THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

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# A COMPARISON OF OPTIONS MODELS IN TIMES OF RISING MARKET VOLATILITY

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# ABSTRACT

This paper compares the effectiveness of the of the Binomial Options Pricing model, Black-Scholes model, and Monte Carlo model in pricing American call options on large-cap non-dividend paying equities. These models are implemented on 32 options contracts that span the relatively volatile fourth quarters of 2011 and 2012. This study did not result in any of the models performing consistently better than the others during these time periods.

# TABLE OF CONTENTS

## LIST OF FIGURES

## LIST OF TABLES

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# Chapter 1

## Introduction

In this paper, I evaluate the effectiveness of the Binomial Options Pricing, Black-Scholes, and Monte Carlo methods for American call pricing of non-dividend paying equities over the periods of increasing volatility in the fourth quarters of 2011 and 2012. The models that I use take historical data as parameters without adjusting for future expectancies. The parameters used in a valuation model are arguably more important than the model used, so it is reasonable to expect that in times of high market volatility that all pricing methods based on historical data produce prices that deviate further from the market price. More model variance implies that there is more opportunity for the models to deviate from each other. Honig (2009) finds no statistically significant difference in the pricing capabilities of these models. In this paper, I check the robustness of his results by examining the pricing ability of each method during periods with rapidly increasing volatility.

In August of 2011, there was a sharp increase in market volatility. There were several causes for this including concerns about a double dip recession in the U.S. following the global financial crisis as well as the potential for Europe to enter a banking crisis. Given that the events of 2008 were not far from investors' minds, weakening economic data, the debt ceiling debate in Washington, and problems with European sovereign debt were enough to cause mass fear and panic in the markets. This fear led to a more than doubling of the CBOE Volatility Index (VIX). In December of 2012, investors once again pushed the VIX higher, albeit not as high as in 2011, when the federal government approached the fiscal cliff where federal spending could have been drastically cut and tax rates increased. In both of these time periods, the VIX increased more than 40% in a matter of months. Since volatility is such an important measure in options pricing models, it is reasonable to assume that the models using inputs based on historical data would perform worse in scenarios where volatility is expected to be much different in the future. This study does not find any of the models to consistently produce more accurate forecasts than the others in the analyzed time periods.

Chapter 2 is a literature review of previous work in the area of options pricing. Chapter 3 is a description of the data collection process. Chapter 4 is an introduction to the options pricing models that I use. Chapter 5 is a discussion of the data analysis techniques used in the paper. Chapter 6 provides the empirical results of the study, and Chapter 7 is a summary of the results.

# Chapter 2

## Literature Review

In their landmark 1973 paper, "The Pricing of Options and Corporate Liabilities," Fisher Black and Myron Scholes published their pricing method for non-dividend paying European options. This paper is foundational in the finance industry because it created a new, scientific way of analyzing options. The Black-Scholes model requires the inputs of the current stock price, strike price, time to expiration, risk-free rate, and volatility. It assumes a log-normal distribution of stock prices. The model allows market participants to create more complex trading and hedging strategies, which led to an increase in options trading at the time of its publication. While the Black-Scholes equation works well for European options on non-dividend paying equities, it becomes less effective for other types of options. As investors realized the model's limitations, many other models were created to price options. Given that the Black-Scholes model is one of the most prominent, if not the most prominent options pricing model, I selected it as one of the models to analyze in my study.

Another widely used model for options pricing is the Binomial Options Pricing model. Cox, Ross, and Rubinstein developed the model in their 1979 paper, "Option Pricing: A Simplified Approach." The Binomial Options Pricing model assumes that the underlying stock price follows a multiplicative binomial process in discrete time. This is unlike the Black-Scholes model, which assumes that stock prices are a log-normally distributed continuous random variable. Despite the differences between models, the model can be constructed such that its underlying volatility and return assumptions match that of the Black-Scholes model. Other assumptions in the Binomial Options Pricing model include no taxes, transaction costs, or margin requirements and the ability to short any stock. Using a no-arbitrage assumption, Cox, Ross, and Rubinstein showed that their model could accurately price options under these conditions. As with most models, the method is not perfect due to imperfect assumptions, however it does provide a relatively simple numerical method.

