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THE SWITCHING POINT WITHIN 401K TAXATION STRUCTURES

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ABSTRACT

THE SWITCHING POINT WITHIN 401K TAXATION STRUCTURES

ADAM RUNK

This paper explores the choice presented to workers with access to both a Roth and Traditional 401k. By using mathematical analysis and financial modeling this paper determines an optimal strategy to maximize an individual's after-tax payouts in retirement while accounting for social security taxation. The strategy is highly dependent upon factors such as the interest rate earned, contribution amount, salary, number of years contributed, number of years in retirement, future tax structures, future social security laws, and the inflation rate. Since there are so many variables, equations and theory are presented in assisting an individual in determining their strategy. Generally, the optimal strategy is to choose a Roth taxation structure in the earlier years of employment when income is lower and contributions will grow the longest and have the largest impact on tax rate at withdrawal. Then, at a certain point termed the "Switching Point", it is beneficial to stop contributions to the Roth 401k, keep the current funds in the Roth 401k, allow them to grow, and then start contributing to a Traditional 401k until retirement.

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Introduction

Generally, most articles regarding Roth and Traditional taxation structures are titled "Roth vs. Traditional" as if there is an upcoming boxing match between the two. These articles list vague circumstances where one should be used over the other and ignore that an investor can have the best of both worlds.

With the Roth 401k becoming ever popular since its inception in 2006, employees with access to both taxation vehicles have had to choose which taxation structure to pick with little guidance as to which one would be more beneficial. The choice between taxation structures can dramatically impact the lifestyle that an individual receives in retirement. This paper formulates a strategy for maximizing the after-tax payouts of a 401k by using both taxation structures.

The formulation of this strategy should be fiscally rewarding to workers with access to both types of 401ks who are early on in their careers. This paper will also be useful to 401k managers, certified financial planners, employers who are considering the benefits of making a Roth 401k available to their workers, and individuals who are generally interested in the topic of retirement taxation.

This paper shows that the optimal solution for a 401k (in most circumstances) is to choose a Roth taxation structure in the earlier years of employment when income is lower and contributions will grow the longest and have the largest impact on tax rate at withdrawal. Then, at a certain point termed the "Switching Point", it is beneficial to stop contributions to the Roth 401k, keep the current funds in the Roth 401k, allow them to grow, and then start contributing to a Traditional 401k until retirement. This strategy allows the investor to gain the most money in retirement at no cost other than the time to learn this strategy and open a Traditional or Roth

401k, which is generally free of cost. The switching point times vary based upon a variety of factors such as salary, amount of years contributing, amount of years in retirement, interest rate, amount contributed annually, and taxable income earned during retirement. The tax rates at contribution as well as the tax rate at retirement are not factors as they are determined by the factors in the prior sentence. The tax rate at contribution is determined by the salary earned in the year of contribution while the tax rate at withdrawal is determined by a variety of factors such as choice of taxation structure, contribution amount, interest rate, length of contribution, and length of withdrawal.

Initial Theory Required for Understanding

Marginal Tax Rate

The following equation determines the average marginal tax rate to use when computing the appropriate pretax contribution:

T = Taxable Income
Tx = Tax using marginal tax brackets

$$\frac{\Delta Tx}{\Delta T} = Marginal\ Tax\ Rate\ at\ contribution = Tc$$

For example, assume Joe makes $36,000 a year in salary. Joe decides to contribute $2,000 to his Traditional 401k which means that his total taxable income would then be $34,000. Comparing the amount of tax on $36,000 had Joe chosen a Roth:

Single 2011 Tax Brackets

Taxable Income	Marginal Tax Rate:
$0-$8,500	10%
$8,500-$34,500	15%
$34,500-$83,600	25%
$83,600-$174,400	28%
$174,400-$379,150	33%
$379,150+	35%

$$\left(\left(\$8{,}500 - \$0\right) \times 0.1 + \left(\$34{,}500 - \$8{,}500\right) \times 0.15 + \left(\$36{,}000 - \$34{,}500\right) \times 0.25\right) = \$5{,}125$$

with $34,000 had Joe chosen a Traditional:

$$\left(\left(\$8{,}500 - \$0\right) \times 0.1 + \left(\$34{,}000 - \$8{,}500\right) \times 0.15\right) = \$4{,}675$$

The average marginal tax rate for this increase in taxable funds can be found by using the equation above:

$$\left(\frac{\$5{,}125 - \$4{,}675}{\$36{,}000 - \$34{,}000}\right) = 0.225 \; or \; 22.5\%$$

Pretax Contribution Comparison

There is no difference between the amount gained in a Roth 401k and a Traditional 401k if the average marginal tax rates at contribution and withdrawal are equal.

