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THE INFLUENCE OF ARISTOTLE'S DEMONSTRATION ON EUCLID

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ABSTRACT

The Greeks existed in a time of unparalleled progress. It was during this age that a synergistic apex of philosophy and mathematics thrived which can not be found anywhere else in the history of human knowledge. This paper will examine a subset of works forged by two of the greatest thinkers during this time. In particular, Aristotle's *Posterior Analytics* and Book I of Euclid's *Elements* will be explored. The objective of this thesis is to consider how Aristotle's scientific method, specifically his concept of demonstration, influenced Euclid.

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Section 1

Introduction

According to Thomas Heath, the Greeks existed in a time of unparalleled progress. Their society was saturated with thinkers and explorers whose salient interest was that of knowledge. In Heath's introduction, he identifies the Greek's "genius for mathematics" with their "genius for philosophy" (3). This apex of philosophy and mathematics cannot be found anywhere else in the history of human knowledge. In this paper, I will examine a subset of the works by Aristotle and Euclid. The works used will be Aristotle's *Posterior Analytics* and Book I of Euclid's *Elements*. The edition of Aristotle's works is a book compiled by Richard McKeon; Geoffrey Reginald Gilchrist Mure translates the *Posterior Analytics* and A.J. Jenkinson translates the *Prior Analytics*. Additionally, the translation of Euclid's *Elements* is by Thomas L. Heath. By examining a subset of their works, I hope to isolate two moments in history that are most revealing of Aristotle and Euclid's intention with respect to the scientific method.

My exploration of these texts is not meant to be exhaustive but rather it is an analysis based on appreciative and historical observations. An analysis based on anything other than a historical evaluation will generally lead to anachronistic inaccuracies. These two thinkers have much in common; both of their works function to organize information and create systematic proofs that produce scientific knowledge. McKeon maintains, "the *Organon* [is] concerned with two major problems, the technique of proof and the principles of proof … this method of constructing demonstrations and sciences was new in philosophy… (xvi)". As I will show, Euclid adopts this 'new' system of demonstration in his definitive treatise, the *Elements*. Moreover, I will discuss whether Aristotle would consider Euclid's proofs as productive of scientific knowledge and I will cite particular proofs from the *Elements* to substantiate my claims. Subsequently, I will discuss professional sentiments and my personal position on the subject. Finally, I will give an exposition of the Pythagorean theorem, which is the culmination of Book I of Euclid's *Elements* and a proof that I believe is most representative of Aristotle's influence on Euclid.

Section 2

Aristotle's Scientific Method

Aristotle's *Posterior Analytics* is the treatise most relevant to this exploration. This work, along with the *Prior Analytics*, is one part of a set of works later known as the *Organon*. These books collectively reveal Aristotle's ideas with respect to his scientific method and logic. The *Posterior Analytics* concerns demonstration and the practical application of its more theoretical partner, the *Prior Analytics*. Aristotle's scientific method relies on deduction and demonstration. The fundamental vehicle for demonstration is his concept of the syllogism, deductive in structure, which is not synonymous with demonstration. Demonstration produces knowledge through the medium of deduction. Aristotle's sense of his scientific method, observed through the *Posterior Analytics*, can be mapped onto Euclid's exposition in Book I of the *Elements*, the commencement of his geometrical science. Aristotle contends that there are three elements in demonstration: "[1] what is proved, the conclusion – an attribute inhering essentially in a genus; [2] the axioms, i.e. axioms which are premises of demonstration; [3] the subject-genus whose attributes, i.e. essential properties, are revealed by demonstration (121)." Unless otherwise stated, all references in this section are credited to Richard McKeon's *The Basic Works of Aristotle*.

This section will serve as a summary and a brief commentary of the *Posterior Analytics*. Not all of this information is relevant to the following section on Euclid but it is pertinent in order to fully understand Aristotle and elaborate on the three elements in demonstration mentioned above. This section can be broken up into the following subsections: form versus matter, the shape of a syllogism, what determines an axiom, what determines a definition, what is the most effective form of an argument, what is the most effective form of demonstration, and Aristotle's four questions that reveal knowledge.

The first subsection deals with form versus matter. This is congruent to the difference between deduction and demonstration, respectively. For Aristotle, if an argument is to be sound it cannot only be deductively valid; the content of the reasoning must also be true. This marks the difference between the shape of an argument and the substance of it. An argument can be deductively legitimate, but flawed because it is based upon false premises. Again, this is fundamentally the distinction between deduction and demonstration. In the *Posterior Analytics*, Aristotle states, "By demonstration I mean a syllogism productive of scientific knowledge". To delineate demonstration, he establishes a criterion for premises that may precede a logical conclusion, "Assuming then that my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause. Unless these basic conditions are satisfied, the basic truths will not be 'appropriate' to the conclusion. Syllogism there may indeed be without these conditions, but such syllogism, not being productive of scientific knowledge, will not be demonstration (112)." This statement and many others similar to it provide evidence of the fact that Aristotle saw deduction as somewhat empty. Deduction is only a vessel until it is filled with true premises. Therefore, to advance scientific knowledge both the method of correct form and matter are necessary.

