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A MODERATE MIGRATION SCENARIO FOR JUPITER TO FORM THE TERRESTRIAL PLANETS

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ABSTRACT

The early solar system contained a gas-dominated protoplanetary disk that could cause the migration of the giant planets. This migration can be in the form of a two-stage migration, including an inward and then outward migration. One of the current favored theories, the Grand Tack theory, states that Jupiter migrates in to 1.5 AU, creating a planetesimal disk truncated at 1 AU to then form the terrestrial planets during the subsequent outward migration of Jupiter. There are reasons to believe that such a large movement by Jupiter may be impractical, namely the disk would need to be massive and long-lived. An exploration of migration parameters that involve smaller migration distances and shorter timescales can shed light on whether such extreme displacements are necessary for the formation of the solar system. We examine more moderate migration simulations, where Jupiter starts near the conjectured location of the ice line (~3-4 AU) and migrates a moderate radial distance inward for a variety of distances and times. After the inward migration, Jupiter moves outwards to its final orbital configuration today. We find that the planetesimal disk need not be truncated at 1 AU to form planets with similar characteristics to those in the solar system. We vary the number and mass of planetesimals in the disk to see how this affects the characteristics of the forming terrestrial planets. We find a number of scenarios that provide systems of terrestrial planets similar to those in the solar system. We thus propose an alternative to the Grand Tack theory where Jupiter's migration is less extreme than proposed in the Grand Tack theory.

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Chapter 1

Introduction

There are two main theories of planet formation: core accretion (Rafikov 2004, Lissauer 1993) and gravitational instability (Boss 1997, Kuiper 1951). In core accretion, smaller planetesimals interact with larger protoplanetary cores among a gas disk to form planets through gravitational interactions. The gravity of protoplanetary cores can cause the smaller bodies to have excited eccentricities and inclinations, but gas drag acts to reduce the eccentricity and inclination of the smaller bodies. Planetesimals less than 1 km in size will move toward the midplane of the nebula to create a vertically thin subdisk. Furthermore, the random velocities of the smaller objects decay as they interact with the more massive planetary cores. These effects can increase the accretion rate of small planetesimals by protoplanetary cores. These cores can then grow relatively fast, especially if over 1% of the mass of the disk is in these smaller planetesimals. Planetesimals larger than 1 km in size typically fragment into smaller pieces upon collisions, which could serve as a replenishment source for the smaller particles. This mechanism of planet growth has been shown to allow for rapid growth on several million year timescales, resulting in fairly massive protoplanetary cores (Rafikov 2004).

One issue with this theory is that it is somewhat difficult for planets to complete their growth to present sizes in a reasonable amount of time. The outer parts of the protoplanetary disk could take as long a 10^8-10^9 years for this mechanism to work, as initially suggested. However, the gas disk typically dissipates on timescales of 10^6-10^7 years. This causes a problem with solving how the giant planets in our solar system accreted their gaseous envelopes. One possible solution to this problem is the theory of runaway growth (Wetherill & Stewart 1989), where there is rapid accretion by the protoplanetary cores. However, this has been shown to only persist for a short amount of time (Ida & Makino 1993 and Kokubo

& Ida 1996, 1998). Having smaller bodies comprise more of the solid matter in the disk allows for a more rapid growth of planetary cores (Rafikov 2004).

An alternative scenario to the core accretion process for planet formation is the idea that planets could form from gravitational instabilities (Boss 1997). It has been hypothesized that gravitational instability present in the outer solar nebula could provide a mechanism for giant planet formation. Three-dimensional hydrodynamic calculations have shown that gas giant protoplanets can form in the protoplanetary disk with locally isothermal or adiabatic thermodynamics (Boss 1997). Furthermore, forming gas giants by gravitational instability has been shown to occur on shorter timescales than the core accretion model (Boss 1997), which could potentially mitigate one the current problems in planet formation theory. In the gravitational instability model, the solar nebula forms clumps of gas and dust, which are giant gaseous protoplanets, due to the self-gravity of the disk. The protoplanets then collapse to form the giant planets. In order for this theory to be supported, the large core masses for gas and ice giants had to be reconciled, but it has been argued that this issue is no longer a problem (Boss 1997).

In order for a theory to be a plausible explanation for the formation of the solar system, it must reproduce several observations (Lissauer 1993):

1) The orbits of the planets, minor planets, and asteroid are nearly coplanar, with the plane close to the plane defined by the Sun's equator. The planets orbit the sun in the same direction as the sun rotates, which is called a prograde orbit.

2) The major planets all fall within 30 AU of the sun. As the distance from the sun increases, the separation between planets also increases. The orbital paths of the major planets do not cross.

3) Comets are observed to have radii of greater than 1 km and orbit the sun at distances of greater than 10^4 AU, making up the Oort cloud. This cloud is isotropic. The Kuiper belt is an additional source of comets, but this is a flattened disk located at greater than 40 AU (Duncan et al 1988, Duncan & Quinn 1993a).

4) The majority of the planets in the solar system rotate in the same direction as their orbit around the sun. Exceptions are Venus, Uranus, and Pluto.

5) Most planets have natural satellites, especially the gas giants. These satellites tend to follow low inclination, low eccentricity orbits but small and distant satellites commonly have larger inclinations and eccentricities.

6) Overall, planets account for only a small fraction of the mass in the solar system, but the planets contain over 98% of the angular momentum in the solar system.

7) The terrestrial planets orbit closest to the sun and are made of dense rocky material. The larger planets have low densities and are composed mainly of hydrogen and helium.

8) The asteroid belt contains many minor bodies on orbits between Mars and Jupiter. The size distribution follows an approximate power law while the eccentricities and inclinations are roughly Rayleigh distributed. Almost all meteorites come from asteroids.

9) The age of solar system as constrained by radioisotope dating of primitive meteorites is 4.56 +/- 0.02 billion years (Tilton 1988). This age represents the oldest rocks known. Differentiated meteorites have ages of 4.4-4.56 billion years while lunar rocks have been dated at 3.1-4.4 billion years. Terrestrial rocks are less than 4.1 billion years old. Ratios of isotopes in the solar system are roughly constant for different bodies in the solar system. Many isotopic variations are explained by mass fractionation or radioactive decay, but additional small variations in the oxygen isotope ratios suggest that the protoplanetary nebula was not completely homogenous on a molecular level. Furthermore, the mineral and crystalline structures seen in primitive meteorites suggests that there were rapid temperature variations, including both heating and cooling. These meteorites also suggest magnetic fields on the order of 1 Gauss during the planet formation process. Finally, many planets and satellites show signs of heavy

> cratering. The current impact rates are insufficient to recreate the frequency of cratering on these surfaces in the age of the solar system. This suggests that the impact rate must have been substantially higher at earlier times (Lissauer 1993).

No matter how the giants planets form, whether through core accretion or gravitational instability, observations of exoplanet systems have shown that giant planets are frequently orbiting very close to their host star (Mayor and Queloz 1995, Gatewood 1996). Estimates of the fraction of solar type stars (F, G, and K dwarfs) in the nearby solar neighborhood hosting so-called “hot Jupiters” are around 1.2% +/- 0.38%, using data from Lick and Keck planet searches (Wright, et al 2012). These planets could not have formed at their current semimajor axis. There exists a so-called “snow line” or “frost line” in each solar system beyond which water and other volatile materials can freeze out. This line is defined as the inner edge of the region where the temperature is lower than the condensation temperature of water. The temperature defining the snow line is likely around 145-170 K, depending on the partial pressure of water vapor in the nebula. (Lecar, et al 2006) Gas giants, according to the core accretion model, grew from smaller bodies until the core reached a large enough mass to accrete a gaseous envelope. The composition of gas giants indicates that they had to form out beyond the snow line where such materials would have condensed. The location of the snow line is important for telling us where these gas giants could have originally formed. Furthermore, the snow line is important for some aspects of planet formation itself. When ice is present, the surface densities are higher, which is thought to be required for making the cores of these gas giant planets before the gas disk dissipates. (Lecar, et al 2006) Dust grains coated with ice may also have a higher stickiness, which could aid coagulation of particles in the formation of planetesimals. (Min, et al 2011) The exact location of the snow line has been the subject of much study. Observations of our own solar system show that the C type asteroids contain some water ice. These asteroids are located at around 2.7 AU, suggesting that this may have been the location of the snow line in our solar system. (Abe et al 2000, Morbidelli et al 2000, Rivkin et al 2002) Models have been used to attempt to determine the location of the snow line. To move the snow line past 2.7 AU, the disk would need to have increased opacities, a higher accretion rate, or more mass.(Lecar, et al 2006). For an optically thin nebula, the snow line was calculated to be around 3 AU (Min, et al 2011). However, the nebula was optically thick in the first 5-10 Myr, which causes several factors with competing effects. The snow line would be moved closer to the star due to shielding provided by the material, but viscous heating would act to push to snow line further out. Recent studies of where the snow line would be in a disk find that the location is extremely sensitive to opacities of dust grains and mass accretion rate in the disk (Min, et al 2011). A minimum-mass disk with typical opacities and a mass accretion rate of 10^{-8} Msun/yr has a snow line around 1.6-1.8 AU (Lecar, et al 2006).

Irrespective of the details of the location of the snow line, the hot Jupiters seen in many exoplanetary systems could not have formed in their current locations because the snow line could not be that close to the star. This indicates that these planets formed elsewhere, namely beyond the snow line, and moved to their current orbital configurations. After the initial discovery of a possible gas giant planet orbiting the solar type 51 Pegasi at a semimajor axis of 0.05 AU, it was suggested that the planet formed through gradual accretion of solids and gas capture at larger radii, which was followed by subsequent inward migration. This migration was thought to be due to interactions with the remaining circumstellar disk. The halting of the planet at its current orbit was hypothesized to be due to tidal interactions with the star or truncation of the inner circumstellar disk due to the magnetosphere of the star. (Lin, et al 1996 Further studies of the effects of the disk on the orbits of planets show that if the protoplanet's mass is relatively small compared to the total mass of disk, where the disk is evolving viscously, the direction and timescale of the migration of the protoplanet is determined by the location of the protoplanet compared to the mass distribution of the disk. Inward migration would occur fairly rapidly if the protoplanet is initially located at smaller radii. If located further out, the protoplanet would tend to migrate outwards until the disk material interior is at least partially depleted. These studies of the evolution of protoplanets and the disk led to the conjectures that Jupiter formed beyond the current radius of Mars (around 1.5 AU), the disk mass exterior to Jupiter's orbit at the time of its formation was not greater than the mass interior to the orbit by more than 0.1 solar masses, otherwise Jupiter would have collided with the protosun in less than a million years. (Lin & Papaloizou 1980)

Additional studies of the migration of planets in a disk showed that two different types of migration can occur. Low mass planets experience Type I migration, which occurs when the gravitational interaction between the planet and gas disk is relatively weak. The surface density of the disk is perturbed by a spiral wave due to the effect of the planet. The planet then feels a gravitational torque from the perturbation, causing a loss of angular momentum and subsequent decay of the orbit of the planet. As the planet grows and takes up more material from the disk, the resonance between the gas disk and planet increases. Angular momentum is exchanged between the two, which causes gas to be removed from the vicinity of the planet's orbit. This forms an annular gap around the orbit of the planet, where there is less material by a factor of approximately >100, in the disk than if no planet were to be there. Type II migration occurs for more massive planets. When the mass of the planet increases beyond a certain mass, the gravitational interaction between the planet and disk form a deep gap in the disk. Some gas will fall into the gap and be captured within the Hill radius of the planet, thus further increasing the mass of the planet. As the mass of the planet increases, the efficiency of mass growth due to gas overflowing the gap decreases. Angular momentum is exchanged between the disk and the planet as determined by viscous evolution of the disk. The planet migrates in the same way as the disk, which is usually inward at small radii. The position of the planet in the gap is retained throughout this migration.

Migration is thought to have occurred in our own solar system in a fairly large extent. Fernandez and Ip (1984) suggested that proto-Uranus and proto-Neptune could migrate on large scales due to exchange of angular momentum with a planetesimal disk of fairly massive size. The gravitational effects of Uranus and Neptune could give many of the planetesimals fairly high eccentricities. If a planetesimal were to get too close to the next inner planet, it could be "handed off" to that planet, which would result in a net angular momentum gain for the first planet. Planetesimals could thus move from Neptune to Uranus to Saturn and then to Jupiter. Jupiter has enough mass to eject planetesimals from the solar system on fairly short timescales. This is thought to be the mechanism for forming the Oort cloud. Jupiter, which is assumed to do most of the scattering, would experience a loss of angular momentum, causing the orbit to decay. The other outer planets would experience an increase in angular momentum. It is unknown if this mechanism is a large driver of migration of planets, but it has been theorized to be a possibility.

One of the currently most plausible theories for the formation of the solar system is the Grand Tack theory (Walsh, et al 2011). In this theory, the giant planets carve gaps in the gas-dominated protoplanetary disk and migrate inward. Saturn migrates at a faster rate than Jupiter, and eventually catches Jupiter in the 2:3 mean motion resonance. This means the orbital period between the two is 3/2. Once this happens, Jupiter and Saturn tend to migrate outwards until the gas disk disappears. Because the disk properties are unknown and the relative timescales of the growth of Jupiter and Saturn are unconstrained, there is ambiguity in the extent of the inward and outward migration of Jupiter. The Grand Tack theory seeks to constrain these unknown quantities by searching for where Jupiter may have "tacked" or reversed migration.

The Grand Tack theory states that the terrestrial planets can best be reproduced when the planetesimal disk they are formed from is truncated at 1 AU (Hansen 2009). This theory further claims that this condition is generated naturally if Jupiter tacked near 1.5 AU. The effect of moving Jupiter in to 1.5 AU is examined, in particular to determine if the asteroid belt can survive the passage of Jupiter and if the masses and semimajor axes of the terrestrial planets can be recovered from these conditions. All of the Grand Tack simulations maintain the fundamental assumption that the tacking point of Jupiter was at 1.5 AU.

Jupiter is assumed to be fully formed and is initially located at 3.5 AU, which is a favorable place for the formation of a giant planet due to the location of the snow line. A 30 Earth mass core of Saturn is placed initially at 4.5 AU. Throughout the course of the first 10^5 years of inward migration, the core grows to 60 Earth masses. During the growth phase of Saturn, it remains at 4.5 AU due to inhibition of

Type I migration in disks with a realistic cooling timescale. Cores of Uranus and Neptune are initially located around 6 and 8 AU and they grow from an initial mass of 60 Earth masses without migration. Saturn begins to migrate inward once it reaches a mass of 5 Earth masses, at a rate much faster than Jupiter's migration rate. Saturn catches Jupiter in the 2:3 resonance when Jupiter is at 1.5 AU in these cases. The authors claim that this causes a reversal of the migration and the giant planets then migrate outwards together. They capture Uranus and Neptune in resonance and pull them outwards as well. Jupiter ends its migration at 5.4 AU and at this time, the other three giant planets have reached their full mass. The gas disk dissipates, causing the migration rate to drastically decrease. This orbital configuration, upon later dynamical evolution, is consistent with the present-day orbits of the planets in the solar system.

