ACOUSTIC CHARACTERIZATION OF COMBUSTORS WITH AZIMUTHALLY RESOLVED ACOUSTIC FORCING

SHAYNA LEVENSON
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Reviewed and approved* by the following:

Jacqueline O’Connor
Dorothy Quiggle Career Development Professorship and
Assistant Professor of Mechanical Engineering
Thesis Supervisor

Hosam Fathy
Assistant Professor of Mechanical Engineering
Honors Adviser

* Signatures are on file in the Schreyer Honors College.
ABSTRACT

Thermoacoustic instabilities are a significant operational issue for low-emissions gas turbines in both the power and propulsion industries. Specifically, gas turbines that use lean-burn technology in an effort to reduce NOx emissions are highly susceptible to instabilities. Within the combustor, thermoacoustic instabilities can couple with a number of acoustic modes. Focusing on annular combustors, common in aircraft and power generation engines, the azimuthal mode that dominates the thermoacoustic feedback process excites the flame asymmetrically, which can lead to unique flame response. Therefore, the goal of this research is to create asymmetric acoustic fields to understand the acoustic field and how it’s affecting the flow and the flame.

In order to better understand these acoustic fields, Comsol models were generated to mimic both a laboratory combustor and a simplified system in which only the combustion chamber is considered. From this analysis, it is clear that the exit boundary condition of the combustion chamber has a large impact on the acoustic field. It is also evident that if the system is forced at a frequency close to one of its natural frequencies, it will experience a stronger response. However, it has also been shown that higher modes of forcing will produce a weaker acoustic response. Finally, the same acoustic modes were visible in both the combustion chamber and full lab experiment at the same eigenfrequencies when acoustic forcing was applied at lower modes. Experimental validation of the Comsol models mimicked the results from both the simplified geometry and laboratory experimental simulations. Additional resonant frequencies were identified through the use of white-noise excitation in the experiment.

The implications of these results are discussed and future work proposed.
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Chapter 1
Introduction

The power and propulsion industries are currently experiencing a pressing need for the production of low-emission gas turbines due to the ever increasing stringency of emission regulations for nitrogen oxide, carbon monoxide, and unburned hydrocarbons. The new gas turbine models, like any other emerging technology, face their own technological challenges. Specifically, low-emission gas turbines are subject to a phenomenon called high-amplitude, combustor pressure oscillations (CPOs), which have been shown to damage the turbine parts [5]. In particular, engines that use lean-burn technology, where the flame temperatures are reduced by reducing the fuel-to-air ratio, are highly susceptible to these instabilities. By reducing the flame temperatures, the flame stability is also reduced and the heat release rate is affected. Therefore, the system experiences CPOs at resonant frequencies. Despite the ever-growing restrictions on air quality control and the high demand for low-emission turbines, combustor pressure oscillations still remain an issue with no “universal fix.”

To better understand how CPOs have affected the gas turbine industry, it is important to explore a case study. For example, CPOs have been a significant operational issue for Solar Turbines Incorporated since they began the production of lean premixed turbines, such as their Centaur Type H and Mars engines, in the mid-1980s. A sketch of an annular combustor liner-fuel injector configuration, such as the one found in the Centaur and Mars engines, can be seen below.
In the initial testing of the engines, CPOs were experienced in magnitude ranging from one to three psi. They caused a lot of damage to the structure of the engine. Namely, the dome’s internal splash plates cracked and the fuel injector’s out barrel fretted all within the first thirty minutes. The dome is the upstream face of the combustor, which experiences significant heating during combustion, and the fuel injector is the portion of the engine that injects fuel and mixes it with air before the flame. Upon further inspection of the pressure wave measurements of the instabilities, it appeared the instability was mixed modal, with both axial and circumferential parts, which made the CPOs even more difficult to properly characterize. Among the early stages of experimentation, one method was discovered to control CPOs: operation of the pilot injector with a small amount of the total engine fuel [5].
It was determined that at approximately 10% of engine fuel used through the pilot injector, CPOs remain less than one percent psi and nitrogen oxide emissions remain below the accepted level of 42 ppmv. While this was a decent strategy to control CPOs, there was still a tradeoff: nitrogen oxide emissions did increase, which isn’t aligned with governmental plans to have emissions below 25 ppmv.

Further testing revealed other methods to reduce the amplitude and occurrence of combustor pressure oscillations. Injector exit velocity, overall fuel-air mixed-ness, and fuel transport time from injection location to the flame were all shown to affect CPOs [5]. Tests were completed on the Mars engine that mapped the ranges of stable/unstable operation as a function of fuel spoke location. The variable that was used to do the mapping was $\tau f_c$, where $\tau$ is the time required to travel from the fuel injection location to the injector exit and $f_c$ is the frequency of the CPOs. High CPOs were shown to occur at values of $\tau f_c$ from 0.30 to 0.48 [5]. Testing conducted on the Centaur engine showed that a reduction of injector axial exit velocity greatly reduced the occurrence of CPOs, while achieving nitrogen oxide levels below 25 ppmv.

Over time, combustion pressure oscillations have developed a different industry-wide name: thermoacoustic instabilities. These instabilities, also often referred to as “combustion instabilities,” are couplings between flame heat-release rate oscillations and resonant combustor acoustics [6]. The coupling of these two mechanisms result in large amplitude pressure and velocity oscillations that can have extremely detrimental consequences, causing potential catastrophic component and/or mission failure. Combustion instabilities are the resultant of a phenomenon called a feedback loop. In the feedback loop, changes in velocity and thermodynamic-state variables cause a change in the heat-release rate. This in turn excites
acoustic pressure oscillations, which cause fluctuations in the velocity and thermodynamic-state variables, thus closing the loop [6]. The feedback loop is summarized in Figure 2.

There are several coupling mechanisms that employ the concept of the feedback loop in order to drive combustion instabilities in gas turbines. For example, in fuel feed line-acoustic coupling, pressure drop in the un-chocked fuel nozzle causes a change in fuel injection rate, which in turn causes oscillations in the heat-release process, thus beginning the feedback loop [6]. As another example, in equivalence-ratio oscillations, pressure oscillation in the combustor yields a change in the mixing process and fuel and/or air supply rates, which in turn varies the equivalence ratio of the mixture with time. The mixture interacts with the flame and causes heat-release oscillations, thus beginning the feedback loop once again [6].

In order to determine the conditions under which the coupling of heat-release rate oscillations and resonant combustor acoustics occur, the Rayleigh Criterion is utilized. Below is the equation for the Rayleigh index.
Equation 1: The Rayleigh Index

\[ R = \int_0^T p'(t) q'(t) dt \]

In the above equation, \( T \) represents the period of oscillation, \( p' \) is the combustor pressure oscillations, and \( q' \) is the heat-addition oscillations. If \( R > 0 \), energy is added to the acoustic field. This means that the rate of energy addition is greater than the rate of energy damping, and combustion instabilities will occur. If \( R < 0 \), none of the components in the feedback loop are excited and no acoustic instabilities occur. Another way to evaluate the conditions of combustion using the Rayleigh Criterion is to look at the magnitude of the phase difference between the pressure and heat-release operations, \( \theta_{pq} \). If \( 0 < |\theta_{pq}| \leq 90^\circ \), energy is locally added to the acoustic field and combustion instabilities occur. If \( 90^\circ < |\theta_{pq}| < 180^\circ \), heat-addition oscillations damp the acoustic field oscillations and no combustion instabilities occur.