Now that we understand why Aristotle argues that there is a difference between deduction and demonstration, we can move onto the next subsection that deals with Aristotle's concept of a syllogism. Though the form of a syllogism can be empty in manifestation, as was just said of deduction, the formation carries some meaning as a consequence of its arrangement. To say that it is empty alludes to the fact that false consequences can be legitimately deduced but not demonstrated. Scientific knowledge is contingent on knowing the cause of the 'thing' in question (113). Even in the very beginning of the *Posterior Analytics*, Aristotle refers to cause and effect. He states, "The premises must be the causes of the conclusion" (112). In a syllogism, the cause is represented as the premises and the conclusion exhibits the effect. So it is the nature of the premise that expresses the cause and leads to a consequential effect. Cause and effect are representative of the relationship present in a syllogism. In his *Prior Analytics*, Aristotle discusses various forms of the syllogism of which there are three figures. Each figure is a syllogism that is operative by way of three terms: two of these steps are premises and both steps contain the middle term in a subject-predicate order corresponding to one of the three figures. "Demonstrative knowledge must be knowledge of a necessary nexus, and therefore must clearly be obtained through a necessary middle term; otherwise its possessor will know neither the cause nor the fact that his conclusion is a necessary connexion" (120). To be of the first figure, the middle term must be the subject of one premise and the predicate of another premise. Concerning the figures, "[the] most scientific is the first. Thus it is the vehicle of the demonstrations of all the mathematical sciences, such as arithmetic, geometry, and optics, and practically of all sciences that investigate causes … Clearly, the first figure is the primary condition of knowledge" (131). The middle term is the connective term in the syllogism and the term most responsible for exhibiting the relation between the cause and the effect. This is analogous to the theory of proportion in which two ratios are similar; it is not simply an equation. A ratio asserts a similarity between two homogenous terms but a proportion among ratios can put into relation the ratios of magnitude and area i.e. non-homogenous terms. Ergo, the middle term is the most significant term in deduction and demonstration. Therefore, this is the shape of Aristotle's syllogism.

This subsection concerns what determines an axiom. An important stipulation for a premise is that it be indemonstrable. This notion is distinct from that of a premise being necessarily primary. To be indemonstrable, a premise must be consistent with Aristotle's sense of an axiom whose truth is self-evident. If the premise is demonstrable then it is not independent of previous proof and cannot be created as basic knowledge. Aristotle further explains the sense of an axiom when he asserts, "A 'basic truth' in a demonstration is an immediate proposition. An immediate proposition is one which has no other proposition prior to it" (113). There are two types of basic truths for Aristotle. There are basic truths, which are the first principles of logic, and those truths are applicable in any science; however, there are also basic truths that must be specific to the subject in order for demonstration to be possible. An axiom is this first type of basic truth. Aristotle never names this second type of basic truth but for Euclid, first principles that are specific to geometry are called postulates. Further, "demonstrative logic must rest on necessary basic truths; for the object of scientific knowledge cannot be other than it is (119) … I call the basic truths of every genus those elements in it the existence of which cannot be proved" (124). An axiom then is a basic truth or a general principle of logic while other basic truths are specific to the subject; Euclid later calls these second basic truths postulates. Both types of basic truth are necessary for demonstration.

This subsection is devoted to determining Aristotle's concept of a definition, another essential phrase. As Aristotle explains what determines a definition, he constantly refers to demonstration as well. So, this concerns the third element mentioned about demonstration at the beginning of this section. Concerning the purpose of a definition, "the basic premises of demonstrations are definitions… definition reveals essential nature, demonstration reveals that a given attribute attaches or does not attach to a given subject" (161). A 'thing' has attributes if it "has a cause that is distinct from itself" so it is through demonstration that this "essential nature is exhibited" (169). To elaborate on this relationship between definition and demonstration, Aristotle claims that definition does not guarantee existence (166) and to "know its essential nature is, as we said, the same as to know the cause of a thing's existence" (167). Furthermore, "not all that is definable is demonstrable nor all the demonstrable definable" (162). The correspondence between definition and demonstration may seem confusing but it amounts to the fact that definitions cannot produce anything by themselves; they need demonstration to be productive. Finally, concerning what a definition is, Aristotle states, "to define is to prove either a thing's essential nature or the meaning of its name, we may conclude that definition, if it in no sense proves essential nature, is a set of words signifying precisely what a name signifies" (166). Without demonstration, proving the existence of a thing, a definition is just a set of words. Therefore, this is what determines a definition for Aristotle.

Now that we have parsed Aristotle's language specific to his scientific method, let's consider the more philosophical perspectives of his technique. This subsection is dedicated to discovering what Aristotle considered to be the most effective type of argument. Aristotle discusses types of argumentation in pairs. For instance, he puts induction in opposition with deduction. Letting 'demonstration' supersede the action of 'deduction', he says, "Demonstration develops from universals, induction from particulars (136)". Although he believes both induction and deduction have their benefits in ascertaining knowledge, he favors deduction because he regards induction as circular in its reasoning (114). The nature of knowledge will be discussed in the last subsection and Aristotle gives a vital quality of scientific knowledge in the context of induction and deduction. Maintaining that scientific knowledge is only functional through deduction, he says, "Scientific knowledge is not possible through the act of perception... for perception must be of a particular, whereas scientific knowledge involves the recognition of the commensurate universal… for the commensurate universal is elicited from the several groups of singulars” (154). Ergo, it is necessary that scientific knowledge participates in the universal i.e. deduction rather than induction. Relating back to the ‘cause’ again, “the commensurate universal is precious because it makes clear the cause; so that in the case of facts like these which have a cause other than themselves universal knowledge is more precious than sense-perceptions and than intuition” (154). The next type of arguments he presents in a pair is dialectical and demonstrative arguments. Aristotle aligns demonstration with the mathematical and dialectic with rhetoric. In the introduction by the compiler, McKeon notes, “…the dialectical method, which was the scientific and philosophic method according to Plato, becomes according to Aristotle a second best method distinct from the method of science” (xvii). Aristotle presents a dialectical proof, saying, “for if there are basic truths, (a) not all truths are demonstrable, and (b) an infinite regress is impossible” but still maintains that it is not rigorous enough (145). He therefore concludes that demonstration is preferable to a dialectical argument. In the previous subsection, we proved that demonstration is superior to deduction, therefore demonstration is the most effective form of argument overall.