While this migration is occurring, the planetesimals are affected by the gravity of the planets. The planetesimal disk inside of Jupiter's orbit has a total of 3.7 Earth masses distributed equally between larger planetary embyros and smaller planetesimals. There are additional planetesimals located outside of Jupiter's orbit. There is an inter-planetary belt and a trans-Neptunian disk. The inner disk bodies are considered to be S type and the outer planetesimals are considered to be C type.

During the inward migration of the giant planets, the inner S type planetesimals were viewed to migrate inward due to resonant trapping, eccentricity excitation, and gas drag. The disk mass interior to 1 AU increases by approximately a factor of 2 for a total mass near 2 Earth masses. Some of the inner disk is scattered out into orbits beyond 3 AU. When the planets migrate back outwards, this material can be scattered back inward in the asteroid belt region. When the giant planets cross the planetesimals in the Jupiter-Neptune formation region, a small amount of that material is scattered into the asteroid belt. The giant planets then encounter the trans-Neptunian material, where an even smaller fraction is scattered into the asteroid belt. By the time the giant planets finish their migration, the asteroid belt has a complete population, but the terrestrial planets need another 30 Myr to complete accretion.

The asteroid belt in these simulations is made up of two distinct populations. The S type material is originally from the inner disk and the C type material is originally from the outer disk. Observations of the asteroid belt today show a distinction between S type and C type asteroids. S type asteroids are more prevalent in the inner belt and C type asteroids make up the main population of the outer belt. The asteroid belt would have been rearranged due to the late heavy bombardment (Gomez, et al 2005), but the orbital configuration of the asteroid belt obtained at the end of the Grand Tack simulations is consistent with the conditions necessary for the LHB models.

Additionally, these models could explain how the Earth got some of its volatile contents. The Grand Tack theory finds that for each C type planetismal that ends up in the asteroid belt, another 11-28 C type planetesimals were placed on high eccentricity orbits and eventually entered the terrestrial planet region. These planetesimals could have delivered water and other volatiles to the proto-planetary Earth.

The Grand Tack theory creates the inner terrestrial planets from a truncated disk at 1 AU in a 150 Myr integration. Earth and Venus were shown to grow from the planetesimals located in the 0.7-1 AU annulus while Mars was formed from scattered embryos beyond the disk truncation location. The planet masses of the newly formed terrestrial planets in this theory successfully recover the mass ratio between Earth and Mars, which has long been an issue because almost all models make Mars too massive. Once the giant planet migration was finished, the evolution of the planetesimals located interior to 2 AU was continued to 150 Myr, with the evolution of Jupiter and Saturn as well. The collisions between the embryos and planetesimals were assumed to be fully accretional. The Grand Tack theory tested a number of migration schemes with varying migration and gas disk dissipation timescales, gas density, and planetesimal sizes. They obtained similar results for the terrestrial planet formation in these cases.

The Grand Tack theory is extremely successful at explaining many of the characteristics of our solar system, including the mass ratio of Earth and Mars, the populations of the asteroid belt, and hypothesis about water and volatile delivery. Nevertheless, this theory makes some fundamental assumptions that may not be true. One of the most substantial assumptions made in this theory is that Jupiter tacks at 1.5 AU so as to create a planetesimal disk truncated at 1 AU. The disk of planetesimals was desgined to range from 0.7-1 AU in order to replicate previous work that showed that this configuration allowed for formation of the terrestrial planets similar to those seen in our solar system today.

There are some uncertainties remaining in the Grand Tack. These include the mechanism causing the Tack to occur and the time and rate of gas accretion of Saturn and Jupiter. In order for the outward migration called upon in the Grand Tack theory to be plausible, the planets must orbit close enough that the annular gaps in the disk overlap. Additionally, the Jupiter-to-Saturn mass ratio must be in the range of 2-4. (Masset & Snellgrove 2001; Morbidelli & Crida 2007) There are ways for these two criteria to be met, but these involve changing the properties of planet migration as the protoplanetary disk evolves (Raymond and Morbidelli, 2014). The ambiguities in the Grand Tack theory have left room for other models to attempt to explain the formation of the solar system. Many of the previous models suffered from the *small Mars* problem (Wetherill 1991; Chambers 2001; Raymond et al 2009), where Mars is formed 5-10 times more massive than Mars in our solar system. Several theories have overcome the small Mars problem, but there are additional issues in these scenarios. The terrestrial planets can be reproduced from an annulus of planetary embryos ranging from 0.7-1 AU (Hansen 2009). In this scenario, Earth and Venus were accreted from the material within the annulus. Mars and Mercury formed from material scattered beyond the edge of the annulus. This produces a smaller Mercury and Mars because the material they are forming from is less abundant. While this theory is successful at forming the terrestrial planets, the physical reason behind the initial conditions is difficult to justify. Protoplanetary disks are typically quite extended objects and there are examples of planets extending beyond the orbit of Mars in exoplanetary systems. This calls into question the assumption that there are no planetary embryos beyond 1 AU.

Another alternative scenario is the Extra Eccentric Jupiter and Saturn (EEJS) model (Raymond et al 2009, Morishima et al 2010). Here, the original eccentricities of the gas giants were in the range of 0.07-0.1, which is larger than the current values of ~0.05. The nu_6 secular resonance at 2.1 AU is stronger in this configuration, such that particles entering this region have their eccentricities increased and are eventually lost from the system. This mechanism acts to remove material from Mars' accretion zone, thus restricting its total mass.

Another similar model, suggested by Nagasawa et al 2005 and Thommes et al 2008, proposes that the nu_5 secular resonance sweeps through the asteroid belt while the disk is dissipating, eventually leaving it at its current location of ~0.7 AU (Ward 1981). The excitation from the sweeping resonance can efficiently clear out the Mars accretion region while not affecting the growth of Earth and Venus.

Both of these theories have additional issues. The sweeping secular resonances would require a gas disk, but this is not consistent with theories of interactions between the planet and disk required for the gas giants to have their current orbits. It would also be very difficult to imagine a situation that could excite Jupiter and Saturn into very high eccentricity orbits.

Another theory suggested by Jin et al 2008 hypothesizes that the viscosity structure of the disk could allow for the mass depletion in Mars' accretion zone to allow for its low mass to be reproduced. In this model, MRI is active in the inner and outer parts of the disk, so the viscosity there is very high. In the middle of the disk, the viscosity is assumed to be low. This produces a deficit in the surface density of the disk that could be consistent with Mars' orbit for certain parameters of the disk. This model requires a specific viscosity structure of the disk and no later migration or eccentricity damping of giant planets. The latter point is contradictory to current thoughts of the alter evolution of the outer solar system.

The Grand Tack theory, in addition to recovering the small mass of Mars, also reproduces other elements of the solar system. The present-day asteroid belt has a distinct architecture (Gradie & Tedesco 1982, Demeo & Carry 2014). The inner asteroid belt is made of material that was originally located interior to Jupiter's orbit, which represent the S-type asteroids. The outer belt alternatively is mainly composed of material that originated outside of Jupiter's orbit, which make up the C-type asteroids. The Grand Tack theory also provides a mechanism for delivering water to the Earth. The inner planetesimals that make up most of the composition of the Earth likely would have been fairly dry, yet we see a large amount of water on the Earth today. In the Grand Tack theory, water-rich planetesimals can pollute the inner material and thus act as a mechanism for water delivery. Detailed studies of the number of planetesimals thought to be scattered into the inner solar system suggest that the Earth's current water budget could be established through this mechanism (Walsh et al 2011, O'Brien et al 2014).

While the Grand Tack theory has many merits, it is not absolutely certain to be the mechanism that formed our solar system. The Grand Tack theory requires that Jupiter moves ~2 AU in 10^5 years, which is quite a substantial amount. Such large amounts of migration would require a long-lived and massive gas disk that may not be plausible in the protoplanetary disk around the sun. Here, we investigate other migration scenarios where Jupiter does not move on as large scales as suggested in the Grand Tack theory. We also do not constrain the tacking point to be at 1.5 AU; rather, we investigate how changing the tack point affects the results of terrestrial planet formation.

Chapter 2

Methods

To carry out these simulations, a version of the code *Mercury* (Chambers 1999), was used. *Mercury* is a hybrid symplectic N-body integrator. Symplectic integration is a numerical integration scheme to solve differential equations in classical mechanisms (Ruth 1983). N body integration uses the position and velocity of each body at a specific time to predict the forces and orbital motions in future times. Variations of the N-body problem include the two body problem and the restricted 3-body problem. The N-body problem consists of N point masses of mass m_i, where I runs from 1 to N. These point masses are in an inertial reference frame and move according to gravitational attraction. Each one of the masses m_i has a position vector q_i.

From Newton's equations of gravity, it is clear that there will be a problem if two bodies get too close to each other. In this case, the potential energy term becomes very large. One way to overcome this problem numerically is to use a hybrid method. In this method, the close encounter is treated with a conventional integrator while the remaining terms are solved with the symplectic integrator. By using a separable potential technique, the hybrid scheme can actually be made symplectic despite the use of a non-symplectic term.

Symplectic integrators have a few specific advantages over other N-body integrators. Namely, they do not have any accumulation of long-term energy error, in addition to the fact that they are much faster for systems where most of the mass is contained in a single body. This type of situation is encountered with planetary dynamics around the sun. Symplectic integrators inherently use a fixed time step, which can make dealing with close encounters difficult. During a close encounter, it would be best if the time step were decreased in order to maintain the accuracy of the overall integration. But, changing the step size of the integrator can also introduce an error. One such solution to this problem is to separate the perturbation terms and give each a separate step size such that stronger perturbations have smaller step sizes. (Duncan, Levison, Lee 1998). An alternative solution is the hybrid integrator used in *Mercury*. This integrator uses both symplectic and non-symplectic terms so that the overall algorithm has the desirable properties of both. (Chambers, 1999)

The modified version of *Mercury* used in these simulations included a subroutine (Mandell 2007) to allow for forced migration of one or more of the bodies. In this case, Jupiter was the body that was forced to migrate by applying drag forces. Additionally, this subroutine applied gas drag to the smaller objects. Both type I and type II migration were included in this calculation. The user could modify a number of parameters controlling the migration types and rates.

In these simulations, the gas-giant planets were assumed to be fully formed at the beginning of the integration, even though this might not necessarily be the case, in order to simplify the calculations and allow for an overall understanding of the process. The inner terrestrial planets were assumed to be unformed at this time. Instead, there was a disk of planetesimals placed in the inner solar system. The exact parameters and characteristics of the disk were varied for different simulations. The total mass in the disk of planetesimals ranged from approximately 2.5 Earth masses to 4 Earth masses. The minimum mass solar nebula concept states that the current distribution of solid and gas, once restored to solar composition is the minimum mass the proto-planetary disk must have had (Weidenschilling 1977, Hayashi 1981). The total mass of the four inner planets is just under 2 Earth masses, so the protoplanetary disk must have had at least this much mass. The disks used in the simulations had higher masses to account for the possibility of scattering of planetesimals, or collisions with the sun or the giant planets. All of these mechanisms act to remove mass from the disk to decrease the amount of material available for making the terrestrial planets.

The various disks tested had the total mass distributed between varying numbers of planetesimals of equal mass. Generally, there were between 1000-2000 planetesimals making up the disk. The semimajor axes, eccentricities, inclinations, arguments of pericenter, longitude of ascending node, and mean anomaly for planetesimals were generated by random numbers, within some constraints. In this project specifically, not too much new code was added to the already existing program, but there were several modifications and debugging processes. Once the code had been modified slightly in order to best solve this problem, numerous simulations with varying disk and migration parameters were run and the results were adjusted and analyzed. The results were used to provide suggestions of further adjustments in order to attempt to get better results in terms of terrestrial planet formation.

Chapter 3

Results and Discussion

The initial set of simulations used a planetesimal disk containing 1500 bodies with a total of 4 Earth masses evenly distributed between them. This means each planetesimal had a mass of 8.01x10^-9 solar masses, or 0.00267 Earth masses. This disk is referred to as Disk 1 hereafter. The planetesimals in disk 1 had semimajor axes ranging out to 2.5 AU. The initial configuration of the disk in terms of semimajor axis and eccentricity can be seen in Figure 1.

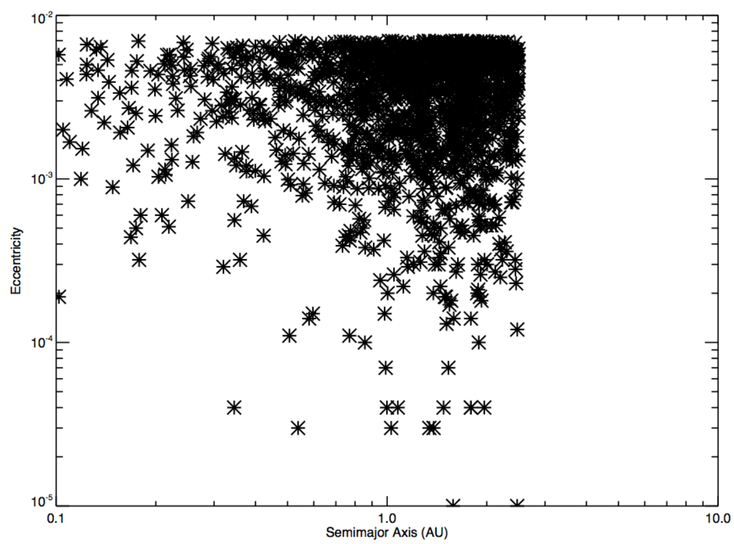


Figure 1: Initial configuration of planetesimals in Disk 1.

The first set of simulations used Disk 1 and initially had Jupiter located at a semimajor axis of 4.40 AU. The timescale for migration was set to 1 Myr and the ratio of the migration time to this timescale was 0.60 for Jupiter. This inward migration was carried out for 200,000 years, after which Jupiter was located at 3.055 AU. The 1500 planetesimals at this point had coalesced into 23 larger proto-planets. This simulation is referred to as Disk 1 inward A. Figure 2 shows the semimajor axes vs. time of the giant planets for this run.