Thermoacoustic instabilities can couple with any of the possible acoustic modes inside the combustor, such as longitudinal, azimuthal, and mixed modes. In a longitudinal acoustic mode, the resulting acoustic pressure propagates back and forth along the axis of the combustor due to acoustic forcing along its length. In an azimuthal acoustic mode, the acoustic pressure propagates as a result of acoustic forcing around the circumference of the combustor. Just as the name suggests, in a mixed acoustic mode, different types of acoustics interact. Below are examples of what these types of modes physically look like inside a combustion chamber.
The circumferential mode, also known as azimuthal modes, present a unique challenge to the gas turbine industry. In annular combustor, the azimuthal mode that dominates the thermoacoustic feedback process excites the flame asymmetrically, which can lead to a very different response. A paper written by O’Connor, Dawson, and Worth [7], explores the effect of azimuthal acoustic fields on flame behavior. In the experimental annular combustor, a pressure node, which is an asymmetric forcing condition, caused acoustic velocity fluctuations. By creating asymmetric velocity fluctuations about the centerline of the annular combustor, an asymmetric response was also created in the flame [7]. The velocity fluctuations caused the shear layers, and thus the flame, to flap periodically, which created an observable phase difference between the left and right hand flame edges. The following images detail the flame response at different times of the standing wave oscillation, which is the driving mode in this experiment.
When comparing the left and right hand sides of the flame at 0° and 200°, the structure is essentially the same, which means that the response of the two sides is in approximate anti-phase [7]. The reason for the flame asymmetry can be attributed to the vortex rollup in the shear layers. The vortex rollup appeared to be alternate vortices shedding from the dump place on either side of the centerline of the flow [7]. Phase-averaged vorticity fluctuations contours were also documented for the left and right hand flame edge response.
The contours also revealed asymmetry in the flame response. It is clear from the images that the vorticity disturbances in both the inner and outer shear layers are out-of-phase, which makes physical sense due to the asymmetric forcing condition resulting from the pressure node.

The goal of our research is to create asymmetric acoustic fields in a controlled manner so that we can quantify the acoustic field and understand how the acoustic field is affecting the flow and flame. First, testing was conducted in the analysis software Comsol under different forcing and boundary conditions to understand how each variable affects the resulting acoustic pressure. Second, testing was conducted on a laboratory setup which was able to both longitudinally and azimuthally force a rig containing a combustion chamber. We were able to alter the outlet boundary condition and force the system at various frequencies to see how this affected the recorded pressure within the combustion chamber. The results of both the Comsol and lab experimentation are presented and analyzed in the following thesis.
Chapter 2
Modeling Framework

Comsol Overview

Comsol Multiphysics is a useful tool that can be used to solve a variety of engineering problems. The software provides built-in physics interfaces that allow the user to conduct various studies, including: stationary and time-dependent studies, linear and nonlinear studies, and eigenfrequency, modal, and frequency response studies [1].

Before a study can be performed, the user must indulge in a series of tasks that include, but are not limited to, building the geometry, defining the material, applying boundary conditions, and choosing an appropriate mesh.

Upon opening the Comsol interface, users are prompted with two options for building geometry: Model Wizard or Blank Model. Because all Comsol modeling was conducted using a combustor built in the Model Wizard mode, that will be the focus for this thesis. There are five space dimension options for building a geometry in the Model Wizard mode: 3D, 2D Asymmetric, 2D, 1D Asymmetric, 1D, 0D. After choosing the space dimension, the user will be asked to choose a physics and a study that are most appropriate for his needs. The specific physics and study platform used for analysis of the combustion chamber and swirl rig are detailed in later sections of the thesis.
Once the user has selected the physics and study, he will be brought to the final workspace in which he can build geometry and apply materials, boundary conditions, and meshes.

Building the Geometry

There are many options for building geometries within Comsol. For example, users may choose from a selection of predefined shapes.

![Predefined shape options offered within Comsol](image)

**Figure 6: Predefined shape options offered within Comsol**

After selecting the shape, the user would define parameters so that Comsol can build the shape in the specified domain. For example, in the case of a cylinder, which is the shape that was used to build the combustion chamber, the user would need to define a radius, height, XYZ
origin position, primary axis, and rotation angle. Once the user is satisfied with the chosen parameters, he can select the “Build Selected” option to produce the geometry within Comsol.

![Graphical interface showing parameters for cylindrical geometry]

**Figure 7: Needed specifications to produce cylindrical geometry**

*Defining the Material*

To define the material of the geometry, the user must right click on the “Materials” option in the Model Builder window and select “Add Material.” The user will then be brought to a material library with various selections to apply to the geometry. Once the appropriate material is located, the user must right click on it and select “Add to Component…”
**Figure 8: Selection of material**

It is possible the user may need to manually input a material property that is not already stored within Comsol. This can be done by editing the “Material Contents” window. If all material properties are not satisfied, a red “X” will appear next to “Materials” in the Model Builder window and the user will not be able to conduct a study.

**Applying Boundary Conditions**

In order to apply a boundary condition to the geometry, the user can right click on the physics within the Model Builder and select from a number of boundary conditions previously built into the software.
Figure 9: Selection of boundary conditions

Comsol automatically partitions the geometry into segments (called domains). The domains make it convenient to select only a portion of the geometry to apply a boundary condition to. For example, in the case of a cylinder, the user can apply a boundary condition to just the top face. The user can then check that the boundary condition is applied to the right domain by checking which domains are “active” in the “Domain Selection” window.

Meshing

Applying a mesh can range from being quite simple to quite complex depending on the geometry the user is working with. In the case of the cylinder, which is considered to be simple geometry, the user can apply a mesh by expanding the arrow next to “Mesh” in the Model Builder window and clicking on the “Size” option. The user can select from a number of predefined meshes including, but not limited to, course, normal and fine.
Choosing a mesh can be a very important step before conducting a study on the geometry. If the wrong mesh is chosen, either the study won’t run or the results can be inaccurate. The user may need to experiment with various meshes until he finds the one that is appropriate for the specific engineering problem at hand. In general, the mesh cell size should be several times smaller than any fluctuation or gradient in the system to allow for full resolution of the behavior of the system.

**The Pressure Acoustics, Frequency Domain Interface**

All simulations conducted on both the combustion chamber itself and the full experimental mock-up were run in the Pressure Acoustics, Frequency Domain physics interface provided by Comsol. This specific interface is useful for capturing the interaction between acoustic waves and physical geometry. All studies conducted using the Pressure Acoustics, Frequency Domain interface will have a solution that measures the acoustic pressure fluctuation...
(p) within the modeled geometry. Measured acoustic pressure characterizes the acoustic variations to ambient pressure [1]. In order to theorize what the acoustic pressure phenomenon would look like within both the combustion chamber and swirl rig, two different study platforms were used: the eigenfrequency study and the frequency domain study.

