive to repay the loan—an individual can only shirk on a loan at the expense of their close friends: any failed loan must be repaid by the other savings group members. Upon successful repayment of the loan, another member of the same savings group becomes eligible to borrow and the process repeats.

During our daily bus travels between communities, I began to challenge the savings group model. If the borrowers successfully repay their loans, then the savings group is content and economic development was fostered. But what if the borrower is unable to repay the loan? To what extent could this banking infrastructure cause damage to the community members in Piriati Embera?

Voicing my concerns the next day, the project leader assured me that these things have been considered. He informed me that by giving out a large variety of loans and using the social pressure mechanism, the failure rate would be minimal. Lastly, the charged interest rates on these loans would cover administrative costs and even the rare instances in which an individual defaults on their loan.

This explanation seemed satisfactory; however, without a quantified explanation, I remained critical. What if something were to happen in the world of Piriati Embera in which not one or two, but all of the borrowers simultaneously defaulted? As a result of growing up during the financial crisis of 2007, I constantly remain critical to the assumption of non-correlation and independence—an external event can make seemingly observed independent events become intertwined. A flood could be such an event.

During the flight home, I hypothesized two scenarios: a loan to a local farmer, and a loan to a small business startup. For the farmer, the loan could purchase seeds and farming equipment at the beginning of a season to be repaid with the harvest at the end of the season. Massive flooding occurs and the loan would default, regardless of the farmer’s virtues or work ethic. For
the business startup, a flood represents two threats—the threat of damaged inventory and property and the threat of a financially devastated customer base. Massive flooding occurs and the loan would default, regardless of the borrowers’ intentions. Both seemingly independent loans could jointly default from the flood.

I thought about the sustainability of rural business growth as well as the proliferating effect that financial leveraging can have on the Panamanian household. I thought about a Panamanian who loses their business, loses their home, and now loses their ability to repay an outstanding loan, because of a routine flood. And where does that leave the other members of the savings groups? Can we use actuarial risk modeling to hedge these risks?

**Purpose**

The purpose of this thesis is to present specific actuarial knowledge that I have learned during my undergraduate studies at Penn State and place it in the context of the Global Business Brigades microfinance project. This thesis will examine the recent history and development of Grameen microfinance, offer a palette of actuarial risk modeling concepts, and discuss their possible implementation.

As with all quantified modeling, it is important to hold a level of skepticism. Our models are only as good as our assumptions and the quality of the data that we input. As quoted by previous Secretary of Defense, Donald Rumsfeld:

"There are known knowns; there are things we know we know. We also know there are known unknowns; that is to say, we know there are some things we do not know. But there are also unknown unknowns – the ones we don’t know we don’t know."
This paper will examine the known knowns and the known unknowns. As with all quantified modeling, confidence in these two realms is never protection against the risks of the unknown unknowns.
Chapter 2  

Literature Review  

2.1 Introduction to Microfinance

Microfinance, originally called microcredit, is a field of finance that started out in the 1960s to provide small loans to impoverished workers in an effort to stir entrepreneurial activity. Microfinance is defined as “the provision of basic financial services, including savings, credit, money-transfer to the poor—or in a broader sense, those unable to access such services due to exclusion by the mainstream retail banking sector” (Jetha, 2007). Launched as a social experiment in Bangladesh, the microfinance operations of The Grameen Bank have exceeded $481 million with an average loan size of only $70 each.

The socioeconomic impact of these microfinance loans is incredible. An impoverished farmer seeking agricultural equipment must typically wait until the end of a harvest before he can use additional profits, if they exist, toward the investment of additional tools. A microfinance institution can provide access to credit which allows the same farmer the ability to obtain tools at the beginning of the season and the resulting crop yield becomes multiples of the otherwise possible harvest. At the end of the season, the farmer can use the income to repay the loan and still take home additional profits. While many of these operations began as non-profit missions, microfinance-funded economic growth yields double-digit and triple-digit returns—returns large enough for for-profit banking models to be successful and self-sustainable.

If this prospect is able to offer such high returns, then why have larger financial institutions remained noticeably absent from rural credit markets? Historically, large banks have not been interested in acquiring customers in this broad demographic: the loan sizes are extremely
small and the probability of success appears to be unattractive at first glance. According to the Grameen Trust, “the poor have not been traditionally welcomed to financial institutions because they have no assets, they have nothing to offer as collateral, they have no business experience, they have no training, they have no credit history, and they have no education.” Contrary to this attitude, well-managed microfinance operations have proven default rates that are as low as 2 percent.

To overcome these issues, Professor Muhammad Yunus launched the Grameen Bank Project in 1976 and opened the doors to a first-of-its-kind bank in 1983. The Grameen Bank’s lending is unique in the sense that its borrowing is based primarily on ‘trust-lending,’ or lending without collateral. Close friends and neighbors arrange themselves into five-person savings groups in which peer support acts as both a deterrent to shirk on repayment as well as a support system for business development. One loan is issued at a time to a member of a savings group with the other four members acting as collective cosigners to the loan liability. Once a loan is successfully repaid, another member of the group is granted the ability to borrow.

This is just one mechanism used by The Grameen Bank to ensure repayment. In addition to the savings group, a ceremonial public repayment component is used to keep a direct and informed relationship with the borrower. Each week, a public meeting is held at the bank in which all savings group members are required to attend. At this time, each borrower makes a public display of payment on their loan in the presence of their savings group. This repayment ritual creates a routine that encourages the borrower to repay the loan on schedule and helps contribute to low default rates.

Microfinance institutions that replicate The Grameen Bank but lack this component have had limited success. A Kenyan microfinance institution experimented with a modified banking model in which borrowers repay directly to the bank via deposits into a bank account. According to the Kenyan bank, the default rate was very high and likely attributable to the lack of direct
contact with the community members (Espisu, 2005). By making the process ceremonial, regularly scheduled, and public, the accountability of the borrower is greatly enhanced.

In the undesirable event that the borrower does default on their loan, the bank imposes a punishment on the entire savings group--the savings group members are unable to borrow until the debt has been fully repaid. This “joint-liability” component encourages friends to support a borrower when their business venture is unsuccessful—after all, failure of a single member revokes the borrowing ability of the entire group. In addition, potential borrowers are given the incentive to choose savings group members that are responsible and trustworthy—by design, potential borrowers who have a poor reputation will struggle to form a savings group. In this sense, the savings group formation is also a mechanism to screen loan applicants, addressing an asymmetric information issue.

2.2 Pre-existing Risk Management Techniques in Impoverished Communities

Our paper focuses on the loan failure possibility and implementing insurance to hedge against uncertainties in the underlying community members’ income used to repay the loan. An important consideration is the existing attitudes of community members toward risk management. Do borrowers have a contingency plan in the event that their business ventures fail? If borrowers already have their own effective risk management techniques in place, perhaps insuring microfinance loans would prove to be redundant!

A review conducted by Harold Alderman, of The World Bank, and Christina H. Paxson, of Princeton University, discusses income risk management in impoverished communities. They identify two particular techniques that individuals can engage in in the absence of formal insurance markets: risk-coping through personal savings and occupation diversification, and risk-sharing through informal remittance from neighbors and friends.
The first technique, risk-coping, is achieved privately. A borrower can save income from different time periods to cover potential losses in future time periods. A borrower can also diversify the type of work that he engages in in order avoid “putting all of his eggs in one basket.” While very natural behavior, risk-coping has its shortfalls. In rural communities, the availability of formal savings institutions can be limited: savings occurs through storing wealth in assets like the additional storage of crop, gold, etc. Frequently, these assets will not be an efficient, secure, or particularly functional store of wealth. By diversifying types of work, such as simultaneously engaging in farming, artisan, and textile production, community members sacrifice economic gains from specialization. Alderman argues that risk-coping is not an efficient way for community members to manage risk.

The second technique, risk-sharing, is achieved through something that resembles insurance. Alderman poses the question: if barriers to entry have prevented formal insurance markets from forming, are community members able to implement informal risk-sharing mechanisms themselves? To test this hypothesis, the authors introduce an economic model that measures individual utility as a function of income, idiosyncratic risk (risks specific to an individual), and joint risk (risks that simultaneously affect the entire community, such as weather).

After creating a formal model, Alderman attempts to fit the model to empirical data. Issues with data accuracy from the studied rural communities and the inability to properly distill risk attitudes from aggregate household behavior made attempts to fit empirical data to this model unsuccessful. In addition, transfers from neighbors, family members, and friends, could be attempts to manage income risk; however, are not technically risk-sharing. These complications make it impractical to unveil an implied risk premium that already exists in these developing communities.
Townsend also studied the concept of risk sharing in developing communities. He proposed a model in which community members fully share their income risk with each other. The strong assumptions of Townsend’s model and the lack of statistical significance when fitting empirical data also leave our understanding of an individual’s income risk management incomplete. Townsend claims that while his model is not statistically significant with the data, it serves as a good benchmark for further research.

Hongbin Cai takes this benchmark model to demonstrate that increased economic development is possible through the offering of formalized insurance. In his paper “Microinsurance, Trust and Economic Development,” Cai investigates the impact of formalized insurance on farmers’ production decisions in rural China. Cai argues that by beginning with the assumption that risk aversion techniques are already adequately in place, such as in the Townsend model, then the availability of insurance should have a minimal impact on farmer production behavior.

To prove this is not the case, Cai introduces an economic framework in which farmers make a two-step decision over three time periods. In the first period, the farmer must choose whether or not to raise the pigs. In the second period the farmer must choose whether or not to purchase insurance covering the raised pigs. In the third period, farmers are able to recognize the success (or failure) of their endeavor by selling their livestock. Cai concludes through set theory mathematics that for farmers who are risk averse, the number of farmers who choose to participate in farming surely increases when insurance is offered. In addition, Cai’s economic model suggests that farmers will purchase insurance unless they have private information that significantly differs from the actuarial fair risk premium.