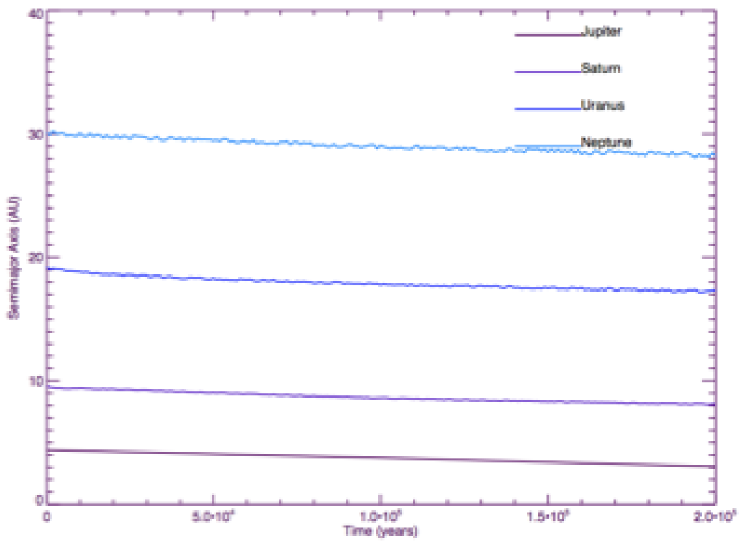


Figure 2: Semimajor Axis vs. Time for giant planets for Disk 1 Inward A.

Another inward migration was carried out using Disk 1 but different migration parameters and a different starting location for Jupiter. This simulation, called Disk 1 inward B, had Jupiter initially located at 3.70 AU. The timescale for migration was kept at 1 Myr but the ratio of the migration timescale of Jupiter to the fiducial timescale was 0.35. This inward migration was only carried out for 100,000 years, after which Jupiter was located at 3.14 AU. At this point, the 1500 planetesimals had coalesced to form 22 larger proto-planets. Figure 3 shows the evolution of the semimajor axes of the giant planets through time.

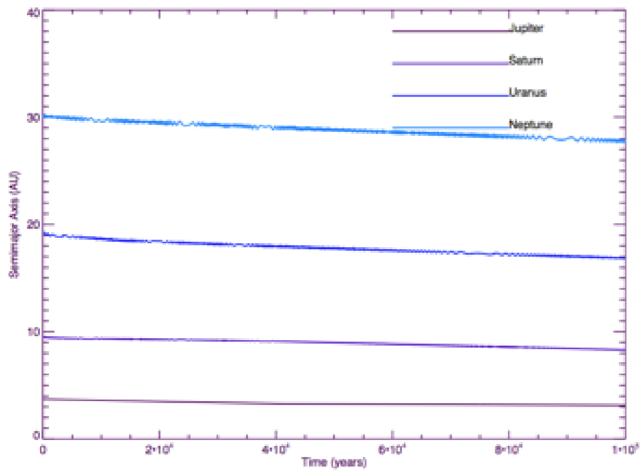


Figure 3: Semimajor axis vs. time for giant planets for Disk 1 Inward B.

In order to view how the planetesimals are interacting, plots of the mass vs. semimajor axis of the planetesimals at various points during the inward A and inward B migrations were created. These are shown in Figure 4, where inward A is shown in purple and inward B is shown in blue.

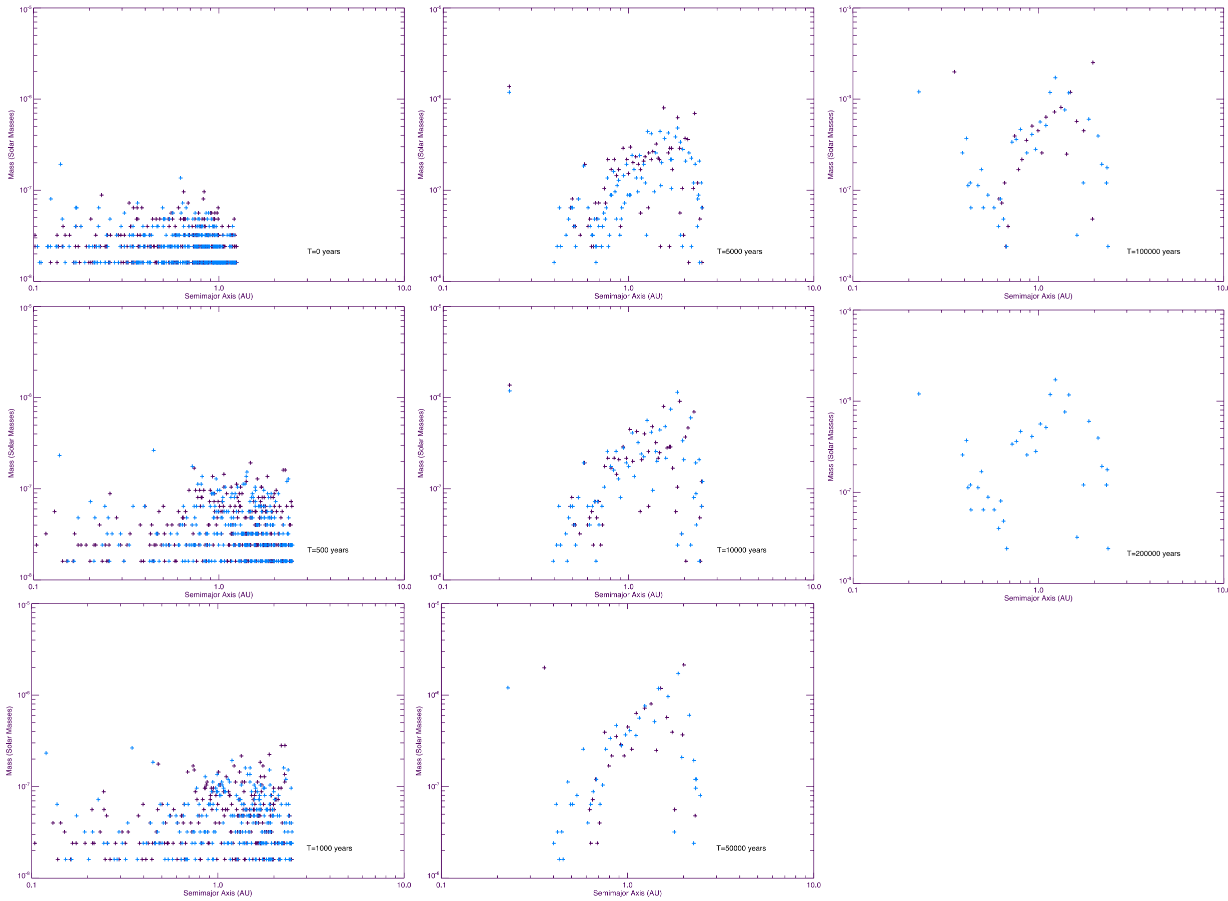
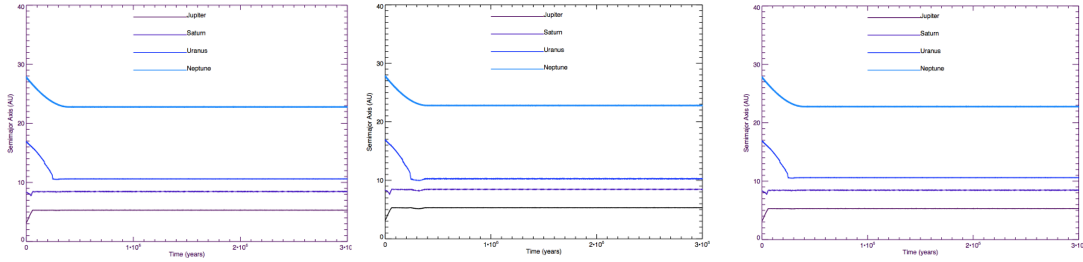


Figure 4: Mass vs. Semimajor Axis for planetesimals during the inward migrations at various points in time. The purple represent Inward A and the blue are Inward B.

For each of the two inward migrations, inward A and inward B, the migration of Jupiter was then reversed and sent back out to its current position. The rate of Jupiter's migration was varied in order to see how this affected the overall formation of terrestrial planets. Three different outward migration rates were tested using the same initial formation of the end of the inward migration. These simulations were run for 100 Myr to assess the long term stability of the resulting systems. At the end of the integrations, Jupiter was located at 5.3 AU, which is close to its present-day location of 5.2 AU.

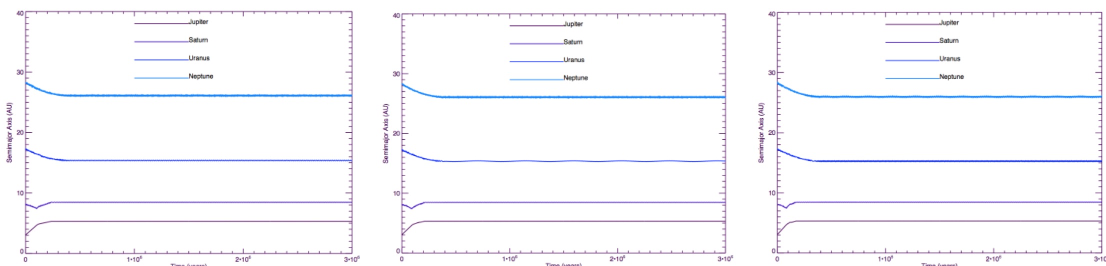
The three outward orbital evolutions of the giant planets stemming from Inward A are shown in Figure 5.



Outward 1 Outward 2 Outward 3

Figure 5: Semimajor Axis vs. Time for giant planets during outward migrations starting from the end of Inward A.

Similarly, the three orbital evolutions of the giant planets when Jupiter is migrated outward from the Inward B configuration are shown in Figure 6.



Outward 1 Outward 2 Outward 3

Figure 6: Semimajor Axis vs. Time for giant planets during outward migration starting from the end of Inward B.

During the course of the outward migrations, the 20 or so planetesimals left at the end of the inward A and inward B migrations continue to interact. By the end of the 100 Myr outward migration of Jupiter, the remaining planetesimals have coalesced to form 4-5 inner terrestrial planets. The evolution of the masses as a function of the semimajor axes of the planetesimals is shown at various points in time during the integration in Figure 7.

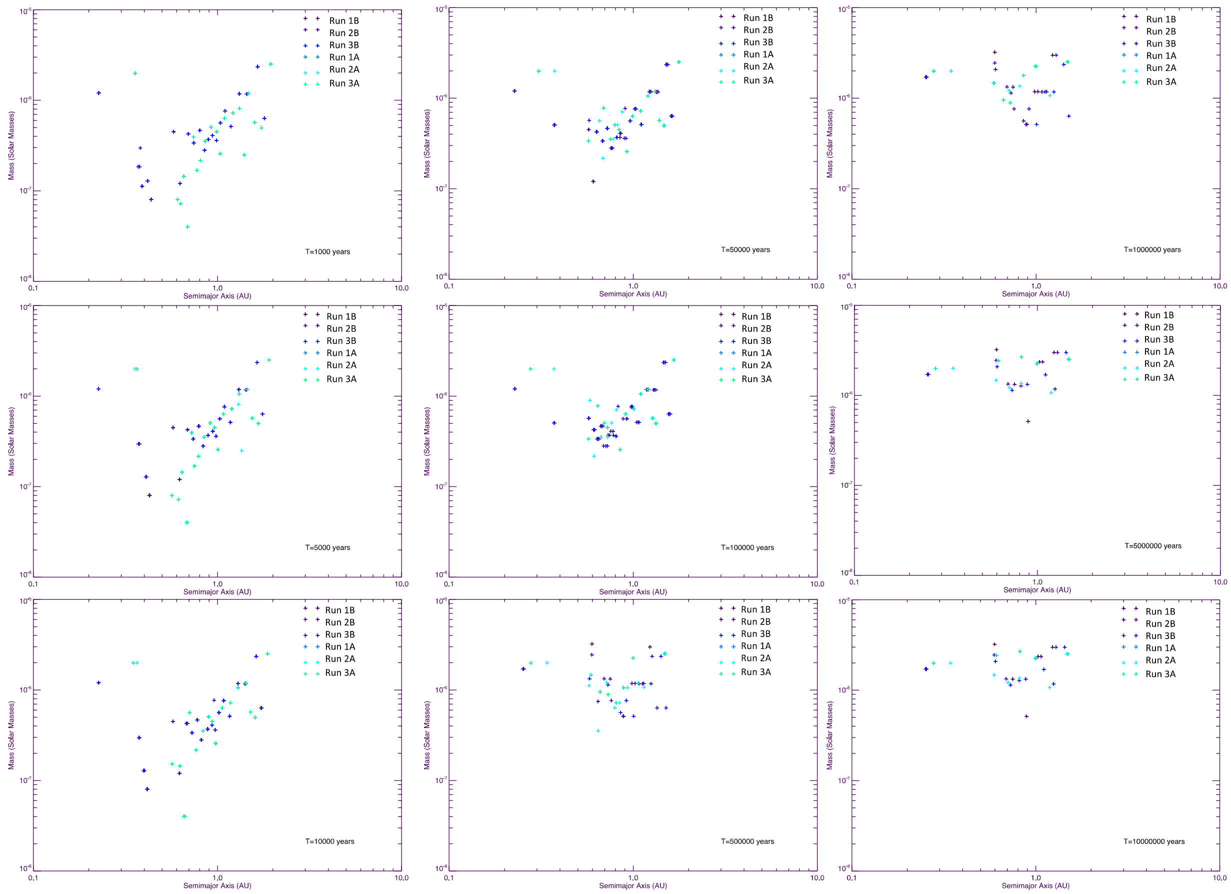


Figure 7: Mass vs. Semimajor Axis for planetesimals during the outward migrations of Disk 1.

In order to put these results in the context of terrestrial planet formation in the solar system, the final planetesimal masses and semimajor axes were plotted along with the planets making up the inner solar system today. Additionally, the overall configurations of the systems, including the giant planets, are plotted with all the bodies in the solar system today. These are shown in Figure 8A and 8B, respectively.

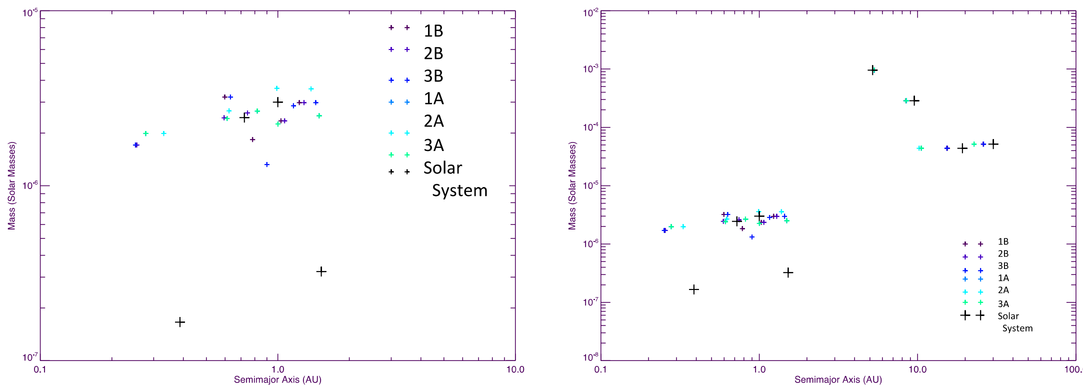


Figure 8: Mass vs. Semimajor Axis for simulations of Disk 1 and the solar system for comparison. Solar system bodies are shown in black while the six various simulation results are shown in colors according to the legends.