For example, suppose Bob has 500 pretax dollars to invest in either a Roth or Traditional 401k. Bob expects to earn 40% in his investments over a 15 year period which will not differ depending upon his taxation vehicle choice. Bob has a salary of $34,000. Therefore Bob will have a taxable income of $33,500 if he contributes $500 to a Traditional 401k and will continue to have $34,000 in taxable income if he chooses a Roth 401k.

$$(\$8{,}500 - 0) \times 0.1 + (\$34{,}000 - \$8{,}500) \times 0.15 = \$4{,}675$$

$$(\$8{,}500 - 0) \times 0.1 + (\$33{,}500 - \$8{,}500) \times 0.15 = \$4{,}600$$

Using the marginal tax rate formula above, one can compute Bob's marginal tax rate at contribution.

$$\left(\frac{\$4{,}675 - \$4{,}600}{\$34{,}000 - \$33{,}500}\right) = 15\%$$

Using the marginal tax rate, the after-tax contribution amount can be determined. As Traditional contributions are not taxed upon contribution, the Traditional contribution will be the same before and after tax while the Roth contribution will be taxed.

$$\$500 \times (1 - 0.15) = \$425 = Roth\ Post\ Tax\ Contribution$$

Imagine that over the next 20 years, Bob earns a 40% on his investments. Bob's after interest amount is:

$$(1 + interest\ rate) \times Post\ tax\ Contribution = After\ Interest\ Contribution\ Amount$$

Assume Bob has the same Marginal Tax Rate at Withdrawal as at Contribution. The after-tax contribution at retirement is the same for a Roth, since withdrawals are not taxed. The After-tax amount for a Traditional is: $(1 - Tax\ Rate) \times Future\ Amount$. Below, the table summarizes the information equated in the above example.

Type	**Roth**	**Traditional**
Salary	$34,000	$34,000
Pre Tax Contribution Amount	$500	$500
Taxable Income after contribution	$34,000	$33,500
Marginal Tax Rate at Contribution	15%	15%
Post-Tax Contribution	$425	$500
Interest Rate	40%	40%
Future Amount	$595	$700
Tax Rate at Withdrawal	15%	15%
After Tax After Interest Amounts	$595	$595

A common mistake is to compare Roth and Traditional vehicles on a post-tax contribution versus a pre-tax contribution basis. The correct way of comparing a Roth and Traditional structure is used throughout the paper.

Effective Tax Rates

Since the U.S. uses a progressive tax bracket, saying that a total amount of income is taxed at 20% is not true if one is using marginal rates. The first $8,500 earned is taxed at a 10%

rate. The next $26,000 is taxed at a 15% and so on. The difference between the marginal tax rates and an effective rate is just a way of showing the actual single rate of tax. Marginal tax rates are used to calculate the total amount of tax on an income. This tax is then divided by the total income being taxed to get the true tax rate. Single effective rates are used in some examples to make tax rates easier to understand. Effective tax rates are calculated using marginal tax rates, so they give the same answer as marginal tax rates.

T = Taxable Income
Tx = Tax using marginal tax brackets

$\frac{Tx}{T} = Effective\ Tax\ Rate.$

Multiplying both sides of the equation by "T" would yield a result of:

$$Tx = Effective\ Tax\ Rate \times T$$

Effective tax rates are basically an "average" tax rate for a given amount of taxable income. For example, assume Tim is single and earns $30,000 annually ignoring any deductions he may receive.

Single 2011 Tax Brackets

Taxable Income	Marginal Tax Rate:
$0-$8,500	10%
$8,500-$34,500	15%
$34,500-$83,600	25%
$83,600-$174,400	28%
$174,400-$379,150	33%
$379,150+	35%

($8,500 – 0) × 0.10 = $850

($30,000 - $8,500) × 0.15 = $3,225

Total Tax = $4,025

True/Effective Tax Rate = $\frac{\$4025}{\$30,000} =$ 0.134167 or 13.4%

Even though Joe is in a marginal tax bracket of 15%, his actual tax rate is 13.4%. For any income above $8,500, Joe's effective tax rate will change even if his marginal tax bracket

does not change. If his income increases from $5,000 to $8,000, his effective tax bracket would remain at 10%. Interestingly enough, the effective tax rate also increases steadily for those who earn over $379,150, the top marginal tax bracket. The effective rate does not become a steady 35% until an income of 44,712,499,957 is earned in one year, which has never been done before in the United States.

$$8{,}500 \times 0.1 + (34{,}500 - 8{,}500) \times 0.15 + (83{,}600 - 34{,}500) \times 0.25 + (174{,}400 - 83{,}600) \times 0.28 + (379{,}150 - 174{,}400) \times 0.33 + (44{,}712{,}499{,}957 - 379{,}150) \times 0.35 = 15{,}649{,}352{,}298.95$$

$$\frac{15{,}649{,}352{,}298.95}{44{,}712{,}499{,}957} = 0.349999492625104 \; or \; 35\%$$

Monthly Amortization

Typically, individuals prefer to receive even, inflation adjusted payouts in retirement. This ensures a constant standard of living in retirement and ensures that an individual will utilize all of their available financial resources. To accomplish this end goal (which also maximizes payouts by decreasing taxes), the amortization method is used. Generally, amortization is commonly used by businesses to pay off debt or depreciate an asset. Amortization is also used by banks and credit card organizations to determine monthly payments on a mortgage or credit card bill. A person new to the concept of amortization may wonder why the principal would not be divided by the number of payment. The answer is simple: interest. While a loan is being paid off, interest is being accumulated every month on the principal. As someone makes a debt payment, they are paying the Interest Payment + Principal until the principal reaches zero. Initially, a high percentage of a person's debt payment will be the interest payment while a low

percentage will be the principal payment. As more of the principal is paid off, the interest payment will become a smaller percentage of the payment while the principal will increase in percentage until the initial relationship is reversed and the person is primarily paying principal near the end of the loan.

So how does amortization, a method commonly used to pay off debt, relate to the payment from a 401k in retirement? The concept of paying off debt is structurally similar to the concept of receiving payments equal payments from a 401k. Think of the amount in the 401k upon retiring as the principal of the loan. The amount in the 401k continues making interest as it is withdrawn. Eventually the individual also wants the full amount to reach 0. A study done by US Trust found that more than half of millionaire baby boomers plan to leave no inheritance to their children (Hamilton).