Having established that demonstration is the most effective form of argument, let’s determine in this subsection what is the most effective form of demonstration for Aristotle. The goal of demonstration is to produce greater knowledge, but what form of demonstration is the most successful i.e. the most concise while also producing the most scientific knowledge? The ‘cause’ is especially revealing of a demonstration’s success. “Since demonstrations may be either commensurately universal or particular, and either affirmative or negative; the question arises, which form is the better? And the same question may be put in regard to so called ‘direct’ demonstration and *reductio ad impossibile* (147).” It is important to note here that proofs by *reductio ad impossibile* are still represented syllogistically. Aristotle says in the *Prior Analytics*, “Syllogisms which lead to impossible conclusions are similar to ostensive syllogisms; they are also formed by means of the consequents and antecedents of the terms in question. In both cases the same inquiry is involved. For what is proved ostensively may also be concluded syllogistically *per impossibile* by means of the same terms; and what is proved *per impossibile* may also be proved ostensively…" (89). The form of demonstration that proves to be the most successful stems from Aristotle's contrast between induction and deduction. Induction is an appeal to particulars, while deduction is an appeal to universals. Aristotle claims that cause is found in universals because a singular event cannot exhibit a reason. "Demonstration is syllogism that proves the cause, i.e. the reasoned fact, and it is rather the commensurate universal than the particular which is causative. Consequently commensurately universal demonstration is superior as more especially proving the cause, that is the reasoned fact (149)." Likewise, affirmation is preferable to negation: "affirmative demonstration excels negative … [And also] there cannot be more than one negative premises in each complete proof (150) … affirmative demonstration is more of the nature of a basic form of proof, because it is *sine qua non* of negative demonstration (152)." Hence, the affirmative is always preferable to the negative in a proof. Furthermore, "since affirmative demonstration is superior to negative, it is clearly superior also to *reductio ad impossibile* (152)." Therefore, the most effective form of demonstration deals in universals and the affirmative.

The last subsection, which like the last two is involved in the philosophical interests of Aristotle, will deal with knowledge and his four questions that reveal knowledge. Let's consider the latter first; in Book 2 of the *Posterior Analytics*, he begins right away by stating that there are four questions, "[1] whether the connexion of an attribute with a thing is a fact, [2] what is the reason of the connexion, [3] whether a thing exists, [4] what is the nature of the thing (158) … these, then, are the four kinds of questions we ask, and it is in the answers to these questions that our knowledge consists (159)". All of these questions seek the 'middle', which refers to the crucial middle term discussed in a previous subsection, but it is not necessarily a term here. Instead, the way in which we can understand this 'middle' he states, "We conclude that in all our inquires we are asking either whether there is a 'middle' or what the 'middle' is: for the 'middle' here is precisely the cause, and it is the cause that we seek in all of our inquiries". For Aristotle, the cause is essential, "to know a thing's nature is to know the reason why it is… it is clear then that all questions are a search for a 'middle' " (160). Now that we have discussed the significance of Aristotle's four questions and what the true purpose of them is, let's move on to the former consideration of this subsection; specifically, to determine exactly what constitutes knowledge. Simply being the converse of opinion does not clarify it; such juxtaposition only provides a silhouette against a negative background. Something cannot be both knowledge and opinion. Corresponding to what Aristotle asserts as the elements of demonstration, "Consequently a proof even from true, indemonstrable, and immediate premises does not constitute knowledge (123) … we think we have scientific knowledge if we have reasoned from true and primary premises. But that is not so: the conclusion must be homogeneous with the basic facts of the science" (124). Through Aristotle's exposition of the form and substance of his demonstrative method, the production of true knowledge is possible. This is important point because for Aristotle, the production of scientific knowledge is the most indispensible task.

In conclusion, through the *Posterior Analytics* Aristotle establishes a rigorous language as a system for producing scientific knowledge. In *A History of Greek Mathematics*, Heath says of Aristotle, "he gives the clearest distinctions between axioms (which are common to all sciences), definitions, hypotheses, and postulates (which are different for different sciences since they relate to the subject-matter of the particular science)" (336). These distinctions are necessary if one wishes to be constructive in the task of obtaining scientific knowledge. Aristotle is equally rigorous concerning all of the subjects discussed in each subsection: I discussed that it is not enough for an argument to be in correct deductive form, the substance contained in the form must meet the criteria that Aristotle has described otherwise demonstration cannot occur. I also discussed the shape of a syllogism and how this transforms an argument into a meaningful endeavor, and what determines an axiom and a definition for Aristotle. Additionally, the philosophical considerations of Aristotle were also discussed i.e. how demonstration is the most effective form of argument and what type of demonstration is the most successful. Lastly, I discussed what constitues knowledge and Aristotle's four questions that seek a 'middle' or a cause. The production of scientific knowledge is of the upmost importance to Aristotle. In the next section, I will analyze Euclid's *Elements* Book I and discover how Aristotle's method influenced Euclid.

Section 3

Evidence in Euclid's Elements

Euclid's masterpiece is comprised of thirteen books. This analysis will concentrate on the first book. Euclid's treatises is not the first 'textbook' thought to exist during Greek times, however it is the most definitive. Another treatise was likely in existence directly before Euclid, around the time of Aristotle, since Aristotle seems often to refer to such a book, which may have been common in Plato's Academy. In the biographical note to Euclid's translation, there is a quote by Proclus, "[Euclid] collected many of the theorems of Eudoxus, perfected many of those of Theatetus, and also brought to incontrovertible demonstration the things which were only loosely proved by his predecessors" (ix). According to Netz, Proclus is most relevant for "a commentary on Euclid's *Elements* Book I" (315). Proclus, by using the word 'demonstration', was indisputably referring to the way in which Aristotle formulated his scientific method. To 'demonstrate' was to invest in a technique pioneered and developed by Aristotle. Unless otherwise stated, all references in this section are credited to Thomas Heath's translation of Euclid's *Elements* located in the Encyclopaedia Britannica's *Great Books of the Western World*.

