

THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

DEPARTMENT OF AEROSPACE ENGINEERING

A GAME THEORY ANALYSIS OF SAILPLANE RACING

NICHOLAS J. GRASSER
SPRING 2014

A thesis
submitted in partial fulfillment
of the requirements
for baccalaureate degrees
in Aerospace Engineering and Mathematics
with interdisciplinary honors in Aerospace Engineering and Mathematics

Reviewed and approved* by the following:

Mark Maughmer
Professor of Aerospace Engineering
Thesis Supervisor & Aerospace Engineering Honors Advisor

Diane Henderson
Professor of Mathematics
Mathematics Honors Advisor

Robert Melton
Professor of Aerospace Engineering
Faculty Reader

George Lesieutre
Professor of Aerospace Engineering
Faculty Reader

* Signatures are on file in the Schreyer Honors College.

ABSTRACT

The sport of soaring allows pilots a multitude of decisions that influence the outcome of a competition. From how fast to fly, what altitude to enter and exit thermals, and who to fly with, each pilot can create a strategy to best fit his skill set and yield the best chance of victory. This paper examines and groups a multitude of tactical decisions into two overarching strategies, conservative flying and aggressive flying. Through simulation of single and multiple day competitions, these strategies are examined to determine which situations necessitate their use. As will be shown, the aggressive strategy has a greater chance of winning a single day competition. As the number of days of the competition increases, the chances of obtaining a high place will increase when using the conservative strategy and will decrease when using the aggressive strategy.

TABLE OF CONTENTS

List of Figures	iii
List of Tables	iv
Acknowledgements.....	v
Chapter 1: Introduction.....	1
The Sport of Soaring	1
Chapter 2: Race Strategy	3
Height Band	3
Speed-to-Fly Between Thermals.....	6
Flying in a Gaggle.....	9
Chapter 3: Analytical Model.....	10
Expected Payoffs.....	10
Modeling the Payoff Distribution	11
Examining Payoff Probabilities	14
Chapter 4: Simulating Single and Repeated Games	17
Single Game Simulation	17
Pure Strategy Repeated Game Simulation	19
Chapter 5: Conclusion and Recommendations for Future Work	20
Conclusion	20
Recommendations for Future Work.....	20
Appendix A Payoff Data.....	22
Appendix B MATLAB Code.....	23
BIBLIOGRAPHY	26

LIST OF FIGURES

Figure 1. Forces Acting on a Sailplane	1
Figure 2. Thermal Soaring.....	2
Figure 3. Height Band vs. Time of Day	4
Figure 4. Altitude vs. Thermal Strength	5
Figure 5. Conservative Height Band.....	6
Figure 6. Aggressive Height Band.....	6
Figure 7. Sink Rate Polar	7
Figure 8. MacCready Speed to Fly	8
Figure 9. Examples of Normal Distributions	12
Figure 10. Examples of Beta Distributions	13
Figure 11. Normal Distribution of Payoffs	13
Figure 12. Beta Distribution of Payoffs	14
Figure 13. Cumulative Distribution Function	15
Figure 14. Probability of Payoff Being Greater Than a Given Value.....	15
Figure 15. Occurrences for Each Place for C and A in a Single Game	18
Figure 16. Occurrences for Each Place for C and A in a Repeated Games	19

LIST OF TABLES

Table 1. Mean and Standard Deviation.....	11
Table 2. Beta Distribution Shape Parameters	12
Table 3. Probability of Yielding a Higher Payoff.....	16
Table 4. Payoff and Place of a Single Game.....	18

ACKNOWLEDGEMENTS

I would like to thank Dr. Maughmer for the continued support that you have given to me throughout this project.

I would also like to thank Dr. Byrne for helping me to establish the strategies and algorithms that used in this thesis.

Chapter 1

Introduction

The Sport of Soaring

The Federal Aviation Administration defines a glider as a heavier-than-air aircraft that is supported in flight by the dynamic reaction of the air against its lifting surfaces, and whose free flight does not depend principally on an engine¹. A sailplane is a glider that is designed to utilize naturally occurring atmospheric phenomena as lift sources to gain altitude. Common examples of these phenomena are thermals, ridge lift, and lee waves. Sailplanes are generally very aerodynamically efficient, a trait that is commonly measured by the lift-to-drag ratio of an aircraft. Today's competition sailplanes typically have a maximum lift-to-drag ratio between 40:1 and 70:1², meaning that the sailplane is able to produce 40 to 70 times more lift than drag at the velocity of minimum drag. Figure 1 shows the forces acting on a sailplane during steady flight.

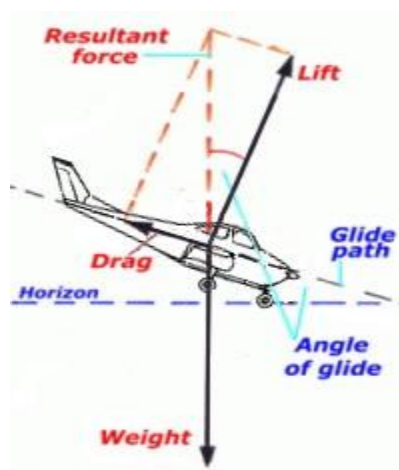


Figure 1. Forces Acting on a Sailplane³

Having a high lift-to-drag ratio is important for sailplanes because it directly correlates to the gliding ability of the aircraft. For example, a sailplane flying in still air with a lift to drag ratio

of 50:1 is capable of traveling 50 miles horizontally for every 1 mile it descends. Thus, an efficient sailplane that is using sources of lift can travel large distances.