The Equation for calculating monthly payments:

- **i** = Interest Rate
- **B** = Balance in Vehicle at Retirement
- **P** = Number of payments (12 if monthly payments, 52 if weekly payments, 365 if daily payments, etc.)
- **Y** = Number of Years you will receive monthly/weekly/daily/annual payments

$$i \times \frac{B \times \left(1 + \frac{i}{P}\right)^{P \times Y}}{\left(1 + \frac{i}{P}\right)^{P \times Y} - 1} = Annual\ Amortized\ Payment\ in\ terms\ of\ Y$$

Inflation

During the course of this paper the reader may wonder where inflation is factored in. All interest rates referred to in the equation are real interest rates. Contributions and payouts are assumed to increase every year with inflation. This paper will assume that both tax brackets as

well as social security payments increase with inflation as is currently the case. Salary is also considered to increase with inflation through annual cost of living raises.

For example, John has the following metrics for 2010 and 2011:

	2009	**2010**	**2011**
Income	30000	30690	31395.87
Interest Rate		5%	6%
CPI		2.3%	2.3%
Salary Increase		2.3%	2.3%
Contribution	$5000	$5115	$5232.645
Effective Tax Rate		0.13636	0.13646

2010 Tax Rates

Tax Bracket	Single
10% Bracket	$0 – $8,375
15% Bracket	$8,375 – $34,000
25% Bracket	$34,000 – $82,400
28% Bracket	$82,400 – $171,850
33% Bracket	$171,850 – $373,650
35% Bracket	$373,650 +

In terms of the equation the inputs factoring out inflation would be:

	2009	**2010**	**2011**
Income	30000	30000	30000
Interest Rate		2.7%	3.7%
CPI		0%	0%
Salary Increase		0%	0%
Contribution	$5000	$5000	$5000
Effective Tax Rate		0.13636	0.13646

Single 2011 Tax Brackets

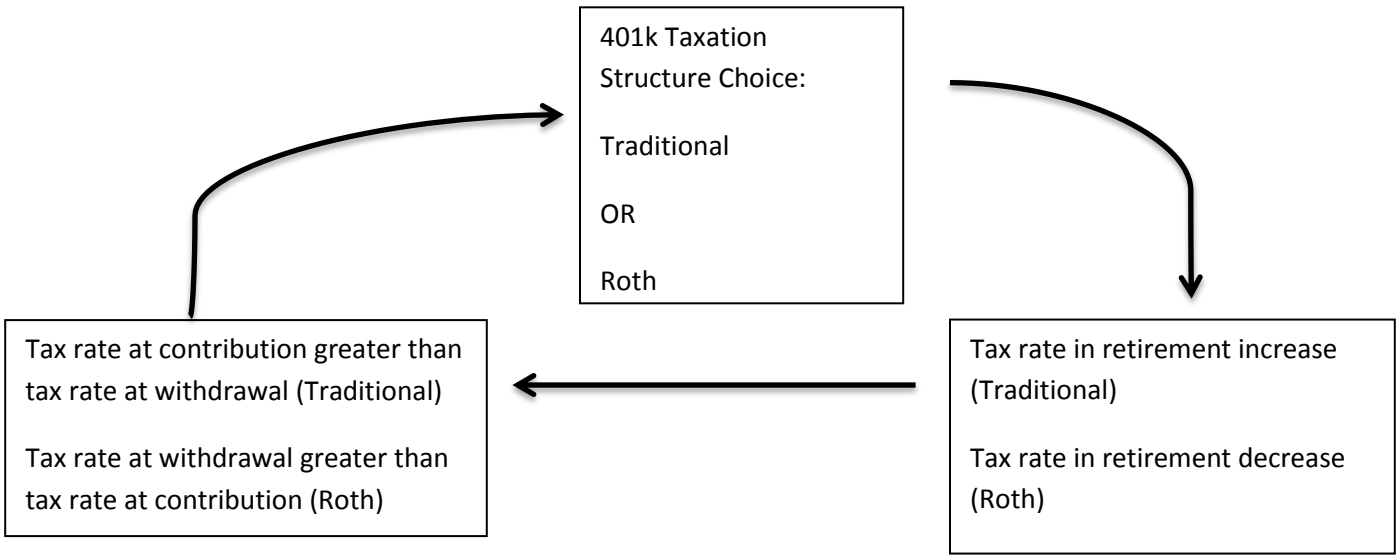
Taxable Income	Marginal Tax Rate:
$0-$8,500	10%
$8,500-$34,500	15%
$34,500-$83,600	25%
$83,600-$174,400	28%
$174,400-$379,150	33%
$379,150+	35%

In 1949, a bottle of Coca Cola cost 5 cents (Olver). Today, a bottle of Coca Cola costs roughly $1.50 in many vending machines. By factoring out inflation throughout the paper, a reader can see figures that will relate to the prices on the shelves today instead of the prices on the shelves years from now when they retire. This methodology will be easier for the reader to relate the answers of equations to the cost of their lifestyle today to determine if they need to save more based upon their assumptions.

Tax Rate at Withdrawal

The choice between a Roth and Traditional 401k depends upon the tax rate. While the tax rate at contribution can be calculated from current income, the tax rate at withdrawal is determined by annual amount of taxable income at retirement. However, the annual amount of taxable income is determined by the amortized amount of taxable income from an individual's Traditional 401k. Unfortunately this is a case of circular logic as the choice of taxation structure determines the tax rate at withdrawal and the tax rate at withdrawal determines the choice of taxation structure.