As computer processing speeds improved, additional numerical models were developed, including the Monte Carlo method which was first described by Boyle in his 1977 paper "Options: A Monte Carlo Approach." This model simulates all potential paths for a stock and calculates options prices based on averages of these paths. The model incorporates the same underlying assumptions as the Black-Scholes model and provides a way to simulate options prices. Taken together, these three models provide a strong base for options models, as they all work with somewhat different assumptions and calculation methods.

Once the theoretical framework for valuing options had been built, it became a comparatively simple task to implement the models in computer algorithms. In their 1998 book, *Implementing Derivatives Models*, Clewlow and Strickland shared algorithms to implement options pricing models including the Black-Scholes, Binomial, and Monte Carlo methods. Additionally, in their 2009 paper “Implementing Binomial Trees,” Gilli and Schumann describe MATLAB code for pricing options using a binomial tree model. The pseudo-code from these papers was foundational in the development of my analytical processes.

Honig (2009) conducts a study that compared the overall effectiveness of the Black-Scholes, Binomial Pricing, and Monte Carlo methods. Honig produces a study of 10 stocks with 10 puts and 10 calls per stock (for a total of 200 contracts). He examines trading data for these options over a 30-day period from January 21, 2009 through March 4, 2009. Honig’s volatility estimates are taken from the stock price performance during the preceding 3 months from the start of data collection. He then compares the actual market prices of the options to his calculated intrinsic values. He finds that different models were more accurate for pricing different types of options at varying levels of moneyness, and that there is not one best model. He does find, however, that the Black-Scholes model is the least effective overall.

Honig (2009) serves as a framework for this paper, as I use very similar methodology. While Honig evaluates options data for approximately one month, my data set spans the entire life of the options that I analyze, approximately 3 months per contract. I also obtain statistics for the option model inputs from the past two years of data, rather than the 3 months that Honig uses. I used similar statistical methods to evaluate which models are most effective in the fourth quarters of 2011 and 2012 where the VIX increased more than 40% over a period of several months.

In addition, other papers have addressed how options pricing is impacted by the type of volatility impact in the model in addition to the volatility in the market. Buraschi and Jackwerth (1999) evaluate the effectiveness of options pricing models that have various volatility assumptions. They deduce that the models using deterministic volatility assumptions, although easier to fit to the market data, do not explain all of the nuances of options pricing. The authors also determine that volatility is generally priced into the market prices of options and that stochastic models tend to perform better than the deterministic models.

With regards to actual options pricing during times of economic distress, Cheng, Fung, and Chan (2000) addresses options pricing efficiency. The authors examine whether put-call-futures parity held during the Asian financial crisis between January 1996 and August 1998.

Cheng et al. analyze the data using a multiple regression analysis, and they conclude that during the crisis, greater mispricing occurred leading to increased arbitrage opportunities.

Cheng et al. (2000) implies that volatility will lead to mispricing of the options models. I can also infer that although the intrinsic value of the options that I calculate from the models will most likely deviate from the observed price in the market, some of this variance will not be due only to the change in forward volatility. It is probable that some of the variation is attributable to lower liquidity and irrational trading behavior from market participants. I do not quantify these factors in the study, however it is important to be aware of the limitations of the models.

The results of these papers and studies both create a foundation upon which to base my study and support the reasonable expectation that the models perform worse in periods of increasing volatility. Although the models that I use are more simplistic than those used by investors, especially with regards to the use of historical performance as a predictor of future performance, they are still used by many market participants.