As previously mentioned, sailplanes use atmospheric phenomena to gain altitude during flight. In the case of a thermal, this is achieved by allowing the rising air in the thermal to carry the sailplane upwards. Every sailplane will naturally lose altitude, or sink, as it flies. This is because of the lack of a power system; the sailplane must convert its potential energy into kinetic energy by trading height for speed. To recover altitude and potential energy, a sailplane must fly in air that is rising at a greater rate than the sailplane is sinking. Figure 2 shows an example of how this is achieved using thermals. In the case of thermal soaring, a sailplane will circle within the column of rising air until it has reached a sufficient altitude or the thermal has lost strength.

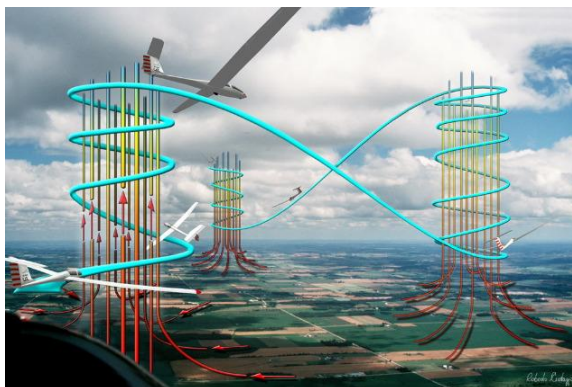


Figure 2. Thermal Soaring⁴

Throughout the last 90 years, soaring competitions have grown in size and quantity, and now incorporate seven racing classes. Standard Class competitions, which will be explored in this paper, require that the wingspan of the sailplane be no greater than 15 meters and prohibit the use of flaps and lift enhancing devices.

Competitions last multiple days, with pilots flying different tasks each day. Races are scored on the basis of speed, with the fastest pilot scoring 1000 points and the finishers scoring points based on the ratio of their speed to the winning speed⁵. When a pilot cannot complete the course, he is said to land out and receives a score based on a distance completed. The pilot with the highest total score at the end of the competition is declared the winner.

Chapter 2

RACE STRATEGY

What separates the winning pilots from the rest of the field are the decisions that they make and the skill with which they execute them. A pilot must make numerous decisions regarding the race in which he is participating if he hopes to complete the task with a respectable score. Since races are all about finishing with the fastest speed, it is imperative that pilots make choices that will not only give them the best chance of completing the course, but also will advance them through the course in the shortest amount of time possible.

To analyze the decisions that pilots make and their effectiveness, a model was developed to simulate a Standard Class competition. This model is based on analysis of the 2011 Standard Class Nationals⁶ competition and it employs two strategies: conservative flying and aggressive flying. While there are many choices that the pilots must make, many appear to be coupled and lead to the same two-strategy model. The metrics used to determine the strategy employed by each pilot on each race day are: range of the height band, speed to fly between thermals and flying in a gaggle. These metrics will be explained in detail in the following sections.

Height Band

The height band is a range of altitudes that result in the strongest average lift and consequently the fastest average speed. This band has both an upper and a lower limit; the lower being derived from an altitude that reasonably strong lift can still be achieved and the upper being a function of the local thermal strength to the maximum thermal strength. Many environmental factors play into the formation of the height band for any particular day, but ultimately the bounds of the height band are decided by the pilot.

Thermal strength is strongly influenced by the time of day¹. As shown in Figure 3, thermals are generally weakest in the mornings and evenings, with nearly constant climb rates in

the middle of the day. It is important to note that the numerical values shown in figure 3 are not significant; the behavior of the thermal as the day progresses is the important trait to glean.

By understanding the structure of a thermal, it is possible to model the lift distribution as a function of height inside of the thermal. This non-uniform distribution yields an altitude range that is known as the height band, as previously mentioned. Figure 3 shows a possible height band. Again, the numerical values are of no significance.

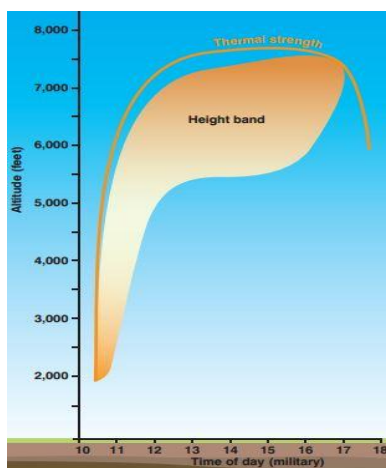


Figure 3. Height Band vs. Time of Day¹

It is worth noting that both the upper and lower limit of the height band increase in altitude throughout the morning and nearly level off in the afternoon, but the two limits quickly converge as evening sets.

It is common practice for a pilot to exit a thermal when the thermal weakens, thus the upper limit of the height band is reached when the climb rate has diminished to some fraction of the maximum achieved in that thermal, perhaps 70%. The lower limit is the lowest altitude at which the thermal is producing reasonably strong lift. A thermal does not have constant strength; rather, it has a range which yields the strongest lift and it is increasingly weak above and below this range. This makeup is shown in Figure 4.

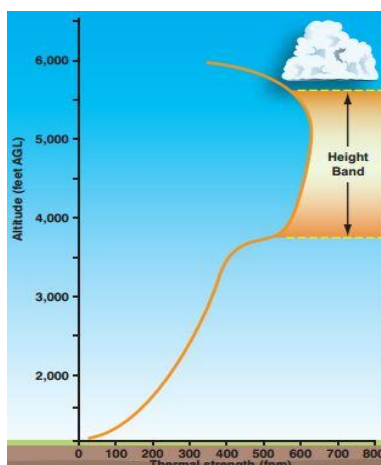


Figure 4. Altitude vs. Thermal Strength¹

In a race, each pilot will choose the height band that he will attempt to fly. Choosing a high upper limit will result in slower climb rates near the top, and choosing a low lower limit will result in slower climb rates near the bottom. Additionally, if the lower limit is moved close to the ground, the chance of not finding a thermal and landing out increases.

