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AN INTRODUCTION TO MULTI-SCHEMA GAME THEORY

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ABSTRACT

The primary purpose of this work is to introduce "schemas" into game theory to produce a new way in which to view Nash Equilibrium. Functioning similarly to enzymes in a chemical reaction, schemas eliminate the need to incorporate motivations into player utility functions, lessen subjective analysis, and allow games to be viewed via an engineering "systems" perspective. After mathematically defining several schemas (i.e. selfish, global, and fair), two solvers were created to predict the equilibrium outcome of real-world problems simplified into 2x2 player games. As predicted, games with new schemas produced different optimal strategies for players in even the most well-known games. Furthermore, a player's schema distribution was predicted from experimental results found in literature. The concepts described herein could lead to the development of a new, more robust game theory model.

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Chapter 1

Introduction to game theory and schemas

1.1 Difficult decisions may force us to go outside our natural preferences

"Where there is no decision there is no life."

\- *J.J. Dewey*

Every muscle in your body is urging you not to make this decision. It is the harder choice, but for some reason you know it is the direction to go. How do you make that difficult decision? Should you, or can you, account for the seemingly unpredictable actions of others?

At some point in time, we have all made a difficult or uncomfortable decision. In retrospect, it can change our character or how we view the world, but that initial decision often involves a leap of faith. Whether you consider yourself decisive or indecisive, a good or bad decision-maker, there is no denying the importance and power of choice. The purpose of this research is to add to our conceptual understanding of decisions, especially in scenarios in which these decisions depend upon and affect the actions of others. Furthermore, this work will lay a mathematical framework in which more quantitative, applicable models might develop.

First, I will address a pervasive assumption in game theory analysis that each person always seeks to maximize their personal utility. To do so, I will introduce games with multiple "schemas" that are analogous to enzymes in a chemical reaction. After showing a new method for examining games, I will point to future research, including the use of chemical kinetics to quantitatively describe "tipping points."[1] In doing so, I hope to contribute to a developing field, "Physics of Community", that uses engineering, chemistry, physics, biology, and ecology to explain and model human decision making.

[1] The "tipping point" is a phenomena popularized by Malcom Gladwell in which he describes the moment in which an idea, trend, or social behavior crosses a threshold, tips, and spreads like wildfire.

1.2 Thermodynamics provides a unique perspective in which to view decisions

Thermodynamics is a branch of natural science that uses mathematical modeling to show the relationship of heat and temperature to energy and work. In the past, the study of thermodynamics was essential to the development of an idealized engine[2] and enhanced our knowledge of and ability to utilize chemical processes. Although thermodynamics is usually thought of in a technical sense, it can also be used to describe a social situation. First, consider a thermodynamic system, or a macroscopic region of the universe that is completely separated from its surroundings via a strictly defined boundary (Figure 1-1). This terminology can be applied to the environment of a decision:

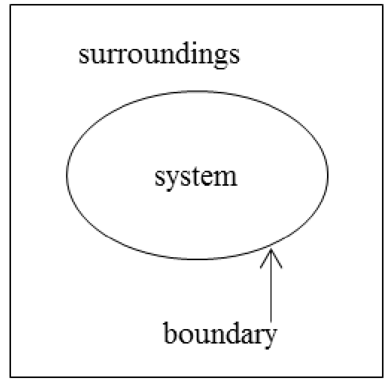

Figure 1-1 Thermodynamic system. Its use can extend to the realm of human decision making.

1. system. A system involves all history, facts, perspectives, choices, outcomes, people, etc. that are relevant to the inputs and outputs of a particular decision.
2. surroundings. The surroundings contain all aspects and influences that are unimportant, unavailable, or unknown to a system for the time being.
3. boundary. A boundary separates the system against potential influences from the environment. The separation can, among others, be physical, spatial, temporal, or knowledge-based.

There are four major laws that characterize, define, and create a complete picture in which to view a thermodynamic system. Of these four thermodynamic laws, the second law provides a particularly unique perspective in which to analyze the system outlined above. The second law of thermodynamics states that the entropy of a closed system (i.e. completely separated from its surroundings) will increase over time, eventually reaching a maximum value.

[2] An idealized engine uses the Carnot cycle, proposed by Nicolas Léon Nicolas Léonard Sadi Carnot in 1824, to show the most efficient way in which to convert thermal energy to work.

Equivalently, the internal energy of a system will decrease and approach a minimum value at equilibrium. Much like physical systems tending to choose states with the lowest free energy, decision-makers tend to pick states with low pain, or high utility. The utility scale is a function of a person's preferences and beliefs between outcomes that result from alternative choices. Defining a utility function requires that the preferences between any two decisions can be established and that these preferences are stable and transitive (Fishburn 1982). Instead of using utility, in this research I create an analogy between the internal energy of a system and the "pain" of a decision. Much like internal energy, pain can be described as the pain in a decision arising from the relative positions and interactions of its parts.[3] Like electricity, the system will tend to flow towards a path of least resistance and equilibrium occurs when pain is minimized.

1.3 This research creates a new way in which to view decision equilibrium

Similar thermodynamic analogies to a social system have been previously studied, starting with J. Willard Gibbs. Gibbs, along with Irving Fisher, published a thesis in 1892, entitled *Mathematical Investigations in the Theory of Value and Prices*, that drew a direct analogy between Gibbsian equilibrium in physical and chemical systems and the general equilibrium of markets. Paul Samuelson, the first American to win the Nobel Memorial Prize in Economics, stated that in understanding prices he was in debt "not to Pareto or Slutsky, but to the great thermodynamicist, Willard Gibbs of Yale." (Samuelson 1986)

Recently, researchers have been applying thermodynamics and statistical physics to the problem of bounded rationality[4] decision-making (Wolpert 2006). To my knowledge, the most recent research in this area has focused on generalizing models of bounded rationality based on variational principles regarding information costs and utility gains (Ortega & Braun 2013). This

[3] Pain, described further in the Methods section, is comparable to a "utility function" used in game theory to describe the favorability of outcomes that arise from various decisions.

[4] Bounded rationality is described further in section 1.3

research considered differences in the free energy (ΔF) of an initial and final state corresponding to a decision before and after deliberation processes. By focusing on changes of free energy rather than absolute values, these researches put a thermodynamic lens on Kahneman's prospect theory (Kahneman et al. 1979), the belief that human decision-makers consider changes in value rather than absolute values.

While research has given significant thermodynamic insight into how a person balances utility gains and information processing costs, it has not focused on what thermodynamic equilibrium really means in such a scenario, and how it might be different for different systems of decision-makers. I believe it is essential to understand decision equilibrium[5] and its surprising uniqueness between individuals. Just as physical equilibrium depends on temperature, decision equilibrium depends on a person's motivations. I attempt to mathematically describe these motivations to give a new way in which to view long-term (meaning enough time has lapsed for available resources to be exhausted), equilibrium decisions. Naturally, this generates two questions: Why equilibrium analysis? Why separate motivations from the utility functions of decision-makers?

First, equilibrium research allows a vast amount of deductions to flow from an accurately described system. By having an idea of what characterizes the equilibrium conditions and variables of a system, further research can extrapolate what happens in the case of disturbances (much like Le Chatelier's principle), or even how the system reached equilibrium in the first place. Furthermore, these analyses could possibly explain divergences from the norm by predicting the conditions that would need to change for equilibrium to occur.

Second, and the element this work adds to equilibrium analysis, is the viewpoint that player motivations should be extricated from utility functions. If true, decisions will eventually be

[5] Equilibrium decisions imply that the relevant system has reached a homeostatic state, enjoying a condition of harmony, stability, or balance.

modeled as a system of complex reactions where the product of these reactions, the final decision, will depend on time and reaction conditions. Therefore, once basic equilibrium reactions that surround decisions are described, this research can be extended to more complex, dynamic systems.