Using Aristotle's first-hand account of his scientific method, I will determine whether and how Aristotle informed Euclid's exposition. Some relevant questions would be: Does Euclid use terminology similar to Aristotle? Are Euclid's proofs syllogistic and deductive in structure? Does Euclid obey Aristotle's sense of premises and conclusion regarding both criteria and their relationship to each other? In particular, how does Euclid treat the middle term? More importantly, are his proofs demonstrative? Lastly, does Euclid execute what Aristotle considers to be the most effective form of demonstration? These questions will illustrate the progress of the scientific method between two great Greek thinkers.

Let's begin with the first question; does Euclid use the same terminology as Aristotle? It is remarkable that Euclid's method is often and almost always described as 'axiomatic', however, Euclid never used the word 'axiom'. How can we resolve this? The translation is by Thomas Heath, the nonpariel authority on Greek mathematics, so it can be certain that this rendering, found in the *Great Books of the Western World*, was neither a mistake nor an oversight but a reality that Euclid never intended to use the word axiom. The only techincal terms Euclid assigns to his catalog are definitions, postulates, common notions, and propositions. The propositions are either followed by a construction or a proof. Those propositions intended to be constructions conclude with either "(Being) what it was required to do" or "Q.E.F." (3,7). Those propositions intended to be demonstrated by proof finish with "Q.E.D." (5). The difference between contruction and proof can be understood by these terminating expressions. The letters Q.E.F. are abridged from the phrase *quod erat faciendum* when translated is "(Being) what it was required to do" while Q.E.D. is abbreviated from the Latin *quod erat demonstrandum* signifying that which was to be demonstrated has been demonstrated. As was mentioned in the beginning of the section, Proclus recognized this demonstration, the significance of which cannot be diminished. Therefore, it stands to reason that Euclid used the phrase *quod erat demonstrandum* deliberately as a throwback to the technique of Aristotle's methodical demonstration. Now that we can be sure that Euclid did in fact intend to proceed in an Aristotelian axiomatotic way, the question still remains, how do the terminologies of these two men intersect? Though Euclid did not use the word axiom he does use the expression 'common notions'. In *A History of Greek Mathematics*, Heath says "Aristotle calls axioms by various terms, 'common (things)', 'common axioms', 'common opinions', and this seems to be the origin of "common notions', the term by which they are discribed in the text of Euclid" (336). Hence, Aristotle's sense of an axiom is plotted to Euclid's common notions. This still leaves three technical terms of Euclid's left to explore, namely definitions, postulates, and propositons. Euclid has many equivalents of Aristotle's defintions and Euclid's postulates, the basic truths specific to a science, "correspond exactly enough to Aristotle's idea of a postulate". Definitions, according to Aristotle, "do not assert existence or non-existence; they only require to be understood" and postulates are something which "the geometer assumes (for reasons known to himself) without demonstration (though properly a subject for demonstration) and without any assent on the part of the learner, or even against his opinion rather than otherwise" (336-337). Evidence that Euclid adopts Aristotle's sense of a definition is witnessed in the fact that Euclid's constructions prove existence and existence cannot be ascertained through a definition alone. These terms, specifically definitions, postulates, and common notions, are the building blocks in the proofs of Euclid's propositions and consequently the culmination of Aristotle's intention in demonstration. Therefore, the terminology of these two men is very similar and actually almost exactant in its implication. This alone does not reveal anything except that these they existed in a similar intellectual climate with similar expectations and conventions. However, I believe the most telling of these terminologies is Euclid's concluding phrase *quod erat demonstrandum* literally meaning to demonstrate or prove by logically valid premises. The technique of demonstration was unique to Aristotle and definitively a significant influence on the logical success of Euclid's *Elements*.

Now that the first question has been satisfied, let's move on to the second question; are Euclid's proofs syllogistic and deductive in structure? This question may not be as simple as one might expect. In Aristotle's work, an argument is usually simultaneously syllogistic and deductive. In the section concerning Aristotle it is established that a syllogism has three parts and that the conclusion is a logical consequence of the first two parts. The first figure is the syllogism most applicable to mathematical proof and this means that the imperative middle term, previously examined, is the subject of the major premise and the predicate of the minor premise e.g. If A is predicated of all B and if B of all C, consequently A is predicated of all C where B is the middle term. However, as one can identify just from glancing at Euclid's proofs, they are not immediately available in three straightforward parts. This may be cause for confusion but if one can conceive that Euclid took liberty with Aristotle's rigid syllogistic structure, these proofs can still be reconciled with Aristotle's convention of deduction. So overall the question must be, are Euclid's proofs logically valid? To be logically sound, the proofs must be deductive but not necessarily demonstrative and have steps that are flexibly syllogistic. As an example, let's look at Euclid's Proposition 6 in detail. For the purposes of this example, Heath's exact translation is used and not put into quotations but rather italicized. The proposition states, *if in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.* The following is Euclid's proof wherein I have added the enumeration of the steps:

(i) *Let ABC be a triangle having the angle ABC equal to the angle ACB;*

(ii) *I say that the side AB is also equal to the side AC.*

(iii) *For, if AB is unequal to AC, one of them is greater.*

(iv) *Let AB be greater; and from AB greater let DB be cut off equal to AC the less;*

(v) *Let DC be joined.*

(vi) *Then, since DB is equal to AC, and BC is common, the two sides DB, BC are equal to the two sides AC, CB respectively; and the angle DBC is equal to the angle ACB;*

(vii) *Therefore the base DC is equal to the base AB, and the triangle DBC will be equal to the triangle ACB, the less to the greater: which is absurd.*