Although it sounds as if a pilot would have an easy decision to simply fly a height band that is centered on the strongest range of the thermal, it is not so. Incomplete information about the upcoming thermals and time required to find the center of a thermal, known as centering the thermal, are factors that affect the height band. It is crucial that the pilot be able to reach the next thermal, but as the distance to the next thermal increases, so does the altitude lost by the sailplane due to its sink rate. Thus, the upper limit must allow the pilot a sufficient altitude to descend and still reach the next thermal at or above the lower limit.

A pilot must choose between a small height band that gives up more time due to centering the thermal and a large height band that yields less climb near the top and bottom.

The height band was factored into the two strategies as a measure of the conservatism or aggressiveness of each flight. Using flight log data of the 2011 Standard Class Nationals competition, IGC Flight Replay⁷ software was used to qualitatively analyze the altitude variation of each flight to determine if the pilot was flying within a large or small height band. Flying a smaller band with a high lower limit is modeled as conservative because the sailplane will always be at a safe altitude to avoid landing out, and also because the pilot must accept the lost time

needed to center a greater number of thermals. Flying a larger band is modeled as aggressive because the pilot looks for only the strongest thermals to try to avoid weak lift. Shown in Figures 5 and 6 are the plots of altitude as a function of time of day for two pilots that are flying the same task. Using the method described, it was determined that the pilot in Figure 5 flew a consistent race and the pilot in Figure 6 flew an aggressive race with respect to the height band. It is important to note that the pilots were not directly compared against each other; rather, they were compared individually against the conditions for the given day. This process was repeated for each pilot and each day of the competition.

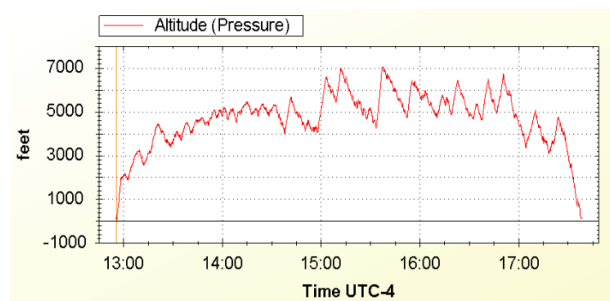


Figure 5. Conservative Height Band

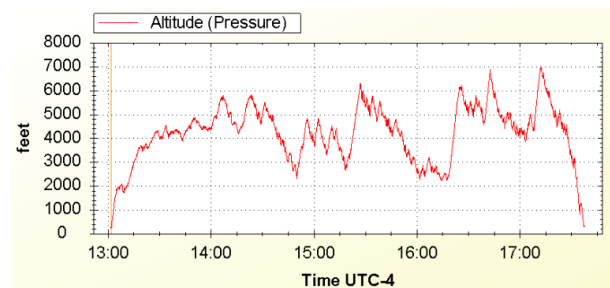


Figure 6. Aggressive Height Band

Speed-to-Fly Between Thermals

Determining what speed should be flown between thermals is a nontrivial exercise; there are pros and cons to varying flight speed, namely the tradeoff between sink rate and velocity. As discussed earlier, each sailplane has a minimum drag, and a corresponding velocity at which this minimum drag occurs. Due to the lack of propulsion on a sailplane, weight must be used to

provide the force to overcome drag, which is done by flying nose down with respect to the horizon. Thus, the sailplane must fly downhill to achieve a constant horizontal velocity. By calculating the drag at each airspeed, the angle of flight becomes readily available and leads to the sink rate: the vertical component of the velocity. Figure 7 shows the relationship of sink rate to velocity for a sailplane.

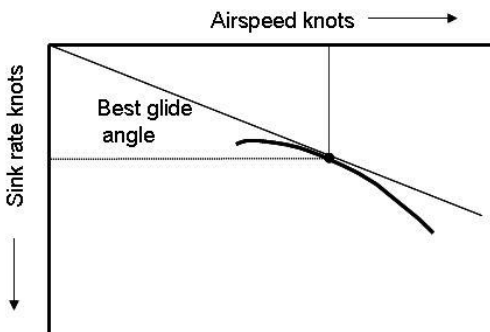


Figure 7. Sink Rate Polar

Most theories on speed-to-fly result in a speed that maximizes cross-country speed. For the purposes of this analysis, the theory developed and implemented by Dr. Paul MacCready in the 1950's will be used to evaluate the decisions made by the pilots. MacCready's theory uses the expected climb rate of the next thermal to determine the cruising speed that will minimize the time required to reach and climb to the top of the next thermal². While this theory does provide an optimum speed, one of its key assumptions is that the pilot will fly directly to the thermal. This is, however, not always the case. Pilots do not always know where they will find the next thermal and do not always fly in a straight line while searching for it. To further the reader's understanding of the theory, Figure 8 shows how the speed to fly is graphically determined using MacCready's theory.

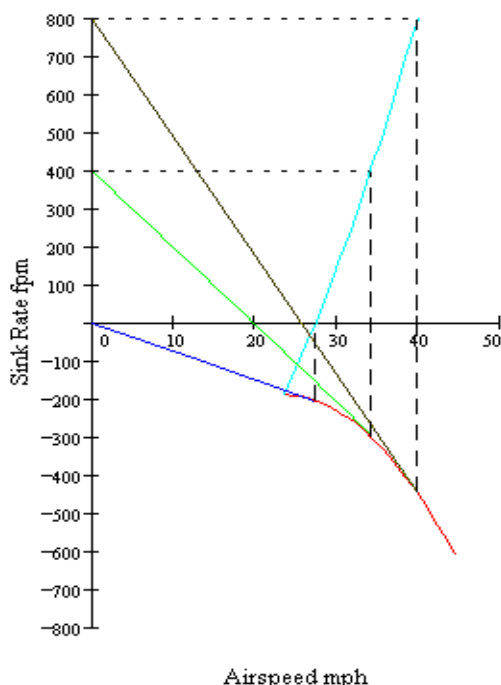


Figure 8. MacCready Speed-to-Fly⁸

Setting the intercept of the vertical axis to the expected climb rate, a line is drawn tangent to the sailplane's sink rate polar shown in red. This point of tangency corresponds to the velocity that minimizes the time required to fly to and climb to the top of the next thermal. Increasing the expected climb rate results in a faster speed-to-fly, but also yields a greater sink rate. The average cross-country speed can also be obtained from Figure 8, as this speed corresponds to the horizontal intercept of the tangent line.