1.4 Game theory uses an integrated approach to model human decision making

The idea of a "social system" (Ashby 1952) and thermodynamic equilibrium can be combined with principles taken from game theory, a field that uses a body of techniques to study conflict, cooperation, and decisions between "intelligent, rational" decision-makers. In application, game theory uses mathematical techniques for analyzing situations where two or more "players" (where players can be individual people, groups of people, companies, countries, etc.) make decisions that influence each other's welfare. Today, game theory can be used to analyze a variety of real-word situations such as military arms races (Intriligator 1982), policy choices for presidential candidates (Downs 1957), salary negotiations (Raiffa 1982), and criminal behavior[6]. (Graetz et al. 1986) In general, game theory has expanded rapidly over the past century, with applications diversifying and usefulness increasing.[7]

One of the strengths of game theory is its ability to combine various methods in which to model human decisions. Until recently, there were typically two main ways in which to research how people make judgments under uncertainty (Maital 2004). On one hand, there was the mathematical. Here, theoretical economists tested the validity of theories by comparing predictions to reality. On the other hand, there was the behavioral. Here, psychologists would observe and carefully generalize behavior. Despite their mutual interest in decision-making, these fields were not always intertwined. In 1959, economist Hebert Simon noticed this flaw and asked,

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[6] This article relates specifically to the criminal behavior of tax compliance.

[7] See for instance: FRIEDMAN, J. W. (1991): *Game theory with applications to economics*. Oxford University Press.

"How have psychology and economics gotten along with little relation in the past?" (Maital and Maital 1984) However, recent research is moving towards a more integrated approach. Sholmo Maital, the Academic Director of Israel's leading executive development institute, predicts that "the confluence of mathematical and behavioral approaches will dominate our profession and will create at last a powerful machine for generating policies that hold water." (Maital & Maital 1984) The usefulness of game theory is continually improving as it constantly combines theoretical, mathematical thought with practical observations. I hope to add to the theoretical realm of the field, while also pointing towards future work to confirm or deny proposed theories.

1.5 Game theory uses three basic components to model decision making

Before developing new theories, it is important to understand the basics of game theory modeling. As far back as 1838, economists such as Auguste Cournot have been developing methods for studying strategic interaction. However, these methods were often case specific and produced no general toolbox for analysis. Starting in the 1900's, mathematicians such as Zermelo, Borelo, and von Neumann began studying mathematical formulations of games. It was not until economist Oskar Morgenstern met von Neumann that modern game theory was born. Von Neumann and Morgenstern's seminal work, *Theory of Games and Economic Behavior*, revolutionized the field of economics by providing unique mathematical insights into economic and social organization theory. To von Neumann and Morgenstern, it was "without a doubt reasonable to discover what has led to progress in other sciences, and investigate whether the application of the same principles may not lead to progress in economics also." (Neumann & Morgenstern 1947) By this, the authors were referring to their introduction of mathematical methods to explain economic behavior.

I possess a similar mentality and believe game theory can benefit from integration with additional sciences. Game theory can be used in a manner similar to that of physics models.

Game theorists, like physicists, seek to understand more complicated scenarios by first understanding simplified, hypothetical examples. Although these models may be highly simplified, they can provide important lessons that lead to further discoveries. In order to simplify a real-life problem into a game theory problem, rules and assumptions must be made. Underlying all games are the following three components:

1. players. There are 2 or more players (e.g., A and B), whose decisions impact each other.
2. choices. Each player has two or more well-specified choices (e.g., A_1, A_2; B_1,B_2).
3. payoffs. Associated with each combination of choices is a known outcome.

The games are played "rationally", such that each player's sole objective is to maximize the expected value of his own payoff (Stearns 2000), which is measured on some utility scale. As mentioned previously, however, I introduce the idea that each player seeks to "minimize pain." I do this, in part, because of the difficulty in accurately creating and using a utility scale. Von Neumann and Morgenstern illustrated this difficulty in *Theory of Games and Economic Behavior*:

> *One of the chief difficulties lies in properly describing the assumptions which have to be made about the motives of the individual. This problem has been stated traditionally by assuming that the consumer desires to obtain a maximum of utility or satisfaction ... The conceptual and practical difficulties among the notion of utility… are well known and their treatment is not among the primary objective of this work.*

I believe the time to address these conceptual and practical difficulties is now. While many researchers also recognize the importance of understanding player motivations, few generalizations have been made regarding its treatment. George Akerlof (1982), Peter H. Huang and Ho-Mou Wu (1992), Vai-Lam Mui (1992) and Julio J. Rotember (1992) have all investigated social motivations, but all with models that tended to be context specific. University of California Professor Matthew Rabin took the first step in developing a general framework for incorporating social emotions by introducing fairness into the equilibrium analysis of games (Rabin 1993). Rabin adopted and generalized the framework developed by John Geanakopolos, David Pearce, and Ennio Stacchetti (1989) in which they modified payoffs by allowing players' utility to depend on beliefs, as well as material payoffs. Recently, game theorists have allowed utility to even depend on the intention behind the action (Falk et al. 2008).

Despite the work of these researchers, the rational assumption underlying game theory has never been truly tested or modified. The current treatment of rational behavior in game theory is described by Figure 1-2.

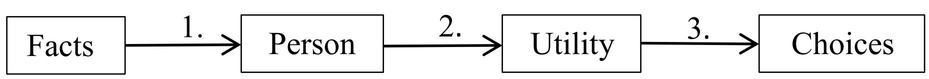

Figure 1-2 Understanding the true meaning of rational behavior in game theory

1. A player receives the facts that are relevant to the game
2. Based on the players preferences, motivations, beliefs, etc., each possible outcome is attached to an expected utility
3. A player chooses a strategy that will minimize personal pain

Game theory rationality concerns itself with how players choose the actions they will play, which is contained only in step 3. In fact, the classical theory of rationality generally ignores information processing costs (steps 1 and 2) and the effort it might take to find an optimal decision (Ortega & Braun 2013). Unlike perfectly rational players, bounded rationality players are subject to limited information processing resources. Because having limited processing resources can be compared to a system with finite energy, bounded rationality has helped eliminate arguments (Easton 1956)[8] that decisions cannot be fully described by thermodynamic equilibrium. If resources available to the system are limited, decision-makers may not always be able to find the optimum option. This limitation, sometimes referred to as satisficing, has been

[8] These arguments tended to focus on the seemingly limitless availability of energy available to any social system.

demonstrated in numerous experiments in behavioral economics (Camerer 2003). Beginning with Simon (1982), bounded rationality has since been studied in psychology, economics, political science, industrial organization, computer science, and artificial intelligence research (Aumann 1997, Gigerenzer 2001, Kahneman 2003, Lipman 1995, Rubinstein 1998, Russell & Surbramanian 1995, Spiegler 2011).

In short, this research will deal with players that are perfectly rational. Players with bounded rationality would, without a doubt, create a more accurate model, but the purpose of this research is more theoretical than practical, at least for the time being. As alluded to previously, I say a perfectly rational player attempts to minimize their personal pain. Rabin, along with other researchers, did not violate this assumption because he incorporated emotions into games by developing frameworks that change player utilities. Although the utilities change, the equilibrium analysis (minimization of pain) stays the same.

While these alterations might create more realistic games, I believe there is a useful distinction to be drawn between changing one's pain matrix, and choosing the lens through which the game is played. The traditional approach assumes that the players are always "selfish"; that is, although they might alter their pain matrix, they still want what is best for themselves while minimizing the incentive for others to change strategy. However, what if we explicitly allowed that a player could develop a pain matrix, and then for instance view it as a player who sought to maximize fairness, or perhaps the greatest good for the greatest number?