From Figure 8, we can see that the planets formed during the various runs have a range of semimajor axes that are consistent with the semimajor axes of the inner terrestrial planets in the solar system. Additionally, the Venus and Earth analogs in the simulations have masses quite similar to those of the actual planets. The formation models tested in these simulations are thus quite successful at reproducing Venus and Earth. The model is less successful at reproducing some of the characteristics of Mercury and Mars, particularly the mass. Most runs produce planets at the appropriate semimajor axes for Mercury and Mars, but these planets turn out to be about ten times more massive than their counterparts in the solar system. This could suggest that the disk of planetsimals being used in these simulations is not accurate for the protoplanetary disk that made the solar system. The initial disk had four Earth masses of material, which is larger than the total mass of the inner terrestrial planets. Since too much mass was converted into actual planets during the simulations, other disks with less mass were investigated next.

Disk 2 had 1350 planetesimals, each of mass 6.00 x 10^-9 solar masses, or 0.002 Earth masses. A number of different inward migrations were carried out with differing migration parameters and successively less bodies in the disk. Disk 2 Inward A had all 1350 planetesimals, while Disk 2 Inward B had 50 planetesimals from the outer region removed, leaving 1300 total, and finally, Disk 2 Inward C had 150 planetesimals from the outer region removed, leaving 1200 total. The masses of these disks for

Inward A, B, and C were 2.7, 2.6 and 2.4 Earth masses respectively. The initial configurations of these disks are shown in Figure 9.

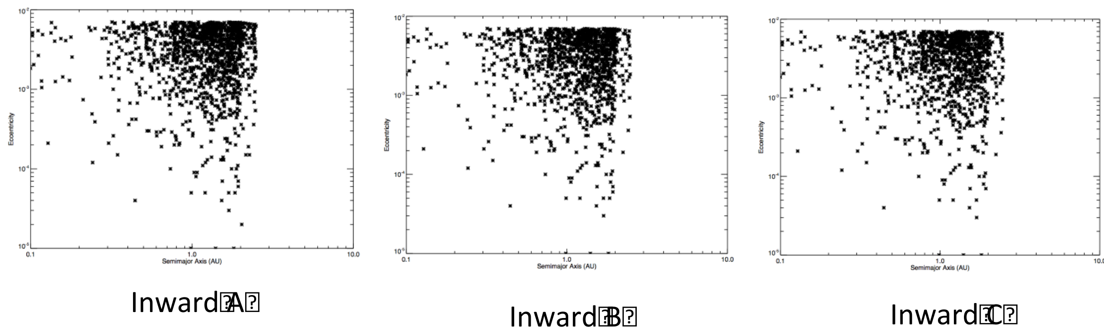


Figure 9: Intitial configuration of Disk 2 for inward runs a, b, and c. The plots show the distribution of planetesimals in eccentricity-semimajor axis space.

These three different disk configurations were used as initial conditions for inward migrations of Jupiter. In all three cases, Jupiter began at a distance of 3.7 AY. Inward A and Inward B both had a ratio of Jupiter's migration timescale to the fiducial timescale of 0.5. Inward C had a slightly faster migration of Jupiter, with a ratio of 0.4. Inward A was run for 300,000 years. After this time, the 1350 original planetesimals had formed 11 larger proto-planets and Jupiter was located at 2.20 AU. Inward B was integrated for 100,000 years, after which Jupiter was located at 3.28 AU and 24 planetesimals remained. Inward C was run for 100,000 years, leaving Jupiter at 2.78 AU and 32 planetesimals remaining. The semimajor axes vs. time for the giant planets in these three simulations are shown in Figure 10.

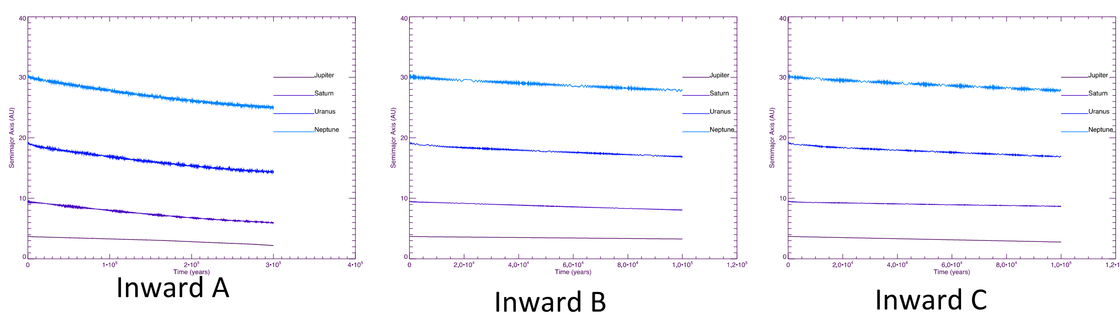
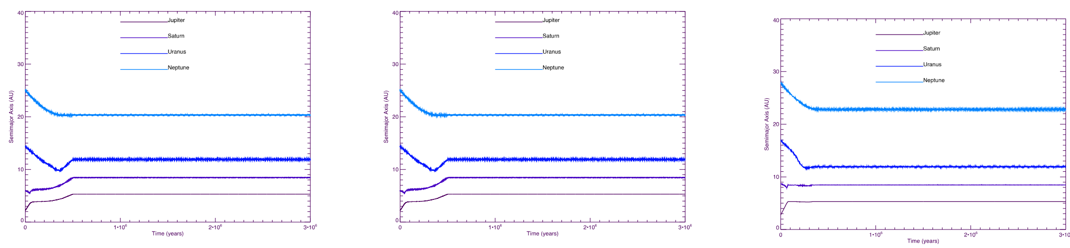


Figure 10: Semimajor Axis vs. Time for the three inward migrations using Disk 2.

Similarly to the runs for Disk 1, the migration of Jupiter was then reversed after the inward migrations. The outward extensions of run A and B were integrated for 100 Myr and four planets were generated in each case. Outward C was also integrated for 100 Myr, but five planets remained at the end of the integration. The orbital evolutions of the giant planets during the outward migration of Jupiter are shown in Figure 11.



Outward A   Outward B   Outward C

Figure 11: Semimajor Axis vs. Time for the three outward migrations for Disk 2.

The masses and semimajor axes of the terrestrial planets formed in these three simulations are shown in Figure 12A. Figure 12B shows the configuration of the inner and outer planets as well in these simulations in comparison to the solar system.

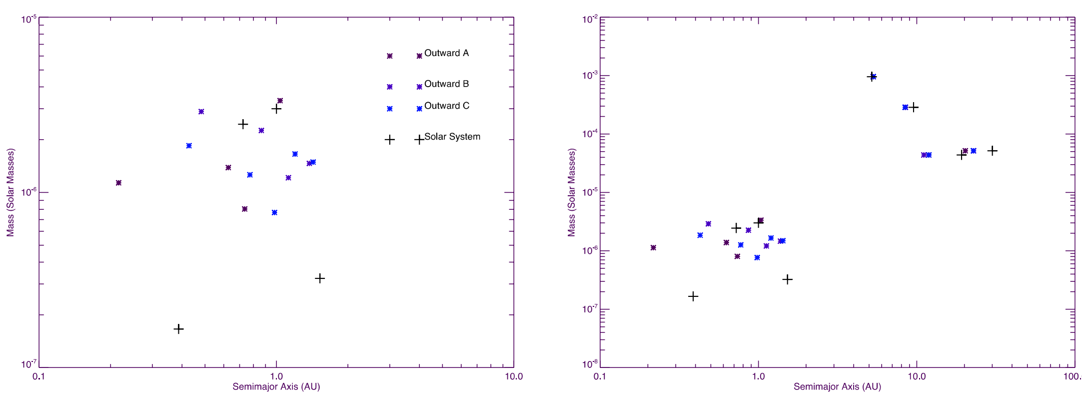


Figure 12: Results of terrestrial planet formation for the Disk 2 simulations. Panel A shows the inner planets in the solar system in black, as well as the planets formed in each run, in color. Panel B shows all the bodies in the systems, including the giants.

From Figure 12, one can see that the semimajor axes of the formed planets are overall consistent with those of the planets in the solar system today. The masses have a larger variation from the bodies in the solar system. In this set of simulations, Venus and Earth tend to be slightly less massive than their counterparts in the solar system, while Mercury and Mars tend to be slightly too massive. Thus, it seems as if decreasing the mass of the disk helped slightly with the problem of forming Mercury and Mars too massive, but not to such a large extent as to solve the problem entirely. The mass taken away from the disk appears to have come largely out of the masses of the forming Venus and Earth rather than Mercury and Mars.

In order to further investigate the effects of the disk characteristics on terrestrial planet formation, the disk was varied again. This time, for the first inward migration, Inward A, the disk was made of 1200 planetesimals, each of mass 8.01 x 10^-9 solar masses, or 0.0027 Earth masses. This is referred to as Disk 3. The total mass in the disk was 3.2 Earth masses. Inward B used the same disk but with 200 fewer planetesimals in the outer regions of the disk. The total disk mass in this case was 2.67 Earth masses. The distribution of planetesimals in semimajor axis and eccentricity in these disks is shown in Figure 13.

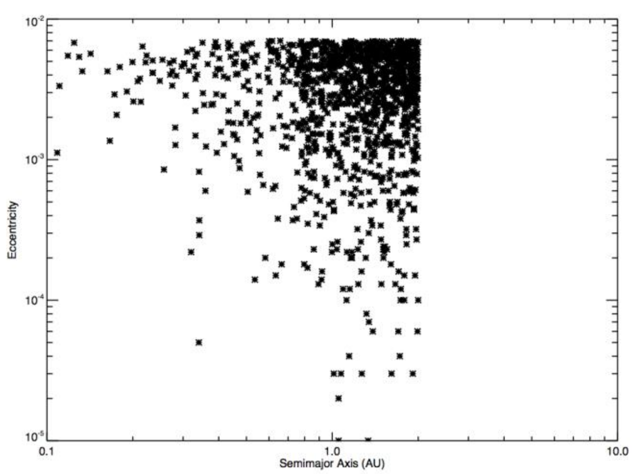
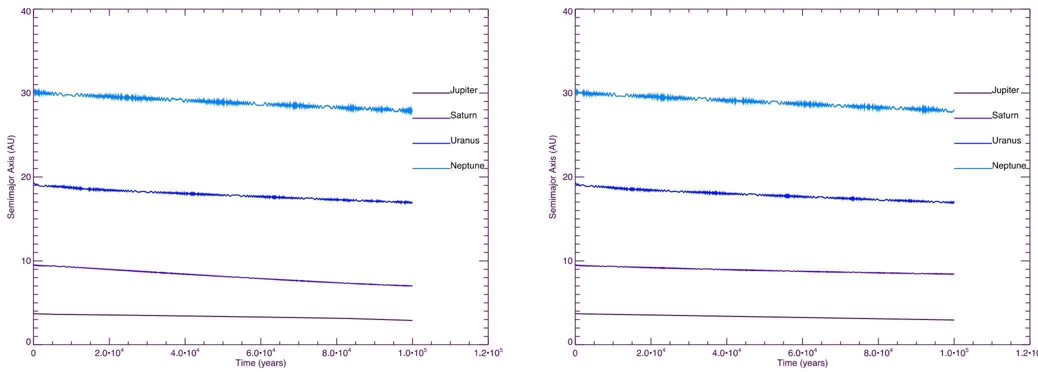


Figure 13: Eccentricity vs. Semimajor axis for planetesimals in Disk 3.

Jupiter was initially located at 3.7 AU in both inward migrations. Inward A was run for 100,000 years. At the end of this integration, Jupiter was located at 2.90 AU and 14 planetesimals remained.

Inward B was also run for 100,000 years, leaving Jupiter at 2.95 AU and 29 planetesimals. The orbital evolutions of the giant planets during these two inward migrations are shown in Figure 14.



Inward A

Inward B

Figure 14: Semimajor Axis vs. Time for the inward migrations of Disk 3.

Next, two different outward migration rates for Jupiter were used for each of the inward migrations. In Disk 3 Outward 1A, 5 terrestrial planets were formed upon the outward migration of Jupiter. This simulations was integrated for 100 Myr total. 5 terrestrial planets were also formed in the 100 Myr integration carried out in Outward 2A. The evolution of the giant planets in these two outward migrations are shown in Figure 15.

Figure 15: Semimajor Axis vs. Time for outward migrations with Disk 3.

Disk 3 Outward 1B formed 4 planets in the 100 Myr integration. The second outward migration, Outward 2B, formed 5 planets over the 100 Myr timescale. Figure 16 shows the orbital evolution of the giant planets during the outward migration for these two simulations.

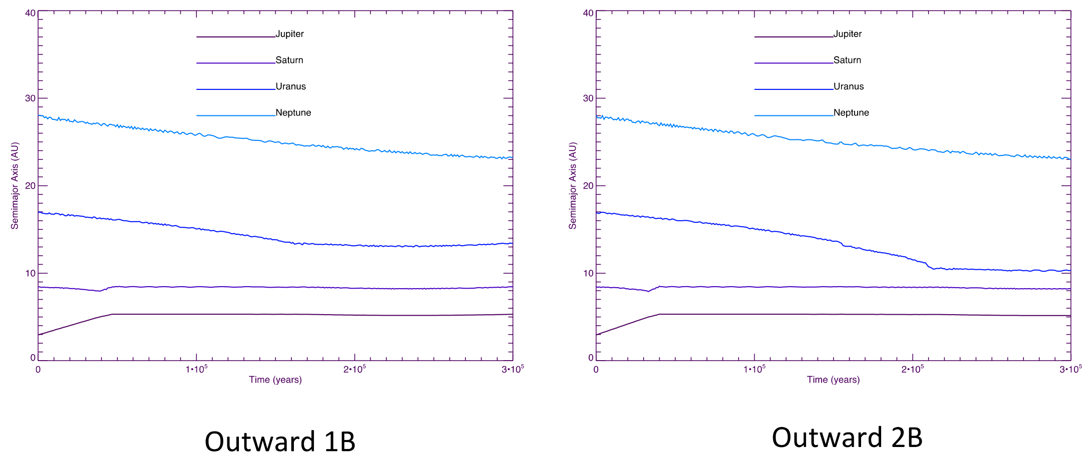


Figure 16: Semimajor Axis vs. Time for outward migrations that are continuations of Inward B for Disk 3.