(viii) *Therefore, AB is not unequal to AC; it is therefore equal to it.*

(ix) *Therefore etc. … Q.E.D.*

First, let's examine the steps of the proof before trying to determine whether it is deductive or syllogistic in nature. In step (i), Euclid is forming a representative triangle with two equal angles. In steps (ii) and (iii), Euclid is asserting a conclusion prior to the actual proof. The purpose of this may be to guide the reader and inform them of the proof's intent. He is essentially saying, 'is it valid to conclude this?'. Step (iv) is the beginning of Euclid's *reductio ad impossibile* proof as he begins with a contradiction of his own premises. Steps (iv) through (vi) are an explanation of this inconsistent triangle. In step (vii) Euclid comes to a subsidiary conclusion and instead of declaring 'this results in a contradiction' Euclid states '[this] is absurd'. Thereby his initial premise must be upheld. In the last step (ix), Euclid maintains this proposition is true of all similar triangles. Now the nature of this proof can be determined. Although this proof is not contained in three steps but rather 9 (give or take personal enumeration) it does contain a syllogism in a sense. Steps (iv) through (vi) are a three-tiered argument beginning with a major premise (iv), a minor premise (v), and a conclusion (vi), which is subordinate to the actual conclusion. Taking even more liberty with the structure of the syllogism, let's imagine that steps (i) through (iii) are the major premise of the superior 'syllogism', while steps (iv) through (vi) are the minor premise and (vii) through (ix) is the necessary conclusion comprised of three conclusions each responding to the inferior syllogism, the superior syllogism, and the universal respectively. However, to be manifest of a true syllogism there must be a subject-predicate relationship and in this sense the proof does not entirely map onto Aristotle's sense of a syllogism. Still there is a sense of the first figure in both the inferior and superior 'syllogisms' and through this series of 'syllogisms' a richer justification is provided then if the argument was presented in a conventional syllogism. Though Euclid did not absolutely conform to Aristotle's concept of a syllogism this does not exclude the possibility that his proofs are deductive. In fact, as Euclid moves from the general to the more specific he is practicing deduction, which Aristotle perceived as the most appropriate manner of reasoning. All of Euclid's definitions, postulates, and common notions deal in the general; it is the deductive proofs that bring the postulates into the specific. Hence, to answer the second question, Euclid's proofs can only be viewed as liberally syllogistic but are still clearly deductive.

The next two questions can be considered together. Does Euclid obey Aristotle's sense of premises and conclusion regarding both criterion and their relationship to each other and more specifically, how does Euclid treat the middle term? As a second example, let's look at Euclid's Proposition 36 in detail. Again, for the purposes of this example, Heath's exact translation is italicized. The proposition states, *parallelograms which are on equal bases and in the same parallels are equal to one another.* The following is Euclid's proof wherein I have added the enumeration of the steps:

(i) *Let ABCD, EFGH be parallelograms which are on equal bases BC, FG, and in the same parallels AH, BG;*

(ii) *I say that the parallelogram ABCD is equal to EFGH.*

(iii) *For let BE, CH be joined.*

(iv) *Then, since BC is equal to FG,*
while FG is equal to EH,
BC is also equal to EH.

(v) *But they are also parallel.*

(vi) *And EB, HC join them; but straight lines joining equal and parallel straight lines (at the extremities which are) in the same directions (respectively) are equal and parallel.*

(vii) *Therefore EBCH is a parallelogram.*

(viii) *And it is equal to ABCD; for it has the same base BC with it, and is in the same parallels BC, AH with it.*

(ix) *For the same reason also EFGH is equal to the same EBCH;*

(x) *So that the parallelogram ABCD is also equal to EFGH.*

(xi) *Therefore etc. … Q.E.D.*

The numeration of this proof was more or less in accordance with the punctuation given in the translation. First, let's examine the proofs before relating the proof to the questions guiding this subsection. In step (i) Euclid is forming two representative parallelograms that have equal bases and opposite sides parallel. In step (ii) he is making a preliminary conclusion as a guide for the intention of the proof, namely that the parallelogram ABCD is equal to EFGH. In step (iii) he draws two lines inside the parallelograms. In step (iv) he uses the second parallelogram to show that the bases of the new and first parallelogram are equal. Step (v) through (xi) is a relation of parallels and parallelograms. It is interesting to note that step (iv) in the proof is a straightforward syllogism where the middle term is FG, the base of the parallelogram EFGH. Another syllogism is extended throughout the second half of the proof in steps (vii) through (x), although the order of the subject and predicate is reversed, the middle term is EBCH, the new parallelogram. So there are two conventional syllogisms contained in the proof. Reconciling this with our previous example, although Euclid at times demonstrates a kind of fidelity to Aristotle's theory of the syllogism, it is still in a series of justifications that result in a much richer proof then accessible through the conventional syllogism. Now back to the questions, does Euclid obey Aristotle's sense of premises and conclusion regarding both criterion and their relationship to each other and more specifically, how does Euclid treat the middle term? These questions are different formulations of the same objective. Let's consider the former question first. In the *Posterior Analytics*, noted in previous section, Aristotle states "…the premises of demonstrated knowledge must be true, primary, immediate, better known then and prior to the conclusion, which is further related to them as effect to cause." For Euclid, and mathematics in general, the middle term cannot be causal in the sense that Aristotle defined it. The middle term for Euclid must be the term that brings the two other terms into rational relation. So it still remains, are the premises true, primary, immediate, and prior to the conclusion? This is certainly true of the syllogisms Euclid applies in this proof but we must also consider the superior series of premises, i.e. the overall proof, not just the syllogistic element contained within it. Let's assume for this proof that the inferior syllogisms are already known to be true so we can reduce it to the conclusions it contains. A reduced deductive proof would look something like this:

I. Let ABCD, EFGH be parallelograms which have equal bases BC, FG, and AH, BG are parallel; Connect BE and CH to create two lines and a new parallelogram contained in the first two, EBCH.