1.6 Nash Equilibrium is used to analyze the outcome of strategic interactions

The importance of these questions lies in how it might influence what is viewed as the equilibrium outcome of a game. After breaking down a complex scenario, one can predict decisions via a solution concept, with the most common solution concepts being known as equilibrium concepts (Harsanyi & Selten 1988)[9]. John Nash, a Nobel Laureate and mathematician whose life was popularized in the 2001 film *A Beautiful Mind*, defined the most widely used and accepted solution concept, now known as Nash Equilibrium (1950). Stated simply, an outcome is a Nash Equilibrium if each player cannot benefit by changing strategies while the other players keep theirs unchanged. The fact that one must take into account the decision-making of others in order to analyze their own choices is the core insight of Nash Equilibrium. To find a Nash Equilibrium, analyses such as backward induction, forward induction, or iterated elimination of strictly dominated strategies can be used (Spaniel 2011).[10]

The modern concept of Nash Equilibrium is defined in terms of mixed strategies, where players choose a probability distribution over possible actions. For any game with a finite set of actions, a mixed strategy Nash Equilibrium must exist (Nash 1951). To account for situations in which misleading predictions are made, Nash Equilibrium has since experienced a number of refinements (Myerson 1978). However, this work will not deal with the adjustments that have been made. Rather, it will focus on how Nash Equilibrium can be described mathematically in multiple ways that are synonymous with thermodynamic equilibrium.

1.7 Main question and objectives

In this paper, I define Nash Equilibrium from an energetic standpoint and present an alternative approach to solving for the equilibrium of games through the use of players who can view games through three lenses, or "schemas." These schemas include selfish (S), where a player minimizes his or her own pain, and global (G), where a player seeks to minimize the total pain of all players, or at least of a sub-group of players, and fair (F), where a player seeks to minimize the variance between the pain of two players. I do this to address one major question:

[9] Provides an extensive review of equilibrium concepts.
[10] Provides a basic introduction to these methods.

Does the introduction of multiple schemas into game theory produce a new, more robust perspective in which to analyze games?

I hypothesize that changing the schema of a player can yield different results for even the most well-known and accepted games. Furthermore, I believe the introduction of multiple schemas will separate player motivations from the utility scale and enhance the ease at which complex situations can be analyzed mathematically. If so, I see the future of game theory incorporating even more schemas, such as fair (F), loyal (L), and vengeful (V) and the even the principles of chemical kinetics. To study these hypotheses, I enlisted the following objectives:

1. Mathematically define different schemas in order to conduct equilibrium analysis
2. Model real-world situations as games for further analysis
3. Study games with a 2x2 solver in Mathematica or Excel containing schemas

Chapter 2

Creating games for analysis

2.1 Game representation and terminology to be used

Games can be represented in either normal or extensive form. Extensive form games are used to formalize situations in which "moves" or decisions are time sensitive. The graphical representation of an extensive game is shown in Figure 2-1. In this game, player 1 chooses first and has choices U or D while player 2 chooses second and has choices L or R. The four possible outcomes are represented by utilities at the end of the tree with player 1 listed first and player 2 listed second.

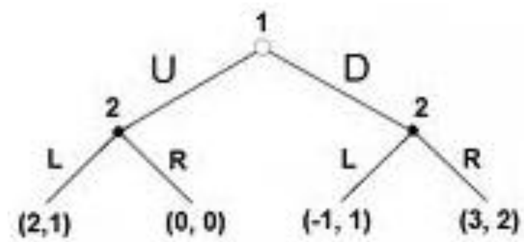

Figure 2-1 2x2 Player extensive form game.

Normal form games are represented by a matrix such as the example shown in Figure 2-2. Each player has two strategies and each square lists the outcome as player 1's utility followed by player 2's utility. When a normal form is used, it is assumed that each player acts simultaneously. Extensive form games can also be shown as simultaneous games by connecting decisions with dashed lines.

Player 2

		Left	Right
Player 1	Up	4,3	-1,-1
	Down	0,0	3,4

Figure 2-2. 2x2 Player normal form game.

Each game can contain a complete or incomplete information set. In a game of complete information, the players, moves, and payoffs are known, but each player might not know all moves that have been previously made by other players (e.g. the initial arrangement of ships in the game Battleship). If all moves that have previously been made are known, the game is said to be of perfect information. In incomplete information games, players may or may not know other players' strategies or payoffs.

This research uses 2x2 non-cooperative, normal form games with perfect information. Although this provides a very simplified version of actual scenarios, I hope it will allow the effects of multiple schemas to be easily seen. This work does not focus on precisely describing the complex variables of a decision system, but rather on illuminating new concepts. Even without accurately quantified social variables, this does not invalidate the usefulness of applying basic mathematics. As von Neumann and Morgenstern put it, “there is no point in using exact methods where there is no clarity in the concepts and issues to which they are to be applied.”(Neumann and Morgenstern 1947) In any case, once the merit of these concepts is determined, modifications and experimentation will produce more applicable models. The matrix and variables used to derive the formulas in this research are shown in Figure 2-3.

B

		B_1	B_2
	A_1	p_{11}^A, p_{11}^B	p_{12}^A, p_{12}^B
A	A_2	p_{21}^A, p_{21}^B	p_{22}^A, p_{22}^B

Figure 2-3. Normal form 2x2 game used in this research

1. players. A, B.
2. choices. 1,2.
3. pains. $p_{11}^A, p_{11}^B, p_{21}^A, p_{21}^B, p_{12}^A, p_{12}^B, p_{22}^A, p_{22}^B$ A positive pain is associated with an unfavorable outcome while a negative pain is associated with a more pleasurable result.[11]

[11] It should be noted that this is a clear distinction from the positive utility scale that most often accompanies the analysis of games.

Each pain is ranked, either positively or negatively, in terms of standard deviations from an average[12] event.

4. probabilities. $\phi_{A1}, \phi_{A2}, \phi_{B1}, \phi_{B2}$ The probabilities that a player chooses an option. The set (ϕ_{A1}, ϕ_{A2}) or (ϕ_{B1}, ϕ_{B2}) represents a player's strategy.

2.4 Modeling real-world scenarios as games

In order to study the effect of multiple schemas on game analysis, real-world scenarios of interest were described by 2x2 player games. Each scenario was modeled after a type of game that is commonly used in game theory (prisoner's dilemma, stag hunt, battle of the sexes, and chicken). Along with each real-world situation, I show the analogous, commonly known game as well as a simplified version with ordinal payoff rankings (where 1 is the outcome with the least pain and 4 is the outcome with the most pain). Although arguments can be made against the assumptions used to formulate the following games, the real purpose lies in demonstrating the use of multiple schemas. It is much more instructive to give examples in terms of understandable and relatable decisions, rather than applying new concepts to confusing situations. Once understood further, multiple schemas can be used to analyze games that are more thoroughly and precisely described.

1. Prisoner's Dilemma

		Prisoner B	
		Quiet	Tell
Prisoner A	Quiet	1,1	12,0
	Tell	0,12	8,8

Figure 2-4. Classic prisoner's dilemma

[12] "Average" can be defined as an outcome that yields neither a favorable or unfavorable result. It is accepted without much physical or emotional consequence.

B

		A_1	B_1
A	A_1	2,2	4,1
	B_1	1,4	3,3

Figure 2-5. Ordinal ranking of prisoner's dilemma

The prisoner's dilemma (Figures 2-4 and 2-5), a game originally formulated by Merrill Flood and Melvin Dresher (1958), is taught in almost all introductions to game theory. In this game, two prisoners (A and B), who have committed a crime together, are taken to two different places for further prosecution. While isolated, they are told that they have two options: keep quiet or betray the other criminal. Furthermore, they are told their prison sentence will depend on what the other criminal decides according to the following rules:

- If A and B both tell, they each serve 8 years in prison
- If A tells B but B remains silent, A will be set free and B will serve 12 years in prison (and vice versa)
- If A and B both remain quiet, both of them will only serve 1 year in prison

Even though the result is not necessarily intuitive, backward induction shows that each prisoner is better off telling, regardless of what the other decides. For example, consider prisoner 2. If prisoner 1 keeps quiet, prisoner 2 will receive 0 years for telling and 1 year for keeping quiet. If prisoner 1 tells, prisoner 2 will receive 8 years for telling and 12 years for keeping quiet. No matter what prisoner 1 decides, prisoner 2 will always be better off telling and the same is true for player 1. This outcome is an example of a pure strategy Nash Equilibrium. A mixed strategy Nash Equilibrium would be an equilibrium outcome in which a player would tell less than 100% of the time. The current definition of Nash Equilibrium will be mathematically described and expanded upon in Chapter 3.