Scenario Analysis



To resolve the above circular reasoning, "if, then" scenarios need to be used to determine tax rates for all combinations of contributions. This can be done by initially assuming all contributions will go to a Traditional taxation structure.

i = Annual Interest Rate
C = Pretax annual contribution to 401k vehicle (Traditional)
K = Post Tax Contribution to 401k vehicle (Roth)
X = Year of First Withdrawal – Year of First Contribution

$$\mathbf{A} = \text{Traditional 401k balance at retirement (if all contributions in Traditional)} = \left(\sum_{n=0}^{X}(C \times (1+\text{i})^{X-0}) + (C \times (1+\text{i})^{X-1}) + \cdots + (C \times (1+\text{i})^{X-n})\right)$$

Now that the Traditional 401k balance at retirement has been determined (assuming all of an individual's contributions are made to the Traditional 401k), other 401k balances from all relevant scenarios must be determined (1st contribution Roth and rest Traditional, 1st and 2nd contributions Roth and rest Traditional, 1st , 2nd , and 3rd contributions Roth and rest Traditional, etc.). The reason the scenarios are not done in reverse (starting with 1st contribution Traditional

and rest Roth) is that an individual's income is assumed to increase over their lifetime. Due to an increasing income, one would assume an increasing effective tax rate if the tax structure were to remain the same. With these assumptions, it does not make practical sense to consider scenarios where an individual would defer his lower taxable contributions from early in his working life (Traditional) for higher rates when he retires while at the same time taking the tax on his higher taxable contributions later in life (Roth) instead of deferring them until retirement.

Balances in Traditional 401k at Retirement – Scenario Analysis

All Contributions Traditional	A
1st Contribution Roth, Rest Traditional	$A - (C \times (1+i)^{X-0})$
1st and 2nd Contribution Roth, Rest Traditional	$A - \sum_{n=0}^{1}\left((C \times (1+i)^{X-0}) + (C \times (1+i)^{X-1})\right)$
1st, 2nd and 3rd Contributions Roth, Rest Traditional	$A - \sum_{n=0}^{2}\left((C \times (1+i)^{X-0}) + (C \times (1+i)^{X-1}) + (C \times (1+i)^{X-2})\right)$
	……
All Contributions Roth	$A - \sum_{n=0}^{X}(C \times (1+i)^{X-0}) + (C \times (1+i)^{X-1}) + \cdots + (C \times (1+i)^{X-n}) = 0$

Now that all possible scenarios have been determined above for a Traditional 401k, the corresponding Roth 401k balances at retirement must be calculated using the same scenarios. In a scenario where all contributions will be allocated to the Traditional 401k, the Roth 401k will have nothing in it. In a scenario where only the 1st contribution is Roth and the rest Traditional, the value in the Roth 401k will be the future value of that 1st contribution. In essence, the Roth 401k balances will mirror the Traditional 401k balances for the scenarios.

Balances in Roth 401k at Retirement – Scenario Analysis

All Contributions Traditional	0
1st Contribution Roth, Rest Traditional	$(K \times (1+i)^{X-0})$
1st and 2nd Contribution Roth, Rest Traditional	$\sum_{n=0}^{1}\left((K \times (1+i)^{X-0}) + (K \times (1+i)^{X-1})\right)$
1st, 2nd and 3rd Contributions Roth, Rest Traditional	$\sum_{n=0}^{2}\left((K \times (1+i)^{X-0}) + (K \times (1+i)^{X-1}) + (K \times (1+i)^{X-2})\right)$
	……
All Contributions Roth	$\sum_{n=0}^{X}(K \times (1+i)^{X-0}) + (K \times (1+i)^{X-1}) + \cdots + (K \times (1+i)^{X-n})$

The balances from the "If, Then" scenario above for both the Traditional and Roth 401k can be amortized using the amortization equation shown under the Amortization and Even Monthly Payouts section above to get an annual amortized amount for each scenario. To determine the total annual taxable income in retirement, any other taxable income (from part time jobs in retirement) would be added to the Traditional 401k amortized amounts for each scenario. Since the 401k taxation structure choice will have no effect on the other taxable income made during retirement in a part-time job, this figure will be the same for all scenarios.

Social Security Taxation

Shortly after social security was enacted, congressmen met to discuss the government-run retirement program. One of the major flaws pointed out by the congressmen was that it was simply not complicated enough. Sure, calculating the social security payment was difficult, but something was lacking. These congressmen decided to tax social security payments depending

upon how much taxable income is earned in retirement. Since withdrawals from a Traditional 401k are taxable, the choice of 401k taxation structure determines whether and to what degree an individual's social security payments are taxable. Below is the complicated equation for determining how taxable social security payments are depending upon the social security benefit and Total Annual Taxable income in retirement. The social security benefit (S) is determined by a series of complex calculations which are largely irrelevant to the purpose of this thesis. The social security benefit(before it is split up for taxation) does not change based upon choice of 401k taxation structure. The Total Annual Taxable income in retirement (T) is the amortized annual amount for the Traditional 401k balance for each scenario plus any other annual taxable income in retirement (earned from a part time job)