To empirically test this framework, Cai designs an experiment in which production decisions in China are measured before and after the availability of an actuarial insurance policy. Cai examines the 2007 state-offered insurance on pigs in the Yunnan and Guizhou provinces of
China. Farmers are offered a slightly actuarial favorable premium of 12 Yuan for 1000 Yuan of coverage on a pig (the true actuarially-fair premium is approximately 20 Yuan). Through a random sample of 480 villages, Cai concluded that there is a statistically significant increase in pig-raising in the presence of the formal state-provided insurance policies than compared to a control group where no insurance was offered.

Through his data research, Cai discovered that participation in the insurance program is largely based upon the farmers’ confidence in the insuring organization. In 2008, an extremely rare snowstorm hit the Guizhou province and the credibility of the insurance program was materially demonstrated: upon the successful completion of a large aggregate payout, uninsured farmers were convinced that joining the program would be worthwhile.

While pre-existing risk management techniques in rural communities are not rigorously understood, Cai’s work suggests that formalized insurance promotes economic development. Our risk-pricing model will conservatively assume that no risk management techniques are currently employed: the priced risk premium will be supplemental to any potential pre-existing risk aversion techniques used by community members.

2.3 Contract Enforceability

Although the Chinese government was successful in its implementation of offering formalized pig insurance to rural Guizhou farmers, proposed insurance in Piriati Embera would not be government-administered. In China, the government had the ability to guarantee contracts, while in Piriati Embera; the enforceability of contracts is greatly diminished.

Rai and Sjostrom (2002) proposed an information-monitoring enhancement to the existing Grameen banking model to improve contract enforceability. Our previously discussed banking model enforces loan contracts through joint-liability savings groups and the implied
element of social pressure. Rai argues that under the Grameen model, a borrower punishment for risk-related failure of a repayment is synonymous to a deadweight loss. If the original intention of the savings group joint-liability model was to prevent borrowers from stealing from the bank, the inevitable punishment that occurs from an honest loan failure is unproductive. Following this reasoning, a microfinance operation should implement an incentive system in which the punishment is minimized to the extent that still deters dishonest borrowing.

Rai states that in a world where state-enforced contracts are possible, individual loans without savings groups would be possible and that individual agents would be able to enter into insurance-type contracts to mitigate their individual risk. In reality, many of these rural communities, Piriati Embera included, lack this crucial element of thorough state-enforced contracts. Rai suggests that by using a monitoring system in which borrowers report information about other borrowers during their repayment process, it would be possible to enforce contracts in the absence of a strong law enforcement agency.

To demonstrate this strategy, Rai introduces a simplified example with two individual borrowers under several scenarios: individual lending, joint-liability lending, and the information monitoring system (referred to as a “message game”). The message game works as follows:

“If [borrower 2] defaults, then borrower [1] receives a harsh punishment only if borrower [2] reports that borrower [1] is withholding some output from the bank. This allows an unsuccessful borrower to threaten her successful partner: ‘Borrower [1], help me repay or I will tell the bank that you refused to help me out and they will impost a harsh punishment on you (but not on me).’”

In the simplified example, the world can take four states: both borrowers fail, only the first borrower fails, only the second borrower fails, or neither borrowers fail. Rai exhaustively demonstrates that there exists a punishment-reward payout design in which each state of the world has a unique behavior outcome of the two borrowers. The bank is then able to witness the
behavior of the two individuals and reveal which borrowers were successful and which borrowers were unsuccessful, in complete absence of a formal contract. This allows the bank to engage in contracts without state-enforcement and allows the bank to overcome asymmetric information barriers.

Although not rigorously tested in real-life, Rai’s methodology is very relevant. It might be risky to try an experimental information-monitoring model in Piriati Embera; however, it would help overcome the inability to enforce formal contracts in the short-term future. In the absence of an effective contract enforcement mechanism, insurance-based loans that remove the joint-liability savings component will have to wait until such an enforcement system develops organically.

2.4 Further Developments in the Grameen Model

In 1983, the Grameen Bank launched its first dispersion of lending: $27 to 42 members. Twenty five years later, over $7.4 billion in volume has been loaned! Impressively, the bank has been able to operate while targeting charged interest rates of only 0-10% higher than the cost of capital! Similar institutions have experienced success with less generous interest rates—the Banco Compartamos, a Mexican bank, operates as a for-profit microfinance operation that charges an effective annual interest rate close to 94%. Critics argue that these extremely high interest rates are an example of loan sharking in an impoverished community, while Banco Compartamos argues that the high interest rate reflects the true high cost of credit in these Mexican communities. Proponents of the Banco Compartamos argue that the availability of credit is of much more importance than the cost.

The current success of the Grameen Bank has attracted similar institutions to open around the world, each facing new demographic-specific challenges. In Kenya, the Jamii Bora Trust,
observed that a large percentage of their loan defaults were caused by the deaths among their borrowers. Upon investigation, the bank discovered that the high death rate in the region was a direct result of a poor health care system. In cooperation with low-cost faith-based hospital missions, Jamii Bora Trust was able to provide better health services for their borrowers. As a result, the institution was able to lower the death rate among its borrowers, and consequently, its default rate. As a side-effect, the lower death rate made way for affordable short-term life insurance premiums. The institution now requires borrowers to purchase short-term life insurance policies that cover the balance of their loans. With low premiums and typically high returns on investment, micro insurance hedged risk and solidified the bank’s operations at a minimal cost to borrowers.

With Jamii Bora Trust’s success, the practice of insuring loans against specific risks is growing in popularity. Other types of insurance have proven beneficial in reducing default rates: health insurance, fire insurance, and livestock insurance. Similar to the short-term life insurance policies, the premiums for these types of coverage are passed on to the borrower (through either higher interest rates or as a separate fee) with the goal of managing default risk.

This thesis will next examine actuarial methodology to price loan default insurance in Piriati Embera and further enhance the Grameen banking model.
Chapter 3

Quantified Risk Models for Panamanian Microfinance

As an actuarial student currently testing through the accreditation process, I have identified several relevant syllabus topics that can be prescribed toward actuarial risk management in Piriati Embara. The Society of Actuaries, an international actuarial accrediting organization, offers five preliminary actuarial modeling exams that broadly cover probability theory, financial mathematics and interest theory, contingency modeling, stochastic financial modeling, and loss and frequency models. The progression of topics includes: time-value of money, probability concepts, and pricing risk premium. We will begin in the realm of certainty and then develop methods to account for uncertainty.

3.1 Interest and Accumulated Value

Interest is a word that often accompanies money and lending, but how do we explain and use it? A basic definition of interest is the additional amount that a borrower repays to a lender in addition to the original amount of a loan.

To illustrate this concept, let us consider a woman named Nancy who borrows from a man named Frank. According to the terms of their loan, Nancy borrows $5 and must repay Frank at some arbitrary date in the future. If Frank is generous and does not charge Nancy any sort of fee for borrowing the money, the amount that Nancy must repay will simply be the original $5. Nancy ultimately decides to repay Frank at the end of 3 years and is shown in the following diagram:
Now let us consider Frank as an investor—he expects to earn some additional money over time in exchange for lending the money today. Instead of allowing Nancy to borrow money at face value, he requests that she repays a fee in addition to the original $5. Frank decides to charge Nancy a flat 4% for every dollar borrowed when she repays him. This equates into a $0.20 fee for the $5 that she borrowed. Again, Nancy arbitrarily chooses to repay Frank at the end of 3 years. Instead of just $5, she must repay a total of $5.20. This “fee” is a crude notion of interest.

\[
$5 + $5(.04) = $5*(1+.04) = $5.20
\]

According to our contract terms, if Nancy pays a flat 4% fee, then she will repay $5.20 regardless of the timeliness in which she repays Frank. Whether she chooses to repay Frank in 3 years or in 1 year or in 10 years, she will still repay the same $5.20. Our current definition of interest is incomplete—our intuition suggests that this borrowing fee should depend on the length of time that the money is borrowed and that the loan should have a predetermined repayment schedule. A repayment in 1 year must be more desirable than an identical repayment in 10 years. But how do we compare the desirability of money in different time periods? More precisely, we need to establish a method to compare different loans that span over different lengths of time.
One way is to standardize everything. We will create a standardized unit of time and then define the fee as a percentage per dollar borrowed per unit of time. For example, suppose that Frank charges Nancy 4% of each dollar borrowed per year. How does a percentage-per-dollar-per-year interest fee compare to a flat percentage-per-dollar interest fee on our $5 loan to be repaid at the end of three years?

At the end of the first year, Nancy owes Frank $5.20. This mirrors our results from earlier: a 4% charge on $5 over a fixed length of time resulted in $5.20. In this case, however, the time interval is on a “per-year” basis as opposed to a three-year basis—and we still have two more years to calculate.

For the second year, the 4% charge is now based upon the outstanding $5.20 amount. To find the amount owed at the end of the second year, we will do the same calculation: $5.20 \times (1.04) = $5.41. To find the final amount, at the end of the third year, we calculate: $5.41 \times (1.04) = $5.62.

The 4% per year charge is called the effective annual interest rate, and is a standard way to express interest rates. In the second year, the notion that Nancy’s interest fee was based upon $5.20 instead of only the original $5 is called “compounding,” or in this case “compounding annually.”

In micro-finance; however, the length of a loan is frequently less than an entire year—typically 3-6 months in length. In this situation, perhaps our standardized unit of time would be more conveniently expressed on a per-month basis. How do we think about a $5 microfinance
loan to be repaid in 5 months? How would we calculate the interest fee charged per month—the effective monthly interest rate? We can solve this problem algebraically.