The prisoner's dilemma is a unique situation that shows why two players might not cooperate, even though it seems to be in their best interest to do so. Similarly, I imagine a republican and democrat deciding on a bill to pass through Congress. For example, parties in Congress have recently been unable to find common ground on ways to help struggling middle-class voters (Mascarao 2014). This situation was modeled in Figure 2-6 with the following assumptions:

- Both parties would prefer that the bill pass according to their own agenda
- Both parties view a passed bill stemming from the other's agenda as the worst outcome
- Both parties would prefer to hold their ground over compromising

		Republican	
		Compromise	Hold Ground
Democrat	Compromise	0.5,0,5	2,-2
	Hold Ground	-2,2	1,1

Figure 2-6. PD and Congress

2. Chicken

		Driver 2	
		Swerve	Straight
Driver 1	Swerve	0,0	1, -1
	Straight	-1,1	5,5

Figure 2-7. Classic version of chicken

		B	
		A_1	B_1
A	A_1	2,2	3,1
	B_1	1,3	4,4

Figure 2-8. Ordinal version of chicken

The principle of chicken (Figures 2-7 and 2-8) is that, although each player prefers not to yield to each other, the worst possible option occurs when neither player gives in. Consider a game in which two teenagers drive towards each other at high speeds on a highway with reckless abandon. If one driver swerves before the other, they will seem cowardly to their friends. If both stay straight, a catastrophic accident will result.

This game, however stupid it may seem, can be compared to two companies (A and B) that are independently deciding on production levels. On one hand, a company can be purely motivated by profit. Therefore, if they have an opportunity to produce more to increase profits they will do so, regardless of the potential effect on the environment. If neither company takes any environmental precautions, an outside regulator might enforce policies that leave both companies worse off than self-regulations. This outcome is similar to a recent sulfur emissions rule released by the Environmental Protection Agency that requires refineries to cut sulfur levels in gasoline by two-thirds in 2017. Based on this regulation, the American Petroleum Institute estimated costs would add 6 to 9 cents a gallon to refiner's manufacturing costs while also requiring $10 billion in capital costs (Fox News 2014). Perhaps such a regulation could have been delayed if manufactures had collectively agreed to lessen sulfur emissions before outside regulations were deemed necessary. A situation in which two companies must decide how much product to manufacture is shown in Figure 2-9 with the following assumptions:

- If company A takes environmental precautions, company B can increase profits substantially by increasing production (and vice versa)
- If both companies take environmental precautions, the effect of decreased profits will be felt fairly and minimally
- If both companies produce just for profit, an outside party will eventually enforce harsh regulations

		Company A	
		Environment & Profit	Just Profit
Company B	Environment & Profit	0.5,0.5	1,-2
	Just Profit	-2,1	2,2

Figure 2-9. Chicken and competing companies

3. Stag hunt

		Hunter 2	
		Stag	Hare
Hunter 1	Stag	5,5	0,3
	Hare	3,0	3,3

Figure 2-10. Classic stag hunt game

		B	
		A_1	B_1
A	A_1	2,2	3,2
	B_1	2,3	1,1

Figure 2-11. Ordinal version of stag hunt

The stag hunt (Figures 2-10 and 2-11) is an important game that describes social cooperation. In this game, two hunters must choose whether to hunt a stag or a hare without knowing what the other will choose. If an individual hunts a stag, he must have the cooperation of the other hunter in order to succeed. An individual is able to hunt a hare by himself, but a hare is worth less than a stag.

Social cooperation is analogous to a consumer and producer's decision on whether or not to want/provide healthy food choices, a scenario that is becoming increasingly important in today's society. For instance, the availability of unhealthy food is a major concern in the fight against obesity, especially as it pertains to kids in school (Linda 2014). Furthermore, restaurants find themselves choosing whether to "disappoint and risk losing customers by not offering healthy menu items, or invest time and money in offering healthy menu choices that are unlikely to draw many customers." (Melnick 2010) Figure 2-12 shows a simplified version of this game with the following assumptions:

- Consumers and producers would prefer to be on the same page in regards to health, but the best possible outcome occurs when both agree on unhealthy food. This is considered the best possible outcome because of ease and inexpensiveness of production.

- For a consumer, wanting to eat healthy and having only unhealthy options is slightly better than wanting to eat unhealthy and having only healthy options. In the latter, the consumer at least has the benefit of being able to buy a cheaper product, despite the product's lack of nutrition. This conclusion can be drawn similarly for a producer.

		Producer	
		Healthy	Unhealthy
Consumer	Healthy	-1,-1	0.5,-0.5
	Unhealthy	-0.5,0.5	-1.5,-1.5

Figure 2-12. Stag hunt and food products

4. Battle of the sexes

		Husband	
		Opera	Football
Wife	Opera	3,2	1,1
	Football	0,0	2,3

Figure 2-13. Classic battle of the sexes game

		B	
		A_1	B_1
A	A_1	1,2	3,3
	B_1	4,4	2,1

Figure 2-14. Ordinal version of BOS

The battle of the sexes (Figures 2-13 and 2-14) game is referred to as a two-player coordination game in which a couple agreed to meet for a date, but forgot whether they decided on attending an opera or a football game. The husband would prefer the football game, the wife would prefer the opera, and they would both prefer to end up together rather than alone. To make matters worse, they are unable to communicate before deciding which event they will attend.

This example is similar to a situation in which a teacher must decide whether to provide a hard or easy exam while a student must decide how hard to study. Studying this coordination game might help teachers understanding testing, its effect on students (Hernadez & Baker 2014), and how to run their classrooms more effectively. This situation is modeled in Figure 2-15 with the following assumptions:

- Similar to battle of the sexes, both the teacher and the student would rather end up at the same place (hard exam and studying or easy exam and no studying), but disagree on which place they would rather end up (teacher prefers difficult exam with studying while student prefers easy exam and no studying).
- The worst possible outcome occurs when the teacher mak es a difficult exam that takes a lot of effort and the student does not study and gets a bad grade.

		Student	
		Study Hard	Don't Study
Teacher	Difficult Exam	-1,-0.5	2,2
	Easy Exam	0,0	-0.5,-1

Figure 2-15 BOS and school testing

Chapter 3

Modeling multiple schemas

3.1 Introduction to schemas and enzyme kinetics

A general introduction of multiple schemas is now given in Table 3-1, along with a mathematical function. This work will focus primarily on (S), (G), and (F).

Table 3-1. Defining schemas, mathematically and conceptually. The objectives are written in terms of player (or set of players) β. Loyal and vengeful are focused on another group of player(s) represented by γ.

Objective	Definition	Function
selfish (S)	"rational," minimizes own energy	$\min E_{\beta\_tot}$
risk taker (R)	gambler, willing to take chance on best choice	$\min E_{\beta 1}$
loyal (L)	Minimizes energy of select group of player(s)	$\min E_{\gamma\_tot}$
vengeful (V)	Maximizes energy of select group of players with no regard to own energy	$\max E_{\gamma\_tot}$
global (G)	Minimizes energy of group	$\min \langle E_{tot} \rangle = \min \sum_{\alpha=1}^{N} E_{\alpha} / N$
altruistic (A)	Minimizes energy of group with no regard to own energy	$\min \sum_{\substack{\alpha=1 \\ \alpha \neq self}}^{N} E_{\alpha} / N$
fair (F)	Minimizes energy of group and deviation between players	$\min s^2 = \min \sum_{\alpha=1}^{N} \left(E_{\alpha} - \langle E \rangle\right)^2 / N$

With these schemas, decisions in game theory can be modeled after an enzymatic reaction. Functioning as a catalyst, enzymes are large biological proteins that decrease the activation energy of a reaction. In the human body, this accelerates the rate and specificity for metabolic processes that sustain and allow life to occur. A single-substrate reaction, shown below, serves as a useful analogy to the decision-making process.

$$E + S \underset{k_r}{\overset{k_f}{\leftrightarrow}} ES \overset{k_{cat}}{\rightarrow} E + P$$

In the first step of the reaction, an enzyme binds to a substrate material to form an enzyme-substrate complex in a reaction where k_f and k_r denote the forward and reverse rate constants respectively. After reacting further, the enzyme-substrate releases the original enzyme intact along with a newly formed product. The decision-making process in game theory can be modeled in a similar manner. To see the analogy, consider the equation below accompanied by further explanations.

$$Schema + Initial\ Player \leftrightarrow Purpose \rightarrow Decision + Schema$$

1. Initial Player / Substrate

   The initial player acts as the main reactant of interest. Each player carries past experiences and perspectives that affect his/her affinity for a certain schema. Furthermore, the initial player is also affected by the possible outcomes of the game. Although pains are commonly considered the product of a game, here they are viewed as reactants.