II. BC is equal to EH and they are parallel lines.

III. Therefore, EBCH = ABCD and EBCH = EFGH.

IV. Therefore, ABCD = EFGH.

Again, please note that this is a reduction for the purposes of an argument and not a claim to improve the proof of Postulate 36. I have condensed the construction of the diagram into step (I) to limit the amount of time wasted assembling such a figure. In any case, this reduced proof can reveal the characteristics of the premises. The premises are true since we have constructed a valid figure, they are primary because they are common to us and unique to this conclusion, they are immediate because there can be no intermediary deductions, and they are prior to the conclusion because we know them better then the consequence. So, the former question has been resolved. Now, let's focus on the latter question. At the beginning of this section, it is confirmed by Proclus that Euclid incorporated many theorems of Eudoxus into the *Elements*. Actually, not much else is known about Eudoxus except through Euclid. Eudoxus and Aristotle were both pupils of Plato, though it is believed the intersection of their lives was minimal and Eudoxus could still technically be considered a predecessor to Aristotle's work. According the Stanford Encyclopedia of Philosophy, "the discovery of universal proofs is usually associated with Eudoxus' theory of proportion". Eudoxus is relevant here because his theory of proportion and ratios informed

Aristotle's concept of the middle term. A syllogism is resembles a ratio or a series of proportions. Furthermore, Euclid was influenced by both of these men and uses the syllogism and its necessary term liberally but with success. Hence Euclid's treatment of the middle term, besides the causal nature, is directly aligned with Aristotle's conception of it. Therefore, Euclid does obey Aristotle's sense of premises and conclusion regarding both criterion and their relationship to each other and abides by the conditions necessary for an effective middle term.

Now that the questions regarding the middle term and the relationship between the premises and conclusion have been satisfied, let's concentrate on the next question. Can Euclid's proofs be regarded as demonstrations? This may be the most important of question of all because valid demonstration is the only way to produce scientific knowledge according to Aristotle. All of the previous questions have provided some evidence for the fact that Euclid is in fact performing demonstration in his proofs. Again, let us not confuse proofs and constructions, as the two are separate and distinct in their formulations and meanings. From the subsections above, we know that Euclid states 'Q.E.D.' at the end of his proofs, from the Latin phrase *quod erat demonstrandum* which literally translates to completing the act of demonstration, and while only liberally syllogistic, Euclid's proofs are certainly deductive in nature. Furthermore, as was just shown, Euclid respects the relationship and criterion of premises and conclusions, particularly in his treament of the middle term. This evidence is an indication that Euclid was in fact demonstrating. Moreover, to truly demonstrate for Aristotle, scientific knowledge must be produced. I contend that the preceeding evidence satisfies this claim and Euclid's proofs are not only deductive but demonstrative and therefore productive of scientific knowledge.

Now that we have determined that Euclid is actually participating in Aristotle's idea of demonstration, let's consider the last question: does Euclid execute what Aristotle considers to be the most effective form of demonstration? From the previous section, the most constructive form of demonstration for Aristotle deals in the universal commensurate and the affirmative; both are superior to the negative and *reductio ad impossibile*. So the question must now be, do Euclid's proofs conform to demonstration by way of universals and the affirmative? It is obvious that this is not always the case and in fact, Euclid tends to use almost all types of demonstrations in his proofs. In my first example, Proposition 6 is not a direct proof but rather a *reductio ad impossibile*. This is not common among Euclid's proofs and he only proves a few propositions in this way, but it is evidence that according to Aristotle, Euclid did not always use the most effective form of demonstration. In my second example, Proposition 36 is a direct and affirmative proof. Though it contains an extended series of 'syllogisms', the inferior syllogisms are ideal and conventional, while the superior 'syllogism' is only liberally labeled so. This is the form that Euclid uses most often when demonstrating propositions in Book I. Therefore, it is obvious that Aristotle's influence is evident in Euclid's proofs.

In conclusion, there is ample evidence in Euclid's *Elements* Book I that Aristotle directly influenced Euclid's method both in form and in function. In this section I was able to prove the following: the terminology that is specific to Aristotle's scientific method can also be found in Euclid's work, the use of meaningful words like 'common notion' and 'definition' is practically identical. For Aristotle the vehicle of deduction is the syllogism but Euclid only liberally participates in this task and establishes instead a series of syllogisms in his proofs. Overall, his proofs are still deductive in nature. Euclid fulfills Aristotle's criteria for premises and conclusion by ensuring that the premises are true, primary, immediate, and prior to the conclusion. This establishes a robust middle term, which is necessary in order to obtain a logical consequence in a proof i.e. to prove a proposition. I also discussed how Euclid does in fact display demonstration in his proofs; again this is not synonymous with his constructions. Additionally, I explored whether or not Euclid used the most effective form of demonstration according to Aristotle. Euclid satisfies all of the above conditions and therefore it is obvious that Aristotle directly influenced Euclid and the method Euclid uses in Book I of the *Elements*. Through demonstration, a technique pioneered by Aristotle, Euclid composed the most definitive and comprehensive treatise regarding geometry and its proofs.

Section 4

Opinions in Academia

This section deals with summaries of those who have provided extensive commentary on Aristotle and Euclid's treatises and a handful of opinions explicitly regarding their connection. Among the authorities designated by the former are Hippocrates Apostle, John Corcoran, and Ian Mueller; while Thomas Heath and Reviel Netz form the latter. Although scholarly assessment will always be secondary to the original sources, I find it indispensible to review the literature presented by these experts to effectively participate in a scholarly conversation and stake out my own position.

The first scholar I will discuss is Apostle, who provides a commentary of Aristotle's philosophy of mathematics. In reference to Aristotle's scientific method, Apostle says, "The method chosen to present Aristotle's philosophy of mathematics is his own. It is discussed in the *Posterior Analytics* and carried out in the *Physics*, *Metaphysics*, *De Anima*, and the rest of the sciences" (vii). Apostle asserts that mathematics is a theoretical science, a science that seeks truth, and it "proceeds according to a method… [and] each of these sciences, having few and definite principles and elements, lends itself easily to demonstration" (32). Furthermore, in mathematics, "definitions and the other principles are laid down at the start, and properties of numbers and magnitudes are demonstrated" (33). His commentary contains a faithful digest of Aristotle's method and the language specific to that method. Finally, Apostle states, "to mathematics belong the task of demonstrating whatever is demonstrable from these principles, for mathematics is a demonstrative science; and the aim of such a science is scientific knowledge, that is, knowledge reached by demonstration" (33). Apostle foregrounds Aristotle's unique but simple approach to mathematics which culminates in a philosophy of mathematics. Therefore, Aristotle's philosophy of mathematics is essential to understanding the scientific method.