**Eigenfrequency Study**

The eigenfrequency study uses the following equation to solve for the eigenfrequencies and eigenmodes of the system:

\[
\nabla \cdot \left( \frac{1}{\rho_c} \nabla p \right) + \frac{\lambda^2}{\rho_c c^2} p = 0
\]

where \( p_c \) and \( c \) are complex-valued damping quantities, \( f \) is the eigenfrequency, \( \omega \) is the angular frequency, and the eigenvalue, \( \lambda = i2\pi f = i\omega \) [1]. This equation is termed the wave equation, and is the governing equation for sound. It is a combination of mass and momentum conservation that accounts for the variation of acoustic pressure in both time and space. The form of the wave equation in Equation 1 is the frequency-domain form, where \( p \) is the pressure fluctuation in the frequency domain. The eigenfrequencies of this equation are the natural resonances of a system.

A user can specify how many eigenfrequencies Comsol should solve for. After the solution is completed, acoustic pressure within the system is depicted using various colors and a corresponding scaled color bar. This type of study is useful to compare how a specific eigenfrequency corresponds to the acoustic pressure within a system. Comsol can be used as a concept verification for the physical experiment. Because the software can predict the eigenfrequencies the system should resonate at, this information can be used to ensure the experiment experiences those frequencies when white noise is applied to the system.
**Frequency Domain Study**

The frequency domain study uses the inhomogeneous Helmholtz equation

\[ \nabla \cdot \left( \frac{1}{\rho_c} \nabla p - q \right) - \omega^2 \frac{p}{\rho_c c_e^2} = Q \]

where \( p_c \) and \( c_e \) are complex-valued damping quantities, \( \omega \) is the angular frequency, \( q \) is the dipole source, and \( Q \) is the monopole source [1].

This study is similar in nature to the eigenfrequency study in that it produces acoustic pressure readings at various frequencies. The key difference is that the user defines specific frequencies instead of letting Comsol solve for the natural eigenfrequencies of the system. A user may either do a frequency sweep or input specific frequencies to determine the forced response of the system at these frequencies. A frequency sweep requires the user to input a range of frequencies he wants the software to test as well as the step between each frequency. If the user wants to know the acoustic pressure of the system at, for example, 1000 Hz, he may also input that single frequency and receive the acoustic pressure solution that corresponds to it.

It is important to note that Comsol can run more than one study at once. The Comsol analysis conducted for this thesis often involved running both the eigenfrequency and frequency domain studies at the same time and comparing the results. These comparisons can be found in the next chapter of the document.
Chapter 3

Acoustic Modes in Combustion Chambers

The Wave Equation and Solutions

Waves are often a hard phenomenon to define. In general, a wave is characterized by a disturbance or deviation from a pre-existing condition. Through the motion of the deviation, information is transported from one point in space to another. Wave motion differs from that of rigid body translation because the speed at which a wave travels is finite, not infinite. Most waves require a medium to travel through, such as air, water, or a solid. However, there are certain waves that require no medium, such as waves of the electromagnetic spectrum. Another characterization of wave movement is that the medium through which it travels does not need to undergo gross movement [2].

The wave equation, in its most idealized form, satisfies the equation:

\[ c^2 \nabla^2 u - \frac{\partial^2 u}{\partial t^2} = 0 \]

Where \( c \) is a constant representing the speed at which the wave travels, \( u \) is a physical property associated with the disturbance or signal, and \( \nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \).

The wave equation is not satisfied by every disturbance in a fluid that is still classified as a wave. For example, when you start factor in medium characteristics such as viscosity, the governing equation becomes much more complicated and mathematically does not satisfy the
wave equation, yet these types of waves go by another name: damped waves [2]. In other words, the wave equation does indeed generally characterize freely propagating waves, but not all waves satisfy the equation. Furthermore, waves are commonly seen as having sinusoidal shapes of an oscillatory nature. However, waves can also take the form of spikes, rectangular pulses, and noise [2]. In the next two sections, detail will be provided on how waves behave in a guide with a specific shape. Namely, the differences between the rectangular waveguide and the cylindrical waveguide (like the laboratory combustion chamber used in the experiment) will be explored. A waveguide is a device that controls the propagation of waves. The guides split the wave into two parts: progressive and standing. A progressive wave is produced in the desired direction of transmission, while a standing wave is usually caused by the presence of the surfaces of the guide [3].

In order to understand the fundamentals of the governing equations of the waveguides, a foundation needs to be laid for wave motion in a bounded region. Let us look to an example of wave motion on a string with fixed ends. The wave equation of the string is written as follows:

Equation 4: Wave Equation for String Motion

\[
\frac{\partial^2 \zeta}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \zeta}{\partial t^2} = 0
\]

where \( \zeta \) is displacement of the string from equilibrium, \( c \) is the wave speed on the string, and \( L \) is the length of the string. If the string has fixed ends, we develop the boundary conditions \( \zeta(0, t) = 0 \) and \( \zeta(L, t) = 0 \). In this scenario, the method of separation of variables is useful to solve the wave equation. Therefore, we set \( \zeta = X(t)T(t) \). The wave equation is then transformed into the following: \( X''T - (1/c^2)XT' = 0 \). This simplifies to \( X''/X = (1/c^2)T''/T = -k^2 \), where \( k \) is defined as the separation constant. We set both sides of the equation equal to the constant \( -k^2 \) to relate the two variables, \( X \) and \( T \). We can now rewrite the \( X \) part of the equation as:
Equation 5: Helmholtz Equation

$$X'' + k^2 X = 0$$

This equation is also referred to as the Helmholtz equation, which was referenced earlier in the thesis. The solutions of this equation is: $X = A_1 \cos kx + A_2 \sin kx$. By employing a similar solution for the T side of the equation, we get: $T = B_1 \cos \omega t + B_2 \sin \omega t$, where $\omega = kc$ or $k = \pm \omega/c$. It is hopefully clear, at this point, why $k$ was a convenient choice as the separation constant in this equation: $K$ turns out to be the wave number.

Substituting the expressions for $X$ and $T$ back into the string displacement equation, it transforms into: $Z = (A_1 \cos kx + A_2 \sin kx)(B_1 \cos \omega t + B_2 \sin \omega t)$. At this point, we employ the boundary conditions to further define the equation. It was said earlier that $\zeta$ must be zero at $x=0$, so we set $A_1 = 0$ to get rid of the $\cos kx$ term of the equation. At $x=L$, $\sin kL$ would have to be equal to 0. This will only occur at certain values of $k$, which are referred to as eigenvalues, such that $k$ is equal to $\pi/L, 2\pi/L, \ldots, n\pi/L$. Furthermore, because $\omega = 2\pi f = kc$, eigenfrequency $f_n = k_n c/2\pi = nc/2L$. The wave equation for wave motion on a string with fixed ends now takes the form:

$$\zeta = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left[ a_n \cos \omega_n t + b_n \sin \omega_n t \right]$$

where all $\zeta$ are referred to as normal modes of vibration of the string.