2. Schema / Enzyme

   A schema can function in two ways. First, a schema can increase the rate at which a decision is made without being consumed or altered. For instance, a person is much more capable of making a decision when holding a clear view or motivation. Therefore, a schema creates a lens through which an initial player sees a game. This lens lowers the activation energy of a decision by providing a distinct way for a player to analyze the necessary information that comes with a game. After the reaction occurs, the schema is released and can be used again and again without being changed. Secondly, the schema does not alter the equilibrium distribution. The equilibrium outcome is determined by thermodynamics, as well as the ratio between the forward and reverse reactions.

3. Purpose / Enzyme-substrate complex

   The enzyme-substrate complex is synonymous with a player that has a clear worldview. Popularized by George Lakoff (1996), worldview is the set of expectations and biases that color the way each person sees the world. Armed with a worldview, a player pays attention differently, contains a unique bias towards information, and interprets images and words in a distinct manner.[13]

4. Deliberation / k_f and k_r

   The forward and reverse rate constants are similar to the deliberation process inherent in any decision-making process. Each player continually analyzes the possible outcomes of a decision while deciding which schema to adopt.

5. Decision / Product

   The main product produced by the reaction is the final decision. Over time, this decision will reach an equilibrium distribution. With limited resources, as is the case with bounded rationality, the decision will deviate from a theoretical distribution. In addition to the decision, the player is also affected by the reaction. Having experienced the pain of the final decision, whether positive or negative, affects the players perspective and affinity towards different schemas when playing new games (i.e. if a player plays selfishly and has a positive outcome, they will be more likely to continue playing selfishly in similar games).

[13] In the book, *All marketers are liars: The power of telling authentic stories in a low-trust world*, Seth Godin outlines the three things worldview changes: attention, bias, and vernacular.

3.2 Modeling equilibrium decisions with schemas and thermodynamics

As mentioned previously, the key insight behind Nash Equilibrium was the realization that it occurs when no player has anything to gain while others keep their strategy constant. In other words, a "rational" player A seeks to minimize player B's motivation to change strategies. Although this may not be how we intuitively make decisions, it is a very powerful explanation of equilibrium outcomes and even describes evolutionary processes (Smith 1982).

The final outcome of each game can be written in terms of "energies" that are dependent upon each player's strategy and pains. These calculations show the energies associated with a 2x2 player game containing a well-defined strategy profile:

Player A:

$$E_{A1} = p_{11}^{A}\phi_{A1}\phi_{B1} + p_{12}^{A}\phi_{A1}\phi_{B2}$$

$$E_{A2} = p_{21}^{A}\phi_{A2}\phi_{B1} + p_{22}^{A}\phi_{A2}\phi_{B2}$$

$$E_{A} = p_{21}^{A}\phi_{A2}\phi_{B1} + p_{22}^{A}\phi_{A2}\phi_{B2} + p_{11}^{A}\phi_{A1}\phi_{B1} + p_{12}^{A}\phi_{A1}\phi_{B2}$$

Player B:

$$E_{B1} = p_{11}^{B}\phi_{A1}\phi_{B1} + p_{12}^{B}\phi_{A1}\phi_{B2}$$

$$E_{B1} = p_{11}^{B}\phi_{A1}\phi_{B1} + p_{12}^{B}\phi_{A1}\phi_{B2}$$

$$E_{B} = p_{21}^{B}\phi_{A2}\phi_{B1} + p_{22}^{B}\phi_{A2}\phi_{B2} + p_{11}^{B}\phi_{A1}\phi_{B1} + p_{12}^{B}\phi_{A1}\phi_{B2}$$

Total

$$\begin{aligned} E &= E_A + E_B \\ &= p_{21}^{B}\phi_{A2}\phi_{B1} + p_{22}^{B}\phi_{A2}\phi_{B2} + p_{11}^{B}\phi_{A1}\phi_{B1} + p_{12}^{B}\phi_{A1}\phi_{B2} \\ &+ p_{21}^{A}\phi_{A2}\phi_{B1} + p_{22}^{A}\phi_{A2}\phi_{B2} + p_{11}^{A}\phi_{A1}\phi_{B1} + p_{12}^{A}\phi_{A1}\phi_{B2} \end{aligned}$$

The current understanding of Nash equilibrium is referred to as "selfish" in Table 1. This schema was combined with the second law of thermodynamics (Section 1.2) to produce the derivation shown below. Following selfish, an alternative schema, "global", is shown. Global follows the same principles of Nash Equilibrium but uses the energy function of the group, rather than the energy function of individual players.

Selfish schema: $\min E_A$

To ensure a strategy is chosen where player B is indifferent to his choice, set $\frac{\partial E_B}{\partial \phi_{B1}} = 0$ to solve for ϕ_{A1}

$$E_B = p_{21}^B (1-\phi_{A1})\phi_{B1} + p_{22}^B (1-\phi_{A1})(1-\phi_{B1}) + p_{11}^B \phi_{A1}\phi_{B1} + p_{12}^B \phi_{A1}(1-\phi_{B1})$$

$$\frac{\partial E_B}{\partial \phi_{B1}} = p_{21}^B - p_{21}^B \phi_{A1} - p_{22}^B + p_{22}^B \phi_{A1} + p_{11}^B \phi_{A1} - p_{12}^B \phi_{A1}$$

$$\boxed{\phi_{A1} = \frac{p_{22}^B - p_{21}^B}{p_{22}^B + p_{11}^B - p_{21}^B - p_{12}^B}} \tag{1}$$

Similarly, ϕ_{B1} can be found by setting $\frac{\partial E_A}{\partial \phi_{A1}} = 0$

$$\boxed{\phi_{B1} = \frac{p_{22}^A - p_{12}^A}{p_{22}^A + p_{11}^A - p_{21}^A - p_{12}^A}} \tag{2}$$

In addition to the selfish schema, I derive two schemas where a player is motivated by a global view and by fairness. The equilibrium strategies of these schemas are shown in Eqns. 3-6:

Global schema: max E

Set $\frac{\partial E}{\partial \phi_{B1}} = 0$ to find ϕ_{A1}

$$E = p_{21}^B (1-\phi_{A1})\phi_{B1} + p_{22}^B (1-\phi_{A1})(1-\phi_{B1}) + p_{11}^B \phi_{A1}\phi_{B1} + p_{12}^B \phi_{A1}(1-\phi_{B1})$$
$$+ p_{21}^A (1-\phi_{A1})\phi_{B1} + p_{22}^A (1-\phi_{A1})(1-\phi_{B1}) + p_{11}^A \phi_{A1}\phi_{B1} + p_{12}^A \phi_{A1}(1-\phi_{B1})$$

$$\frac{\partial E}{\partial \phi_{B1}} = p_{21}^{B} - p_{21}^{B}\phi_{A1} - p_{22}^{B} + p_{22}^{B}\phi_{A1} + p_{11}^{B}\phi_{B1} - p_{12}^{B}\phi_{A1}$$

$$p_{21}^{A} - p_{21}^{A}\phi_{A1} - p_{22}^{A} + p_{22}^{A}\phi_{A1} + p_{11}^{A}\phi_{A1} - p_{12}^{A}\phi_{A1}$$

$$\boxed{\phi_{A1} = \frac{-(p_{21}^{A} + p_{21}^{B}) + (p_{22}^{A} + p_{22}^{B})}{(p_{11}^{A} + p_{11}^{B}) - (p_{12}^{A} + p_{12}^{B}) - (p_{21}^{A} + p_{21}^{B}) + (p_{22}^{A} + p_{22}^{B})}} \qquad (3)$$