As mentioned, if the pilot does not know where the next thermal will be, MacCready's theory can result in a slower average speed than predicted. When searching for a thermal while flying at the MacCready speed-to-fly, the sailplane will lose more altitude than if flying at the speed which maximizes the lift-to-drag ratio. This effect can be seen in Figure 8, and is best explained by the fact that the sink rate increases as the cruising speed increases.

In the two strategies, flying slower than the MacCready speed corresponds to a more conservative flight. This is due to the recognition of the need for more range to search for the next thermal. In contrast, flying at the MacCready speed corresponds to a more aggressive flight because the sailplane will sink at a higher rate and will be at a lower altitude when entering the next thermal, leading to a greater chance of landing out.

IGC Flight Replay was used to determine the climb rates and cross-country speeds for each flight, and the MacCready speed-to-fly values were found using a speed-to-fly calculator available from St. Olaf College⁹.

Flying in a Gaggle

When multiple pilots fly together, it is known as a gaggle. Flying in a gaggle is assumed to be beneficial when trying to find thermals because there are multiple sailplanes searching in one area, making it more likely to find a thermal and less likely to land out. It is also assumed that thermalling in a gaggle is less efficient than thermalling alone because of the additional need to focus on the other sailplanes to ensure safety. Flying in a gaggle is modeled as a conservative strategy due to these two assumptions.

To determine whether or not pilots were flying in a gaggle, their flight data were analyzed using IGC Flight Replay. A preliminary analysis used the plots of altitude as a function of time of day and the start and finish times to compare two or more pilots' flight paths. Once it had been determined which altitude plots and start and finish times were similar, the flights were visually inspected using the flight replay feature of the software. From this analysis, it could be confirmed that pilots had flown together.

Chapter 3

Analytical Model

Expected Payoffs

Using the methods described above, each flight was classified as either a conservative or aggressive flight based on the adherence to these strategies as defined above. In the case of the 2011 Standard Class Nationals, it was determined that 40% of the flights used the conservative strategy and 60% of the flights used the aggressive strategy. The reader should note that from this point, the conservative strategy will be represented by C, and the aggressive strategy will be represented by A.

Using the official points as shown on the Soaring Society of America⁶ website, an average payoff, μ shown in Eq. (1), and standard deviation, σ shown in Eq. (2)¹⁰, were computed for each of the two strategies using

$$\mu = \frac{1}{n} \sum_{i=1}^n (x_i) \quad (1)$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2} \quad (2)$$

where the x_i values are the scores (payoffs) of each flight.

Table 1 shows the results for the 2011 Standard Class Nationals competition.

	C	A
μ	913.2	873.4

σ	47.1	88.1
----------	------	------

Table 1. Mean and Standard Deviation

These results show that C has a higher average payoff than A, but the standard deviation of A is greater than C. The standard deviation is a measure of the variation of a set, equivalently the dispersion from the average value. The fact that A has a greater standard deviation than C indicates that the payoffs from A are more spread out; they are less dense around the mean but occur in higher quantities away from the mean.

Modeling the Payoff Distribution

To create a model for this data, a probability distribution was fit using MATLAB¹¹ software. The model first assumed a normal distribution over the payoffs. A normal distribution is characterized by a bell curve and is symmetric about the mean value. Additionally, one standard deviation to the left and to the right of the mean value represents the range in which 68 percent of the data lie¹². The flaw of using a normal distribution for this model is that the distribution of payoffs is not symmetric about the mean because the maximum payoff of 1000 results in a cluster of data points above the mean and a larger spread of data below the mean.

A beta distribution was then used to model the data. A beta distribution can be used to model a phenomenon with a set of possible values in some closed interval $[c, d]$ ¹². This closed interval can be mapped to any interval with finite values c and d , namely $[0, 1000]$, which is the range of payoffs of a single day of a sailplane competition. The beta distribution is parameterized by two positive shape parameters, α and β , which are obtained via Eq. (3) and Eq. (4)¹³.

$$\mu = \frac{\alpha}{\alpha + \beta} \quad (3)$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \quad (4)$$

Thus, for known values of μ and σ , the values of α and β can be obtained.

	C	A
α	34.766	13.369
β	3.306	1.937

Table 2. Beta Distribution Shape Parameters

To illustrate the differences of the two distributions, Figures 9 and 10 show examples of several normal distributions and several beta distributions, respectively. The x-axis represents a random variable, x , and the y-axis represents the probability density function, $PDF(x)$. The probability that the random variable will occur within a given range of values is equal to the integral of the probability density function over that range of values¹².