S = Social Security Benefit
T = Total Annual Taxable income in retirement
St = Taxable portion of **S**
Sn = Non-taxable portion of **S**

$$St = MIN\left(S * 0.85 \textbf{ OR } MIN\left(\frac{S}{2} \textbf{ OR } \frac{MIN\left(MAX\left(0 \textbf{ OR } T + \frac{S}{2} - 25000\right) \textbf{ OR } 9000\right)}{2}\right)\right.$$

$$\left. + MAX\left(0 \textbf{ OR } MAX\left(0 \textbf{ OR } T + \frac{S}{2} - 25000\right) - 9000\right) * 0.85\right)$$

$$Sn = S - St$$

The equation above is based upon two taxation thresholds that the government has arbitrarily decided upon: $25,000 – 50% and $34,000 – 85%. The 9000 shown above is the difference between the taxation thresholds. If an individual makes below $25,000 in taxable income during retirement, then none of their social security benefit is taxable. If an individual

makes between 25 and 34 thousand, then 50% of their social security benefit is taxable. If an individual makes above 34 thousand, then 85% of their social security benefit is taxable. These figures are factored in incrementally. If an individual were to have 34,001 in taxable income, her entire social security benefit would not be taxed at 85%, only the 1 dollar over would be taxed at 85%. Additionally when the government determines the taxable income using the thresholds, they consider taxable income plus one half of the social security benefit to be the figure that determines how much an individual's benefit is taxable. Disturbingly, these two taxation thresholds (25,000 and 34,000) are not indexed for inflation and have not been indexed since they were enacted. This means that as time goes on and inflation increases Americans will be increasingly taxed on their social security benefits until everyone's social security benefit is 85% taxable unless these taxation thresholds are adjusted for inflation at some point.

Total after-tax annual payout per each scenario

Using the total annual taxable income in retirement (the Traditional 401k amortized amount + other annual taxable income + taxable portion of social security benefit), the amount of taxes paid for each scenario can be computed by using the progressive tax scale. This tax scale can be changed depending upon how an individual thinks taxes will change in the future. The tax determined for each scenario's total taxable income can be subtracted from the total taxable income to get the total annual taxable income after-tax. The total annual taxable income after-tax is added to the corresponding 401k annual balance for each scenario to get the total after-tax annual payout for each scenario.

R = Nontaxable Roth 401k annual amortized payout for each corresponding scenario
T = Taxable Traditional 401k annual amortized payout for each corresponding scenario
Sn = Nontaxable portion of annual social security payment
St = Taxable portion of annual social security payment
O = Other Taxable Income in retirement
X = Effective Tax Rate = $\frac{Z}{(T+O+St)}$
Z = Total Tax (determined by feeding (T+O+St) through the assumed marginal tax brackets)

$$\big((T + O + St) \times (1 - X)\big) + R + Sn = Total\ aftertax\ annual\ payout\ for\ each\ scenario$$

After applying this formula to each scenario, the scenario with the largest after-tax annual payout is the most superior choice. While these equations answer the question of where the switching point exists within any given scenario, they don't answer whether the change between each scenario is due to social security taxation or to the superiority of one taxation vehicle over the other.

Annual Switching Point Factors

There are two main annual switching point factors that add up to equal the scenario to scenario difference between total after-tax annual payout per each scenario. As there previously have been no names for these items they have been termed as: "Roth Factor" and "Social Factor" for the purposes of clarity throughout the paper.

Roth Factor

Roth is likely the most intuitive factor out of the group. It is simply the after-tax amortized future value upon withdrawal of a Roth taxation structure minus the after-tax amortized future value upon withdrawal of Traditional taxation structure. In any year of contribution, an individual can choose to contribute their funds to a Roth 401k or a Traditional 401k. The scenarios are set up to explore the differences between the two. Imagine a man named Joe who contributes to his 401k for 10 years. His options are as follows assuming an increasing income:

0 Contributions Roth, All 10 Contributions Traditional

1st Contribution Roth, 9 Contributions (2^{nd}, 3^{rd}…. 10^{th}) Traditional

1^{st} and 2nd Contributions Roth, 8 Contributions (3^{rd}, 4^{th}… 10^{th}) Traditional

1^{st}, 2^{nd}, and 3^{rd} Contributions Roth, 7 Contributions (4^{th}, 5^{th}…..10^{th}) Traditional

4 Contributions Roth (1^{st}, 2^{nd} … 4^{th}), 6 Contributions (5^{th}, 6^{th}… 10^{th}) Traditional

5 Contributions Roth (1^{st}, 2^{nd} … 5^{th}), 5 Contributions (6^{th}, 7^{th}… 10^{th}) Traditional

6 Contributions Roth (1^{st}, 2^{nd} … 6^{th}), 4 Contributions (7^{th}, 8^{th}… 10^{th}) Traditional

7 Contributions Roth (1^{st}, 2^{nd} … 7^{th}), 3 Contributions (8^{th}, 9^{th}, 10^{th}) Traditional

8 Contributions Roth (1^{st}, 2^{nd}… 8^{th}), 2 Contributions (9^{th}, 10^{th}) Traditional

9 Contributions Roth (1^{st}, 2^{nd} … 9^{th}), 1 Contributions (10^{th}) Traditional

All 10 Contributions Roth, 0 Contributions Traditional

The difference between the 1st and 2nd scenarios

0 Contributions Roth, All 10 Contributions Traditional
1st Contribution Roth, 9 Contributions (2nd, 3rd…. 10th) Traditional

is that the 1st contribution is either allocated as a Traditional type (1st scenario) or a Roth type (2nd scenario). Similarly the differences between the 2nd and 3rd scenario

1st Contribution Roth, 9 Contributions (2nd, 3rd…. 10th) Traditional
1st and 2nd Contributions Roth, 8 Contributions (3rd, 4th… 10th) Traditional

is that the 2nd contribution is either allocated as a Traditional type (2nd scenario) or a Roth type (3rd scenario).