Another important commentator on the subject of Aristotle's work is Corcoran; in *Aristotle's Demonstrative Logic*, he discusses and defends Aristotle's method of demonstration. He also establishes the differences between deduction and demonstration. Corcoran summarizes: "Demonstrative logic, the study of demonstration as opposed to persuasion, is the subject of Aristotle's two-volume Analytics. Many examples are geometrical. Demonstration produces knowledge… persuasion produces opinions" (1). It is important for Corcoran to severely distinguish between knowledge and opinions because for Aristotle sometimes one believes one has knowledge but it is not so; it is only by rigorous method that one can ascertain whether he or she has knowledge or opinion. He beautifully recapitulates the method of demonstration, "According to [Aristotle], a demonstration, which normally proves a conclusion not previously known to be true, is an extended argumentation beginning with premises known to be truths and containing a chain of reasoning showing by deductively evident steps that its conclusion is a consequence of its premises" (1). It is essential to understand that the conclusion is a necessary 'consequence' of its premises; this is related to the discussion in the section on Aristotle concerning the relationship between cause and effect. Finally, Corcoran points out a subtle nuance of Aristotle's work, " [His] general theory of demonstration required a prior general theory of deduction presented in the *Prior Analytics*… To demonstrate his general theory of deduction, [Aristotle] presented an ingeniously simple and mathematically precise special case traditionally known as the catergorical syllogistic" (2). Corcoran emphasizes the simplicity of Aristotle's scientific method while also drawing attention to the insight of his demonstration.

Let's move on to a scholar who has provided commentary on the Euclid's method. In the *Philosophy of Mathematics and Deductive Structure in Euclid's Elements*, Ian Mueller provides a dissection of Euclid's treatise. He indicates that Book I concerns plane rectilineal geometry. Beginning with an itemization of Euclid's proceedings, Mueller states "[he] begins directly with a list of assertions divided into three groups and labelled 'definitions', 'postulates', and 'common notions'. He then proceeds directly to the statement and proof of 48 propositions, which divide into two kinds: those which describe a task (1-3, 9-12, 22, 23, 31, 42, 44-46) and those which make an assertion (the rest). At some time, perhaps after Euclid, the Greeks developed a terminology to mark this distinction, calling the tasks problems and the assertions theorems" (11). This last claim corrsponds to the difference between a construction and a proof, encountered in the section on Euclid. Though Mueller's analysis is helpful, ultimately it proves prejudiced with an inclination to rest in the modern perspective by incessantly comparing Euclid to Hilbert and even featuring Hilbert's geometry as the very first section of his book. Such an assessment cannot be a fair representation of Euclid's circumstances or situation. For these reasons, I believe Mueller's biased appraisal fails its own intentions.

Let's now consider what the some of these authorities articulate regarding the connection between Aristotle and Euclid instead of their individual properties. Thomas Heath's faithful translations and unparalleled commentaries concerning the Greeks are an indispensible resource for any scholar wishing to delve into the discipline of ancient mathematics. Heath maintains that there is a significant relationship between Aristotle and Euclid. In *A History of Greek Mathematics*, he says, "The works of Aristotle are of the greatest importance to the history of mathematics and particularly the *Elements* (335)." Furthermore, Aristotle came just before Euclid, which Heath proposes is a fortuitous succession with respect to investigating the specific influences of Aristotle on Euclid versus Euclid's own ideas. This of course can never be an exact inference but it still reveals a reasonable correlation. In *Mathematics in Aristotle*, Heath observes that, "The importance of a proper understanding of the mathematics in Aristotle lies principally in the fact that most of his illustrations of scientific method are taken from mathematics" (1). Therefore, it is essential to notice that Aristotle's account of a scientific method is inseparable from, and depends on, his mathematical examples.

In *The Shaping of Deduction in Greek Mathematics*, Reviel Netz claims, "The solid starting-point for Euclidean-style geometry is neither Euclid nor Autolycus, but Aristotle. His use of mathematics betrays an acquaintance with mathematics whose shape is only marginally different from that seen in Euclid; the marginal difference can, as a rule, be traced to the fact that Aristotle uses this mathematics for his own special purposes" (275). The persuasive evaluation by Netz is substantiated by his ability to isolate the Greek condition and disregard contemporary perspectives. Netz recognizes that the 'shape' of deduction in Aristotle is analogous to that of Euclid. More importantly, he appreciates the style of Euclid's treatise must be indebted to Aristotle.

In conclusion, these opinions in academia provide an important matrix through which Aristotle and Euclid can be understood. However, it is important to balance these modern sentiments with those of the ancient thinkers. The commentary of Aristotle's work by Apostle and Corcoran provide a faithful exposition of Aristotle's simple yet elegant scientific method, which serves to produce knowledge through demonstration. For the purposes of my paper I did not find the commentary by Mueller very constructive since this paper maintains a historical evaluation, which was in direct opposition to Mueller's criticisms of Euclid through a comparison to the relatively modern Hilbert. Lastly, Heath and Netz, who are both nonpareil authorities of mathematics during the time of the Greeks, justify a direct and meaningful connection between

Aristotle and Euclid. Netz, especially, observes that Euclidean-style geometry, based in an axiomatic method, necessarily starts with Aristotle. The commentaries that I found most helpful were based on a historical evaluation and resisted comparing the Greeks to modern mathematicians and logicians. Most scholars who concentrate on such a comparison were often reductive in their analysis of Aristotle and Euclid. Some criticisms of Euclid included that he was not exhaustive enough, depended too heavily on diagrams, and included definitions that he never used probably for the sake of tradition. However, Euclid's *Elements* remains the primary text concerning geometry and proof since it was written circa 300 B.C. Discussing the observations and conclusions of various scholars seeking to understand the same subject I am invested in has provided me with a richer account of the connection between Aristotle and Euclid.