Set $\frac{\partial E}{\partial \phi_{A1}} = 0$ to find ϕ_{B1}

$$\boxed{\phi_{B1} = \frac{-(p_{12}^{A} + p_{12}^{B}) + (p_{22}^{A} + p_{22}^{B})}{(p_{11}^{A} + p_{11}^{B}) - (p_{12}^{A} + p_{12}^{B}) - (p_{21}^{A} + p_{21}^{B}) + (p_{22}^{A} + p_{22}^{B})}} \qquad (4)$$

Fair schema: $\min \sum_{\alpha=1}^{N} \left(E_{\alpha} - \langle E \rangle\right)^{2} / N$

To solve for $\partial\phi_{A1}$: $\frac{\partial((E_A - E_B))^2}{\partial \phi_{B1}} = 2(E_A - E_B)(\frac{\partial E_A}{\partial \phi_{B1}} - \frac{\partial E_B}{\partial \phi_{B1}}) = 0$ (5)

To solve for $\partial\phi_{B1}$: $\frac{\partial((E_A - E_B))^2}{\partial \phi_{A1}} = 2(E_A - E_B)(\frac{\partial E_A}{\partial \phi_{A1}} - \frac{\partial E_B}{\partial \phi_{A1}}) = 0$ (6)

These derivations (further derivations of Fair were carried out in Wolfram Mathematica as shown in Appendix A) were used to define a new Nash Equilibrium of a 2x2 game in following manner:

1. Define the schema of players A and B.
2. Find a strategy for player A, subject to the constraint $0 \le \partial\phi_{A1} \le 1$, that creates a situation in which player B has no propensity to change decisions (i.e. if player B is selfish, set $\frac{\partial E_B}{\partial \phi_{B1}} = 0$). In the games defined in this research, this step will only result in one strategy for player A. However, in more complex games where there are several

solutions for $\frac{\partial E_B}{\partial \phi_{B1}} = 0$, it will be necessary to choose the strategy that best satisfies player A's schema.

3. Find a strategy for player B, subject to the constraint $0 \le \partial \phi_{B1} \le 1$, that creates a situation in which player A has no propensity to change decisions (i.e. if player A is global, set $\frac{\partial E}{\partial \phi_{A1}} = 0$). If setting $\frac{\partial E}{\partial \phi_{A1}} = 0$ results in multiple solutions, choose the strategy that optimizes player B's schema..

3.4 Modeling procedure

Outlined below are the steps that were taken to produce data for analysis:

1. A Mathematica solver for a 2x2 player was developed (Appendix A)
2. The pains from each real-world game were entered into the solver to give $\partial \phi_{A1}$ and $\partial \phi_{B1}$
3. The strategy resulting from a typical game was compared to strategies resulting from use of fair or global schemas
4. Use experimental results to demonstrate how multiple schemas could be used to predict the schema composition of a player

Chapter 4

First observations taken from multi-schema games

4.1 Results gathered from a multi-schema solver

Table 4-1. 2x2 Player game results. Each player was modeled as 100% (S), (G), or (F).

	Selfish vs Selfish		**Global vs global**		**Selfish vs global**		**Selfish vs Fair**	
Game	ϕ_{A1}	ϕ_{B1}	ϕ_{A1}	ϕ_{B1}	ϕ_{A1}	ϕ_{B1}	ϕ_{A1}	ϕ_{B1}
PD (Congress)	0	0	0.67	0.67	1	0.67	1	1
Stag Hunt (Food)	0.80	0.80	0.60	0.60	0.80	0.60	0.80	0.80
BOS (Testing)	0.29	0.71	0.21	0.79	0.29	0.79	0.29	0.75
Chicken (Production)	0.33	0.33	0.83	0.83	0.33	0.83	0.33	0.33

Democrat \ Republican	Compromise	Hold Ground
Compromise	<u>0.5,0.5</u>	2,-2
Hold Ground	-2,2	**1,1**

Consumer \ Producer	Healthy	Unhealthy
Healthy	**<u>-1,-1</u>**	0.5,-0.5
Unhealthy	-0.5,0.5	-1.5,-1.5

Company B \ Company A	Environment & Profit	Just Profit
Environment & Profit	<u>0,0</u>	1,-2
Just Profit	-2,1	**2,2**

Teacher \ Student	Study Hard	Don't Study
Difficult Exam	-1,-0.5	2,2
Easy Exam	**<u>0,0</u>**	-0.5,-1

Figure 4-1. Most probable outcomes of multi-schema games. In each game, the bolded and underlined results signify the most probable (S) v (S) and (G) v (G) outcomes respectively.

Using a Mathematica solver (Appendix A), equilibrium strategies were predicted (Table 4-1) according to the games developed in Chapter 2. Figure 4-1 shows the most likely outcome of the (G) v (G) (pure global) and (S) v (S) (pure selfish) games. Using multiple schemas, even at such a basic level, predicted outcomes that are seen in real life. For instance, the chicken game, modeled after competing companies, showed two differing results between the purely global and purely selfish game. If we assume that the companies are completely self-interested, as commonly thought, the equilibrium outcome predicts the need for external regulations. Playing the game with players who are completely global causes each company to change their decision. The prisoner's dilemma game, modeled after congress, shows a nearly identical result.

On the other hand, pure global and pure selfish predicted the same outcome for both the stag hunt and battle of the sexes games. Despite predicting the same most likely outcome, each game produced a different equilibrium strategy. Lastly, in each game the "fair" player simply adopted whatever strategy the opponent was playing. As hypothesized, players with new schemas led to product strategies differing from commonly known predictions.

4.2 Multiple schemas produce a model robust model

A likely hesitation around the usefulness of this research will be in explaining the purpose behind using multiple schemas, rather than simply changing payoffs. While it is possible to yield the same predictions by accounting for player motivations within a utility function, this method limits the capabilities of the model and contradicts what we observe in everyday life.

Accounting for player motivations within a utility function is a painstaking and sometimes unnecessary process. Imagine two situations of the prisoner's dilemma. In one game, a wrongly accused citizen plays with a stranger. Next, the same citizen plays again, but this time with a known relative. The potential prison sentences surrounding the game stay the same, but the lens in which the citizen chooses to view the game likely changes.

This new lens can be incorporated into the game by changing the utility function of the citizen, but this creates the need for more subjective analysis. By using multiple schemas, pains (or utilities) can be described in a more concrete manner. For example, pain functions could be developed that depend on time, money, or opportunity cost, rather than on beliefs and values.

At best, incorporating motivations into utility functions will yield the same results as using multiple schemas. However, further pursuit of this route would severely limit potential developments that could stem from a "system" mentality. Using schemas, game theorists would be able to take and expand upon existing scientific principles, rather than continually invent new ones.

4.3 Pitfalls and deviations from reality

Because, to my knowledge, this is the first introduction of multiple schemas into game theory, a lesson was taken from Von Neumann:

> *"What is important is the gradual development of a theory, based on a careful analysis of the ... facts. ... Its first applications are necessarily to elementary problems where the result has never been in doubt and no theory is actually required. At this early stage the application serves to corroborate the theory."*

To reiterate, the main question of this research was, "Does the introduction of multiple schemas into game theory produce a new, more robust perspective in which to analyze games?" At this point, I cannot give a resounding yes, but I can conclude that multiple schemas produce a new perspective that results in new predictions. I believe that this result, along with the concept itself, should merit further work (see Chapter 5).