**The Rectangular Waveguide**

Now consider a three-dimensional system like a rectangular waveguide. The coordinate system in one dimension is similar to that of the string problem in the previous section, but now the oscillating medium (air) is allowed to fluctuate in three dimensions. We extend this analysis to
waveguides, but the equations are similar; the main difference is that we now need to consider the properties of a fluid medium instead of a string. In the following equations, $\varphi$ represents the velocity potential, whose derivation is not shown in this thesis, $u$ is the particle velocity, and $p$ is the particle pressure.

\[
\begin{align*}
    u &= \nabla \varphi \\
    p &= -\rho_0 \frac{\partial \varphi}{\partial t}
\end{align*}
\]

The wave equation can be rewritten in terms of $p$ as follows:

**Equation 6: The Wave Equation in Terms of Particle Pressure ($p$)**

\[
\Delta^2 p = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}
\]

To find the governing equation of the rectangular waveguide, let’s look at a rigid-wall duct of rectangular cross section, height $a$, and width $b$.

![Figure 11: Rigid-wall rectangular duct](image)

In order for there to be vanishing normal component of particle velocity at the walls of the duct, the following must be true:

\[
p_x = 0 \text{ at } x=0,a
\]
Let’s also say at \( z = 0 \), there is a source emitting a signal equal to \( e^{jwt} \). In order to relate the source signal to the sines and cosines used in the previous section to describe waves, Euler’s Formula is used. In this specific example, \( j \) is used as the imaginary number as opposed to \( i \).

**Equation 7: Euler’s Formula**

\[
e^{-j\omega t} = \cos\omega t + jsin\omega t
\]

Rearranging the wave equation in terms of particle pressure into a more useful form, we get

\[
p = \{cosqx\}\{cosry\}\{e^{ijz}\}\{e^{j\omega t}\},
\]

where we set \( q = m\pi/a \) and \( r = n\pi/b \). Substituting in \( q \) and \( r \) into the wave equation, we arrive at an expression of particle pressure for the \( mn \)th traveling wavemode of the waveguide, which takes the form: \( p_{mn} = A_{mn}\cos(m\pi x/a)\cos(n\pi y/b)e^{j(\omega t - \beta_{mn}z)} \). The complete solution for the sum of all the wavemodes is written below.

**Equation 8: Solution of Particle Pressure for all Wavemodes in Rectangular Waveguide**

\[
p = \sum_{m,n} A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{j(\omega t - \beta_{mn}z)}
\]

where \( A_{mn} \) is determined from source conditions [3].

**The Cylindrical Waveguide**

The propagating portion of the wave in a cylindrical waveguide is represented by the same function as in the case of the rectangular waveguide, \( e^{j(\omega t - \beta_{mn}z)} \), where \( z \) is along the axial direction of the cylinder. In the following set of equations, we assume the cylindrical guide has radius \( a \).

In this case, we arrange the solution to the wave equation to have the following form:
\[ p = \{J_m(kr)\} \{\cos m\theta\} \{e^{jlz}\} \{e^{jot}\}, \]
where \( J_m \) is a Bessel function of the first kind. The purpose of \( J_m \) is to ensure that fluctuations go to zero at the centerline in order to preserve axisymmetry. In order for the boundary condition at the wall of the waveguide, \( u^{(1)}(kr) \) to equal zero, we must choose \( kr \) to be equal to \( \alpha mn'/a \), where \( \alpha mn' \) is the \( n^{th} \) zero of \( J'_m \). We can then derive an equation for pressure at the \( mn^{th} \) mode: 
\[ p_{mn} = J_m(\alpha mn'r/a) [A_{mn}\cos m\theta + B_{mn}\sin m\theta] e^{j(\omega t - \beta mn z)}. \]
The complete solution for the sum of all wavemodes is written below.

**Equation 9: Solution of Particle Pressure for all Wavemodes in Cylindrical Waveguide**

\[ p = \sum_{m,n} J_m \left( \frac{\alpha'_mn r}{a} \right) [A_{mn}\cos m\theta + B_{mn}\sin m\theta] e^{j(\omega t - \beta mn z)} \]

Below are some plots of the solution to this equation for two different combustion chamber conditions that were tested: hardwall all around (case 1) and hard wall-open top (case 2). A hardwall boundary condition ensures that the pressure fluctuation is maximum there, and an open or “pressure release” boundary condition forces a zero pressure fluctuation at the boundary. These plots will be compared against the Comsol generated results in the upcoming section.
Figure 12: Pressure fluctuation for hardwall all around condition

Figure 13: Pressure fluctuation for hardwall-open top condition
Cylindrical Waveguide

The base model of the combustion chamber in Comsol consists of an eight inch tall, four inch wide cylindrical build. Three different cases were tested, each having different boundary conditions applied to the model depending on what real life conditions that model was to depict. In case 1, the combustion chamber is completely sealed off. To simulate this in Comsol, all surfaces of the combustor chamber had a hard boundary condition (wall) applied to them.

In case 2, the combustion chamber has a top that is open to the atmosphere. To simulate this in Comsol, the top surface of the cylinder has a pressure release boundary condition applied to it that sets the pressure fluctuations at that surface equal to zero. All other surfaces of the cylinder have a sound hard boundary condition applied to them similar to case 1. In case 3, the combustion chamber has a top on it that is partially open to the atmosphere, which is something that is modeled in the actual lab experiment along with the open top condition. To model this partially open top in Comsol, a smaller cylinder with the same height dimension and a width of approximately 2 and one eighths inches was built concentrically on top of the base cylinder. The zero pressure fluctuation boundary condition was applied to only the smaller cylinder at the top surface, and the hardwall boundary condition was applied to the rest of the model.
Figure 14: Comsol models with various boundary conditions applied

Eigenfrequency Comparison of the Three Cases

Table 1: Eigenfrequency comparison of different modeled boundary conditions

<table>
<thead>
<tr>
<th>Combustion Chamber Conditions</th>
<th>Hard Wall All Around (Case 1)</th>
<th>Hard Wall/ Open Top (Case 2)</th>
<th>Hard Wall/ Partially Open Top (Case 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Eigenfrequencies</td>
<td>1.134672e-5+2.932218e-5i</td>
<td>422.246642</td>
<td>382.802841</td>
</tr>
<tr>
<td></td>
<td>844.525913-0.012349i</td>
<td>1266.749664</td>
<td>1151.551431</td>
</tr>
<tr>
<td></td>
<td>1689.519225+1.604176i</td>
<td>2024.257106</td>
<td>1927.759921</td>
</tr>
<tr>
<td></td>
<td>1979.545452+1.410186i</td>
<td>2024.257129</td>
<td>2010.514936</td>
</tr>
<tr>
<td></td>
<td>1982.255562+0.453053i</td>
<td>2111.245932</td>
<td>2010.520116</td>
</tr>
<tr>
<td></td>
<td>2152.380308+0.243989i</td>
<td>2350.320217</td>
<td>2253.736041</td>
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<tr>
<td></td>
<td>2152.544195-4.885464i</td>
<td>2350.320311</td>
<td>2253.766294</td>
</tr>
<tr>
<td></td>
<td>2533.913101-1.469619i</td>
<td>2894.288747</td>
<td>2707.093774</td>
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<td></td>
<td>2599.9912-0.498296i</td>
<td>2894.289344</td>
<td>2707.14756</td>
</tr>
<tr>
<td></td>
<td>2604.21899-2.207533i</td>
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<td>2711.559667</td>
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<tr>
<td></td>
<td>3214.95583+0.789602i</td>
<td>3311.196538</td>
<td>3298.913895</td>
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<tr>
<td></td>
<td>3219.425587-0.868723i</td>
<td>3311.199369</td>
<td>3298.920239</td>
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<td></td>
<td>3277.445979-4.500759i</td>
<td>3520.034921</td>
<td>3303.342329</td>
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<tr>
<td></td>
<td>3284.491126+2.203778i</td>
<td>3520.038053</td>
<td>3303.386635</td>
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<tr>
<td></td>
<td>3377.55086-0.217529i</td>
<td>3557.630648</td>
<td>3435.706169</td>
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<td>3390.033667+2.578883i</td>
<td>3557.633009</td>
<td>3435.744281</td>
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<td></td>
<td>3392.433671-2.875971i</td>
<td>3800.456</td>
<td>3497.790567</td>
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<td></td>
<td>3600.764808-2.241731i</td>
<td>3904.377363</td>
<td>3739.689875</td>
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<tr>
<td></td>
<td>3692.548042+1.828074i</td>
<td>3904.384075+5.501407e-7i</td>
<td>3739.74365</td>
</tr>
<tr>
<td></td>
<td>3915.558197-3.790945i</td>
<td>4141.865669-9.917943e-7i</td>
<td>3984.227923</td>
</tr>
</tbody>
</table>
The above table is a comparison of the eigenfrequencies, or resonant frequencies, for each of the three different cases. Comsol was told to output a total of twenty eigenfrequencies and acoustic pressure solutions, or eigenmodes, for each case. The solutions show that the eigenfrequencies do differ slightly from case to case. This proves that outlet boundary condition (closed top, open top, or partially open top) will have an effect on the acoustic pressures experienced within the combustion chamber.