The overall planet formation results for the four Disk 3 models is seen in Figure 17. Figure 17A shows the inner terrestrial planets formed in these simulations compared to the solar system inner bodies while Figure 17B compares all the bodies for the models and the solar system.

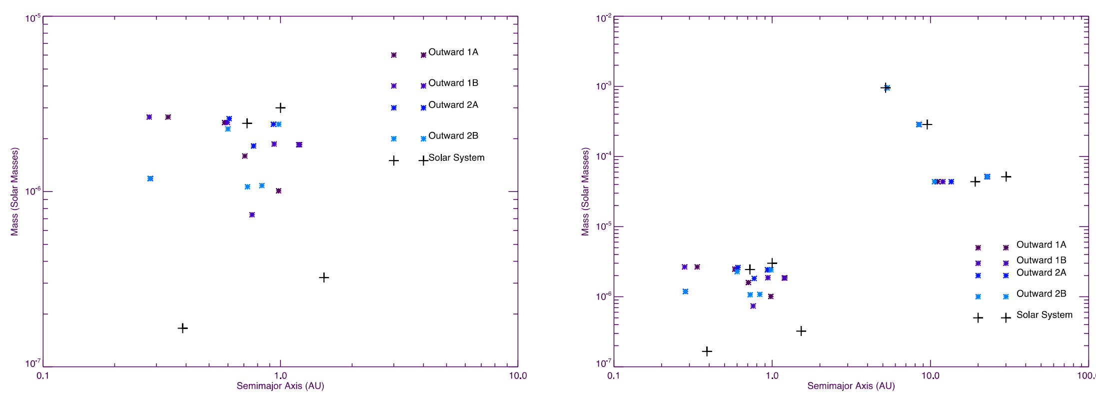


Figure 17: Mass vs. Semimajor Axis of bodies in the simulations and solar system. Panel A shows the inner terrestrial planets formed during the simulations while B shows all the bodies.

Another disk tested, Disk 4, had 1500 planetesimals of mass 4.20 x 10^-9 solar masses. The total mass in this disk was 2.10 Earth masses. The distribution of planetesimals is shown in Figure 18.

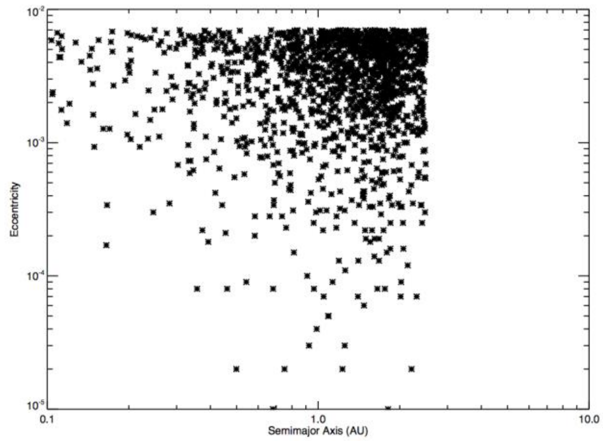


Figure 18: Eccentricity vs. Semimajor Axis for planetesimals in Disk 4.

Figure 19 shows the inward migration of Jupiter's semimajor axis as a function of time.

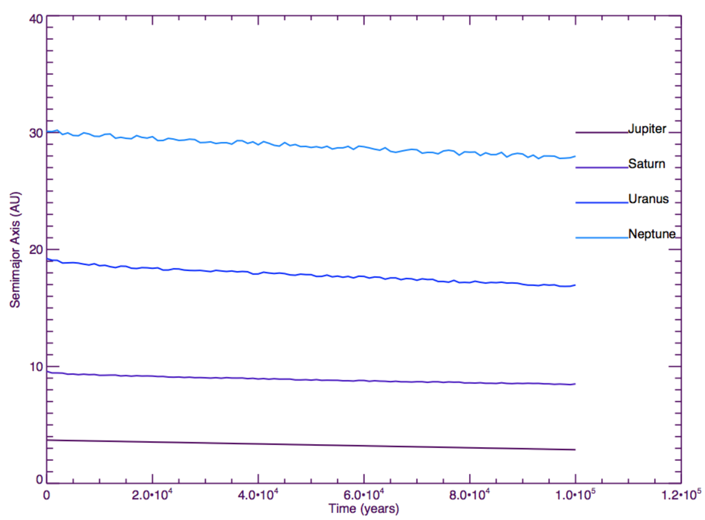


Figure 19: Inward migration semimajor axis vs. time for Disk 4.

After 100,000 years, the 1500 planetesimals had collided to form 33 larger proto-planets. Then, the migration of Jupiter was reversed and sent back outwards. After 100 Myr of integration, four terrestrial planets remained. The evolution of the outer planets during this integration is shown in Figure 20 while the results for planet formation are shown in Figure 21.

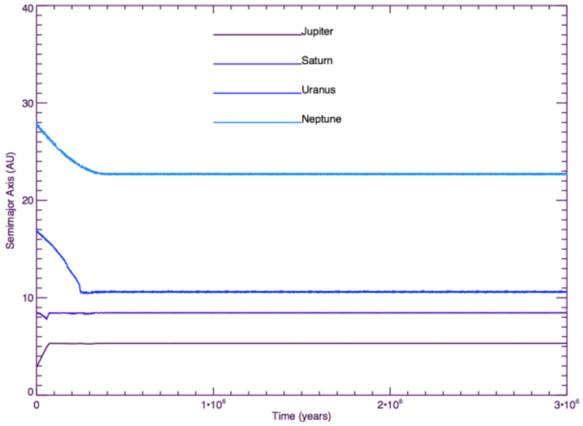


Figure 20: Outward migration of giant planets for Disk 4.

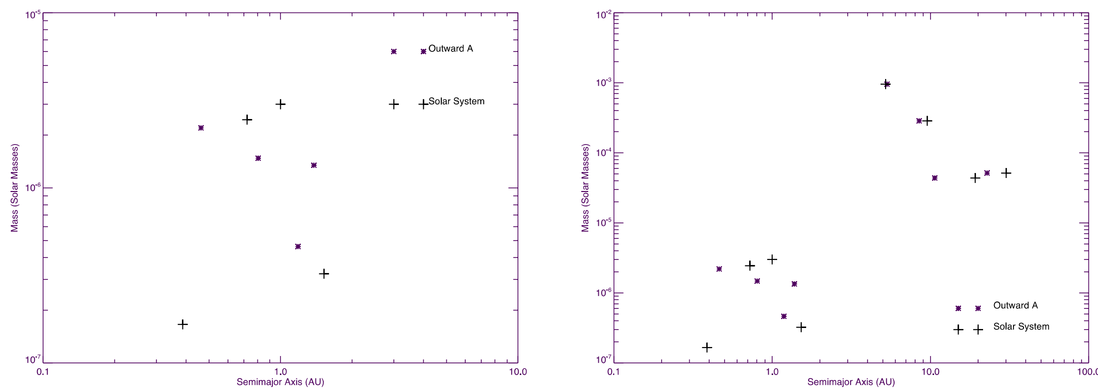


Figure 21: Overall configuration of the Disk 4 system after inward and subsequent outward migration of Jupiter.

This simulation showed that it is possible to make an inner planet with substantially less mass than the others. However, in this case, the smaller planet was the third one in the system, rather than the fourth like Mars is. Nevertheless, this simulation shows that some disk characteristics and migration parameters can indeed allow for a truncated mass of one of the forming planets.

Disk 5 consisted of 900 planetesimals of mass 8.01x10^-9 solar masses. This gives a total disk mass of 2.4 Earth masses. The distribution of planetesimals in disk 5 is shown in Figure 22.

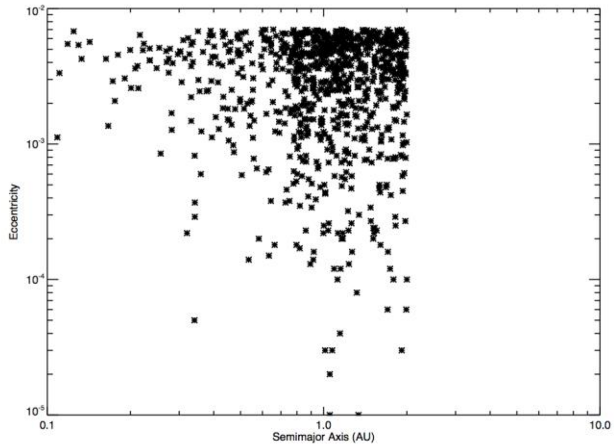


Figure 22: Eccentricity and semimajor axis of planetesimals in Disk 5.

Jupiter had an initial semimajor axis of 3.70 AU. After 100,000 years of inward migration, it ended at 2.98 AU. The planetesimals had formed 25 larger protoplanets at this time. The orbital evolution of the gas giants during this inward migration is shown in Figure 23.

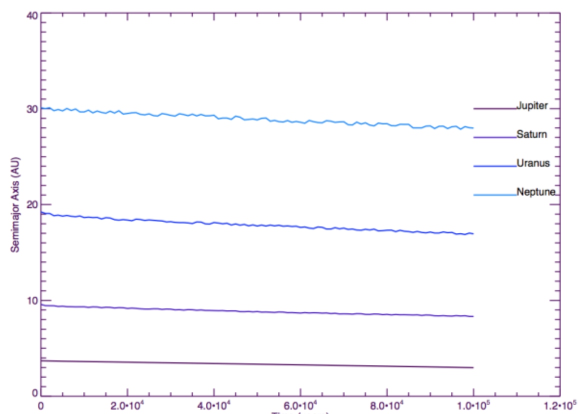


Figure 23: Inward migration of Jupiter for Disk 5.

The final configuration of the planets in the simulation is displayed in Figure 24, along with the solar system configuration so as to aid comparison.

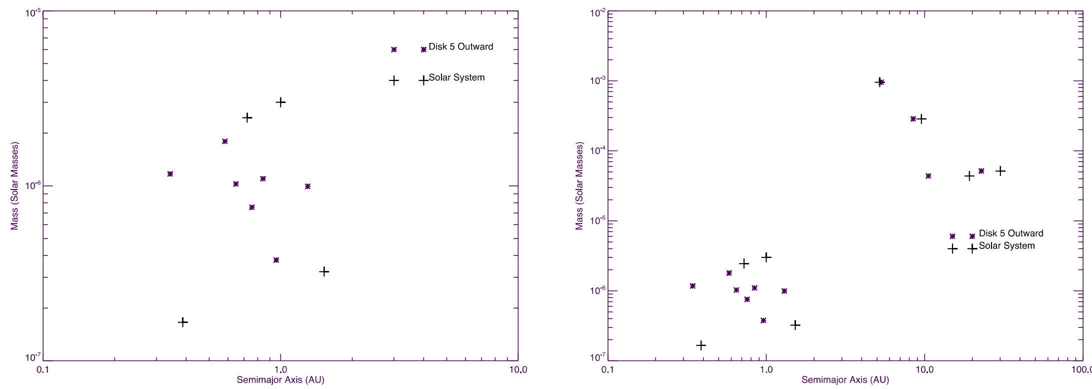


Figure 24: Mass vs. Semimajor axis for the planets formed from Disk 5. Panel A zooms in on the terrestrial planets while panel B shows all the planets.

This simulation had seven planets remaining at the end of the outward migration. The lack of coalescence into fewer planets could potentially be due to the fact that fewer planetesimals were initially used, but they had larger masses in order to still have a reasonable mass in the disk.

Disk 6 contained 1500 planetesimals of mass 6.00 x 10^-9 solar masses. The planetesimals extended out to 2.5 AU. The total mass in Disk 6 Inward A was 3 Earth masses. Inward B had 100 planetesimals removed from the outer regions, leaving a total disk mass of 2.8 Earth masses. Inward C had some additional planetesimals placed in the middle of the disk for a total disk mass of 3.15 Earth masses. The distribution of planetesimals in semimajor axis-eccentricity space is shown in Figure 25.

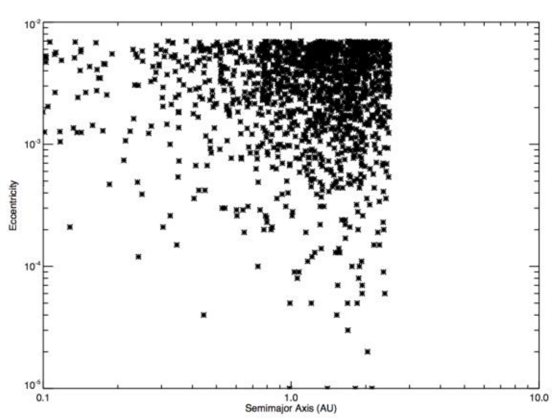


Figure 25: Eccentricity vs. Semimajor Axis for planetesimals making up Disk 6.

Jupiter was initially located at 3.7 AU in all three of the inward migrations. Inward A was run for 100,000 years. At this point, Jupiter was located at 3.02 AU and 24 planetesimals remained. Inward B had a slower migration rate for Jupiter. After 100,000 years, Jupiter was located at 3.09 AU and 19 planetesimals remained. Inward C had a slightly slower migration rate of Jupiter, leaving it at 3.12 AU after 100,000 years. 24 planetesimals were left at this point. Figure 26 shows the inward migration of Jupiter for the three scenarios.

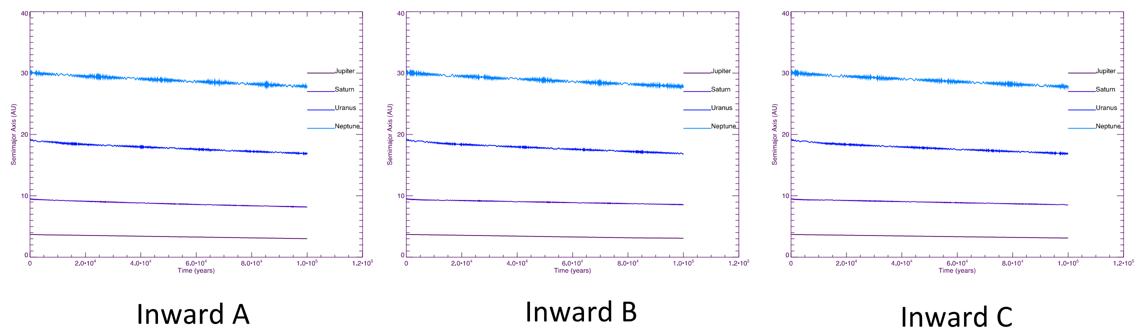


Figure 26: The three inward migrations using Disk 6 but different migration parameters.