Section 5

Interpretations

This section is a final exegesis to the preceding sections. McKeon discusses how a scholar can recognize the influence of one intellectual on later generations of thinkers:

"The influence of a philosopher may be found in forms of speech, distinctions, and information anonymously imbedded in a later civilization as truly as in the explicitly labeled doctrines which a given age attributes to him. The distinctions, terms, and guiding principles taken over from his philosophy and used or opposed by his contemporaries may with familiar use be transmuted, together with the conclusions and doctrines of his philosophy, into the accustomed materials of a culture and tradition. When, on the other hand, men return in later ages to read a philosopher's works, their study seldom yields them concrete knowledge or useable information, unless it be history or philology, but often suggests directions of inquiry and methods inspired by the manner and form of that philosophy … [Aristotle's] philosophy contains the first statement, explicit or by opposition, of many of the technical distinctions, definitions, and convictions on which later science and philosophy have been based (xi)."

Aristotle's influence has pervaded Western culture in immeasurable ways; however, the influence that Aristotle had on Euclid, by supplying him with "directions of inquiry and method", is direct, explicit, and measurable. This paper is dedicated to evaluating the measure of that influence. Aristotle's examples are often geometrical and his treatise is undoubtedly mathematical in nature. One must be careful not to assume Euclid simply reproduced Aristotle. Indeed, Euclid was a revolutionary and most of his proofs indicate that they are originals. Still, it is beyond a doubt that Aristotle influenced Euclid and his development of the *Elements*. Aristotle's use of demonstration and his rigorous technique of deduction from first principles are evident in Euclid's work. Liberally, one can see the structure of Aristotle's proofs but overall the structures of Euclid's proofs are his own. This difference among their proofs cannot be understated. In many ways Euclid supersedes Aristotle. Aristotle can be regarded as the theory of science while Euclid can be understood as the application. Aristotle truly develops a science, more specifically a method and a philosophy of that method.

As a final example of Euclid's demonstration, I will recount the proof of Proposition 47, the Pythagorean theorem. If the previous examples in this paper served to substantiate the claim that Euclid was practicing Aristotle's conception of demonstration, then this example shows that this proof is the culmination of the *Element*'s Book I and therefore a high point in Euclid's application of Aristotle's axiomatic method. Additionally, it is the link to Books II through XIII by persisting as the inception of proportion and ratio theory in geometrical proofs. This theory, originating with Eudoxus, is central to Aristotle's formulation of demonstration and hence integral in the work of Euclid. The proposition states, *in right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.* The following is Euclid's proof wherein I have added the enumeration of the steps:

(i) *Let ABC be a right-angled triangle having the angle BAC right;*

(ii) *I say that the square on BC is equal to the squares on BA, AC.*

(iii) *For let there be described on BC the square BDEC, and on BA, AC the squares BFGA, AHKC;*

(iv) *Through A let AL be drawn parallel to either BD or CE, and let AD, FC be joined.*

(v) *Then, since each of the angles BAC, BAG is right it follows that with a straight line BA, and at the point A on it, the two straight lines AC, AG not lying on the same side make the adjacent angles equal to two right angles;*

(vi) *Therefore CA is in a straight line with AG*

(vii) *For the same reason BA is also in a straight line with AH.*

(viii) *And, since the angle DBC is equal to the angle FBA: for each is right: let the angle ABC be added to each;*

(ix) *Therefore the whole angle DBA is equal to the whole angle FBC.*

(x) *And, since DB is equal to BC, and FB to BA, the two sides AB, BD are equal to the two sides FB, BC respectively;*

(xi) *And the angle ABD is equal to the angle FBC;*

(xii) *Therefore the base AD is equal to the base FC, and the triangle ABD is equal to the triangle FBC.*

(xiii) *Now the parallelogram BL is double the triangle ABD, for they have the same base BD and are in the same parallels BD, AL.*

(xiv) *And the square GB is double the triangle FBC, for they again have the same base FB and are in the same parallels FB, GC.*

(xv) *[But the doubles of equals are equal to one another.] Therefore the parallelogram BL is also equal to the square GB.*

(xvi) *Similarly, if AE, BK be joined, the parallelogram CL can also be proved equal to the square HC;*

(xvii) *Therefore the whole square BDEC is equal to the two squares GB, HC.*

(xviii) *And the square BDEC is described on BC, and the squares GB, HC on BA, AC*

(xix) *Therefore the square on the side BC is equal to the squares on the sides BA, AC.*

(xx) *Therefore etc. … Q.E.D.*

In step (i) through (iii) Euclid constructs a right triangle and forms 3 squares on each side of the triangle. In step (iii) through (xvi) he constructs interior lines, an auxiliary construction, which creates parallelograms and triangles inside the first construction. Using a theory of proportions, he deduces that ratios among the length and area of bases, triangles, parallelograms, and squares, stand in necessary relations through the establishment of middle terms in the proportions between those ratios. In steps (xvii) through (xix) it is the culmination of those proportions of constructed figures; therefore, the square i.e. the area of the square on the line BC (double an equal is its square) is equal to the sum of the square of the other two sides. Therefore, $BC^2 = AB^2 + AC^2$. This proof of the Pythagorean theorem is the culmination of Euclid's use of proportions, i.e. the use of middle terms to establish a series of logical relations and his expertise in demonstration that was unique to Aristotle.

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Papers

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