Modal Comparison of the Three Boundary Conditions

Table 2: Eigenmode comparison among different modeled boundary conditions

<table>
<thead>
<tr>
<th>Eigenmode</th>
<th>All hard wall</th>
<th>Hard Wall/ open end</th>
<th>Hard Wall/ Partially Open End</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Longitudinal</td>
<td><strong>844.525913 – 0.012149i</strong></td>
<td><strong>1266.740964</strong></td>
<td><strong>1151.551431</strong></td>
</tr>
</tbody>
</table>

![1st Longitudinal Eigenmode](image)
<table>
<thead>
<tr>
<th>Mode Level</th>
<th>Mode Type</th>
<th>Frequency</th>
<th>Complex Mode</th>
<th>Frequency</th>
<th>Complex Mode</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td>Longitudinal</td>
<td>1689.519225 + 1.604176i</td>
<td>2111.245932</td>
<td>1927.759921</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>Circumferential</td>
<td>1979.545452 + 1.410186i</td>
<td>2024.257106</td>
<td>2010.514936</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>Circumferential Degenerate</td>
<td>1982.255562 + 0.453053i</td>
<td>2024.257219</td>
<td>2010.520116</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The above table lists the 1st circumferential, the 1st circumferential degenerate mode, the 1st longitudinal, the 2nd longitudinal, the 1st mixed-mode, and the 2nd mixed-mode eigenmodes for each of the three cases along with the eigenfrequency at which they occur. Across every case, the 1st type of mode occurs at the same number eigenfrequency. For example, the 1st longitudinal mode occurs at the second eigenfrequency in each of the eigenfrequency lists. Similarly, the 1st circumferential mode occurs at the fourth eigenfrequency and the 1st mixed mode occurs at the sixth eigenfrequency in each of the three cases.

This Comsol analysis can also be used as a type of concept verification for the physical experiment. If the swirl rig is forced at any frequency in the table above, then the resulting mode, whether
it be circumferential, longitudinal, or mixed-mode, should match the mode predicted by Comsol at the same frequency.

In order to receive a more in depth view of the eigenmodes experienced within the combustion chamber at varying outlet boundary conditions, sliced views have been provided at the 1st longitudinal, 1st circumferential, and 1st mixed mode for each of the three boundary conditions below.

Table 3: Slices of eigenmodes at different modeled boundary conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>1st Longitudinal</th>
<th>1st Circumferential</th>
<th>1st Mixed Modal</th>
</tr>
</thead>
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<td>B.C. 1</td>
<td><img src="image" alt="Graph" /></td>
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<td>B.C. 2</td>
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<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
Based on the slices from the table, it is clear that the outlet boundary condition has an effect on the acoustic pressure within the combustion chamber. In the colored pictures, the light green is representative of a zero pressure. For the 1st longitudinal eigenmode, the complete hardwall boundary condition model experienced zero pressure in the center of the combustion chamber, extending radially throughout the width. Furthermore, the maximum acoustic pressure was experienced at the very top and bottom of the chamber. However, in boundary condition cases 2 and 3, zero pressure can be seen near the top of the combustion chamber, specifically where the chamber is open to the atmosphere. This is driven by the boundary condition applied to the open top and partially open top that forces acoustic pressure fluctuations there equal to zero.

Comparing the Comsol results to the theoretical solution of the wave equation in Figure 13 and Figure 14, it is clear that the results make physical sense. In the theoretical solution of the hardwall condition for the first longitudinal mode, the maximum pressure fluctuations were experienced at the beginning and end of the combustor, which is consistent with the Comsol solutions. In the theoretical solution of the hardwall-open top condition for the first longitudinal
mode, pressure fluctuations are maximized at the bottom of the combustion chamber and go to zero at the top, which also is consistent with Comsol results.

For the 1st circumferential eigenmode, the boundary condition case 1 model experienced zero acoustic pressure in the center of the combustion chamber, extending longitudinally along the length. When the combustion chamber is open to the atmosphere in boundary condition cases 2 and 3, the zero acoustic pressure region appears wider towards the open top. For the mixed eigenmode, it appears that the boundary condition has less of an effect on the acoustic pressure, thus the zero pressure areas remain the same in each boundary condition case.
Azimuthally Resolved Forcing of a Cylindrical Waveguide

For this portion of the Comsol analysis, the combustion chamber was altered to better mimic what would be happening in the physical experiment. Specifically, eight panels were added to the bottom of the chamber to represent the eight speakers attached to the experiment to force acoustics on the system. In order to simulate the forced acoustics, an inward acceleration boundary condition was added to the eight panels at a magnitude of 10 m/s^2. Four different cases were explored: all panels forced in phase, half the panels forced in phase and the other half out of phase, two panels forced in phase and the next two forced out of phase, and every other panel with different phasing.

![Figure 15: Different cases of panel phasing](attachment:figure15.png)
The results of the forced acoustics and varied phasing on the hardwall-open top condition (case 2) are summarized in the table below.