Next, the inward migrations of Jupiter were reversed for each of the runs. Outward A had the slowest outward migration rate for Jupiter while Outward B had the fastest. The profiles for gas giant orbital evolution during the outward migration are shown in Figure 27.

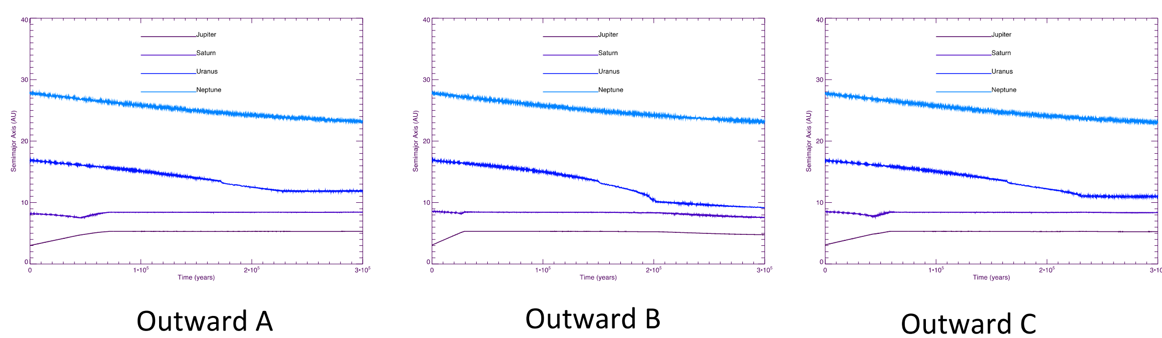


Figure 27: Evolution of semimajor axis over time for giant planets during the three different outward migrations using Disk 6.

Figure 28 shows the planet formation results for these three simulations. Again, Mars seems to come out slightly too massive, though only by a factor of around 3-4 in some cases. Simulation A was seen to have too much mass in the planets formed, which is why some of the planetesimals were removed from the disk for the B simulation. However, after seeing the planet formation results from the B set, it appeared as if now Venus and Earth were not massive enough and Mars remained too massive. Thus, additional planetesimals were placed in the middle of the disk in order to try to boost the overall masses of Venus and Earth. This was moderately successful in Run C, but Mars still turned out to be too massive. Nevertheless, this set of simulations had an interesting set of implications for planet formation of the solar system.

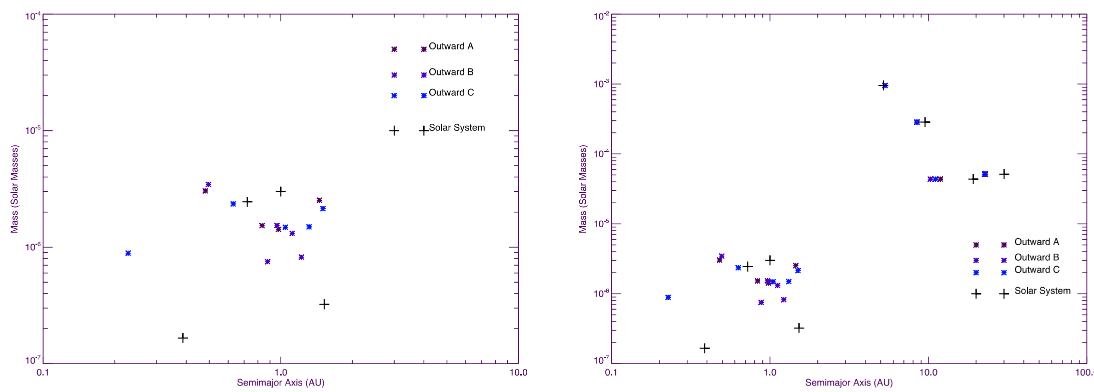


Figure 28: Mass vs. Semimajor Axis for the planets formed during the Disk 6 simulations, plotted with that of the solar system bodies.

Disk 7 had 1350 planetesimals with mass 6.00x10^-9 solar masses. The total mass of the disk was 2.7 Earth masses. The planetesimals extended out to 2.5 AU. The distribution of planetesimals in Disk 7 is shown in Figure 29.

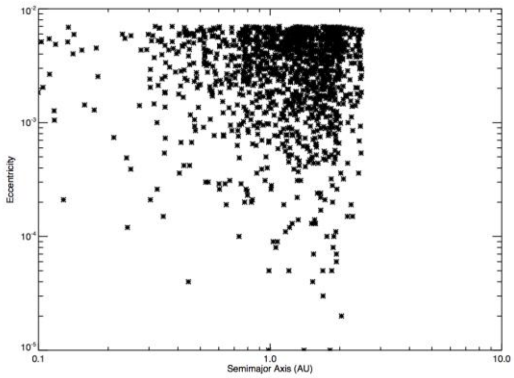
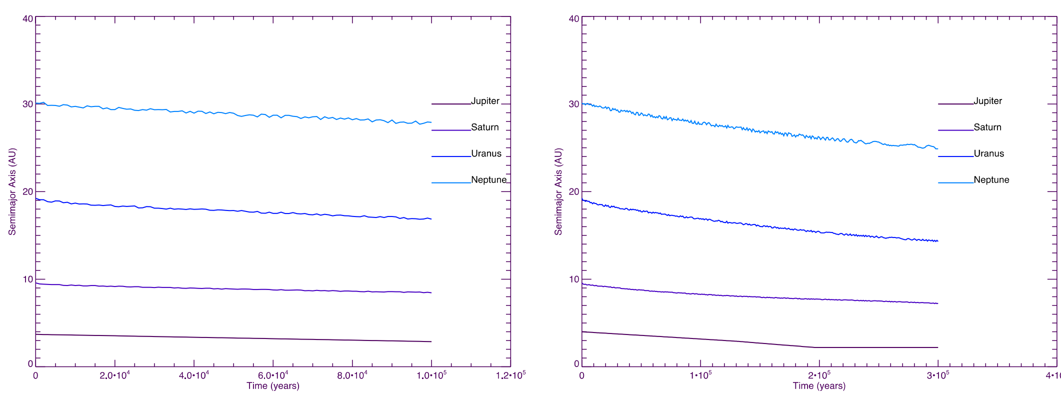


Figure 29: Eccentricity vs. Semimajor Axis for Disk 7.

Two different inward simulations were run from Disk 7. In Inward A, Jupiter started at 3.7 AU, but Inward B had a starting position of 4.0 AU for Jupiter. Inward A was run for 100,000 years while Inward B was run for 300,000 years. At the end of Inward A, 24 planetesimals remained. Inward B only had 9 planetesimals at the end of the integration. The evolution of the semimajor axes of the giant planets as a function of time can be seen in Figure 30.



Inward A　　　　Inward B

Figure 30: Semimajor Axis vs. Time for giant planets during the inward migrations using Disk 7.

Next, the migration of Jupiter was reversed. Jupiter was sent back outwards from the A simulation results. This simulation was integrated for 10 Myr and 4 inner planets remained. Jupiter was migrated outwards from the Simulation B results with three different rates. All three were integrated for

10 Myr. Four planets remained at the end of 1B, 2B, and 3B. The evolution of the semimajor axes of the giant planets as a function of time can be seen in Figure 31.

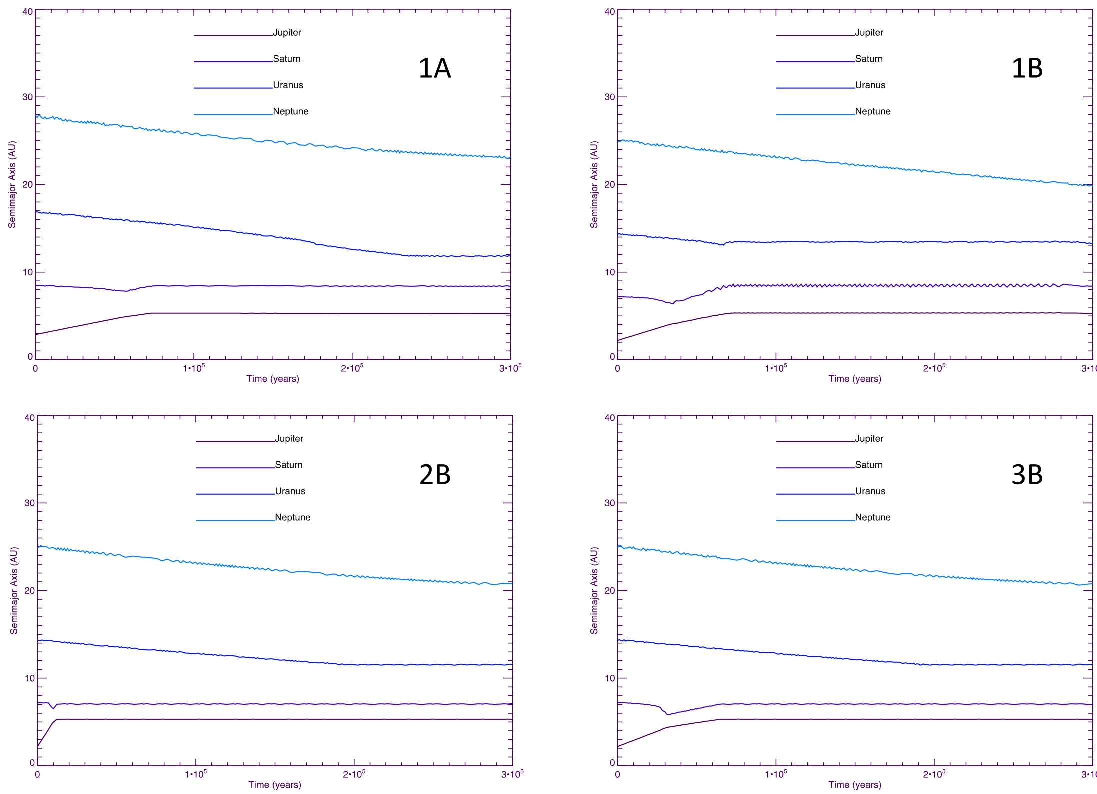


Figure 31: Semimajor Axis vs. Time for outward migrations in Disk 7.

Run 1B had the slowest outward migration of Jupiter, while 2B had the quickest outward migration rate. The overall planet formation from these four simulations as compared to the solar system can be seen in Figure 32.

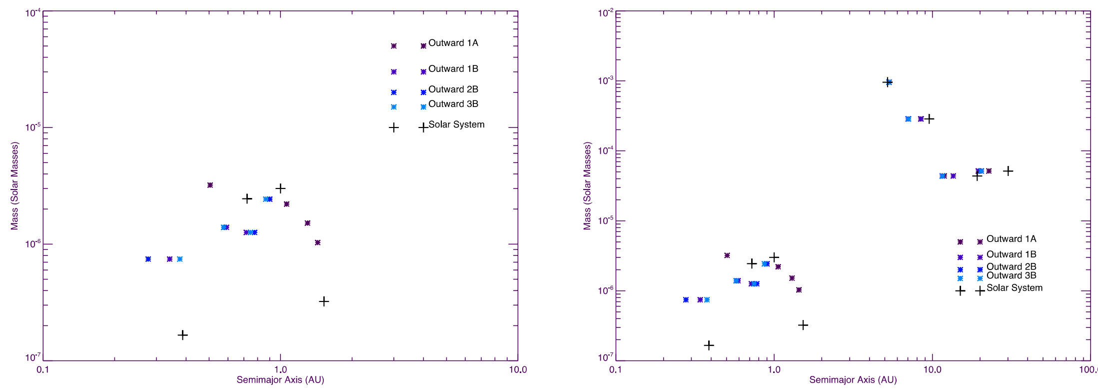


Figure 32: Mass vs. Semimajor Axis of planets formed during Disk 7 simulations plotted with solar system bodies. Panel A shows a close up of the terrestrial planet zone while Panel B shows the overall architecture of the systems.

In this set of simulations, Run 1A shows a promising decrease in mass of formed planets at larger semimajor axes. It thus seems reasonable that similar migration parameters used in this simulation could potentially form an object closer to the mass of Mars at the appropriate distance. The B simulations tend not to form any planets out beyond 1 AU. These systems tend to be more tightly packed, having four planets within 1 AU. While this doesn’t necessarily help to solve the small Mars problem, it is intriguing in itself.

Going back to Disk 1, further analysis was done on how the planetesimals evolve during the migration of the giant planets. In particular the eccentricity and inclination of planetesimals as a function of semimajor axis were measured. Figure 33 shows the eccentricity vs. semimajor axis for the Disk 1 planetesimals during the two different inward migrations at various times.

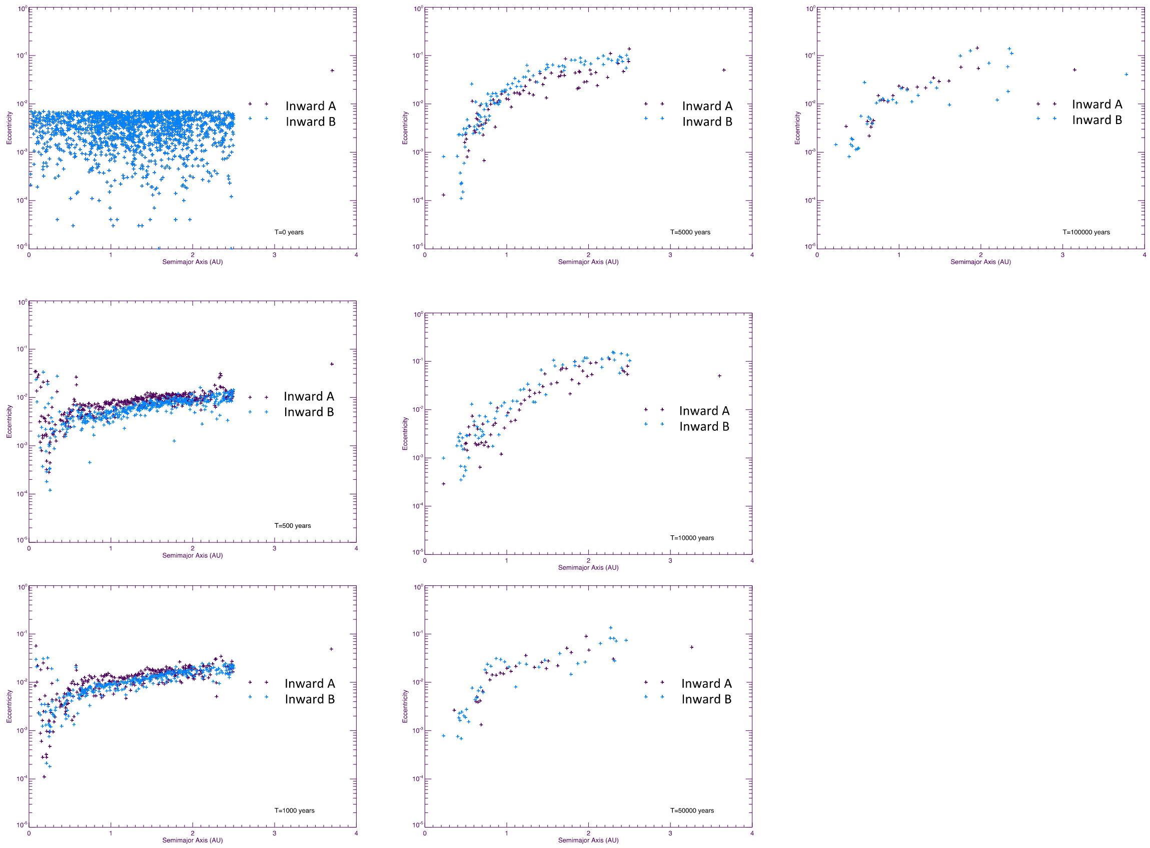


Figure 33: Eccentricity vs. Semimajor Axis for planetesimals at various times during the inward migrations for Disk 1.