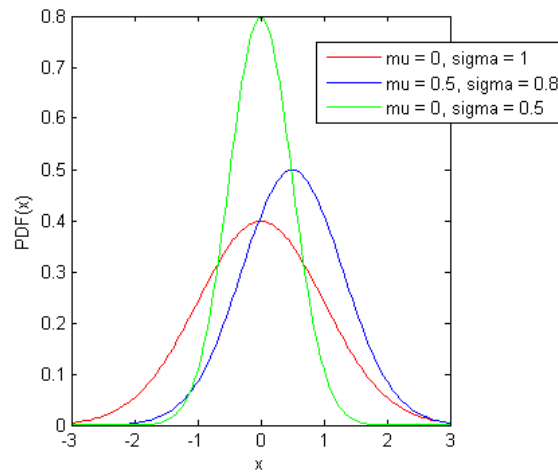


Figure 9. Examples of Normal Distributions

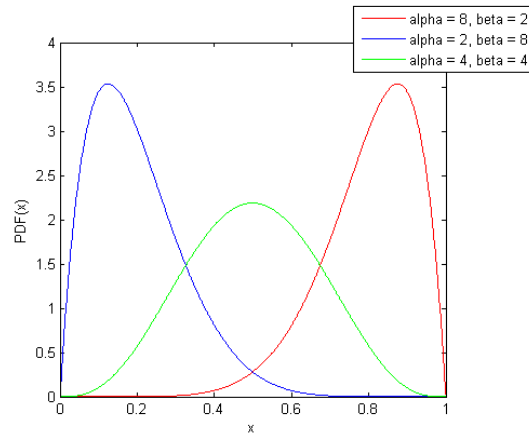


Figure 10. Examples of Beta Distributions

Of the distributions shown above, the shape of the red curve of the beta distribution was hypothesized to best model the data gathered from the competition. This curve is not only bounded due to the requirements of the distribution, but also resembles the frequency of occurrence of the payoffs of C and A.

Using the calculated values of the mean and standard deviation for each strategy, the probability density functions for both a normal and a beta distribution were computed. The results are shown in figures 11 and 12, respectively.

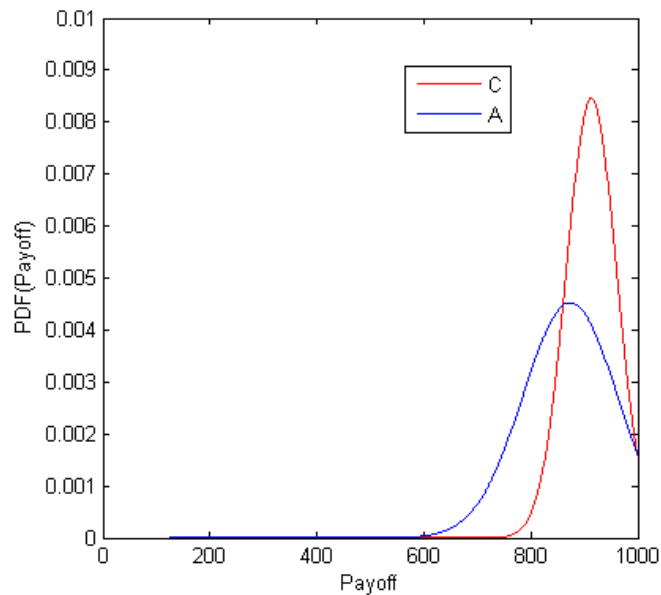


Figure 11. Normal Distribution of Payoffs

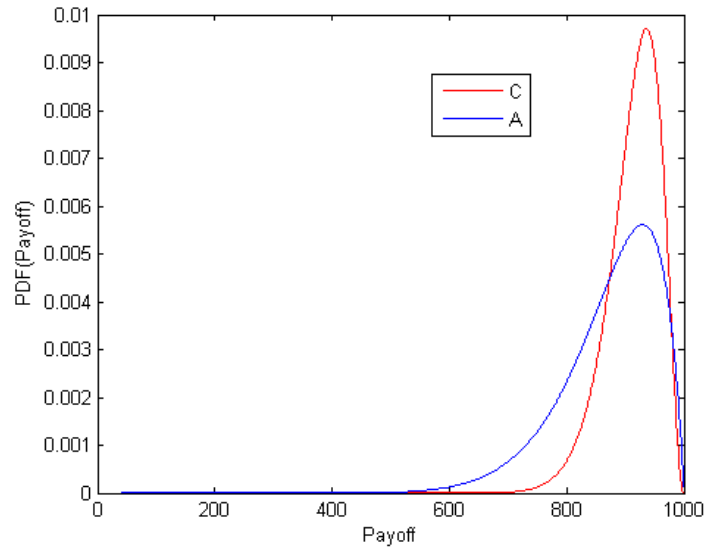


Figure 12. Beta Distribution of Payoffs

The differences between the normal and the beta distributions are now evident. In the case of the normal distribution, the probability density function is symmetric and does not capture the effects of a clustered distribution above the mean and a more spread out distribution below the mean. On the other hand, the beta distribution does capture these effects.

Examining Payoff Probabilities

In probability theory, the cumulative distribution function (CDF) gives the probability that a random variable, x , will be found at a value *less than or equal to* x^{12} . The CDF is applicable to this model because it directly enables the computation of the probability that the payoff is *greater than* a specific value. This relationship exists because, with a probability of 1, a random variable will take some value within its domain or sample space. Thus, for the closed domain $[0, 1000]$, there is a probability of 1 that the payoff will be less than 1000. (The reader should note that this is yet another strike against using a normal distribution because the normal distribution allowed positive probabilities for payoffs greater than 1000.) To compute the probability that the payoff will be greater than a given value, one must simply subtract the corresponding value of the CDF from 1.

Figures 13 and 14 show the CDF and a plot of the probability that the payoff will be greater than a given value, respectively.