Table 4: Eigenvalue and eigenmode comparison for different panel phasing

<table>
<thead>
<tr>
<th>Panel Phasing</th>
<th>Eigenvalues</th>
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<th>1st Circumferential</th>
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</tr>
<tr>
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<td><img src="image4.png" alt="Image" /></td>
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<tr>
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</tr>
<tr>
<td>Half Phase</td>
<td>Quarter Phase</td>
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</tr>
<tr>
<td>----------------------------------------</td>
<td>--------------------------------------------</td>
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<tr>
<td>422.246591 1266.740714 2024.25697 2024.257 2111.244577 2350.319856 2350.320372 2894.289547 2894.290863 2955.786695 3311.196627 3311.198487 3520.36958 3520.389867 3557.630705 3557.634484 3800.434251 3904.386452+4.283952e-7i 3904.389238 4141.876751</td>
<td>422.246591 1266.740714 2024.257 2024.25697 2111.244577 2350.320372 2350.319856 2894.289547 2894.290863 2955.786695 3311.196627 3311.198487 3520.36958 3520.389867 3557.630705 3557.634484 3800.434251 3904.386453+4.612875e-6i 3904.389238+4.323319e-7i 4141.876751</td>
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<tr>
<td>Every Other Phase</td>
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</tbody>
</table>

From the above table, two conclusions can be drawn. Firstly, forcing the panels on the combustion chamber doesn’t change the eigenfrequencies of the system. Secondly, the eigenmodes corresponding to the eigenfrequencies do not experience a change either. Specifically, the 1st longitudinal and 1st circumferential eigenmodes look the same in all cases of phasing and they still occur at the same eigenfrequency.
The next Comsol analysis involved employing the frequency domain study, which was discussed in an earlier section. The frequency domain study forces the system at specific user defined frequencies and produces corresponding acoustic pressure solutions. Specifically, 200, 1000, and 1800 Hz were explored in more depth, as these frequencies could be achieved in the physical experiment. First, the relationship between the forced frequencies and the panel phasing was explored for the hard wall-open top condition (case 2). These results can be seen in the figures below.

![Acoustic pressure when all panels are in phase](image)

**Figure 16: Acoustic pressure when all panels are in phase**
Figure 17: Acoustic pressure when panels are half in phase

Figure 18: Acoustic pressure when panels are quarter in phase
There are two important observations to draw from the above comparisons. Firstly, more of an acoustic response is apparent in the combustion chamber when forced at 1800 Hz as opposed to the other two frequencies. This is especially apparent when the panels are all in phase or half in phase. For the quarter phasing and every other phasing, the response is too tiny to see the differences between the forcing frequencies. The reason for the response being stronger at 1800 Hz is because that frequency is closest to one of the natural eigenfrequencies of the system (most likely 2024 Hz). From the above table, it is also evident that higher modes of phasing produce less of an acoustic response in the system. The magnitude of the acoustic response when the panels are in quarter phase and in every other phase is much less than the response when the panels are all in phase or half in phase.
A frequency domain study was also performed at the specific eigenfrequencies in which the longitudinal, circumferential, and mixed eigenmodes appeared within the combustion chamber. The acoustic pressure plot from the forced frequency was then compared against the original eigenmode to ensure there wasn’t a change. The results from the comparison of the hardwall-open top condition (case 2) are summarized in the table below.

**Table 5: Comparison of eigenfrequency and frequency domain study**

<table>
<thead>
<tr>
<th>Eigenmode</th>
<th>Eigenfrequency Study</th>
<th>Frequency Domain Study (All in Phase)</th>
<th>Frequency Domain Study (Half Phase)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Longitudinal</td>
<td>1266.740964</td>
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<td>1266</td>
</tr>
<tr>
<td>1st Circumferential</td>
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</tbody>
</table>
The comparison was completed for the hardwall-open top condition for two different panel phasing cases: when all the panels were in phase and when half were in phase and half were out of phase. It is interesting to see how the results differ depending on the phasing of acoustic forcing. The 1\textsuperscript{st} longitudinal mode occurs in the combustion chamber at about 1,266 Hz without any panel forcing. Note that even though the colors are flipped in these two images, the solution is the same. However, if the combustion chamber were forced at 1,266 Hz at the two different conditions of panel forcing, only when they are all in phase will a similar longitudinal eigenmode appear. This is an example of how the axisymmetry of acoustic forcing impacts the response of the system. If a system, which resonates with an axisymmetric mode like that at 1,266 Hz, is forced axisymmetrically, the response is quite strong. However, if it is forced asymmetrically, the response is minimal even though the frequency of forcing is the same as the eigenfrequency. If we turn to the 1\textsuperscript{st} circumferential and 1\textsuperscript{st} mixed mode eigenmodes, it can be seen from the above table that similar eigenmodes will only appear in the combustion chamber with panels forced half in phase and half out of phase. Here, the opposing half and half forcing of the panels more easily allow for the non-axisymmetric eigenmodes to appear as opposed to the longitudinally propagating eigenmodes.
Comparison of Chamber Only and Full Experiment Comsol Simulations

The swirl rig experiment was modeled in Comsol as a cylindrically-shaped base of approximately 20 inches tall and a width comparable to that of the combustion chamber itself. The combustion chamber is modeled on top of a post connected to the base. Eight speaks are attached to the post that connect the combustion chamber to the base of the experiment.

![Figure 20: Comsol model of full experiment](image)

To begin, the eigenfrequencies in which the combustion chamber and full experimental setup experienced their 1st longitudinal eigenmode were compared for the both hardwall-open top boundary condition (case 2) and the hardwall-partially open top boundary condition (case 3). Comsol was only able to locate the 1st longitudinal mode within the swirl rig because at higher frequencies, the eigenmodes were experienced in the body of the experiment as opposed to the combustion chamber itself. The results are organized in the table below.
### Table 6: Comparison of combustor only and full experiment for 1\textsuperscript{st} longitudinal eigenmode

<table>
<thead>
<tr>
<th>Mode</th>
<th>Full Model</th>
<th>Combustor Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} Longitudinal (B.C. Case 2)</td>
<td><img src="image1.png" alt="Full Model Image" /></td>
<td><img src="image2.png" alt="Combustor Only Image" /></td>
</tr>
<tr>
<td></td>
<td>1108.10121</td>
<td>1266.740964</td>
</tr>
<tr>
<td>1\textsuperscript{st} Longitudinal (B.C. Case 3)</td>
<td><img src="image3.png" alt="Full Model Image" /></td>
<td><img src="image4.png" alt="Combustor Only Image" /></td>
</tr>
<tr>
<td></td>
<td>1108.10121</td>
<td>1151.551431</td>
</tr>
</tbody>
</table>
The eigenfrequencies in which the 1\textsuperscript{st} longitudinal mode appear for both the open top and partially open top combustion chambers (1266 and 1151 Hz, respectively) are similar in magnitude to the eigenfrequency in which the eigenmode appears in the full experiment (1108 Hz).