Figure 34 shows the same parameters, but for the outward migrations for Disk 1.

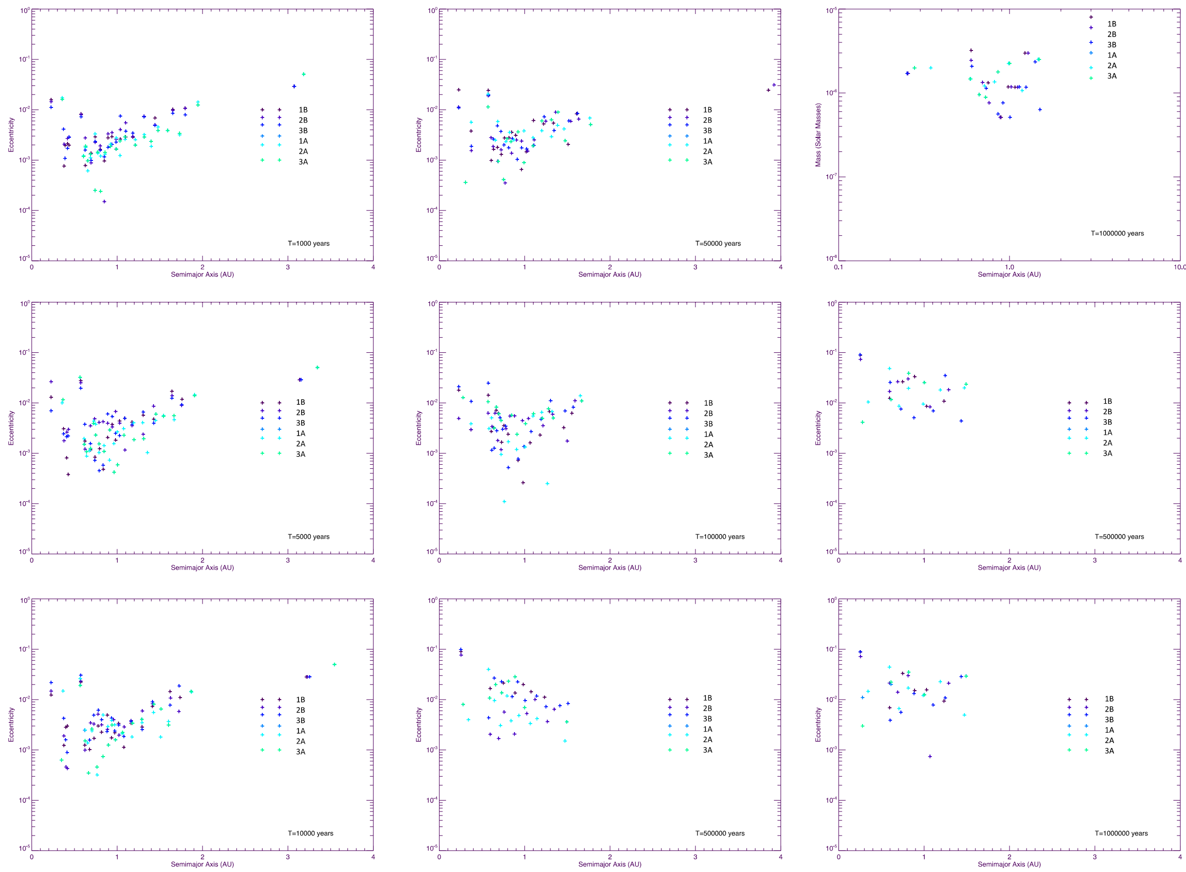


Figure 34: Eccentricity vs. Semimajor Axis for planetesimals at various times during the outward migrations in Disk 1.

Figure 35 shows the inclination and semimajor axis of planetesimals during the outward migrations for Disk 1.

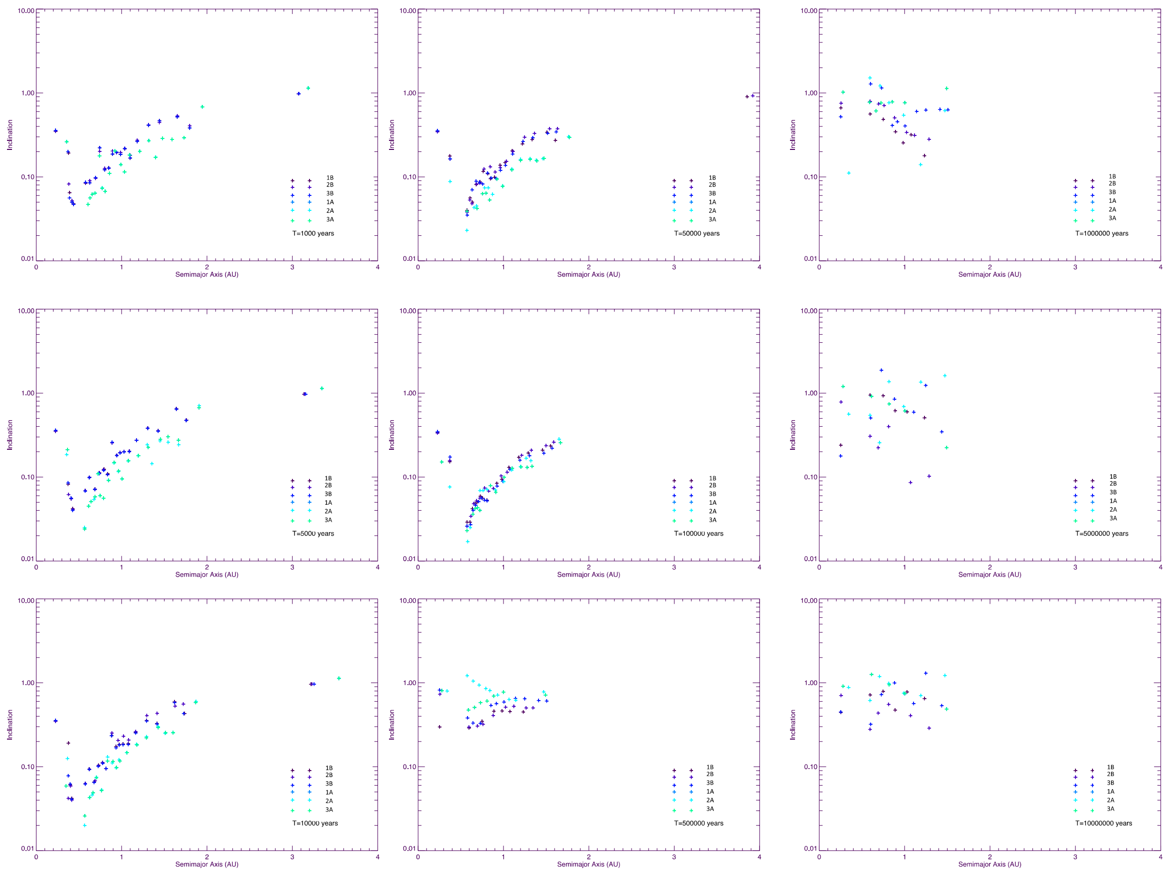


Figure 35: Inclination vs. Semimajor Axis of planetesimals during the outward migrations of Disk 1.

From these figures, it can be seen that over time, the planetesimals located at smaller semimajor axes will have their eccentricities damped. Alternatively, those at larger orbital radii tend to have their eccentricities excited, especially during the inward migrations. During the outward migrations, the eccentricities tend to even out for planetesimals located at both large and small semimajor axes. The inclinations also seem to follow a similar trend. The bodies at smaller semimajor axes initially have lower inclinations while those are larger semimajor axes have increased inclinations. Past a certain point, around 500,000 years in this case, the inclinations seem to become fairly constant. A similar pattern was seen for the other migrations, so this analysis was only done for Disk 1.

Chapter 4

Conclusions

Overall, seven different disks were studied with varying total masses and numbers of planetesimals. Each disk had a number of different migration parameters of Jupiter tested, both in inward and outward migrations. Most outward migrations were carried out for 100 Myr to determine if the systems produced were stable. In all cases, the systems were found to be stable of 100 Myr timescales. Through the two-stage inward-then-outward migration of Jupiter, a disk of typically >1000 planetesimals was seen to interact. The planetesimals collided together to form usually 20 or so larger proto-planets at the end of the inward migration. Upon reversal of the migration, Jupiter was sent back to its current day position and the remaining planetesimals coalesced to form 4-5 inner terrestrial planets. A wide range of masses and semimajor axes were found for these forming planets. Generally, Mercury and Mars turned out to be more massive than their actual counterparts in the solar system today, but some simulations showed better results for their masses than others. A number of interesting characteristics and trends could be found in these simulations. For example, some of the most promising planet formation occurred with a moderate migration rate of Jupiter. When Jupiter's migration rate was large, occasionally fewer planets would form that were found to be stable. When the migration rate of Jupiter was too small, it occasionally would not make it back out to its present-day orbit. Thus, many times, the moderate migrations rates for Jupiter were the most promising for terrestrial planet formation.

When comparing these results to the Grand Tack Theory, clearly the mass of Mars continues to be an issue. The Grand Tack Theory is quite successful at reproducing the Earth to Mars mass ratio, which was still an issue in many of these results. Nevertheless, these results show that there could potentially be a solution to the small Mars problem that doesn't require a tacking point of 1.5 AU for

Jupiter. None of these simulations had Jupiter moving in to 1.5 AU. The closest it typically got was around 2.2 AU. We have thus investigated planet formation in scenarios where the migration of Jupiter is not on as large scales as suggested in the Grand Tack Theory. Essentially, we find that the tacking point of Jupiter need not be constrained to occur at 1.5 AU in order to form systems of planets resembling the solar system. With a tacking point not at 1.5 AU, we did not completely solve the small Mars problem. In these simulations, Mars still turned out to be too massive, but the mass was down by a factor of a few occasionally. This suggests that with further tests of parameter space and disk characteristics, it might be possible to fully recover the small mass of Mars without constraining the tacking point to 1.5 AU. With more tests of different disks and migration parameters, it seems likely that we could solve the small Mars problem without this constraint of having Jupiter migrate in as far as 1.5 AU. The initial distribution of planetesimals in the protoplanetary disk is unconstrained at this point, so testing various different numbers and distributions of planetesimals and their effects on terrestrial planet formation is a viable option. There is an extraordinary amount of parameter space to explore in this problem as well as numerous variations on initial disk architecture that could potentially give rise to systems of planets resembling our solar system. Our results have shown potential for solving the small Mars problem without constraining the tacking point, so it is possible that there is a clear solution to the small Mars problem without this constraint on the tacking point. Nevertheless, the Grand Tack Theory is remarkably successful in predicting certain aspects of the solar system. We use this work to suggest that the Grand Tack Theory, while successful in reproducing some observations, may not necessarily be the only way to form the solar system.

Future work in this area is essential. These results have shown potential for an alternative to the Grand Tack Theory for forming the solar system, but a clearly successful solution to the small Mars problem has yet to be found. Further exploration of various migration parameters and disk architecture would help to constrain which scenarios could plausibly form the terrestrial planets in our solar system. Additionally, further work could include adding planetesimals in the outer solar system and as trans-

Neptunian objects. These planetesimals are likely to be carriers of many volatile compounds and analyzing the amount of these outer planetesimals that collide with the inner planets would give an estimate as to volatile delivery through impacts. This could go a long way towards explaining how the Earth received many of its volatile compounds and could additionally have implications for prebiotics and the origin of life on Earth.