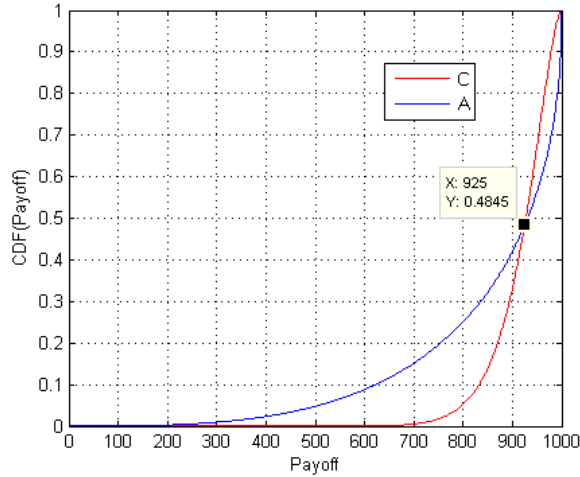


Figure 13. Cumulative Distribution Function

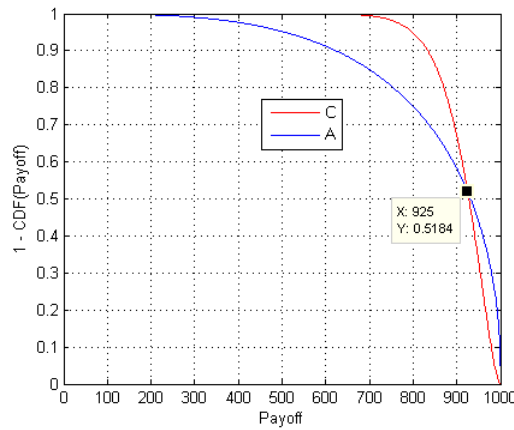


Figure 14. Probability of Payoff Being Greater Than a Given Value

Examining Figure 14, there is about a 0.5 probability that the payoff will be above 925 for both C and A. In fact, Table 3 shows the probability of payoffs greater than certain values for both C and A.

Payoff	Probability C yields higher	Probability A yields higher

	Payoff	payoff
700	0.9950	0.8506
750	0.9834	0.8069
800	0.9500	0.7515
850	0.8651	0.6800
875	0.7878	0.6359
900	0.6770	0.5839
925	0.5266	0.5212
950	0.3385	0.4420
960	0.2568	0.4033
970	0.1754	0.3582
980	0.0996	0.3036
990	0.0372	0.2314

Table 3. Probability of Yielding Higher Payoff

Chapter 4

Simulating Single and Repeated Games

A single game is a game that is played only once. A repeated game is a single game that is played multiple times. In the case of sailplane racing, a regional competition needs three days of racing to be official and can run for six or seven. A national competition needs four days and can run for ten. Thus, regional and national competitions can be modeled as repeated games. This thesis will first simulate a single day (single game) before simulating multiple day competitions (repeated games).

Single Game Simulation

The simplest way to simulate a single game is to assume a total number of players and assign either the C or A strategy to each of the players. In this analysis, a total of 15 players were assumed, of which 6 played C and 9 played A. This models the distribution of strategies within the 2011 Standard Class Nationals competition.

For each player, a random number generator was then used to generate a payoff using the previously computed probability density functions of C and A.

Table 4 shows an example of the resulting payoffs when the game is played once. The third column shows an absolute score that is obtained for each player by dividing the player's score by the highest score of any player, then multiplying by 1000.

Place	Payoff	Absolute Payoff	Strategy
1	999.4	1000	A
2	987.0	988	A
3	975.4	976	C
4	961.8	962	A
5	955.5	956	C

6	933.2	934	C
7	930.9	932	A
8	924.9	926	A
9	877.5	878	C
10	871.1	872	A
11	868.7	869	A
12	861.1	862	C
13	856.1	857	A
14	828.7	829	A
15	753.8	754	C

Table 4. Payoff and Place of a Single Game

This single game was then played 10,000 times to generate a distribution of finish places for each strategy. The results are shown in Figure 15.

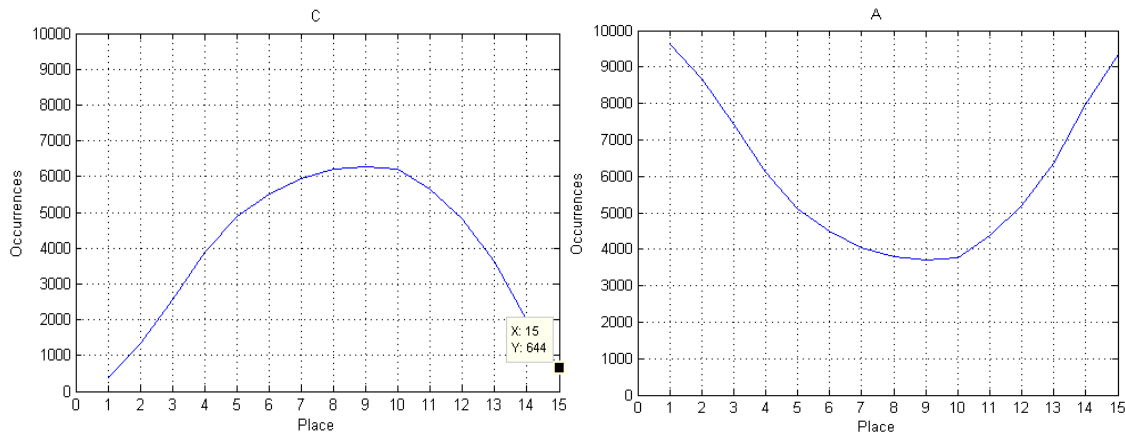


Figure 15. Occurrences for Each Place for C and A in a Single Game

The use of 10,000 repetitions allowed for a better representation of outlying data points. For example, Table 4 shows that in one outcome of a single day competition, a player playing C came in last place; however, after executing 10,000 repetitions, only six percent of the games had C in last place.

As shown in Figure 15, A takes both 1st and 15th about 95% of the time, while C takes 5th through 11th more than half of the time.

Pure Strategy Repeated Game Simulation

To simulate a multiple day sailplane competition, a repeated game is necessary. In the analysis, it will be assumed that a player plays the same strategy for every day of the competition, thus making this a pure strategy analysis. Below, the results for C and A are shown for two through ten day competitions in Figure 16. As with the single game, each repeated game was played through 10,000 repetitions.