The next comparison made between the combustion chamber and swirl rig looked at how the acoustic pressure inside the chamber differed at the forcing frequencies (200, 1000, and 1800 Hz) for the four different modes of phasing applied to the panels in the combustion chamber and to the full experiment. In the case of the swirl rig, the acceleration boundary condition was applied to the surface of the eight speakers. The results of this comparison for the hardwall-open top boundary condition (case 2) can be viewed in the four tables below. One table is provide for each case of speaker phasing.
Table 7: Comparison of combustor only and full experiment: all panels in phase

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Full Model</th>
<th>Combustor Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>![Full Model Image]</td>
<td>![Combustor Only Image]</td>
</tr>
</tbody>
</table>
Table 8: Comparison of combustor only and full experiment: half panels in phase

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Full Model</th>
<th>Combustor Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td><img src="image1.png" alt="Full Model Image" /></td>
<td><img src="image2.png" alt="Combustor Only Image" /></td>
</tr>
</tbody>
</table>
Table 9: Comparison of combustor only and full experiment: quarter panels in phase

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Full Model</th>
<th>Combustor Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>![Full Model Image]</td>
<td>![Combustor Only Image]</td>
</tr>
</tbody>
</table>
Table 10: Comparison of combustor only and full experiment: every other panel in phase

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Full Model</th>
<th>Combustor Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td><img src="53" alt="Image" /></td>
<td><img src="53" alt="Image" /></td>
</tr>
</tbody>
</table>
Some observations can be drawn from the comparison above. The eigenmodes displayed in only the combustion chamber don’t match up as well to the eigenmodes displayed in the combustion chamber of the swirl rig at the higher frequencies of 1000 and 1800 Hz. Yet, at a forcing frequency of 200 Hz, the eigenmodes matched up in both the Comsol models for all cases of phasing. Furthermore, in the quarter phasing and every other phasing tables, there are acoustic oscillations in the post connecting the combustion chamber to the base at the higher frequencies. The higher acoustic energy is driving other parts of the system besides the combustion chamber, particularly the nozzle upstream of the combustor. This may be one of the reasons as to why the eigenmodes aren’t matching up at the higher frequencies.
Swirl Rig Set-Up

The swirling test rig can be dissected into three main sections: the stagnation chamber, the swirler, and the combustion chamber. The flow enters the stagnation chamber through a 2” flexible metal hose. The stagnation chamber itself is 6” in diameter and has two sections. Dowel pins were inserted into the flanges of the chamber to help with alignment. Between the two sections of the stagnation chamber are two perforated plates which are used to break up the
turbulence of the incoming flow. The stagnation chamber and the swirler sections are connected by a ¼” steel shaft. This shaft is operated by a stepper motor, which is able to change the swirl number inside the swirler. The shaft and stepper motor are connected with a coupler in order to ensure minimal leakage around the shaft. This section contains three outlets, two of which are dedicated to Galls police sirens that are used throughout the longitudinal forcing testing. Further downstream of the stagnation chamber, there is a 6” to 3” pipe reducer, which creates clean flow field as the flow enters the swirler.

The swirler is unique to this experiment because it is controlled by the stepper motor, and the angle of the blades can be changed. The stepper motor has an encoder wheel that is able to output a precise swirl number. The swirler is composed of a top and bottom plate. The bottom plate has precisely machined straight slots and the top plate has curved slot, which guide eight NACA 0025 airfoils. The airfoils are positions at 45 degrees around the swirler axis to provide symmetry to the swirler.

After the swirler, the flow travels through an annulus with an outer diameter of 1” and an inner diameter of ½” towards the dump plane. Right before the dump plane are eight, 1 mm holes. In this section of the experiment, acoustic speakers are attached at these eight holes to provide excitation at the dump. These holes expand to an inner diameter of ¼” like a conical horn design. Attached to the holes are approximately 1’ of ¼” pipe, which lead to 1” thick piece of Acetal that change the diameter to 5”, which match the diameter of the acoustic speakers. The speakers used in the experiment are Dayton Audio PA130-8 full range speakers. While operating these speakers, some vibration was seen in the test rig.

The combustion chamber at the top of the experiment has an inner diameter of 5”, an outer diameter of 5 and ½”, and a length of 8.25”. A top with a 2” diameter pipe was created to
put on the top of the combustion chamber that would change the exit condition of the flow. The dimensions of the Comsol-modeled combustion chamber, with an open diameter of 4”, a partially open diameter of 2 and 1/8”, and a height of 8”, are very close to that of the combustion chamber used on the swirl test rig.

There are some differences between the Comsol-modeled swirl rig and the physical rig itself that could account for some discrepancies in a comparison of the results. The Comsol model does not account for the air pipe being fed into the experiment, the swirler portion, or the screens in the flanges of the rig. Furthermore, Comsol does not take into account the perforated plates or the viscous losses along the boundaries of the experiment.
Siren Testing: Axial and Circumferential Placement of Pressure Transducers

In the axial portion of testing, the pressure transducers on the swirling test rig were arranged in a longitudinal manner on the side of the combustion chamber.

![Image of pressure transducers](image1.jpg)

**Figure 22: Axial placement of pressure transducers**

The Gall sirens were used to provide acoustic excitation to the system. Testing was conducted across a frequency sweep of 200-3000 Hz with both an open top and partially open top condition. For the purpose of Comsol comparison, the power spectral density of the acoustic pressure at each pressure transducer and the phase difference in comparison to pressure transducer 1 at frequencies of 200, 1000, and 1800 Hz are reported below.
In the circumferential portion of testing, the pressure transducers on the swirling test rig were arranged in a circumferential manner along the bottom circumference of the combustion chamber.

Figure 23: Circumferential placement of pressure transducers

The Galls sirens were used to provide acoustic excitation to the system. Testing was conducted across a frequency sweep of 200-3000 Hz with both an open top and partially open top condition. For the purpose of Comsol comparison, the power spectral density of the acoustic pressure at each pressure transducer and the phase difference in comparison to pressure transducer 1 at frequencies of 200, 1000, and 1800 Hz are reported below.

Hardwall-Partially Open Top

Figure 25 through Figure 28 show the amplitude and phase of the spectra in the combustion chamber at three forcing frequencies: 200 Hz, 1000 Hz, and 1800 Hz, with a partially open top
configuration. The amplitude and phase of the acoustic modes in both the axial and circumferential directions are provided. All these cases are longitudinally, or axisymmetrically, forced.

Figure 24: Axial power spectral density of acoustic pressure

Figure 25: Circumferential power spectral density of acoustic pressure
In the plot of the axial power spectral density, the 1800 Hz plot appears to take the shape of a second longitudinal mode shape. Revisiting the Comsol analysis, the second longitudinal mode appears in the model with a partially open top boundary condition at an eigenfrequency of approximately 1928 Hz - which is extremely close to 1800 Hz (see: Table 2, Row 2, Column 3). Furthermore, in the plot of the circumferential power spectral density, the 1800 Hz plot shows a first circumferential mode shape. Combining both the axial and circumferential power spectral densities, we can conclude that 1800 Hz is a mixed mode within the swirl rig.
Figure 25 through Figure 28 show the amplitude and phase of the spectra in the combustion chamber at three forcing frequencies, 200 Hz, 1000 Hz, and 1800 Hz, with an open top configuration. The amplitude and phase of the acoustic modes in both the axial and circumferential directions are provided. All these cases are longitudinally, or axisymmetrically, forced.