However, it is important to at least mention, on some level, the results that seem contrary to reality. The predictions with the most glaring differences from reality were the strategies found in the battle of the sexes and stag hunt games. Both the global and selfish schemas landed on a healthy-healthy and study-easy test scenario as an equilibrium strategy. From experience, it is obvious that this result is not always true, and likely not an equilibrium outcome. We can also say from experience that hardly any decision has a completely selfish or completely global motive.

Therefore, the introduction of more schemas, such as those listed in Table 4-1, might yield results more closely aligned with (and able to predict) reality.

4.4 Experimental results from *Prisoners and their Dilemma* and schema predictions

To further test the usefulness of multiple schemas, I gathered experimental results from a recent experimental study (Khadjavi & Lange 2013) in which actual prisoners and students participated in a version of the prisoner's dilemma according to the matrix[14] in Figure 4-2. Instead of prisons sentences, the players were playing for rewards in the form of cash (in the case of students) or coffee and tobacco packs (in the case of the prisoners). The summarized results from this study are shown in Table 4-2.

		Player 2	
		Quiet	Tell
Player 1	Quiet	7,7	1,9
	Tell	9,1	3,3

Figure 4-2. Game matrix from *Prisoners and their Dilemma*

Table 4-2. Experimental results from *Prisoners and their Dilemma*

	Students	**Prisoners**
Participants	46	36
Simultaneous PD: **Individual cooperation rate (in %)**	36.97 (17/46)	55.56 (20/36)

The authors of this work speculated that the results seemed to indicate that "criminal behavior does not appear to be a self-selection process by which purely self-interested individuals are more likely to commit crimes than socially orientated individuals." By this, the authors were concluding that the expected preferences of the prisoners and students did not align with the results seen experimentally. I hypothesize, however, that the students and prisoners were using different schemas independent from their innate preferences.

[14] Only the results relevant to this research were taken from *Prisoners and their Dilemma*. Refer to this source to see additional results or experimental methods.

Using the Excel solver developed in Appendix B, the global and selfish distribution of the students was calculated to be 37% and 63% while the prisoner distribution calculated to be 56% and 44% respectively. These distributions match the experimental cooperation rates because experimental game matrix would produce a pure strategy for a pure global or selfish player (i.e. a cooperation rate of 100% would mean a player was 100% global, a cooperation of 0% would mean a player was 100% selfish). Once additional schemas are developed, more meaningful inferences could be made about the potential schema distribution of a player or group of players. Finding the schema distribution of a player could help explain why, in situations such as these, players make decisions in contrast to predicted or assumed preferences.

Chapter 5

Conclusions and future work

5.1 Main conclusions

The main conclusions of this work can be summarized as follows:

1. The introduction of multiple schemas into 2x2 player games produced equilibrium strategies that differ from strategies anticipated by typical Nash Equilibrium analysis.
2. Multiple schemas can incorporate player motivations into games, eliminating the need to change utility functions. It is also important to note, however, that we can only identify schema that satisfy the equilibrium. In cases where multiple schemas give the same equilibrium, there would need to be calculations of uncertainity.
3. Schemas can be used to predict equilibrium strategy and equilibrium strategies can be used to predict the schema distribution of a player.
4. Future mathematical and conceptual development should be combined with experimental research to further test and improve the robustness of multiple schema game theory.

5.2 Future work and connections to Physics of Community

Darrell Velegol, a Distinguished Professor of Chemical Engineering at The Pennsylvania State University, has recently started developing a field called "Physics of Community" (PC) that pursues questions in learning, creativity, motivation, trust and deceit, courage, and other social science ideas using results from physics, chemistry, biology, and chemical engineering. I believe the future of multiple-schema game theory may catalyze a rapid development of this field. To expand upon this viewpoint, I give a brief framework of potential research and connections to PC:

1. More schemas and experimental research.

   Before making connections to additional scientific principles, the schemas outlined in Table 3-1 should be explicitly defined and then added to the Mathematica and

Excel solvers (which might use linear programming techniques). Once developed, additional research should first center on 2x2 player games in which players have multiple objectives.

2. Chemical kinetics.

Examine the different time scales in which it takes to a make a decision. With this research, it might be possible to define rate constants for different schemas.[15] We might find that a selfish schema has a large rate constant, and thus fast kinetics.

In addition, this analogy could be extended to enzyme inhibition. There are several broad categories of inhibitors such as competitive (substrate and inhibitor bind to the same site, increasing Km but leaving Vmax unchanged), uncompetitive (inhibitor binds to complex, decreasing both Vmax and Km), and non-competitive (Vmax decreases but there is no effect on Km). In this way, we would learn more about how schemas might interact with each other.

3. Microcanonical ensemble, Shannon's entropy, and codon degeneracy.

As more information is gained on how to model decisions in a time-based manner, it may be possible to study the inverse problem – given a decision, can we predict a player's schema distribution? This is similar to the microcanonical ensemble of statistical thermodynamics in which multiple microstates can model one macrostate. Here, Shannon's entropy might predict the most likely schema distribution. Furthermore, an important aspect of the microcanonical ensemble, which aligns with the principle of codon degeneracy, is that the microstate can never truly be known. In codon degeneracy, we can know the amino acid structure of a protein, but in many cases we cannot know the exact DNA sequence that caused its formation.

[15] I see this potentially expanding on Daniel Khaneman's *Thinking, Fast and Slow* (2011).

4. Information theory.

    Once kinetics and equilibrium analyses come to form, research might tie in information theory. This could add to Kahneman's prospect theory, and help better describe how players interpret and receive facts and information.

5. Tipping point and applications.

    After developing 2x2 player games, research might expand to games that contain N players with M choices. In such situations, what is the minimum distribution of schemas that are necessary to achieve a certain outcome? At what distribution will the current outcome "tip" to a different result? Kinetics in combination with equilibrium analysis would be useful here.

6. Schema distribution assessment.

    Many companies are now requiring potential employees to take leadership or personality tests to assess what role they might play on a team. A similar test, using multiple schema game theory, might give companies insight on a person's selfish, global, fair, loyal, altruistic, and entrepreneurial nature.

Amazingly, these six areas apply principles from chemical reaction kinetics, biology, applied mathematics, and thermodynamics. If they can contribute to the advancement of multiple schema game theory, PC might expand quickly to other fields and applications.

APPENDIX A

Mathematica 2X2 Solver

```
(*PC solver
 2014apr01
 DV PS TV
 We will list several games, and then solve
 them as below for selfish (S), global (G), and fair (F).
  The games to solve are then all combinations, including
 SS, SG, SF, GG, GF, FF, GS, FS,
FG.*)

(*List of games*)
painPD = {{{-0.5, -0.5}, {2.0, -2.0}},
   {{-2.0, 2.0}, {1.0, 1.0}}};
painStagHunt = {{{-3, -3}, {0, -2}},
   {{-2, 0}, {-1, -1}}};
painChicken = {{{0.0, 0.0}, {1.0, -1.0}},
   {{-1.0, 1.0}, {10.0, 10.0}}};
painBoS = {{{-2.0, -1.0}, {0.0, 0.0}},
   {{0.0, 0.0}, {-1.0, -2.0}}};

(*PD game*)
Clear[x, y, Ea, Eb, dEadA, dEbdB, pain]; (*x = phiA1,
y = phiB1 *)
pain = painChicken;
Ea = x*y*pain[[1, 1, 1]] + x*(1 - y)*pain[[1, 2, 1]] +
    (1 - x)*y*pain[[2, 1, 1]] + (1 - x)*(1 - y)*pain[[2, 2, 1]];
DEaa = D[Ea, x];
DEab = D[Ea, y];
Eb = x*y*pain[[1, 1, 2]] + x*(1 - y)*pain[[1, 2, 2]] +
    (1 - x)*y*pain[[2, 1, 2]] + (1 - x)*(1 - y)*pain[[2, 2, 2]];
DEba = D[Eb, x];
DEbb = D[Eb, y];
gAB = Ea + Eb; (*global*)
DgABa = D[gAB, x];
DgABb = D[gAB, y];
fAB = (Ea - Eb)^2; (*fair*)
DfABa = D[fAB, x];
DfABb = D[fAB, y];

(*solutions*)
ss = FindRoot[{DEbb == 0.0, DEaa == 0}, {x, 0.2}, {y, 0.2}];
gg = FindRoot[{DgABb == 0.0, DgABa == 0.0}, {x, 0.2}, {y, 0.2}];
ff = FindRoot[{DfABb == 0.0, DfABa == 0.0}, {x, 0.2}, {y, 0.2}];
sg = FindRoot[{DEbb == 0.0, DgABa == 0.0}, {x, 0.2}, {y, 0.2}];
sf = FindRoot[{DEbb == 0.0, DfABa == 0.0}, {x, 0.2}, {y, 0.2}];
{ss, gg, ff, sg, fs}

{{x → 0.9, y → 0.9}, {x → 1., y → 1.},
 {x → 0.2, y → 0.2}, {x → 0.9, y → 1.}, {x → 0.9, y → 0.9}}
```