The difference between the 1st and 2nd scenario is simply the difference of the amortized future value of the 1st contribution if it was allocated as a Traditional type and the amortized future value of the 1st contribution if it was allocated as a Roth type.

Above in the Marginal Tax Rate section, the formula for the average marginal tax rate at contribution (Tc) is equated. The marginal tax rate at withdrawal (Tw) is equated using the same formula.

T = Taxable Income
Tx = Tax using marginal tax brackets

$$\frac{\Delta Tx}{\Delta T} = Marginal\ Tax\ Rate\ at\ withdrawal = Tw$$

The change in taxable income is the change in the amortized annual pre-tax amount in a Traditional 401k plus the change in the amount of taxable social security within a given scenario.

i = Interest Rate
C = Pretax annual contribution to 401k vehicle (Traditional)
K = Post Tax Contribution to 401k vehicle (Roth)
X = Number of Years the contribution will grow from present period until first withdrawal
T_w = Marginal Tax rate at withdrawal
T_c = Marginal Tax rate at contribution
Y = Number of Years you will receive monthly/weekly/daily/annual payments

$$\left(i \times \frac{C \times (1+i)^X \times \left(1+\frac{i}{P}\right)^{P*Y}}{\left(1+\frac{i}{P}\right)^{P*Y} - 1} \times (1 - Tw) \right) - \left(i \times \frac{K \times (1+i)^X \times \left(1+\frac{i}{P}\right)^{P*Y}}{\left(1+\frac{i}{P}\right)^{P*Y} - 1} \right) = Annual\ Amortized\ Roth\ Factor$$

The above equation can also be simplified into a much easier equation:

$$(Tw - Tc) \times C = Present\ Value\ Roth\ Factor$$

$$(Tw - Tc) \times C \times (1+i)^X = Future\ Value\ at\ withdrawal\ Roth\ Factor$$

$$i \times \frac{(Tw-Tc) \times C \times (1+i)^X \times \left(1+\frac{i}{P}\right)^{P*Y}}{\left(1+\frac{i}{P}\right)^{P*Y} - 1} = Annual\ Amortized\ Roth\ Factor$$

There are two important pieces of information that can be garnered from the above equations. First, the Roth Natural factor is the difference between the after-tax amortized future value of Roth and Traditional taxation structures that have the same pre-tax contribution, interest, rate, years of growth, and periods of amortization. Second, the present value difference between the two is simply the spread of the two tax rates multiplied by the pre-tax contribution made in the period. If social security were non-existent, this equation alone would determine whether an individual should choose Roth or Traditional within a given year.

Social Factor

As discussed previously, social security is taxable based upon an individual's taxable income in retirement. Therefore, an individual's 401k taxation structure will affect if and to what degree their social security is taxable. This factor quantifies the year to year change of the total, after-tax social security benefit an individual receives based upon the choice of 401k taxation structure. As shown in the Roth Factor section, the difference between the scenarios is simply how a payment is allocated, Roth or Traditional.

St = Taxable portion of annual social security payment
$\mathbf{T_w}$ = Marginal Tax rate at withdrawal

$$\Delta St \times Tw = Social\ Factor$$

Example

Throughout the paper, equations are created to assist the reader in determining when the optimal switching point in retirement. While there are examples for most equations, there is not a unified example that takes the reader through all the steps. Below are simple examples which utilize all equations and mathematically prove that a switching point can be more beneficial than picking just Traditional or Roth 401k for every contribution.

Imagine a man named Bob who works and contributes to his 401k for 10 years and retires for 5 years. There is no inflation in Bob's world and social security does not exist. Bob makes $50,000 a year at his job and contributes $15,000 (pretax) to his 401k. Bob expects to earn an interest rate of 20% every year. In retirement, Bob will take out money from his 401k on a monthly basis. Bob assumes that the current United States 2011 tax rates will not change from now until he dies in 15 years.

First, the post-tax contribution must be determined. As Traditional contributions are not taxed upon contribution it remains $15,000. For Roth contributions, the marginal tax rate must be determined. The taxable income for the Roth is merely the salary ($50,000) while the taxable income for a Traditional is the salary minus the contribution ($35,000).

Single 2011 Tax Brackets

Taxable Income	Marginal Tax Rate:
$0-$8,500	10%
$8,500-$34,500	15%
$34,500-$83,600	25%
$83,600-$174,400	28%
$174,400-$379,150	33%
$379,150+	35%

The tax on $50,000 is $8,625 while the tax on $35,000 is $4,875.

$$(\$8{,}500 - 0) \times 0.1 + (\$34{,}500 - \$8{,}500) \times 0.15 + (\$50{,}000 - \$34{,}500) \times 0.25 = \$8{,}625$$

$$(\$8{,}500 - 0) \times 0.1 + (\$34{,}500 - \$8{,}500) \times 0.15 + (\$35{,}000 - \$34{,}500) \times 0.25 = \$4{,}875$$

$$\frac{\Delta Tax}{\Delta Taxable\ Income} = Marginal\ Tax\ Rate\ at\ contribution = Tc$$

$$\frac{\$8{,}625 - \$4{,}875}{(\$50{,}000 - \$35{,}000)} = 25\%$$

The after-tax contribution for the Roth is:

$$(1 - Tc) \times Pretax\ Contribution\ (Traditional) = Aftertax\ Contribution\ (Roth)$$

$$(1 - 0.25) \times \$15{,}000 = \$11{,}250$$

In preparation for the scenario analysis, the Traditional 401k balance at retirement must be determined assuming that all contributions go to a Traditional 401k.