![Graph showing axial power spectral density of acoustic pressure](image)

**Figure 28:** Axial power spectral density of acoustic pressure
Figure 29: Circumferential power spectral density of acoustic pressure

Figure 30: Axial pressure transducer phase difference
Figure 31: Circumferential pressure transducer phase difference

From studying the plots of power spectral density and phase difference of the hardwall-open top condition, it is clear the exit boundary condition of the combustion chamber has an effect on the acoustic field within. According to the power spectral density graphs, the readings of the acoustic pressure for the hardwall-open top condition is noticeably weaker than that of the hardwall-partially open top condition, by a power of 10. Furthermore, the mode shapes in the hardwall-open top condition do not align as well with the Comsol results as the mode shapes from the hardwall-partially open top condition. Because the readings from the hardwall-open top study were not as strong, the graph of acoustic pressure power spectral density, taken from pressure transducer 1 in axial placement, was studied further for each of the three frequencies.
Figure 32: Power spectral density for 200 Hz

Figure 33: Power spectral density for 1000 Hz
Figure 34: Power spectral density for 1800 Hz

The power spectral density graph of all thee frequencies (Figure 29) shows a zero amplitude at pressure transducer 1, the bottom of the combustion chamber, which doesn’t make physical sense. However, upon inspection of the power spectral density graphs of each frequency, the reasoning behind this discrepancy becomes clear. The pressure transducers utilized in the swirl rig testing have a resolution of $10^{-6}$, meaning readings from the pressure transducers are physically meaningful only if the outputted signal has peaks above $10^{-6}$. From the above graphs, only the 1800 Hz frequency produced a peak above $10^{-6}$, and only just barely, which may account for the peculiar behavior of the frequencies in Figure 29. Furthermore, the individual graphs of the power spectral densities show how a stronger frequency will in turn create a stronger signal for the pressure transducers to read. As you can see from the graphs of 200 and 1000 Hz, there is a lot of noise surrounding the main peak, which occurs at the frequency in question. Yet in the graph of 1800 Hz, there is one clear peak with little to no noise surrounding it.
**White Noise Testing**

In the white noise portion of testing, the swirl rig was forced with white noise is an effort to pick out the natural resonances of the system. Data was pulled from pressure transducer 1 (at the bottom of the combustion chamber) in axial placement because it receives the strongest acoustic pressure readings. After finding the power spectral density of the acoustic pressure, an ensemble average was performed on the signal in order to bring the noise floor down and highlight possible resonances of the system. The results of the white noise study for hardwall-partially open top and hardwall-open top conditions are shown below.

![Frequency spectrum for hardwall-partially open top condition](image)

**Figure 35: Frequency spectrum for hardwall-partially open top condition**
The ensemble average revealed some peculiar characteristics of the swirl rig. The natural resonances of the system appear to occur at very high frequencies, such as 6,000 and 7,000 Hz. Comsol testing of the swirl test rig revealed eigenfrequencies of the system that range from 1,100 to 22,000 Hz, with a big jump from 1,200 to 12,000 Hz. Therefore, the natural resonances of the system revealed by the white noise testing do appear in the eigenfrequencies outputted in Comsol.

Furthermore, according to the Figures above, the system resonates at the same frequencies regardless of the outlet boundary condition of the combustion chamber. A reason for this discrepancy may be the resolution of the pressure transducers. When the system was subjected to white noise, the acoustic pressure readings of the pressure transducers were below that of the manufacturer-specified resolution. This calls into question the accuracy of the recorded data for white noise testing, and therefore, the accuracy of the outputted natural resonances of the system.
Chapter 5

Conclusions and Future Work

Combustion instabilities have come under much scrutiny in the gas turbine industry. As the need for lower emission turbines increases, so does the occurrence and severity of thermoacoustic instabilities. One of the main attributes of these instabilities are asymmetric acoustic fields. Therefore, it was the goal of this research to create asymmetric acoustic fields, in both computational and experimental settings, to quantify the acoustic field and understand how it’s affecting the flow and flame. In order to try and better understand the acoustic field, different parameters of the system were altered, such as outlet boundary condition, forcing frequency, and phase of forcing. The first round of experimentation was conducted computationally, in the software Comsol, and focused mainly on a simplified model of a combustion chamber. Physical experimentation was conducted on the swirl rig that measured acoustic pressure inside the combustion chamber under similar conditions to those modeled in Comsol. Not all the modeled Comsol conditions were tested on the physical experiment and included in this thesis because of time constraints and other limitations. However, the tests that were completed on the swirl rig were compared to the corresponding Comsol results.

Comsol analysis showed that the exit boundary condition of the combustion chamber has a large impact on the acoustic field. It was also shown that when acoustic forcing was imparted on the system at various modes, neither the eigenfrequencies nor the acoustic modes within the combustion chamber experienced any change. The Comsol analysis also proved that if the system is forced at a frequency that is close to one of its natural frequencies, it will experience a stronger response. However, it simultaneously proved that a higher mode of forcing will produce
less of an acoustic response. A final observation of the Comsol study is that the same acoustic modes were visible in both the combustion chamber and swirl rig at the same eigenfrequencies when acoustic forcing was applied at lower modes.

From experimentation conducted on the laboratory swirl rig, it can be concluded that the Comsol results are a rather accurate depiction of the acoustic field within the combustion chamber. Despite the physical factors Comsol did not take into account during combustor analysis, the same mode shapes still appeared in the physical experiment at similar frequencies to those revealed in the Comsol eigenfrequency study. It also became clear that some of the experimental results have discrepancies because of the resolution of the pressure transducers attached to the swirl rig. Stronger frequencies, such as the 1800 Hz signal, outputted better acoustic pressure results when compared to lower frequencies, such as the 200 Hz signal or the white noise. Moving forward with further testing, pressure transducers with increased sensitivity may be purchased to obtain more viable experimental data.

There is still a lot of physical experimentation left to complete and compare to Comsol results. Because asymmetric acoustic fields contribute to combustion instabilities, a large portion of Comsol analysis was dedicated to acoustically forcing the system at different speaker phases in an effort to create asymmetry in the acoustic field. Therefore, testing will be conducted that force the eight speakers attached to the swirl rig in different phases (all in phase, half in phase, quarter in phase, and every other in phase) with open and partially open combustor boundary conditions. In the future, next steps for experimentation involve forcing a non-swirling jet to see if the flow can be excited by using the azimuthally resolved forcing system employed by the swirl test rig.
Appendix A

Swirl Rig Solidworks Drawings

Figure A-1: Complete swirl rig solidworks model

Figure A-2: Stagnation chamber base
Figure A-3: Stagnation chamber

Figure A-4: Swirler casing
Figure A-5: Airfoil assembly in bottom curved slot plate

Figure A-6: Combustion chamber
BIBLIOGRAPHY


[9] Figure 5. Phase-averaged vorticity fluctuations. Adapted from “Effect of Excitation Amplitude on Disturbance Field of a Transversely Forced Swirl Flow and Flame,” by Jacqueline O’Connor. The Pennsylvania State University.
ACADEMIC VITA

Shayna Levenson
299 Vista View Drive
Mahwah, NJ 07430
shynalevenson21@gmail.com

Education:
Bachelor of Science in Mechanical Engineering
The Pennsylvania State University, Spring 2015
Honors in Mechanical Engineering
Thesis Title: Acoustic Characterization of Combustors with Azimuthally Resolved Acoustic Forcing
Thesis Advisor: Jacqueline O’Connor

Professional Experience:
The Boeing Company Intern
Lord Corporation Intern
Public Speaking for Engineering Students Teaching Assistant

Association Memberships/Activities:
Phi Sigma Rho Sorority (Penn State Chapter)
Undergraduate Teaching and Research Experiences in Engineering
Tau Beta Pi Engineering Honor Society (Penn State Chapter)

Honors and Awards:
Dean’s List Graduate
New York Times Civic Engagement Public Speaking Contest (First Place)
Leonard Center Public Speaking Competition (Honorable Recognition)