Our goal is to find an equivalent charge per month, and then work the $5 loan amount forward through 5 months of time. We know that over a 1-year period, the annual effective interest rate would compound exactly 1 time, while an effective monthly interest rate would compound exactly 12 times. If we call this effective monthly interest rate \( x \), then we can solve for it in the following equation:

\[
(1 + x) \times (1 + x) \times \ldots \times (1 + x) = (1 + .04)
\]

\[
(1 + x)^{12} = (1 + .04)
\]

\[
(1 + x)^{12/12} = (1 + .04)^{1/12}
\]

\[
(1 + x)^1 = (1.04)^{1/12}
\]

\[
1 + x = (1.04)^{1/12}
\]

\[
x = (1.04)^{1/12} - 1
\]

\[
x = 0.00327374
\]

Frank would charge Nancy .327374\% per month. Mirroring our previous example, we can move the $5 loan through time. At the end of three months, Nancy would have to repay $5.049.
3.2 Present Value

We have successfully described a technique to take a fixed amount of money today and find its equivalent accumulated value at some point in the future. What if we wanted to calculate the opposite—that is, to take a fixed amount of money at some point in the future and calculate its equivalent present value. While the concept might seem unnatural at first, the algebra is exactly as we would expect: instead of multiplying by \((1+i)\) to move money 1 time period into the future, we divide by \((1+i)\) to bring money 1 time period toward the present.

Returning to Frank and Nancy, suppose that Nancy enters into a contract in which she agrees to pay Frank exactly $1 at the end of 2 years. How much is this contract worth today? Equivalently, how much money must Nancy save today, to ensure having exactly $1 to pay Frank in 2 years?

If we define \(X\) as the amount of money Nancy must set aside today to repay Frank at the end of 2 years, we can manipulate our accumulated value framework from before. If we assume an interest rate of \(i = .06\), we can solve the following equation:

To create an accumulated value of $1 in \(t\) years:

\[
X \times (1 + i)^t = 1
\]

Setting our parameters \(i = .06\) and \(t = 2\) years.

\[
X \times (1 + .06)^2 = 1
\]

Solving for \(X\) we obtain:

\[
X = \frac{1}{(1 + .06)^2} = .8899
\]

We interpret this as the amount of money that Nancy must deposit $0.8899 today to ensure that she will have an accumulated value of $1 at the end of two years. By this logic, Frank would willingly accept $0.8899 today in lieu of $1 in two years, despite the present value being a smaller face amount. The $0.8899 will accumulate into the full $1 at \(i = .06\) in \(t=2\) years.
Previously, to accumulate a present value to some point that is $t$ time periods into the future, we would repeatedly multiply by $(1 + i)$ for a total of $t$ times, or simply multiply by $(1 + i)^t$. To bring a future cash flow from some point $t$ time periods in the future to the present, we divide by $(1 + i)^t$.

3.3 Euler’s Constant and Continuous Interest

Previously, we calculated effective annual interest rates and their equivalent effective monthly interest rates. Using the same methodology, we could compute equivalent semiannual, quarterly, monthly, weekly, or even daily interest rates if we desired.

To formalize our technique, we will introduce the “$m$thly” interest rate, $i^{(m)}$, where $m$ is the number of times that the interest rate is compounded in a single year. Keeping $i$ as the effective annual rate, the following relationship is always true:

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

For example, to calculate the quarterly interest rate, $i^{(4)}$, we would solve for an $i^{(4)}$ such that

$$1 + i = \left(1 + \frac{i^{(4)}}{4}\right)^4$$

Choosing a specific value for $i$, say $i = 0.06$:

$$1 + .06 = \left(1 + \frac{i^{(4)}}{4}\right)^4$$

$$1.06^{\frac{1}{4}} = \left(1 + \frac{i^{(4)}}{4}\right)^{\frac{4}{4}}$$

$$1.06^{\frac{1}{4}} - 1 = \frac{i^{(4)}}{4}$$
\[
\left(1.06^{\frac{1}{4}} - 1\right) \times 4 = i^{(4)}
\]

\[
i^{(4)} = 0.05869
\]

Notice that \(i^{(4)} = 0.05869\) is not the effective quarterly interest rate, but that \(\frac{i^{(4)}}{4} = 0.01467\) is the effective quarterly rate in which we accumulate while compounding 4 times per year.

If we wanted to take an mthly interest rate and find the equivalent effective annual interest rate, we algebraically solve the same relationship:

\[
1 + i = \left(1 + i^{(m)}\right)^{\frac{m}{m}}
\]

If we know that \(i^{(4)} = 0.05869\) and want to solve for the effective annual interest rate \(i\):

\[
1 + i = \left(1 + \frac{0.05869}{4}\right)^4
\]

\[
i = \left(1 + \frac{0.05869}{4}\right)^4 - 1
\]

\[
i = 0.06
\]

This confirms our results from before.

In a theoretical context, what if we chose a fixed mthly interest rate, such as \(i^{(m)} = .04\), and wanted to examine the effect of increasing the compounding frequency—that is, what is the effect of an increasing \(m\)? What is 4\% compounded quarterly \((m = 4)\)? Monthly \((m = 12)\)? Or even daily \((m = 365)\)? Let’s keep going—by the hour \((m = 8,760)\)? By the minute \((m = 525,600)\)? By the second \((m = 31,536,000)\)? What about by the half-second? What happens to our interest rate as \(m\) becomes larger and larger—as \(m\) approaches infinity? To answer this, we introduce Euler’s constant, referred to as \(e\). Its relationship with interest rates is very natural.

The Euler constant is defined as:
\[ e^c = \lim_{n \to \infty} \left(1 + \frac{c}{n}\right)^n \]

When \( c = 1 \):

\[ e^1 = e = 2.71828183 \]

While the notation is different, this looks very similar to the limit of our mthly interest rate expression as \( m \) increases toward infinity:

\[ \lim_{m \to \infty} \left(1 + \frac{i(m)}{m}\right)^m \]

Adjusting our notation:

\[ e^{i(m)} = \lim_{m \to \infty} \left(1 + \frac{i(m)}{m}\right)^m \]

where \( i(m) \) is an unchanging constant.

Our mechanics work the same as before: to accumulate an amount of money from the present-value to \( t \) years into the future, we multiply by \( e^{i(m) t} \) times, or equivalently, \( e^{i(m) + t} \). To find the present value of an amount of money occurring one year into the future, we divide by \( e^{i(m) t} \) times, or equivalently, divide by \( e^{i(m) + t} \). This is also equivalent to multiplying by \( e^{-i(m) t} \).

### 3.4 Present Value and Accumulated Value Functions

In this section, we will add one final layer of sophistication to our present value and accumulated value framework and create a formal function. Although we will use the function in its most simplified form, it is worth previewing a more dynamic framework in case the reader is interested in expanding the sophistication of our models in the future. If you do not like calculus, skip to the last part of this section.
The final moving piece of present value and accumulated value is the interest rate itself. In real life, interest rates are very rarely stable—they are constantly changing every day. In the previous section, we defined Euler’s constant, $e^{(m)\Delta t}$, as a way of continuously accumulating money through time, with the value of $(m)$ not depending on any specific relationship with the time of the year.

To create a dynamic interest rate model, we will create a function that defines the force of interest that is unique to each period of time. For each value of $t$, we want to create a function that will define the associated $(m)$. To describe the unique interest rates over time, we introduce the force of interest, $\delta (t)$.

The force of interest can be conceptually thought of as the “continuous rate of interest” that is continuously changing through time. Our goal is to set our growth rate $(m)$ in our Euler’s constant to be equal to some equivalent constant. We define $\delta (t)$ as:

$$i^{(m)} = c = \int_a^b \delta (t) \, dt$$

where and b are two distinct points in time.

We incorporate this component into our original framework and we have the present value and accumulated value functions:

$$PV (X, \delta (t), a, b) = X * e^{-\int_a^b \delta (t) \, dt}$$

$$AV (X, \delta (t), a, b) = X * e^{\int_a^b \delta (t) \, dt}$$

For most of our work we will make the following assumptions, that $a=0$ (meaning that our first reference point is today) and $b=t$ (renaming the parameter to $t$ out of convenience), and that the force of interest is constant, $\delta (t) = \delta$.

A constant force of interest simplifies the integral to
Our simplified functions become:

\[ PV(X, \delta, t) = X \cdot e^{-\delta t} \]

\[ AV(X, \delta, t) = X \cdot e^{\delta t} \]

Recall that \( \delta \) is a constant, and our simplified function is similar to our \( e^{i(m) \cdot t} \) where \( i(m) = \delta \).

### 3.5 Multiple Cash Flows

In practice, loans are rarely repaid in full. Instead, they are repaid in several installments. In the case of our proposed Panamanian microloans, installment periods occur monthly over a 3-6 month period. Our framework up to this point has only discussed single flows of money—how do we value multiple cash flows at the same time? We value each stream of money individually, and then add them all up to calculate a total!

Returning to our beloved Frank and Nancy, we consider a contract in which Nancy agrees to pay Frank $10 per month for six months, assuming a constant force of interest of \( \delta = 0.06 \). How much is this contract worth today?

We know how to find the present value of the first payment using the present value function. The first payment occurs at the end of the first month. Plugging these values into our Present Value function:

\[ PV(10, \delta = 0.06, t = 1/12) = 10 \cdot e^{-0.06 \cdot \frac{1}{12}} = 9.950 \]

Frank would accept $9.95 today in lieu of a payment one month from now of $10.
But we aren’t finished yet—there are still five more payments. The second payment occurs at the end of the second month. Plugging this into the Present Value function:

$$ PV(\$10, \delta = .06, t = \frac{2}{12}) = \$10 \times e^{-0.06 \times \left(\frac{2}{12}\right)} = \$9.900 $$

We continue this for the payment that occurs at the end of the third, fourth, fifth, and sixth months.

$$ PV \left( \$10, \delta = .06, t = \frac{3}{12} \right) = \$10 \times e^{-0.06 \times \left(\frac{3}{12}\right)} = \$9.851 $$

$$ PV \left( \$10, \delta = .06, t = \frac{4}{12} \right) = \$10 \times e^{-0.06 \times \left(\frac{4}{12}\right)} = \$9.801 $$

$$ PV \left( \$10, \delta = .06, t = \frac{5}{12} \right) = \$10 \times e^{-0.06 \times \left(\frac{5}{12}\right)} = \$9.753 $$

$$ PV \left( \$10, \delta = .06, t = \frac{6}{12} \right) = \$10 \times e^{-0.06 \times \left(\frac{6}{12}\right)} = \$9.704 $$

Now that we have calculated the equivalent dollar amount that we would accept today opposed to a payment of $10 at some month in the future, the question still remains: how much is the contract worth today? Since we know how much Frank would accept for each individual payment, the amount he would accept for all of the payments is simply the sum of these calculated present values. Taking the sum:


A single payment of $58.961 today is the equivalent to a stream of $10 payments over the next sixth months, valued at the constant force of interest of $\delta = .06$.