APPENDIX B

Excel 2x2 Solver

GAME THEORY 2 PLAYER SOLVER (MULTIPLE SCHEMAS)

Enter schema distributions:		Selfish	Global	TOTAL
	Player A	100%	0%	100%
	Player B	50%	50%	100%

ENTER PAIN:

	B_1	B_2
A_1	-1 / -0.5	2 / 2
A_2	0 / 0	-0.5 / -1

To solve 2x2 player game:
1. Enter the pains associated with each outcome in the shaded box
2. Enter the schema distribution of each player

USING SCHEMA	S vs S	G vs G	S vs G	G vs S
ϕA1	0.285714	0.214286	0.285714	0.214286
ϕB1	0.714286	0.785714	0.785714	0.714286

ϕ_{A1}	0.28571429	ϕ_{B1}	0.75
ϕ_{A2}	0.71428571	ϕ_{B2}	0.25
E_A	-0.1607143	E_B	-0.142857
E_{tot}	-0.30357		

Selfish solver:

ϕ_{A1} = IF(OR((E13-D13)/(E13+D11-E11-D13)>1,(E13-D13)/(E13+D11-E11-D13)=0,(E13-D13)/(E13+D11-E11-D13)<0,E13+D11-E11-D13=0),IF(D10<D12,1,0),(E13-D13)/(E13+D11-E11-D13))

ϕ_{B1} = IF(OR((E12-E10)/(E12+D10-E10-D12)>1,(E12-E10)/(E12+D10-E10-D12)=0,(E12-E10)/(E12+D10-E10-D12)<0,E12+D10-E10-D12=0),IF(D11<E11,1,0),(E12-E10)/(E12+D10-E10-D12))

Global solver:

ϕ_{A1} = IF(OR((-C12-C13+D12+D13)/(C10+C11-D10-D11-C12-C13+D12+D13)>1,(-C12-C13+D12+D13)/(C10+C11-D10-D11-C12-C13+D12+D13)<0,(-C12-C13+D12+D13)/(C10+C11-D10-D11-C12-C13+D12+D13)=0,C10+C11-D10-D11-C12-C13+D12+D13=0),IF(C10+C11<C12+C13,1,0),(-C12-C13+D12+D13)/(C10+C11-D10-D11-C12-C13+D12+D13))

ϕ_{B1} = IF(OR((D13+D12-D11-D10)/(C10+C11+D12+D13-C12-C13-D10-D11)>1,(D13+D12-D11-D10)/(C10+C11+D12+D13-C12-C13-D10-D11)<0,(D13+D12-D11-D10)/(C10+C11+D12+D13-C12-C13-D10-D11)=0,C10+C11+D12+D13-C12-C13-D10-D11=0),IF(C10+C11<D10+D11,1,0),(D13+D12-D11-D10)/(C10+C11+D12+D13-C12-C13-D10-D11)

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ACADEMIC VITA

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ACADEMICS

The Pennsylvania State University, Schreyer Honors College — **Graduation: May 2014**
Bachelor of Science in Chemical Engineering

WORK EXPERIENCE

Merck & Co. — **Summit, NJ; Elkton, VA**
Pharmaceutical Commercialization Technology Engineering Intern — *Summer 2013*
- Designed experiments, collaborated with technical experts, and directed trial-runs with operators in order to cut the time needed to scale between development machinery by 50%
- Communicated final observations in a 20 page document and 30 minute presentation to department management

Merck Manufacturing Division Engineering Intern — *Summer 2012*
- Led two cross-functional teams via Toyota Kata and Lean Six Sigma problem solving methodologies
  - Determined the root-cause of product loss and obtained machinery quotes to save $250k in product material
  - Developed a recovery process and guided interaction between experts to save $3 million in product material
- Diagnosed the effect a process unit operation on process downtime in an antibiotic dry filling process
  - Worked with process engineers and operators to quantify production loss to 386 vials per day
  - Eliminated unit operation as an area of immediate concern and summarized findings in a presentation to process experts

Penn State Chemical and Nuclear Engineering Laboratories — **University Park, PA**
Research Assistant — *Fall 2012, Summer 2011*
- Developed and conducted experiments to determine the best technique to prepare nuclear reactor materials for surface analysis

Honors Thesis in Chemical Engineering Undergraduate Studies — **University Park, PA**
Scalable Assessment in Massive Open Online Courses (MOOCs): A Chemical Engineering Perspective — *2012-Present*
- Currently analyzing assessment from a 100k student MOOC through the perspective of a chemical engineer

Suhey Peppers, Inc. — **State College, PA**
Marketing and Distribution Manager — *Fall 2010-Present*
- Facilitated the launch of a family business in the specialty food industry, currently operating at profit with $30k in revenue
- Established a business relationship with 3 specialty food retailers and currently spend 10 hours per week communicating with 7 local retailers in order to ensure timely product deliveries and customer satisfaction

LEADERSHIP EXPERIENCE

Apollo THON — **University Park, PA**
President, a student philanthropy which raises funds and awareness for pediatric cancer — *2012-2013*
- Initiated strategic fundraising and recruiting efforts that more than tripled fundraising total from $37k to $122k in one year
- Led weekly executive strategy meetings and general organizational meetings (110 involved members)
- Instituted data recording and standardization methods to facilitate current and future organizational efforts

Family Relations Chair — *2011-2012*
- Scheduled, planned, and coordinated events between organization and a family with a child suffering from pediatric cancer
- Created and managed 3 subordinate "captain" positions

The Schreyer Honors College Consulting Group — **University Park, PA**
Volunteer, The Malini Foundation, non-profit providing a safe environment for orphaned girls in Sri Lanka — *Summer 2013-Present*
- Currently working with 3 students and a Deloitte consultant to develop a self-sufficiency business plan in which merchandise sales in the U.S. would cover 50% of the foundation's operating costs within 3 to 5 years

Penn State Engineering Ambassadors — **University Park, PA**
Member, represent College of Engineering through tours and presentations to students, alumni, and industry partners — *2013-Present*

HONORS AND AWARDS

Skull and Bones Honorary Society — **University Park, PA**
One of 12 seniors selected in recognition of service and leadership to the University community — *2013 - Present*

Civic Engagement Speaking Competition — **University Park, PA**
Finalist - Top 10 out of 1,500 Penn State students — *Spring 2011*

Deloitte Consulting Undergraduate Case Competition — **University Park, PA**
Finalist – Team of 4 placed 2nd out of 30 teams — *Spring 2013*

Phi Kappa Phi Honor Society — **University Park, PA**
Membership awarded to the top 7.5% of the junior class based on academic achievement — *Spring 2013*

PERSONAL INTERESTS

- Avid golfer (single-digit handicap), triathlons (recently completed half-ironman), public speaking