# Chapter 3

## Data Description

The process that I use to select stock and options contracts is described in this chapter. I first collect a current (as of October 2014) list of stocks in the S&P 500 along with each stock's dividend yield. I then sort the stocks to find those with no dividend yield. Then, to select the stocks, I only select stocks which had been public since at least 2009 and had not paid dividends in the past. This requirement is important, as I needed to be able to obtain stock trading data beginning in 2009 due to my use of 2 year historical trading data. Additionally, I make sure to select companies that operated in different industries to account for the possibility that the options models act differently on different types of stocks. The final list of stocks is not an exhaustive list of stocks that fit the above criteria.

Next, to select the options contracts, I obtain stock price data from each of the stocks prior to October 2011 and December 2012 and search the Bloomberg historical options database for out of the money American call options that fulfilled two criteria: 1) have sufficient trading volume such that the market price could be assumed to be a relatively accurate intrinsic value, and 2) have daily price data for market trading days. Table 1 includes the options analyzed in this study.

Based on the above criteria, the stocks that I analyze were CELG, CMG, EBAY, ETFC, FOSL, GOOG, MNST, PCLN, and UA. Due to a lack of data availability, ETFC and UA contracts for December 2012 are excluded from the analysis. Using this methodology, I select 32 contracts to analyze. The sources of my data are the CRSP database for stock returns, Bloomberg for options prices, and Kenneth French's website for risk free rate returns.

*Table 1: Selected Options*

| Ticker | Maturity | Strike |
|---|---|---|
| **CELG** | Oct-11 | 60 |
| **CELG** | Oct-11 | 65 |
| **CMG** | Oct-11 | 350 |
| **CMG** | Oct-11 | 300 |
| **EBAY** | Oct-11 | 30 |
| **EBAY** | Oct-11 | 35 |
| **ETFC** | Oct-11 | 10 |
| **ETFC** | Oct-11 | 15 |
| **FOSL** | Oct-11 | 95 |
| **FOSL** | Oct-11 | 105 |
| **GOOG** | Oct-11 | 600 |
| **GOOG** | Oct-11 | 500 |
| **MNST** | Oct-11 | 95 |
| **MNST** | Oct-11 | 80 |
| **PCLN** | Oct-11 | 500 |
| **PCLN** | Oct-11 | 450 |
| **UA** | Oct-11 | 80 |
| **UA** | Oct-11 | 70 |
| **CELG** | Dec-12 | 75 |
| **CELG** | Dec-12 | 80 |
| **CMG** | Dec-12 | 250 |
| **CMG** | Dec-12 | 300 |
| **EBAY** | Dec-12 | 50 |
| **EBAY** | Dec-12 | 45 |
| **FOSL** | Dec-12 | 90 |
| **FOSL** | Dec-12 | 80 |
| **GOOG** | Dec-12 | 700 |
| **GOOG** | Dec-12 | 650 |
| **MNST** | Dec-12 | 50 |
| **MNST** | Dec-12 | 45 |
| **PCLN** | Dec-12 | 600 |
| **PCLN** | Dec-12 | 650 |

# Chapter 4

## Models

Common inputs for each of the three models are:

- S – Current stock price
- K – Strike price of the contract
- r – Risk free rate (one month Treasury bill)
- T – Number of days until maturity/365
- σ – Annualized volatility

### Binomial Options Pricing Model

This model uses a risk-neutral approach and finds the intrinsic value of an option by finding the option's final payoff at maturity then discounting it by the risk free rate. Using this model, an option's value at time t is the discounted expected value of the option at time t + 1. I use daily time steps for the model. The formula for the intrinsic value is applied starting at the day before maturity until the value at time 0 is reached. The final payoff for the option at maturity is $\max(S_T - K, 0)$. At any given node that is not maturity (a node is defined as a given combination of up and down movements that can be in any point of time from [0,T]), the value of the option using the daily step model is:

$$V = \max[e^{-r/(30*12)}[pV_u + (1-p)V_d], S_t - K]$$

Where:

$V_u$ = value of option with an up movement in the stock

$V_d$ = value of option with a down movement in the stock

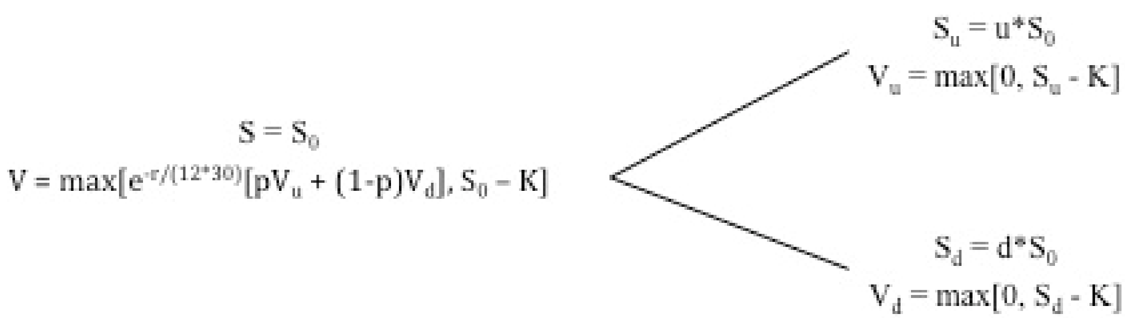
$u = e^{\sigma * \sqrt{1/365}}$ - daily percent stock price change in an "up" movement

$d = 1/u$ - daily percent stock price change in a "down" movement

$p = \frac{(e^{r/(12*30)} - d)}{(u-d)}$ - risk-neutral probability of an "up" movement

Figure 1 below is a visualization of a one step binomial tree. This process can be repeated for any number of steps, with each node acting as its own one step model.

*Figure 1: One Step Binomial Tree Visualization*




If we assume a stock with inputs:

$S_0 = 10$

$K = 11$

$r = 0.002$

$T = 60/365$

$\sigma = 0.30$

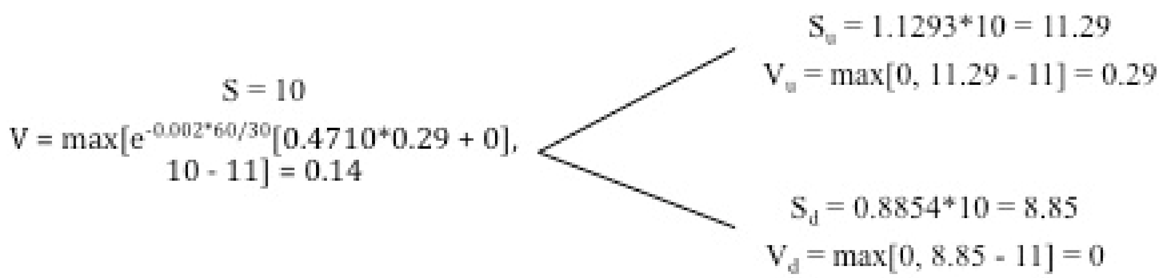
Then the one step binomial tree calculation for the valuation of this option is:

$u = e^{0.30 * \sqrt{60/365}} = 1.1293$

$\mathrm{d} = 1/1.1293 = 0.8854$

$$\mathrm{p} = \frac{(\mathrm{e}^{0.002*60/365} - 0.8854)}{(1.1293 - 0.8854)} = 0.4710$$

*Figure 2: One Step Binomial Tree Example*




**Black-Scholes Model**

It is important to note that this model is used with the assumption that American calls on non-dividend paying stocks will never be exercised before maturity. Therefore, the same model can be used for both European and American call options. The model calculates a price as follows:

$$\mathrm{V} = \mathrm{S} * \mathrm{N}(\mathrm{d}_1) - \mathrm{K}\mathrm{e}^{(-\mathrm{r}*\mathrm{T})} * \mathrm{N}(\mathrm{d}_2)$$