We can use the exact same method with the Accumulated Value function to find the amount that Frank would accept at the end of six months in lieu of the monthly payments along the way. Similar to before, we will accumulate each of the individual payments and then add them all up!
To find the Accumulated Value of our first payment, we want to accumulate the payment made 1 month from now forward to the end of 6 months. The length of the time interval we are accumulating is $\frac{6}{12} - \frac{1}{12} = \frac{5}{12}$ years.

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{1}{12}\right) = 10 \times e^{0.06 \times \frac{5}{12}} = 10.253$$

The two values are equivalent: either $10 to be received one month from now, or $10.253 to be received 6 months from now.

What about the payment two months from now? Notice that to take a payment two months from now to six months from now, the length of the time is now only $\frac{6}{12} - \frac{2}{12} = \frac{4}{12}$ years.

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{2}{12}\right) = 10 \times e^{0.06 \times \frac{4}{12}} = 10.202$$

To bring forward the other payments six months from now we will follow the same process:

$10 to be received three months from now accumulated three more months:

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{3}{12}\right) = 10 \times e^{0.06 \times \frac{3}{12}} = 10.151$$

$10 to be received four months from now accumulated two more months:

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{4}{12}\right) = 10 \times e^{0.06 \times \frac{2}{12}} = 10.101$$

$10 to be received five months from now accumulated one more month:

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{5}{12}\right) = 10 \times e^{0.06 \times \frac{1}{12}} = 10.050$$

$10 to be received six months from now accumulated “zero” more months:

$$AV \left(10, \delta = .06, t = \frac{6}{12} - \frac{6}{12}\right) = 10 \times e^{0.06 \times \frac{0}{12}} = 10$$

After accumulating each individual payment, we add up the individual payments to find the total accumulated value:
\[ 10.253 + 10.202 + 10.151 + 10.101 + 10.050 + 10 = 60.757 \]

Frank would be willing to accept a single payment of $60.757 in six months in lieu of a stream of $10 payments over the next six months. In the context of loan repayments, we can think about this as repaying the loan all-at-once at the end of the contract, or repaying a small part each month while accounting for interest along the way.

Since each payment is moving from a point in the future to another point even further in the future, it is sometimes tedious to calculate the Accumulated Value. One computational trick is to convert the stream of payments into a single present value payment. Once we have a single present value payment, we can bring the single cash flow forward using the AV function only once. In our previous example, plugging in the present value for the dollar amount, taking it six months into the future:

\[ AV (58.961, \delta = .06, t = \frac{6}{12}) = 60.757 \]

This confirms our work from before. If we already have the present value, this method will find the single repayment at the end of the loan with significantly less work.

### 3.6 Actuarial Present Value

Up until this point, we have assumed that every single payment agreed upon actually gets made. In real life, this is far from the truth. To account for this uncertainty, we will combine time-value of money concepts with some basic probability theory. We will use a concept called “expected value” to modify our valuation of these cash payments.

The concept of expected value is most easily explained through example. Suppose that we create a game for rolling a fair six-sided die: if the die rolls a 2, the player receives $20, if the
die rolls any other number, the player receives nothing. If we played this game many times, what’s the average amount of money the player can expect to walk away with each round?

To answer this question, we will create an exhaustive list of potential scenarios and then weight them by the probability that they actually occur. The first scenario is to win $20 by rolling a 2. The second scenario is to win $0 by rolling a 1, 3, 4, 5, or 6. The probability of the first scenario, rolling a 2 on a fair 6-sided die and winning $20, is one out of six, or $\frac{1}{6}$. The probability of the second scenario, rolling any other number \{1, 3, 4, 5, 6\} and winning $0, is 5 out of 6, or $\frac{5}{6}$.

With this information, we can compute the expected value:

$$\frac{1}{6} \times \$20 + \frac{5}{6} \times \$0 = \$3.333$$

This value is called the expected value of the game—we took an exhaustive list of scenarios, and then weighted them by the probability that they occur. In some sense, we can think of this as the amount of money that we would charge someone to play this game.

To bring this into our time-value framework, we assign probabilities to the success of receiving each individual payment. If we believe there is only a 55% chance that we will receive a specific payment, and a 45% chance we will receive nothing, we should adjust our present value, now called our Actuarial Present Value, to reflect this uncertainty.

Consider the following example: Nancy owes Frank a payment of $25 in exactly one year from now. As a farmer, there is a 45% chance that Nancy’s crops will fail because of bad weather. If Nancy’s crops fail, she will be unable to repay Frank. How do we value this payment today?

Previously, we would use the Present Value function. Assuming a constant force of interest, $\delta = .06$:

$$PV($25, \delta = .06, t = 1) = \$25 \times e^{-0.06 \times (1)} = \$23.544$$
This is the equivalent dollar amount for a guaranteed payment-- but we know there is a 45% chance that we receive nothing (or equivalently, a 55% chance that we will receive the payment). To reflect this in the valuation, we will use our probability-weighted expected value framework to calculate the expected value of the present value of Nancy’s repayment. As before, we multiply each scenario’s dollar value by the probability that that scenario occurs:

\[ 0.45 \times \$0 + 0.55 \times \$23,544 = \$12.949 \]

Given the interest rates (our constant force of interest of \( \delta = 0.06 \)) and probabilities (45% chance of failure, 55% chance of success), we would accept $12.949 guaranteed today in lieu of a payment with a 55% chance of success, exactly one year from today.

For multiple payments, we take the same approach: find the present value of the individual payment, then compute the expected value using probabilities. To calculate the value of a contract with multiple payments, we take the sum of each payment’s individual expected present value.

3.7 Variance

We have identified the expected present value as a way to describe the value of uncertain loans. When probabilities are similar and situations are not extreme, expected value might seem like a complete description. What happens when the probabilities are not similar or the outcomes are extreme? Let us consider two hypothetical worlds—one extreme and one mundane. First, an extreme case: Frank receives a $1,000,000 payment with a probability of 1% or a $0 payment with a probability of 99%. Under our previous framework (still assuming \( \delta = 0.06 \)), we can calculate the expected present value of this contract:

\[ 0.99 \times \$0 + 0.01 \times \$1,000,000 \times e^{-0.06\times1} = \$9,417.64 \]
Now let us consider a mundane world: Frank receives an $11,000 payment in exactly one year with 50% probability or only a $9,000 payment with 50% probability. Again, we can use our previous framework to calculate the expected value:

\[0.50 \times \$11,000 \times e^{-0.06 \times 1} + 0.50 \times \$9,000 \times e^{-0.06 \times 1} = \$9,417.64\]

Notice that our expected value is identical in both worlds. But would Frank really be indifferent to the equivalent expected values? In the mundane world, Frank knows that he will receive at least $9,000 but maybe $11,000 depending on the scenario—an expected value of $9,417.64 might serve as a reasonable description. In the extreme world; however, there is a large probability that he will receive absolutely nothing and a very small chance that he will receive a payment—our calculated $9,417.64 figure does not give a complete picture. Our expected value of $9,417.64 might not be a complete description in this situation. It is also clear that our two described worlds are very different from each other, but using our expected value framework, we would describe them identically: $9,417.64.

To add another layer of description to our framework, we will introduce a statistical concept called the variance. The variance is defined as:

\[\text{Variance} [\text{Present Value of Losses}] = \]

\[\text{Pr}[\text{Scenario 1}] \times \text{Present Value of Losses in Scenario 1}^2 + \cdots + \text{Pr}[\text{Scenario N}] \times \text{Present Value of Losses in Scenario N}^2 - \text{Expected Value of Present Value Losses}^2\]

Sometimes statisticians prefer to use the term standard deviation. This is the square root of the calculated variance:

\[\text{Standard Deviation} = \sqrt{\text{Variance}}\]

Let us return to the dice rolling example. Suppose that we invent a game in which a player roles a fair six-sided die. After rolling the die, the player scores the number of points equal to the die’s value. For example, if a player roles a “4,” that player will score 4 points.
To calculate our standard deviation we need to first calculate our variance, and to calculate our variance we need to first calculate our expected value:

\[
E[Die] = \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4 + \frac{1}{6} \times 5 + \frac{1}{6} \times 6
\]

\[
E[Die] = 3.5
\]

In our calculation of the variance, we will need to use the square of the expected value:

\[
E[Die]^2 = 3.5^2
\]

\[
E[Die]^2 = 12.25
\]

Now to calculate the variance:

\[
\text{Variance}[\text{Present Value of Losses}]
\]

\[
= Pr[\text{Scenario 1}] \times \text{Present Value of Losses in Scenario 1}^2 + \cdots
\]

\[
+ Pr[\text{Scenario N}] \times \text{Present Value of Losses in Scenario N}^2
\]

\[
- \text{Expected Value of Present Value Losses}^2
\]

\[
\text{Variance}[\text{Present Value of Losses}]
\]

\[
= \frac{1}{6} \times 1^2 + \frac{1}{6} \times 2^2 + \frac{1}{6} \times 3^2 + \frac{1}{6} \times 4^2 + \frac{1}{6} \times 5^2 + \frac{1}{6} \times 6^2 - 12.25
\]

\[
= 2.91667
\]

The standard deviation is therefore:

\[
\text{Standard Deviation} = \sqrt{\text{Variance}}
\]

\[
= \sqrt{2.91667}
\]
Returning to our example in the beginning of this section, we can calculate the variance and standard deviation of our extreme world and of our mundane world. These are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Expected Value</th>
<th>Variance</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extreme World</td>
<td>$9,417.64</td>
<td>9,900,000,000</td>
<td>99,498</td>
</tr>
<tr>
<td>Mundane World</td>
<td>$9,417.64</td>
<td>1,000,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

As we can see, the standard deviation in the extreme world (99,498) is greater than the standard deviation in the mundane world (1,000). This follows our intuition that the extreme world’s scenarios (either $1,000,000 or $0) are much further spread apart than in the mundane world (either $11,000 or $9,000). Looking only at the expected value, we might think that these two worlds are identical. By using the variance and standard deviation, we are able to give a more complete description in which we can identify the extreme world as riskier than the mundane world.