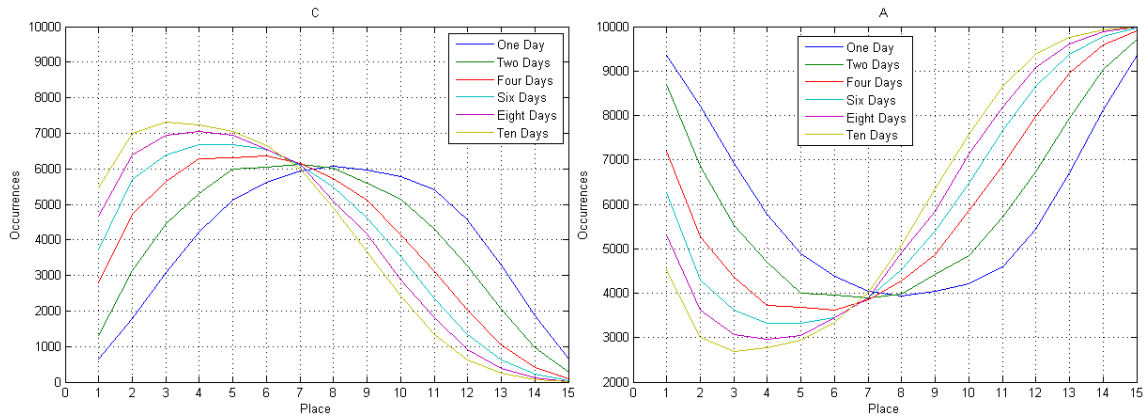


Figure 16. Occurrences for Each Place for C and A in Repeated Games

As can be seen by examining Figure 16, the number of times in which C takes first place in a competition increases as the number of days of the competition increases. In longer contests, C players are more likely to place well than are A players. Conversely, A players placed worse as the number of days of the competition increases.

Chapter 5

Conclusion and Recommendations for Future Work

Conclusion

The strategies C and A, developed from flight data of standard class sailplane races, can model the results of a sailplane race quite well. As expected, the conservative strategy yielded a high chance of a top place in longer competitions and the aggressive strategy was more likely to win on a single day.

In the 2011 Standard Class Nationals competition that was analyzed, the top pilots played C more often than A over the duration of the eight days of racing. In the eight day repeated game simulation, C players took more top places than did A players. Also in the 2011 Standard Class Nationals, all nine winning flights flew A in the single day race (day 5 saw a tie for first place). In the single day simulation, A won 94% of the time.

Recommendations for Future Work

While the analysis presented in this thesis brings about interesting results, there are many possible ways to further the analysis.

An additional simulation that could model the percentage of wins as a function of the number of pilots using each strategy would be welcomed. This analysis would require the number of days of the competition to be held constant while the number of conservative pilots is varied from zero to the total number of pilots in the competition.

Also, simulating single day and multiple day competitions with pilots using a mixed strategy profile would be enlightening. This analysis would require a determination of the percentage of flights that a pilot flies each strategy, then fitting that data to probability

distribution. A strategy set could then be modeled by using this probability distribution to determine a strategy for each pilot. The competition could be simulated as was done in this thesis, and the number of wins as a function of each mixed strategy profile could be computed.

Appendix A

Payoff Data

Using the 2011 Standard Class Nationals competition, the following payoff sets were computed for strategies C and A.

C = {	789	819	836	844	856	866	868	869	872	872	874
	883	884	884	887	889	893	893	893	897	898	898
	900	902	902	908	910	916	919	922	935	936	936
	947	948	948	949	954	956	956	958	962	967	973
	986	997	998	998}							

A = {	114	245	354	360	471	572	770	800	813	823	832
	840	840	840	841	847	849	850	856	865	866	871
	877	878	879	885	886	896	897	898	900	902	908
	911	911	913	914	915	916	918	919	921	921	922
	924	928	930	934	936	937	938	941	946	947	954
	967	969	974	977	991	993	998	998	998	1000	1000
	1000	1000	1000	1000	1000	1000}					

Appendix B

MATLAB Code

The simulations shown above were generated using MATLAB R2009b. The following code was used.

```

%%% Written by Nick Grasser
%%% Used to simulate single and multiple day sailplane races

clear
clc

c = load('c_payoff.txt');
A = load('A_payoff.txt');

c = c/1000;          % MATLAB requires the interval to be (0, 1)
A = A/1000;

x = c';
x = x-0.000001;    % Must subtract very small number to keep an open
                  % interval (0, 1) as required by MATLAB
PD_c = fitdist(x, 'beta');
x = x+0.000001;
c = x';

x = A';
x = x-0.000001;
PD_A = fitdist(x, 'beta');
x = x+0.000001;
A = x';

players = 15;
players_c = 6;
players_A = players - players_c;

games = input('# games = ');

wins = zeros(players,2);
pay_C = zeros(players_c,games);
pay_A = zeros(players_A,games);

repetitions = input('# repetitions = ');

for k = 1:repetitions

```

```

pay_C = zeros(players_c,games);
pay_A = zeros(players_A,games);

for i=1:games
    rc = 1000*random(PD_c,players_c,1);
    rA = 1000*random(PD_A,players_A,1);

    A = [rc; rA];
    for j = 1:size(A)
        if j>size(rc)
            pay_A(j,i) = A(j);

            else
                pay_C(j,i) = A(j);
            end
        end
    end
end
pay_C;
pay_A;

expected_C = 0;
expected_A = 0;

for i = 1:games
    expected_C = expected_C + pay_C(:,i);
    expected_A = expected_A + pay_A(:,i);
end
expected_CONSERVATIVE = sort(expected_C/games);
expected_AGGRESSIVE = sort(expected_A/games);

Final = zeros(players_A,2);

for i = 1:players
    if max(expected_A) < max(expected_C)
        [value,dummy] = max(expected_C);
        Final(i,1) = value;
        expected_C(dummy) = 0;
    else
        [value,dummy] = max(expected_A);
        Final(i,2) = value;
        expected_A(dummy) = 0;
    end
end

end

for i = 1:players
    if Final(i,1)>0
        wins(i,1) = wins(i,1)+1;
    end
    if Final(i,2)>0
        wins(i,2) = wins(i,2)+1;
    end
end