Appendix A

Migration Subroutine

Below is the subroutine MFO_USER, which carries out the forced migration of Jupiter. This code is not part of the public download and was written by Avi Mandell. It has not been published elsewhere.

```
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c
c      MFO_USER.FOR    (ErikSoft   2 March 2001)
c
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c
c Author: Avi M. Mandell
c
c Applies drag forces to giant planet to simulate migration, and gas drag
c to small objects.
c
c If using with the symplectic algorithm MAL_MVS, the force should be
c small compared with the force from the central object.
c If using with the conservative Bulirsch-Stoer algorithm MAL_BS2, the
c force should not be a function of the velocities.
c
c N.B. All coordinates and velocities must be with respect to central body
c ===
c------------------------------------------------------------------------------
c
      subroutine mfo_user (time,step,nbod,nbig,m,rphys,x,v,ausr)
c
      implicit none
      include 'mercury.inc'
      include 'user.inc'
c
c Input/Output
```

```
      integer nbod, nbig
      real*8 time,step,m(nbod),x(3,nbod),v(3,nbod)
      real*8 ausr(3,nbod),rphys(NMAX)
      real*8 dfrac
c
c Local
      integer j,k,nT2
c
      real*8 R,RJUP,SMA,SMAT2(nmig+1),Rhill(nmig+1)

      real*8 tT2,maxstart,aT2(3,nbod)

      real*8 rho0,z0,rhogas,mdrag,rdrag,kdrag,ndrag,rhodrag,rhoperc

      real*8 vk,nu,vgas,xth,vrel(3,nbod),adrag(3,nbod)

      real*8 SDgas,Om,ecc,inc,sinc,tT1e,tT1i,twave

      real*8 vth,vrad,arad,aT1(3,nbod)

      real*8 tmp1,tmp2,tmp3,tmp4
c------------------------------------------------------------------------------
c
c  SETUP
c
      do j = 2,nbod
        aT2(1,j) = 0.0d0
        aT2(2,j) = 0.0d0
        aT2(3,j) = 0.0d0
        adrag(1,j) = 0.0d0
        adrag(2,j) = 0.0d0
        adrag(3,j) = 0.0d0
        aT1(1,j) = 0.0d0
        aT1(2,j) = 0.0d0
        aT1(3,j) = 0.0d0
      enddo
c
c  GIANT PLANET MIGRATION DRAG
c
      if (tfid.eq.0.d0) then
          nT2 = 0
          goto 200
      else
          nT2 = nmig
      endif
```

```
      maxstart = startfact(1)
      if (nT2.gt.1) then
         do j = 2,nT2
            maxstart = max(maxstart,startfact(j))
         enddo
      endif

      if (aint(time).ge.aint(tfid*(1.0d0+maxstart))) goto 200

      do j = 2,2+nT2
        if (j.gt.1+nT2) goto 199
        if (time.lt.startfact(j-1)*tfid) goto 199
        if (time.ge.(1.0d0+startfact(j-1))*tfid) goto 199

c
c  SNR: make comparison with final semimajor axis, otherwise damping
c      happens preferentially at apoastron and get increase in e
c
        call mco_x2a(m(1)+m(j),x(1,j),x(2,j),x(3,j),v(1,j),v(2,j),
     %         v(3,j),SMA,R,tmp1)
        if (SMA.le.stopdist) goto 199
c         R = sqrt(x(1,j)**2 + x(2,j)**2 + x(3,j)**2)
c         if (R.le.stopdist) goto 199

        if (aint(time).lt.aint(startfact(j-1)*tfid +
     &    tfid*switch(j-1)*ratefact(j-1))) then
          tT2 = ratefact(j-1)*tfid*(1.0d0+startfact(j-1))
        else
          tT2 = ratefact(j-1)*tfid*(1.0d0+startfact(j-1))*
     &        ((1.0d0-switch(j-1))/
     &         (1.0d0-((time-(startfact(j-1)*tfid)+0.0001)/tfid)))
     &             **(gamma(j-1))
        endif

        aT2(1,j) = -0.5d0*v(1,j)/(tT2-time+0.0001)
        aT2(2,j) = -0.5d0*v(2,j)/(tT2-time+0.0001)
        aT2(3,j) = -0.5d0*v(3,j)/(tT2-time+0.0001)

 199    continue
      enddo
 200  continue
c
c------------------------------------------------------------------------
c
c  SMALL OBJECT GAS DRAG
c
```

```
      if (rhoconst.eq.0.0d0) goto 300
      if ((gastime.ne.0.0d0).and.(aint(time).ge.aint(gastime))) goto 300

      if (gastime.gt.0.0d0) then
        rhoperc = (gastime - time)/gastime
      else
        rhoperc = 1.0d0
      endif

      if (nT2.gt.0) call mce_hill (nT2+1,m,x,v,Rhill,SMAT2)

      RJUP = sqrt(x(1,2)**2 + x(2,2)**2)

c
c BIG LOOP OVER ALL BODIES FOR AERODYNAMIC AND TYPE 1 DRAG STARTS
HERE
c             (No drag for migrating giant)
c
      do j = 2+nT2,nbod
        if (m(j).eq.0.0d0) goto 250
        dfrac = 1.0

        call mco_x2a(m(1)+m(j),x(1,j),x(2,j),x(3,j),v(1,j),v(2,j),
     %        v(3,j),SMA,R,tmp1)
        R = sqrt(x(1,j)**2 + x(2,j)**2) !AU
c
c  No drag in magnetospheric cavity
c
        if (R.lt.magcav) goto 250
c
c  IF giant planet is also present then 1) no drag within Hill sphere,
c        2) decreased drag interior (~dfrac) to giant planet (RJUP)
c
        if (nT2.gt.0) then
c          do k = 2,nT2+1
c            if ((R.gt.(SMAT2(k)-Rhill(k))).and.
c    %          (R.lt.(SMAT2(k)+Rhill(k)))) goto 250
c          enddo
          if (abs(R-RJUP).le.Rhill(2)) goto 250
          if (R.le.RJUP) then
            dfrac = innerdrag
          endif
        endif

        rho0 = rhoconst*rhoperc*((R)**(-rhoexp))          !g/cm^3
        z0 = zconst*((R)**(zexp))                  !AU
```

```
      rhogas = rho0*exp((-1.0d0*(x(3,j)**2))/(z0**2))          !g/cm^3

      if (j.le.nbig) then
        mdrag = (m(j)*MSUN)/K2
      else
        mdrag = meff
      endif

      ndrag = (m(j)*MSUN)/(K2*mdrag)
      rdrag = (rphys(j)*AU)/(ndrag**(1.0/3.0))
      rhodrag = (3.0*mdrag)/(4.0*PI*(rdrag**3))
      kdrag = dfrac*(AU*3.0d0*rhogas*cdrag)/(8.0d0*rhodrag*rdrag)  !1/AU

      vk = ((m(1)*R*R)**(1./2.)) /
   %       (((R**2)+(x(3,j)**2))**(3./4.))                     !AU/day
      nu = (PI/16.0d0)*(rhoexp-1.0d0)*((z0/R)**2)
      vgas = vk*(1.0d0 - nu)                                  !AU/day

      xth = atan(x(2,j)/x(1,j))
      if (x(1,j).lt.0.0d0) then
        xth = PI + xth
      else
        if (x(2,j).lt.0.0d0) xth = (2.0d0*PI) + xth
      endif

      vrel(1,j) = v(1,j) - (-1.0d0*vgas*sin(xth))
      vrel(2,j) = v(2,j) - (vgas*cos(xth))
      vrel(3,j) = v(3,j)

      do k = 1,3
        adrag(k,j)=-1.0d0*kdrag*sqrt(vrel(k,j)**2)*vrel(k,j) !AU/day^2

        if (adrag(k,j)/(-1.0d0*vrel(k,j)/step).gt.1.0d0)
   %       adrag(k,j) = -1.0d0*vrel(k,j)/step
      enddo

c
c
c Type I migration: Damping from Cresswell 08
c
      if ((j.gt.nbig).or.(j.lt.nT2+1).or.
   %   (econ.eq.0.0d0).or.(icon.eq.0.0d0)) goto 250

      R = sqrt(x(1,j)**2 + x(2,j)**2)
      SDgas = dfrac*sqrt(PI)*rhoconst*rhoperc*zconst*AU* !sol.masses/AU^2
   %      (R**(zexp-rhoexp))*(AU*AU/MSUN)
```

```
          Om = 1.d0/(2.*PI*sqrt((SMA**(3.))/m(1))) !days
        call mco_x2el(m(j)+m(1),x(1,j),x(2,j),x(3,j),v(1,j),v(2,j),
     %        v(3,j),tmp1,ecc,inc,tmp2,tmp3,tmp4)
        sinc = sin(inc)

        twave = (m(1)/m(j))*(m(1)/(K2*SDgas*SMA*SMA))*((z0/R)**(4.))*
     %    (1.d0/Om)
        tT1e = twave*(econ/0.78)*
     %    (1. - 0.14*((ecc*R/z0)**(2.)) + 0.06*((ecc*R/z0)**(3.))
     %     + 0.18*((ecc*R/z0)*(sinc*R/z0)**(2.)))
        tT1i = twave*(icon/0.544)*
     %    (1. - 0.30*((sinc*R/z0)**(2.))+0.24*((sinc*R/z0)**(3.))
     %    + 0.14*((sinc*R/z0)*(ecc*R/z0)**(2.)))

        vth = atan(v(2,j)/v(1,j))
        if (v(1,j).lt.0.0d0) then
          vth = PI + vth
        else
          if (v(2,j).lt.0.0d0) vth = (2.0d0*PI) + vth
        endif

        vrad = sqrt(v(1,j)**2 + v(2,j)**2)*cos(vth-xth)
        arad= -2.0d0*vrad/((R**2)*tT1e)
        aT1(1,j) = arad*cos(xth)
        aT1(2,j) = arad*sin(xth)
        aT1(3,j) = -1.0d0*v(3,j)/tT1i

 250    continue
      enddo
c
c------------------------------------------------------------------------
c
 300  continue

      do j = 2,nbod
        ausr(1,j) = aT2(1,j) + adrag(1,j) + aT1(1,j)
        ausr(2,j) = aT2(2,j) + adrag(2,j) + aT1(2,j)
        ausr(3,j) = aT2(3,j) + adrag(3,j) + aT1(3,j)
      enddo
      return
      end
          c
```

Appendix B

Include File for Migration Subroutine

The user.inc file called in the migration subroutine above is shown below. This file contains many of the parameters that can be adjusted manually to allow for different migration scenarios to be tested. This is just one example of the file; many different parameter values were used throughout this project.

```
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c
c      USER.INC    (ErikSoft   4 March 2001)
c
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c
c Author: Avi Mandell (and Sean Raymond in a couple places)
c
c Parameters that govern migration:
c
c   NMIG = # of migrating planets, in order in BIG file (must be 1 or greater)
c   TFID = Timescale for fiducial linear migration rate, in days
c         (set to 0.d0 to disable T2 mig)
c   RATEFACT = Ratio of  migration time to TFID for each planet;
c       if two planets have MIGFACT in the same ratio as their distances,
c       they will migrate with the same inward velocity
c   STARTFACT = Starting time, as a fraction of TFID
c   SWITCH = Time to switch to expon. mig fade-out, as a fract. of total time
c   GAMMA = Var. mig exponent
c   STOPDIST = Dist. to stop mig at, if reached
c
c   Gas Drag Formulas From Thommes03:
c   GASTIME = Time at which gas drag ceases, in days
c   RHOCONST = Gas density const, in g/cm^3
c   RHOEXP = Gas profile expon.
c   ZCONST = Vert. gas density scale height const., in AU
c   ZEXP = Vert. gas profile expon.
```

```
c   CDRAG = Drag constant
c   MEFF = Effective part. mass, in g
c
c   Other parameters for gas disk
c   MAGCAV = radius of inner magnetospheric cavity (in AU)
c   INNERDRAG = fraction of drag to be applied interior to giant planet's
c      orbit.  This can vary between 0 (if no gas is assumed to cross
c      the gap) and 1 (if inner region is undepleted).  Lubow & D'Angelo (2006)
c      suggest the value should be 0.1-0.25.  This decreases the strength of
c      aerodynamic and type 1 gas drag for all bodies interior to RJUP.
c
c Flag for how much data to store in close encounter (ce.out) files:
c   ceflag = 0 to store info for all close encounters
c            = 1 to only store info for encounters between two "big" bodies
c             (as well as all collisions)
c          = 2 to only store collision information
c
      integer nmig, ceflag
      parameter (nmig = 1)

      real*8 ratefact(nmig),startfact(nmig),switch(nmig),gamma(nmig)
      real*8 tfid,stopdist,maxdist
      real*8 econ,icon
c
      real*8 gastime,rhoconst,rhoexp,zconst,zexp
      real*8 cdrag,meff,gconst,magcav,innerdrag
c
      parameter (tfid = 1.0d6*365.25)        !SET TO 0.D0 TO DISABLE T2 MIG.
      data startfact /0.00d0/
      data ratefact /0.55d0/
      data switch /0.33d0/                   !Stan.:0.333
      data gamma /0.6d0/                                      !Stan.:0.6
      parameter (stopdist = 2.2d0)
      parameter (maxdist = 5.3d0)

      parameter (econ = 0.005d0)             !SET TO 0.D0 TO DISABLE T1 MIG.!was
0.005
      parameter (icon = 0.0035d0)

      parameter (gastime = 4.0d5*365.25)
      parameter (rhoconst = 0.5*1.4d-9)        !Thommes:1.4d-9
      parameter (rhoexp = (11.0/4.0))        !Thommes:11/4
      parameter (zconst = 0.0472d0)          !Thommes: 0.0472
      parameter (zexp = (5.0/4.0))           !Thommes: 5/4
      parameter (cdrag = 1.0d0)
      parameter (meff = 1.26d16)                  !1e-23 M_e
```

```
parameter (magcav = 0.05)        !0.05
parameter (innerdrag = 0.25)

parameter (ceflag = 1)
```

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Education

Pennsylvania State University
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Expected Date of Graduation: Spring 2015
B.S. Astronomy & Astrophysics; Graduate Studies Option
B.S. Biochemistry and Molecular Biology; Biochemistry Option
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International Experience: Travel to Germany in May 2011 as part of a study abroad trip to visit automobile manufacturing plants and cultural landmarks.
Relevant Courses:

- Astronomical Methods and the Solar System
- Astronomy of the Distant Universe
- Observational Astronomy Laboratory
- Stars and Galaxies
- Planets and Planetary System Formation
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- Nebulae, Galaxies and Cosmology
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- Theoretical Mechanics
- Intro to Quantum Mechanics I
- Thermal Physics
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Experience

Penn State Astrobiology Research Center
Researcher University Park, PA Summer 2011-Present

- Study the origin of life by experimenting with methyl thioacetate, a potentially important chemical.
- Explore potential fossilization on Triton, a moon of Neptune.
- Manipulate computer models to study potential theories about the formation of the solar system.

Space Telescope Science Institute
Summer Intern Baltimore, MD June 2014-August 2014

- Study white dwarfs in the local universe to look for signs of metal absorption through spectroscopy.
- Study photometry of local sample of white dwarfs to look for infrared excesses indicative of dust disks.

Organic Chemistry Stockroom
Worker University Park, PA 2012-Present

- Assist in organic chemistry laboratory classes by providing equipment to students, refilling solvents and materials, preparing for lab and cleaning up after lab.
- Aid manager in variety of projects to improve the organic laboratory experience for students.

General Chemistry II (Chem 112) Learning Assistant
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- Facilitated group work among students in lectures.
- Helped students understand chemistry concepts and learn problem-solving skills.
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Skills

Computer Skills:

- UNIX/Linux experience
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- LaTeX
- Microsoft Word
- Microsoft Excel
- Microsoft PowerPoint
- Microsoft Publisher

General Skills:

- Team projects
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Publications/Presentations

- Todd, Zoe R and House, Christopher H. Astrobiology. October 2014, 14(10): 859-865. Doi:10.1089/ast.2014.1185
- Todd, Z.R. and House, C.H. Vesicles Protect Activated Acetic Acid. American Astronomical Society 223rd Meeting Poster Presentation.
- Todd, Z.R. and House, C.H. Vesicles Protect Activated Acetic Acid. Conference for Undergraduate Women in Physics 2014. Poster Presentation.
- Todd, Z.R. and Debes, J. The Search for Dusty White Dwarfs. Natonal Capital Area Disks Meeting. Poster Presentation.
- Todd, Z.R. and Sigurdsson, S. A Moderate Migration Scenario for Jupiter to form the Terrestrial Planets. American Astronomical Society 225th Meeting Poster Presentation.

Honors

Schreyer Honors College Scholar (2011-2015)
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Activities

Astronomy Department Outreach Volunteer	2012-Present
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Astrobiology Primer 2.0 Accessibility Reviewer	June 2012
Astronomy Club	2011-2014
Physics and Astronomy for Women	2011-2014
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