3.8 Forecasting Losses

Let us continue to develop our framework more specifically to the needs in Piriati Embera. For each and every loan that the microfinance operation issues there is a risk that the loan will not be repaid. There is a risk that the borrower is dishonest. There is a risk that the
borrower is honest but that a drought causes their business venture to fail. Or maybe a flood. It could even be some unidentified macroeconomic supply shock.

We will begin by considering a simplified case. Suppose that our microfinance operation issued 10 loans of $50 each. The risk that these loans default can be distilled into two distinct categories, through our own design: the probability that the borrower is unsuccessful in repaying the loan based upon specific circumstances unique to that particular borrower, and the probability that all borrowers are unsuccessful in repaying the loan because of a catastrophic event—like a flood.

3.8.1 Individual Risk

Let us imagine two microfinance loans issued today with monthly payments—one loan of $50 with monthly payments issued to Frank and one loan of $50 with monthly payments issued to Nancy. Suppose that Frank’s loan of $50 fails in the second month before the second payment. Nancy’s loan fails after 5 months, right before the final payment. What is the present value of these imperfect loans?

First, we must calculate the payment amounts. We will call these level payments $P$, and set the present value of these 6 payments equal to the loan’s current value ($50$).

Assuming $\delta = .06$ the present value of the loan is:

$$50 = P e^{-0.06 \times \frac{1}{12}} + P e^{-0.06 \times \frac{2}{12}} + P e^{-0.06 \times \frac{3}{12}} + P e^{-0.06 \times \frac{4}{12}} + P e^{-0.06 \times \frac{5}{12}} + P e^{-0.06 \times \frac{6}{12}}$$

Factoring out the $P$:

$$50 = P \left( e^{-0.06 \times \frac{1}{12}} + e^{-0.06 \times \frac{2}{12}} + e^{-0.06 \times \frac{3}{12}} + e^{-0.06 \times \frac{4}{12}} + e^{-0.06 \times \frac{5}{12}} + e^{-0.06 \times \frac{6}{12}} \right)$$

Dividing both sides:
\[ P = \frac{50}{\left(e^{-0.06 \cdot \left(\frac{1}{12}\right)} + e^{-0.06 \cdot \left(\frac{2}{12}\right)} + e^{-0.06 \cdot \left(\frac{3}{12}\right)} + e^{-0.06 \cdot \left(\frac{4}{12}\right)} + e^{-0.06 \cdot \left(\frac{5}{12}\right)} + e^{-0.06 \cdot \left(\frac{6}{12}\right)}\right)} \]

Evaluating:

\[ P = 8.480 \]

To repay a $50 loan at \( \delta = 0.06 \) with equal payments, each monthly payment would be equal to $8.48. To find the loss on the loan, we find the difference between the present value of the loan and the present value of the sum of the individual payments received:

\[
\text{Loss} = \text{Present Value of Loan} - \text{Present Value of Payments}
\]

\[
\text{Loss} = 50 - 8.48e^{-0.06 \cdot \left(\frac{1}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{2}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{3}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{4}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{5}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{6}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{7}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{8}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{9}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{10}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{11}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{12}{12}\right)}
\]

Evaluating:

\[
\text{Loss} = 0
\]

Unsurprisingly, if we are fully repaid for the loan, there should be no loss!

Now let’s consider Frank’s loan. The present value of Frank’s loan is the same as before: $50. Similarly, we calculate the present value of the payments. In this situation; however, Frank’s payments failed just before the second payment. We model this as:

\[
\text{Loss} = 50 - 8.48e^{-0.06 \cdot \left(\frac{1}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{2}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{3}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{4}{12}\right)} - 8.48e^{-0.06 \cdot \left(\frac{5}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{6}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{7}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{8}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{9}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{10}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{11}{12}\right)} - 0e^{-0.06 \cdot \left(\frac{12}{12}\right)}
\]

Evaluating:

\[
\text{Loss} = 41.562
\]

The present value of the losses in this scenario would be $41.562.

Now we examine Nancy’s loan. Nancy’s loan fails after 5 months, just prior to the final payment. The present value of the losses on Nancy’s loan:
We can see that both loans failed; however, depending on the timing of the loss, the total present value of the losses is significantly different. This can be attributed to two reasons: the later the failure occurs, the less it is worth today because of the time-value of money, but also that the later the failure occurs, the greater number of repayments have already been made.

To model this progression of repayment failure, we will consider the possible present value of losses in all possible scenarios. In line with our calculations from before, we can describe the impact of the loan as one of the following scenarios:

\[
\text{Loss} = 50 - 8.48e^{-0.06\cdot\frac{1}{12}} - 8.48e^{-0.06\cdot\frac{2}{12}} - 8.48e^{-0.06\cdot\frac{3}{12}} - 8.48e^{-0.06\cdot\frac{4}{12}} - 8.48e^{-0.06\cdot\frac{5}{12}} - 0e^{-0.06\cdot\frac{6}{12}}
\]

\[
\text{Loss} = 8.229
\]

We label the scenarios A-G. For example, Scenario A would be a situation in which the loan fails before any payments are made, in which case the entire $50 is lost. Scenario D would be when the borrower repays three payments, then is unable to pay the rest, in which case the present value of losses on the loan would be $24.81. Scenario G describes 6 successful payments—in other words, successful and complete repayment, in which case the present value of losses on the loan would be $0.
Now that we have exhaustively listed out all of the possible scenarios and their respective present value of losses, we need to assign probabilities. It might be difficult to directly assign a probability to any scenario—how do we know if a loan is more likely to fail in the fourth month versus the sixth month?

To simplify this process, we will break down repayment into a step-by-step process assigning probabilities to the question “will the next payment be successful?” This binary process can be used to model the repayment process.

We begin with the first payment and assign a probability of success. If the payment is successfully made, then we assign a probability of success to the second payment, given that the first payment has already been successfully made. We will continue this process until we have assigned probabilities for each step along the way. Once established, these conditional probabilities can be strung together to calculate the unconditional probabilities of each scenario. The probability of failure before the first repayment is simply the probability that the borrower fails the first payment. The probability of failure just before the second payment is the probability that the first payment was made successfully multiplied by the probability that the second payment is unsuccessful. The probability of failure just before the third payment is the probability that the first payment is successful multiplied by the probability that the second payment is successful multiplied by the probability that the third payment is unsuccessful. These unconditional probabilities are the probabilities that we use to calculate the expected value.

To simulate these 6 payments, we can hypothesize the following table. The probabilities here are arbitrarily fictitious, and calibrating these probabilities to real-life will be discussed in Chapter 4.
We can also define the following potential scenarios for our borrower:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>“Fail to Repay Before First Payment: 0 Successful Payments”</td>
</tr>
<tr>
<td>B</td>
<td>“Fail to Repay Before Second Payment: 1 Successful Payments”</td>
</tr>
<tr>
<td>C</td>
<td>“Fail to Repay Before Third Payment: 2 Successful Payments”</td>
</tr>
<tr>
<td>D</td>
<td>“Fail to Repay Before Fourth Payment: 3 Successful Payments”</td>
</tr>
<tr>
<td>E</td>
<td>“Fail to Repay Before Fifth Payment: 4 Successful Payments”</td>
</tr>
<tr>
<td>F</td>
<td>“Fail to Repay Before Sixth Payment: 5 Successful Payments”</td>
</tr>
<tr>
<td>G</td>
<td>“All payments made successfully: 6 Successful Payments”</td>
</tr>
</tbody>
</table>

In order to calculate the associated unconditional probabilities in our second table, we will string together the conditional probabilities from the first table:

For Scenario A

From our table we know that the probability of failing the first payment is 10%.

Computationally:

\[ \Pr["\text{Scenario A}"] = \Pr["1st\ Payment\ Unsuccessful"] \]

\[ \Pr["\text{Scenario A}"] = 0.1 \]
For Scenario B

To have a series of events that will result in scenario B we need to successfully make the first payment, but then fail to make the second payment, given that the first payment was already made.

\[
\text{Pr}[\text{"Scenario B"}] = \text{Pr}[\text{"1st Payment Successful"}] \times \text{Pr}[\text{"2nd Payment Unsuccessful"}]
\]

\[
\text{Pr}[\text{"Scenario B"}] = 0.9 \times 0.2
\]

\[
\text{Pr}[\text{"Scenario B"}] = 0.18
\]

For Scenario C

To have a series of events that will result in scenario C, the borrower needs to successfully make the first payment, then successfully make the second payment, and then fail to make the third payment.

\[
\text{Pr}[\text{"Scenario C"}] = \text{Pr}[\text{"1st Payment Successful"}] \times \text{Pr}[\text{"2nd Payment Successful"}] \times \text{Pr}[\text{"3rd Payment Unsuccessful"}]
\]

\[
\text{Pr}[\text{"Scenario C"}] = 0.9 \times 0.8 \times 0.3
\]

\[
\text{Pr}[\text{"Scenario C"}] = 0.216
\]

We continue this calculation for each Scenario. The results are summarized in the following table, along with our previously calculated present value losses:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Present Value of Losses</th>
<th>Probability of Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td>B</td>
<td>41.56215387</td>
<td>0.18</td>
</tr>
<tr>
<td>C</td>
<td>33.16639167</td>
<td>0.216</td>
</tr>
<tr>
<td>D</td>
<td>24.81250352</td>
<td>0.1512</td>
</tr>
<tr>
<td>E</td>
<td>16.50028055</td>
<td>0.14112</td>
</tr>
<tr>
<td>F</td>
<td>8.229514967</td>
<td>0.10584</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0.10584</td>
</tr>
</tbody>
</table>
Our last step is to calculate the expected value of our losses. To calculate this, we multiply the loss from each scenario by the probability that it actually happens. Then we take the sum of all of these values:

\[
= \text{Pr["Scenario A"]} \times \text{Loss in Scenario A} + \text{Pr["Scenario B"]} \times \text{Loss in Scenario B} + \cdots \\
+ \text{Pr["Scenario G"]} \times \text{Loss in Scenario G}
\]

\[
= 0.1 \times \$50 + 0.18 \times \$41.5621 + \cdots + 0.1058 \times \$0
\]

\[
= \$26.59
\]

What does this number mean? This means that if we make a large number of independent loans that faced identical probabilities on page 35, the average loss we would expect per loan given out is $26.59.