```

```
end

end
wins

figure(1)
bar([1:1:players],wins(:,1))
grid on
xlabel('Place')
ylabel('Frequency')
title('C')

figure(2)
bar([1:1:players],wins(:,2))
grid on
xlabel('Place')
ylabel('Frequency')
title('A')
```

BIBLIOGRAPHY

1. United States of America. FAA. *Glider Handbook*. Web. Accessed 26 Mar. 2014.
2. Thomas, Fred. *Fundamentals of Sailplane Design*. Trans. Judah Milgram. College Park, MD: College Park, 1999. Print.
3. *Aerodynamics of Gliding*. Digital image. *Pilot Friend*. Web. Accessed 26 Mar. 2014.
4. "What Is Soaring?" *Saskatoon Soaring Club*. Web. Accessed 26 Mar. 2014.
5. "RACING SAILPLANES RULE." *Soaring Society of America*. Web. Accessed 26 Mar. 2014.
6. "Standard Class Nationals." *Soaring Society of America*. Web. Accessed 26 Mar. 2014.
7. *IGC Flight Replay*. Computer software. YWTW. Vers. V1.0. Web. Accessed 26 Mar. 2014.
8. Huffman, Larry, and Pete Lehmann. *A Speeds To Fly Toolkit For The Mathematically Challenged*. Digital image. *Speed To Fly*. Web. Accessed 26 Mar. 2014.
9. Hanson, Bob. *Glider Speed-To-Fly Calculator*. Computer software. *St. Olaf College*. Web. Accessed 26 Mar. 2014.
10. "Standard Deviation." *Wolfram MathWorld*. Web. Accessed 26 Mar. 2014.
11. *MATLAB*. Computer software. *The Language of Technical Computing*. Vers. R2009b. MathWorks.
12. Ross, Sheldon M. *A First Course in Probability*. 8th ed. Upper Saddle River, NJ: Pearson Education, 2010. Print.
13. "Beta Distribution." *Wolfram MathWorld*. Web. Accessed 26 Mar. 2014.

ACADEMIC VITA

Nicholas J. Grasser
1123 E. Springfield Dr.
Bellefonte, PA 16823

Education: **The Pennsylvania State University**

- Schreyer Honors College scholar
- B.S. Aerospace Engineering candidate
- B.S. Mathematics candidate
- Expected graduation date: May 2014

Projects and Experience: **Undergraduate Honors Thesis**

- Advisors: Dr. Mark Maughmer, Professor of Aerospace Engineering, and Dr. Diane Henderson, Professor of Mathematics
- Interdisciplinary thesis exploring strategies and payoffs for sailplane competitions using game theory analysis

Aerospace Engineering 404H Teaching Intern

- Teaching Intern for the Penn State Department of Aerospace Engineering 404H course, Spring 2014 semester
- Work with the course professor, Dr. Mark Maughmer, and the graduate teaching assistant
- Responsible for a 4 hour commitment per week to supervise and ensure the safety of students during lab hours

Aerospace Engineering 306 Teaching Intern

- Teaching Intern for the Penn State Department of Aerospace Engineering 306 course, Spring 2014 semester
- Work with the course professor, Dr. Mark Maughmer, and the graduate teaching assistant
- Responsible for a 10 hour commitment per week to attend classes, hold office hours, and grade assignments

Penn State College of Engineering, Academic Excellence Center

- Tutor for undergraduate engineering students, Fall 2012 and Spring 2013 semesters
- Assisted students in Engineering Mechanics, Physics, and Mathematics courses

Penn State Design/Build/Fly (DBF) Team 2013 – 2014

- Design Team Lead for a 6 member undergraduate team
- Responsible for generating the detailed design of the competition airplane
- Responsible for establishing and meeting deadlines for design and fabrication
- Responsible for managing underclassmen during the fabrication of the aircraft

- Will compete in the 2014 AIAA DBF national competition in Wichita, Kansas

Penn State DBF Team 2012 – 2013

- Participated on a 4 member undergraduate team
- Responsible for the fabrication of the aircraft and for the detailed design of the payload system
- Constructed 4 prototype airplanes that were tested and analyzed
- Co-authored a 60 page final design report
- Competed in the 2013 AIAA DBF national competition in Tucson, Arizona (placed 18th out of 81 teams)

Columbia Helicopters, Inc., Portland, Oregon

- Student Intern during the summer of 2013
- Worked with engineers to analyze structural components of Chinook helicopters
- Revised a substantiation report for FAA approval detailing structural integrity of the interior side facing seats of Chinook helicopters
- Used SolidWorks software to design parts for Chinook and Sea Knight helicopters

Activities:	Member- American Institute of Aeronautics and Astronautics	2010- present
	Member- Schreyer Honors College Student Council	2011-2012
	Member- Penn State Club Cross Country Team	2010- 2012
	Eagle Scout- Boy Scouts of America, Troop 34	2010
	President- Bellefonte Area High School National Honor Society	2009- 2010
	President- Bellefonte Area High School Academic Decathlon Club	2009- 2010

Awards:	Penn State Dean’s List- Fall 2010, Spring 2011, Fall 2011, Spring 2012, Fall 2012, Spring 2013, Fall 2013
	Recipient of the Penn State Academic Excellence Scholarship (4 years)
	Recipient of the Francis S. and E. Keith Anderson Scholarship in Engineering (4 years)
	Recipient of the GBU Foundation Scholarship (4 years)
	Recipient of the GBU District 2000-ULS Scholarship (3 years)