$\mathbf{A}$ = Traditional 401k balance at retirement (if all contributions in Traditional) =

$$\left(\sum_{n=0}^{X-1}(C \times (1 + \mathrm{i})^{X-0}) + (C \times (1 + \mathrm{i})^{X-1}) + \cdots + (C \times (1 + \mathrm{i})^{X-n})\right)$$

$\mathbf{A}$ = Traditional 401k balance at retirement (if all contributions in Traditional) =

$$\left(\sum_{n=1}^{9}(\$15{,}000 \times (1 + 0.2)^{10-0}) + (\$15{,}000 \times (1 + 0.2)^{10-1}) + \cdots\right.$$

$$\left. + (C \times (1 + \mathrm{i})^{10-9})\right) = \$467{,}256.28$$

Balances in Traditional 401k at Retirement – Scenario Analysis

All Contributions Traditional	$\$467{,}256.28$
1st Contribution Roth, Rest Traditional	$\$467{,}256.28 - (15000 \times (1+0.2)^{10-0}) = \$374{,}380.23$
1st and 2nd Contribution Roth, Rest Traditional	$\$467{,}256.28 - \sum_{n=0}^{1}\left((15000 \times (1+0.2)^{10-0}) + (15000 \times (1+0.2)^{10-1})\right) = \$296{,}983.53$
1st, 2nd and 3rd Contributions Roth, Rest Traditional	$A - \sum_{n=0}^{2}\left((15000 \times (1+0.2)^{10-0}) + (15000 \times (1+0.2)^{10-1}) + (15000 \times (1+0.2)^{10-2})\right) = \$232{,}486.27$
1st -4th Contributions Roth, Rest Traditional	$178,738.56
1st -5th Contributions Roth, Rest Traditional	$133,948.8
1st -6th Contributions Roth, Rest Traditional	$96,624
1st -7th Contributions Roth, Rest Traditional	$65,520
1st -8th Contributions Roth, Rest Traditional	$39,600
1st -9th Contributions Roth, Rest Traditional	$18,000
All 10 Contributions Roth	$A - \sum_{n=0}^{X-1}(C \times (1+i)^{X-0}) + (C \times (1+i)^{X-1}) + \cdots + (C \times (1+i)^{X-n}) = \$0$

Balances in Roth 401k at Retirement – Scenario Analysis

All Contributions Traditional	$0
1st Contribution Roth, Rest Traditional	$(11{,}250 \times (1+0.2)^{10-0}) = \$69{,}657.03$
1st and 2nd Contribution Roth, Rest Traditional	$\sum_{n=0}^{1}\left((K\times(1+i)^{X-0})+(K\times(1+i)^{X-1})\right) = \$127{,}704.56$
1st, 2nd and 3rd Contributions Roth, Rest Traditional	$\sum_{n=0}^{2}\left((K\times(1+i)^{X-0})+(K\times(1+i)^{X-1})+(K\times(1+i)^{X-2})\right) = \$176{,}077.50$
1st -4th Contributions Roth, Rest Traditional	$216,388.29
1st -5th Contributions Roth, Rest Traditional	$249,980.61
1st -6th Contributions Roth, Rest Traditional	$277,974.21
1st -7th Contributions Roth, Rest Traditional	$301,302.21
1st -8th Contributions Roth, Rest Traditional	$320,742.21
1st -9th Contributions Roth, Rest Traditional	$336,942.21
All 10 Contributions Roth	$\sum_{n=0}^{X-1}(K\times(1+i)^{X-0})+(K\times(1+i)^{X-1})+\cdots+(K\times(1+i)^{X-n}) = \$350{,}442.20$

Now that the balances at retirement have been found, they must be amortized to provide annual figures. The numbers would be fed through the amortization equation.

$$i\times\frac{B\times\left(1+\frac{i}{P}\right)^{P*Y}}{\left(1+\frac{i}{P}\right)^{P*Y}-1} = Annual\ Amortized\ Payment\ in\ terms\ of\ Y$$

$$0.2\times\frac{Amount\ being\ amortized\times\left(1+\frac{0.2}{12}\right)^{12\times5}}{\left(1+\frac{0.2}{12}\right)^{12\times5}-1} = Annual\ Amortized\ Payment$$

The amortized Traditional 401k scenarios are then fed through the marginal tax rates to get an after-tax amortized amount.