We can also calculate the variance of the loss:

\[
\text{Variance [Present Value of Losses]}
\]

\[
= \text{Pr[Scenario A]} \times \text{Present Value of Losses in Scenario A}^2 + \cdots \\
+ \text{Pr[Scenario G]} \times \text{Present Value of Losses in Scenario G}^2
\]

\[
- \text{Expected Value of Present Value Losses}^2
\]

\[
0.1 \times \$50^2 + 0.18 \times \$41.5621^2 + 0.216 \times \$33.16^2 + 0.1512 \times \$24.81^2 + 0.14112 \times \$16.50^2
\]

\[
+ 0.1584 \times \$8.23^2 + 0.10584 \times \$0^2 - \$26.59^2
\]

\[
\text{Var[Present Value of Losses]} = 229.85
\]

The standard deviation is therefore

\[
\text{Standard Deviation} = \sqrt{\text{Var[Present Value of Losses]}}
\]

\[
= \sqrt{229.85}
\]

\[
= 15.16079
\]

We can use these calculations to calibrate a normal distribution in Section 3.10.
3.8.2 Common Shock Model

We have now accounted for the individual risk in our lending—probabilities based upon the circumstances that an individual will be unable to repay the loan. Perhaps it is because of bad luck—maybe a dog destroyed a storage of crops, or maybe the borrower became ill. For whatever the cause, these risks only apply to an individual person, and in this sense, are independent from each other—one failure does not indicate anything about another failure. For a large number of loans to fail simultaneously due to individual risk factors, would be an extremely large coincidence with a very low probability.

Nonetheless, we still need to consider the risks that affect everybody—the concern is for the scenarios in which a massive flood wipes out the entire loan base. A challenge that we face is that each individual loan elapses every six months; however, a catastrophic flood might be much less frequent than that. We could model this risk similarly to our “individual risk model”—that is to create a list of scenarios and assign probabilities to each scenario. In this case, we are only interested in two scenarios: Scenario A in which no flood occurs and we assume that everybody repays (recall that our individual default risk is accounted for separately in our individual risk model), or Scenario B, in which a flood occurs and we assume that everybody defaults.

As with our individual model, it can be difficult to arbitrarily assign probabilities to Scenario A and Scenario B. It is not easy to wake up on a random day and forecast the probability of a flood occurring within that day. What is observable; however, is the number of floods over a long period time. We will use a Poisson framework to model flooding over long periods of time, and then relate this framework back to Scenario A and Scenario B.

The Poisson distribution is governed by the following description:

\[
\Pr[\text{"x number of event occurrences in a fixed time period"}] = \frac{\lambda^x e^{-\lambda}}{x!}
\]
While the formula looks daunting at first glance, the essential idea behind the Poisson distribution is that we define some rate, \( \lambda \), as the average number of events occurring within a fixed time period. We will begin by looking at a long time horizon, and then relate it back to our short 6-month intervals.

To begin, we will define \( \lambda \) as the average number of floods per six months. Once we choose a value for \( \lambda \), we can calculate the probabilities of floods occurring.

Suppose that we estimate that on average, we will see one flood every 2 years. Scaling this description down to our six-month periods, we could say that we expect to see 0.25 floods every six months. Therefore we will set \( \lambda = 0.25 \). Now we are ready to calculate probabilities:

Generally:

\[
\Pr["x \text{ number of event occurrences in a fixed time period}"] = \frac{0.25^x e^{-0.25}}{x \times (x-1) \times \ldots \times 1}
\]

Let’s calculate some specific probabilities:

\[
\Pr["x=0 \text{ occurrences in the next 6 months}"] = \frac{0.25^0 e^{-0.25}}{1} = 0.7788
\]

\[
\Pr["x=1 \text{ occurrences in the next 6 months}"] = \frac{0.25^1 e^{-0.25}}{1} = 0.1947
\]

\[
\Pr["x=2 \text{ occurrences in the next 6 months}"] = \frac{0.25^2 e^{-0.25}}{2 \times 1} = 0.02433
\]

\[
\Pr["x=3 \text{ occurrences in the next 6 months}"] = \frac{0.25^3 e^{-0.25}}{3 \times 2 \times 1} = 0.00202
\]

We could continue calculating more probabilities, but we see that as \( x \) increases, the probabilities become very small. This matches what we expect in real life—nearly a zero probability that we could witness 3 floods in a 6-month period.

In the case of our loan portfolio; however, we can make a further assumption. From the perspective of the lender, it doesn’t matter if there is one flood or two floods or three floods—the
result is the same: massive default. The only two cases we are interested in is the probability of no floods versus the probability of any floods.

To accomplish this, we will assign the probability of Scenario A as the probability of X=0 floods occurring in the 6-month time interval: 0.7788. To find the probability of any flooding occurring (1 or 2 or 3, etc.) we will use the following fact:

\[ \Pr[0\; \text{floods occurring}] + \Pr[\text{Any floods occurring}] = 1 \]

Intuitively, this is saying that the probability of no floods occurring or any flood occurring has to be 100%. Either a flood happens or it doesn’t. Since we already assigned a value to the probability of zero floods occurring, we can solve for the probability that any flood occurs:

\[ 0.7788 + \Pr[\text{Any floods occurring}] = 1 \]

\[ \Pr[\text{Any floods occurring}] = 1 - 0.7788 = 0.2212 \]

Incorporating our previous example of loan portfolios (100 loans of $50 each), we summarize our results into the following table:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Present Value of Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario A – No Floods Occur, All Loans Repaid</td>
<td>0.7788</td>
<td>$0</td>
</tr>
<tr>
<td>Scenario B – Flooding Occurs, No Loans Repaid</td>
<td>1 - 0.7788 = 0.221199</td>
<td>$5,000</td>
</tr>
</tbody>
</table>
We find the expected value as before, assuming that the $5,000 is repaid at the end of the 6-month period:

\[
\text{Pr[Scenario A]} \times \text{Present Value of Losses in Scenario A} + \text{Pr[Scenario B]} \times \text{Present Value of Losses in Scenario B} = 0.788 \times 0 + 0.2212 \times 5,000 = 1,105.99
\]

For a series of 6-month periods, we would expect an average total loss of $1,105.99 per period over our $5,000 portfolio.

3.9 Percentiles

Up to this point we have used probabilities to calculate the expected value of our present value losses. In our individual risk models we calculated the expected value of losses per loan based upon probabilities. In our group shock model, we calculated the expected value of losses based upon probabilities that an entire portfolio would default.

Recall our example from before depicting different scenarios of present value of losses depending on the time of failure to repay:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Present Value of Losses</th>
<th>Probability of Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td>B</td>
<td>41.56215387</td>
<td>0.18</td>
</tr>
<tr>
<td>C</td>
<td>33.16639167</td>
<td>0.216</td>
</tr>
<tr>
<td>D</td>
<td>24.81250352</td>
<td>0.1512</td>
</tr>
<tr>
<td>E</td>
<td>16.50028055</td>
<td>0.1412</td>
</tr>
<tr>
<td>F</td>
<td>8.229514967</td>
<td>0.10584</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0.10584</td>
</tr>
</tbody>
</table>

We calculated the expected value of losses in this scenario as $26.5963. We interpreted this as “on average, we could expect to lose $26.593” on a loan. To be prudent, we might suggest
a strategy of setting aside $26.5963 on a loan to cover our losses. Our discussion of variance suggested that this might not be enough. If we set aside only $26.5963 and the failure occurs before the first payment (Scenario A), we are still out an additional $26.403. If Scenario B occurred, we would be out $14.96. If Scenario C occurred, we would be out $6.56. In each of these scenarios, we failed to effectively protect ourselves from risk.

To improve our risk management strategy, we will introduce cumulative probability and percentiles (also called the “Value at Risk”). Instead of examining the probability of a specific event occurring, cumulative probability looks at the probability of a range or set of specific events occurring. More precisely, \( Pr[ X \leq K ] \) is the probability that the random variable \( X \) takes on a value less than or equal to some constant, \( K \). For example, if we were to consider rolling a random six-sided die, and set out constant \( K=4 \), then the cumulative probability at \( K=4 \) would be the probability the probability of rolling a value less than or equal to 4. Exhaustively, this is the probability of rolling a 1, rolling a 2, rolling a 3, or rolling a 4. We can calculate this as:

\[
\]

\[
Pr[Die Roll \leq 4] = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
\]

\[
Pr[Die Roll \leq 4] = \frac{4}{6}
\]

Instead of dice rolling; however, we return to our microfinance loan losses. The percentile, sometimes referred to as the Value at Risk is a closely related concept. We want to find an amount of money to set aside so that we are covered most of the time, say 95%. This percentage of the time is called the percentile, \( p \). After choosing a \( p \) we try to find a \( K \) that satisfies this condition. We are asking: “what fixed amount of money, \( K \), can we expect our losses to be less than, 95% of the time.” It turns out that in a world with a limited number of possible scenarios, a precise answer might not always be possible. In these situations, we choose the next closest value.
Returning to our example, we will reorder our scenarios from smallest loss (Scenario G) to greatest loss (Scenario A). We will then calculate the cumulative probabilities for each person and use these to determine the appropriate percentiles.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Present Value of Losses</th>
<th>Probability of Scenario</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0</td>
<td>0.10584</td>
<td>0.10584</td>
</tr>
<tr>
<td>F</td>
<td>8.229514967</td>
<td>0.10584</td>
<td>0.21168</td>
</tr>
<tr>
<td>E</td>
<td>16.50028059</td>
<td>0.14112</td>
<td>0.35280</td>
</tr>
<tr>
<td>D</td>
<td>24.81250352</td>
<td>0.1512</td>
<td>0.50400</td>
</tr>
<tr>
<td>C</td>
<td>33.16639167</td>
<td>0.216</td>
<td>0.72000</td>
</tr>
<tr>
<td>B</td>
<td>41.56215387</td>
<td>0.18</td>
<td>0.90000</td>
</tr>
<tr>
<td>A</td>
<td>50</td>
<td>0.1</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

Suppose that we wanted to cover all but the worse 10% of our loss scenarios for each person. Mathematically, this is equivalent to setting aside the same amount of money as our 90th percentile of losses. Reading our table, we see that the 90th percentile is $41.5621. If we set aside $41.5621 for our $50 loan, we should be able to cover our losses with 90% probability.

It is important to keep in mind that the probabilities in our example are fictitious. In real life, if we had to set aside $41.56 for every $50 loan to protect ourselves, we probably would not give out the loan in the first place.
3.10 Pooling Risk and Normal Approximation

3.10.1 Portfolio of Loans

Our analysis thus far has focused on individual loans—but our microfinance operations will consist of numerous loans! How do we calculate the expected value of losses for an entire portfolio? How do we calculate the variance of losses for an entire portfolio?

The expected value is straightforward—the expected value of losses in a portfolio is the sum of the individual expected values:

\[
E[\text{Portfolio of Loans}] = E[\text{Present Value of Losses on Loan 1}] + E[\text{Present Value of Losses on Loan 2}] + \cdots + E[\text{Present Value of Losses on Loan } n]
\]

If each loan is of the same amount and faces the same probabilities, then we can simplify this equation to:

\[
E[\text{Portfolio of Loans}] = n \times E[\text{Present Value of Losses on a Generic Loan}]
\]

Returning to our previous example, we calculated the expected value on a single loan to be $26.59. To find the expected value of present value losses in our \(n=100\) loan portfolio:

\[
E[\text{Present Value of Losses for Entire Portfolio}] = \$26.59 + \$26.59 + \cdots + \$26.59
\]

Simplified with \(n=100\):

\[
E[\text{Present Value of Losses for Entire Portfolio}] = 100 \times \$26.59
\]

\[
= 100 \times \$26.59
\]

\[
= \$2,659.00
\]
Previously, we calculated the expected present value of losses on an individual $50 loan to be $26.59, we calculated that the total expected present value of losses on a 100-loan portfolio of these identical loans to be $2,659.00.

Variance is not as simple—if loan failures are not independent of each other, we must introduce a covariance matrix to calculate the variance. We will assume that our loan failures are independent, an assumption that we will discuss in the next section. Assuming that loan failures are independent of each other, then their variances become additive:

\[
\text{Var} [\text{Present Value of Losses for Entire Portfolio}] = \text{Var} [\text{Present Value of Losses on Loan 1}] + \text{Var} [\text{Present Value of Losses on Loan 2}] + \cdots + \text{Var} [\text{Present Value of Losses on Loan n}]
\]

If the variance of each present value of losses is identical, we can simplify the calculation to:

\[
\text{Var} [\text{Present Value of Losses for Entire Portfolio}] = n \times \text{Var} [\text{Present Value of Losses on Generic Loan}]
\]

We calculated the variance of the present value of losses on a single generic loan to be 229.85. We can now find the variance of the present value of losses for an entire portfolio:

\[
\text{Var} [\text{Present Value of Losses for Entire Portfolio}] = 229.85 + 229.85 + \cdots + 229.85 = 229.85 \times 100 = 22,985
\]

The standard deviation for the portfolio would be the square root of this value:

\[
\text{Standard Deviation} = \sqrt{22,985} = 151.6
\]

It is important to note that you cannot directly add standard deviations—instead you must add the variances and then take the square root.
3.10.2 Normal Approximation

An interesting phenomenon in statistics occurs when we sum a large number of independent risks. While each individual loan follows some probability distribution that we defined earlier (the different scenarios and their respective probabilities), the distribution of the sum of these individual loans grouped together follows a different behavior—it follows the normal distribution.

The normal distribution is a bell shaped distribution with two parameters: the mean, $\mu$, and the standard deviation, $\sigma$. As before, these two parameters define the location and spread of the distribution. The mean, $\mu$, is the center of the normal distribution, while the standard deviation parameter determines how far spread out the probabilities are.

The normal distribution, and more specifically the standard normal distribution, is a very popular and well-studied type of distribution. The standard normal distribution is a normal distribution with parameters $\mu = 0$ and $\sigma = 1$, and its values, called z-values, have been calculated and published. Any other normal distribution is a scaled and shifted version of this distribution. To find the $p$th percentile of a normal distribution, based upon the standard normal distribution, we will use the following formula:

$$ pth\ percentile = \mu + z_p \cdot \sigma $$

Returning to our example, suppose that we want to find the 90th percentile of the present value of losses for our entire loan portfolio. We will use our formula with the following parameters:

$$ \mu = E[Present\ Value\ of\ Losses\ of\ Portfolio] = 100 \times 26.59 = 2,659 $$

$$ \sigma = \sqrt{Var[Present\ Value\ of\ Losses\ of\ Portfolio]} = \sqrt{100 \times 229.84} = 151.60 $$

$$ z_{0.90} = 1.28 \ (extracted\ from\ the\ Normal\ Distribution\ Table) $$

Our 90th percentile is then:
Recall that this amount represents the amount of money we would have to set aside to be able to cover our losses with 90\% probability.

Why is this intriguing? Recall our 90\textsuperscript{th} percentile for an individual loan: $41.56. If we were to insure each of the 100 loans up to the 90\textsuperscript{th} percentile individually (using their individual probabilities), we would need to set aside $41.56 \times 100 = $4,156. By pooling these loans together in a portfolio and using the normal distribution, we are able to achieve the same result for only $2,853.05! This is more than 30\% less than covering these losses individually!

Taking our $2,853.05 amount and dividing it by the number of loans we can calculate a “per-loan” premium. Dividing by 100 loans, we would set aside $28.53 per loan.

We have now successfully described a framework in which we can protect the Panamanian loan portfolio from risk. In the next chapter, we will discuss calibration and assumptions.
Chapter 4 Calibration

We have developed a mathematical framework to quantify different types of risks—namely individual loan risk characteristics and group risks. The quantified framework can appear logical and elegant; however, difficulties arise when trying to apply this model in real life. One of the biggest challenges we face is accurately setting our parameters—how do we choose an interest rate? How do we choose loan default probabilities?

While the task remains challenging, actuaries have methods to tackle these problems. These issues will be discussed below.

4.1 Choosing Interest Rates

We have discussed the time-value of money, but if we choose to set money aside it does not “magically” grow on its own. How do we make $1 accumulate interest? Invest. Some financial institutions actively manage stock, bond, and equity portfolios. Their return on investment is how their money accumulates.

Investing in stocks, bonds, and other financial instruments have their own component of risk—the stock price could go down in which case the investment would earn “negative interest.” It might seem strange to allow fluctuations in the stock market directly impact the success or failure of our microfinance operations. If our goal is to manage risk, it would seem counterintuitive to take on financial market risk along the way. To avoid this risk, we need to invest in something that is risk-free: government bonds. The idea behind government bonds being risk-free is that if the government defaults, the world will break into chaos and bigger issues will arise besides financial market risk.
The U.S. government publishes the interest rate return on their bonds daily:

<table>
<thead>
<tr>
<th>Date</th>
<th>1 Mo</th>
<th>3 Mo</th>
<th>6 Mo</th>
<th>1 Yr</th>
<th>2 Yr</th>
<th>3 Yr</th>
<th>5 Yr</th>
<th>7 Yr</th>
<th>10 Yr</th>
<th>20 Yr</th>
<th>30 Yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/1/2013</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
<td>0.14</td>
<td>0.23</td>
<td>0.36</td>
<td>0.76</td>
<td>1.23</td>
<td>1.86</td>
<td>2.70</td>
<td>3.08</td>
</tr>
</tbody>
</table>

Source: U.S. Treasury

As you can see, the interest rate is different for each length of bond. Actuarial firms will typically use sophisticated software to perfectly match maturities to specific calendar days. The associated cost of hiring a firm to implement such techniques greatly exceeds the needs of our microfinance project. Instead, we can make some assumptions and use a simpler interest model.

Since our loans of relatively short duration (less than 1-year), we can take the 6-month rate and assume that it is constant. Since the treasury yield rate is an effective annual rate, we can solve for the associated \( \delta \), we take the natural log of the \( 1 + \) the treasury yield rate:

\[
e^{\delta t} = (1 + \text{Treasury Yield Rate})^t
\]

Taking the natural log:

\[
\delta t \ln(e) = t \ln(1 + \text{Treasury Yield Rate})
\]

\[
\delta t * 1 = t \ln(1 + \text{Treasury Yield Rate})
\]

\[
\delta = \ln(1 + \text{Treasury Yield Rate})
\]

We can then use the \( \delta \) in our previous framework.

This \( \delta \) is not the true force of interest; however, it is a simplified estimate. When the values of \( t \) are small, the effect of error will also be small, but not non-existent.