Scenarios	(Pre-Tax)Traditional 401k Amort. Payment	(After-Tax)Traditional 401k Amort. Payment	Roth 401k annual Amort payment	After-tax Amortized Payouts
All Contributions Traditional	$148,553.20	$113,341.31	$0	$113,341.31
1st Contribution Roth, Rest Traditional	$119,025.40	$92,081.31	$22,145.82	$114,227.14
1st and 2nd Contribution Roth, Rest Traditional	$94,418.96	$74,364.65	$40,600.68	$114,965.33
1st , 2nd & 3rd Contributions Roth, Rest Traditional	$73,913.57	$59,310.18	$55,979.72	$115,289.90
1st -4th Contributions Roth, Rest Traditional	$56,825.74	$46,494.31	$68,795.59	$115,289.90
1st -5h Contributions Roth, Rest Traditional	$42,585.89	$35,814.42	$79,475.49	$115,289.90
1st -6th Contributions Roth, Rest Traditional	$30,719.34	$26,536.44	$88,375.40	$114,911.84
1st -7th Contributions Roth, Rest Traditional	$20,830.55	$18,130.97	$95,791.99	$113,922.96
1st -8th Contributions Roth, Rest Traditional	$12,589.89	$11,126.41	$101,972.48	$113,098.89
1st -9th Contributions Roth, Rest Traditional	$5,722.68	$5,150.41	$107,122.89	$112,273.30
All 10 Contributions Roth	0	$0	$111,414.90	$111,414.90

Next, the marginal tax rate upon withdrawal must be determined for each scenario.

$$\frac{\Delta Tax}{\Delta Taxable\ Income} = Marginal\ Tax\ Rate\ at\ withdrawal = Tw$$

$$\frac{(\$35{,}221.90 - \$26{,}944.12)}{(\$148{,}553.20 - \$119{,}025.40)} = 28\%$$

$$\frac{(\$35{,}221.90-\$26{,}944.12)}{(\$148{,}553.20-\$119{,}025.40)} = 28\%$$

After-tax Amortized Payouts	Change between After-tax Amortized payouts	Marginal Tax Rate Contribution (Tc)	Marginal Tax Rate Withdrawal (Tw)
$113,341.31			
$114,227.14	$885.83	25%	28%
$114,965.33	$738.19	25%	28%
$115,289.90	$324.57	25%	26.58285%
$115,289.90	$0	25%	25%
$115,289.90	$0	25%	25%
$114,911.84	-$378.07	25%	21.81402%
$113,922.96	-$988.88	25%	15%
$113,098.89	-$824.07	25%	15%
$112,273.30	-$825.59	25%	12.97784%
$111,414.90	-$858.40	25%	10%

Since there is no social security in this simplistic example, the Roth Factor equation should yield the change between after-tax amortized payouts. Remember that the marginal tax rate found above when determining the post-tax contribution for a Roth 401k is used in the equation below. Recall also that the pretax contribution given above was $15,000

$$(Tw - Tc) \times C = Present\ Value\ Roth\ Factor$$

$$(Tw - Tc) \times C \times (1 + i)^X = Future\ Value\ at\ withdrawal\ Roth\ Factor$$

$$i \times \frac{(Tw-Tc)\times C\times(1+i)^X\times\left(1+\frac{i}{P}\right)^{P*Y}}{\left(1+\frac{i}{P}\right)^{P*Y}-1} = Annual\ Amortized\ Roth\ Factor$$

$$(0.28 - 0.25) \times 15000 = 450$$

$$450 \times (1 + 0.2)^{10} = 2786.28$$

$$i \times \frac{2786.28\times\left(1+\frac{0.2}{12}\right)^{12*5}}{\left(1+\frac{0.2}{12}\right)^{12*5}-1} = 885.83$$

These numbers will continue for the differences between all scenarios which will be equal to the actual differences as $885.83 found using the Roth Factor Equation is equal to the annual after-tax payout between the first and second scenarios ($\$114{,}227.14 - \$113{,}341.31$). The benefit of this example is that it mathematically shows the benefit of switching ($115,289.90) over that of picking either pure Roth ($111,414.90) or Pure Traditional ($113,341.31). While the differences may only be a few thousand dollars, note that this is a few thousand dollars a year for retirement, which in this case is 5 years. Additionally, this example corresponds to the observation from the Roth factor formula: mathematically, the optimal switching point exists when the marginal tax rate at contribution equals the marginal tax rate at withdrawal.

Tax Diversification

The optimal solution for 401k taxation structure also provides a bonus intangible benefit that hasn't previously been discussed. As the government is constantly switching tax policy from year to year, there is difficulty in predicting what future tax brackets will be in the near future, let alone at retirement. If an individual were to choose a Roth taxation structure for all contributions, they would benefit in terms of opportunity cost from higher tax rates at retirement while they would have lost money in opportunity cost if tax rates declined at retirement. If an individual were to choose a Traditional taxation structure for all of his contributions, he would benefit from decrease tax rates at retirement and would lose money from increasing tax brackets at retirement. As government information is uncertain, it is beneficial to diversify taxable funds to minimize the impact of a governmental tax bracket change. Since the 401k switching point theory advocates the allocation of both Roth and Traditional 401ks, retirement funds are diversified in terms of taxability.

Conclusion

In conclusion, the initial hypothesis regarding choosing a Roth taxation structure in the earlier years of employment then, stopping contributions to the Roth 401k, keeping the current funds in the Roth 401k, allowing them to grow, and then start contributing to a Traditional 401k until retirement was optimal in certain circumstances. There are situations that exist when a person would be better off choosing a pure Traditional or pure Roth taxation structure. The equations formulated will make it clear if contributing purely to a Traditional or Roth is more beneficial than utilizing the strategy presented in this paper. The optimal switching point exists where the average marginal tax rate at contribution is equal to the average marginal tax rate at withdrawal. Using this strategy to maximize after-tax annual payouts for retirement diversifies an individual's taxable income if a switching point exists. This intangible benefit reduces the impact from a future change in governmental tax bracket changes.

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