In an effort to be more organic, it might make sense to use Panamanian treasury bills rather than U.S. It will be up to the financial officer to make the decision as to whether or not the bond is stable and that the associated yield is adequate. The collected insurance premiums could then be invested in these alternative investments to allow the money to accumulate with interest until a default payout occurs.
4.2 Building a Decrement Table

One of the largest implementation challenges is determining the probabilities of default. Our framework suggests that we either know the inherent probabilities of default, or that we have sufficient data. More than likely, we have neither.

If we begin collecting data, we can construct a decrement table for our loans—we begin with our total amount of issued loans and record the times at which each fail. Our table would look like:

<table>
<thead>
<tr>
<th>Month</th>
<th>Surviving Loans</th>
<th># of Failed Loans with</th>
<th># of Remaining Loans</th>
<th>Conditional Probability of Failure</th>
<th>Conditional Probability of Success</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>B</td>
<td>A-B</td>
<td>B/A</td>
<td>(A-B)/A</td>
</tr>
<tr>
<td>1</td>
<td>A-B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As we continue to collect more data, we could continue to update our table. We can use our last two columns to “string together” our unconditional probabilities and complete our framework as before.
4.3 Fitting a Poisson Distribution

When modeling our flood risk, we discussed using a Poisson distribution model. To choose the appropriate $\lambda$ parameter for this model, we will take historical flooding data and find the average number of floods per loan length time interval. The interval length is in our work is typically 6 months.

Unfortunately, limited public data is available for flooding in Panama, with no information specific to Piriati Embera. According to the International Charter for Space and Major Disasters, there has been exactly one flood in Panama in the last 10 years (Flood in Panama, 2010). Anecdotal evidence from my visit to Panama in the Spring of 2012 contradicts this information.

To obtain historical flooding data, our best hope might be to interview community members in Piriati Embera and ask them for their recollection of flooding over the past decade. Once a good estimate for the number of floods in Piriati Embera over the last $N$ years, $\lambda$ as follows:

$$\lambda = \frac{Number \ of \ Floods \ in \ the \ last \ N \ Years}{N \ years} \times Loan \ Length \ (Measured \ in \ Years)$$

If we were able to estimate the number of loans in the last $N=10$ years, the $\lambda$ for our 6-month loans would look like:

$$\lambda = \frac{Number \ of \ Floods \ in \ the \ last \ 10 \ Years}{10 \ years} \times 0.5$$

An important consideration when choosing the value for $\lambda$ is the quality of the data used. If written records have not been accurately kept or maintained, the quality of the $\lambda$ parameter is limited, and the accuracy of all subsequent calculations are limited as well.
Chapter 5

Decisions

Now that we have thoroughly covered several techniques for modeling risk in Piriati Embera, we must challenge the validity of our model. To the extent that our assumptions hold in real life, our model will be able to calculate risk—and conversely, its shortcomings will limit its impact on the microfinance system.

5.1 Assumptions

As with all modeling, a base-level set of assumptions is made. Since it is nearly impossible to account for every aspect, every possibility, every molecule of the real world, we can think of assumptions as pragmatic simplifications. In some sense, these are also the limitations of a quantified model. Our quantified risk model can only accurately measure risk to the extent that it captures the phenomena of the real world. As we make microfinance decisions, it is important to keep these shortcomings in mind—that the very specific number that the model spits out is far from a perfect representation of the real risk.

5.3.1 Static Properties

Our framework assumes that the world of the borrowers is mostly static—that many characteristics are not changing over time. This means that the size of the loans, the purpose of the loans, the attitude of the borrowers, and the fundamental characteristics of Piriati Embera remain the same. Is the data from the past world an accurate representation of the world today?
In the short-run, it is unlikely that the locals will wake up to find a drastically different world outside their window. Loans purposes and community dynamics will likely not change overnight. In the long-run; however, things can change. If we collect data about $50 loans, our model will be ineffective at modeling losses for brand new $1,000 loans. If borrowers shy away from agricultural loans toward fundamentally new types of business ventures, our models will not properly capture their respective risk. If Panamanians’ attitudes towards borrowing change over time, our models will not properly capture the risk. If macroeconomic trends of the world fundamentally change the markets for the goods produced in Piriápi Emberá—whether through in fluctuations in world demand or changes in international trade dynamics, our model’s accuracy will be limited.

Growth in developing communities can become rampant, changes that happen over decades in developed societies can happen over only months in developing societies. It is important to keep this consideration in the back of the risk manager’s conscious.

5.1.2 Independent Loans and Normal Distribution

A large assumption behind our risk model is that borrowers are independent and identically distributed. This means that one borrower’s individual risks do not interfere with another borrower’s individual risks. In real life, all members live in a shared community rather than complete isolation—the reality that friends and neighbors interact with each other every day does not 100% satisfy our assumption.

The use of the Central Limit Theorem and the Normal Distribution to reduce variance (and therefore percentiles) is a key component to advantageous risk pricing. If the borrowers do not truly satisfy these conditions, then our pricing methodology is no longer useful.
As an important assumption, the nature of loan independence is another consideration that the risk manager must consider when making decisions.

5.1.3 All Risks Are Accounted For

We defined two sources of risk: individual (and assumed independent) risk specific to individual borrowers, and group shock risk—risk that all members of the community face at the same time. Our individual risk fully captures our individual risk by definition—we defined this as any reason that an individual defaults.

Our joint risks; however, might not be fully captured. We identified floods as a potential source of this joint failure risk; it is possible however, that other risks exist but have not been identified or quantified. We have not priced insurance for wildfire, for tornadoes, or for war, just to name a few. Many of these risks are impractical to model. This does not mean that they do not exist. For any event that causes joint failure that we did not identify, its associated cost will not been accounted for.

5.1.4 The Introduction of Insurance Will Not Adversely Affect Default Probabilities

An important assumption is that the implementation of insurance will not create a moral hazard problem. This means that by having protection, borrowers are not given any new incentive to shirk on their loans. If borrowers are protected in the event of default, will they become complacent in the implementation of their business ventures?
5.2 Funding the Risk Premium

If our assumptions are met, and our calculations are correct, we still need to fund the risk premium. The funding of the risk premium will require two components: funding of the individual risk and funding of the common group risk.

Funding of the individual risk can be passed on directly to the borrower through an additional fee. Passing on the cost to the borrower is arguably the “fairest” implementation because it shows the borrower the true cost of taking on a loan without any Global Brigade funded subsidy. The significance of this is that the economic development is mostly in the hands of the community and not partially owned by Global Brigades—there is no unsustainable “free gift” transfer to the borrowers involved.

The funding of the risk premium for the flood risk; however, is more difficult. Because all of the loans are exposed to the same source of risk, the organization is not able to diversify in isolation. As one single risk, we must calculate the percentiles based upon our Poisson framework rather than through the cost-reducing normal approximation that we were able to capitalize with the individual risk. Without this cost-reducing component, the cost of the risk premium would be too high to be useful in Piriati Embera.

There are several options to fund this risk premium. The most direct solution would be to pay a third party to manage this risk: this could be in the form of working with an insurer (or reinsurer) to purchase stop-loss insurance on the entire community, or purchasing an over-the-counter weather derivatives contract from an investment bank. The main concept is to “pay somebody else to bear the risk.” The financial viability of these solutions will need to be evaluated based upon the third party’s own evaluation of the risk premium.

Global Brigades could also attempt to pool this risk through its own administration. This technique would require multiple independent communities that are exposed to independent flood
risks with similar frequencies. This could be other communities in Panama, so long as their respective probabilities of flooding are independent from the probabilities of flooding in Piriati Embera. For smaller flooding, this might seem reasonable; however, catastrophic flooding will probably affect a large geographical portion of the country. Global Brigades could pool risks of communities in different countries in which it operates, so long as the characteristics and probabilities of catastrophe are similar. If Global Brigades is able to pool a sufficiently large number of similar communities (at least 25), then they can use the normal approximation to fund the risk premium. This will be on a cheaper per-community basis.

For both individual and community risk premiums, a large moving component of the calculation is the actual probability of default—it should be clear that the microfinance organization should make every effort possible to actively minimize these probabilities. This means screening risky borrowers, providing extra financial education, and constantly communicating with borrowers to minimize default.

5.3 Conclusion and Future Considerations

I have identified quantified techniques to model Panamanian microfinance. By understanding the risk premiums associated with doing business in Piriati Embera, Global Brigades can create a sustainable and hedged operation. The funding of this risk premium can be self-administered by Global Business Brigades, or can be contracted out to a third party—an insurer, reinsurer, or an investment bank.

The greatest gain by using our actuarial methodology is to the borrowers in the community. By using an insurance-based approach, savings group members will no longer have to bear the burden of an honest-effort failed loan within their group. Rather than forcing the burden of a defaulted loan onto an entire group, the borrower pays the risk premium personally
and upfront. As discussed in Chapter 2, production decisions are enhanced and output increases as producers become insured. By insuring these positions, the community is making an effort to protect itself from depression-esque scenarios in which massive default occurs. Under the currently proposed operation, there is no guarantee that everything will operate smoothly forever.

After examining some of our model’s assumptions, we can conclude that some implementation logistics still need to be worked out. In the absence of strong contract enforcement, the microfinance operation has several choices.

Global Business Brigades could attempt to create a payout structure as outlined in Rai’s work and implement contracts through information monitoring and payout structure. Given the limited real-life experience with such strategies, this might be difficult for the microfinance project to immediately implement.

Pragmatically, Global Business Brigades could implement the insurance strategy as an additional layer to the existing Grameen strategy. Rather than radically changing the structure of the microfinance program, the insurance would serve as a supplement to the existing operations.

Ultimately, the final decision lies in the hands of the microfinance organization. Offering insurance is one mechanism to improve sustainability for economic development in the Piriati Embera community; however it is not exclusive to other supplementary techniques for risk management. While the discussed actuarial techniques provide opportunity to effectively manage risk, it does not supersede common sense and judgment by the leaders working in Piriati Embera.


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