Where:

$$\mathrm{d}_1 = \frac{\ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right) + \left(\mathrm{r} + \frac{\sigma^2}{2}\right)\mathrm{T}}{\sigma\sqrt{\mathrm{T}}}$$

$$d_2 = \frac{\ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$N(d_n)$ = standard normal cumulative density function

Using the same inputs as the example in the binomial tree section, we find the following intrinsic value using the Black-Scholes model.

$$d_1 = \frac{\ln\left(\frac{10}{11}\right) + \left(0.002 + \frac{0.30^2}{2}\right) 60/365}{0.30\sqrt{60/365}} = -0.7201$$

$$d_2 = \frac{\ln\left(\frac{10}{11}\right) + \left(0.002 - \frac{0.30^2}{2}\right) 60/365}{0.30\sqrt{60/365}} = -0.8417$$

$$V = 10 * N(-0.7201) - 11e^{\left(-0.002*\frac{60}{365}\right)} * N(-0.8417) = 0.16$$

**Monte Carlo Model**

I implement the model described by Clelow and Strickland for European call options. Using the same assumption as described above, the model for European and American options for non-dividend paying stocks is the same. The model uses the same assumptions as the Black-Scholes model, but performs a simulation in discrete time. Instead of finding a closed-form solution, the model simulates a certain number of paths and then takes an average of the discounted options value to find an intrinsic value. I perform 1,000 simulations for each contract, each using daily steps for stock price movements. The evolution of the stock prices is simulated with the formula:

$$\ln(S_t) = \ln(S_{t-1}) + \nu * \frac{1}{365} + \sigma * \sqrt{\frac{1}{365}} * \varepsilon$$

Where:

$$\nu = (r - 0.5\sigma^2) * 1/365$$

$$\varepsilon \sim N(0, 1)$$

The option payoffs at maturity are then calculated, averaged together, and then discounted with the risk free rate to calculate the mode’s intrinsic value.

I performed this process with 1,000 simulations and the parameters in the examples in the previous model sections and found an intrinsic value of 0.16.

# Chapter 5

# Data Analysis Methodology

## Mean Squared Error

The mean squared error (MSE) of a model is the average squared difference between a model's output and the actual value of the metric that the model is attempting to forecast. In the case of this paper, the MSE for a model is the average of the squared difference between the options model and the market value of the contract over the life of the contract. This metric is useful in comparing errors across models, however, it is important to note that the absolute value of the MSE is relatively meaningless, as a model that forecasts the value of a contract with a high price will tend to have a higher MSE than a contract with a lower price due solely to the larger dollar amount. A MSE close to 0 implies that a model is more accurate. The formula for the MSE is:

$$\mathrm{MSE} = \frac{1}{\mathrm{n}} * \sum_{i=1}^{n} (\hat{\mathrm{x}}_i - \mathrm{x}_\mathrm{i})$$

Where:

n = number of observations (number of days in contract life)

$\hat{\mathrm{x}}_i$ = model's predicted value

$\mathrm{x}_i$ = market value of the option

I calculate the MSE for all 32 contracts then rank the MSE for each contract from smallest to largest and count the number of each model that ranked the smallest. This process is designed to rank the models from best to worst by their prediction error. If one model consistently produces forecasts with the smallest MSE, then it would be the best using this metric.

I then perform a relative Theil MSE decomposition, breaking the MSE into its relative bias, variance, and noise terms. These three terms are important as they provide us with further insight into the errors of the model. The bias term measures the difference in the average of the forecasts against the average of the predicted series' value. In the case of this paper, I am comparing the average predicted model value against the average of the contract's market value over the course of its life. The decomposition of MSE provides a squared term, so it will always be a positive number, and the size demonstrates a systematic mispricing of a contract. An unbiased model has a bias of 0, meaning that the average model forecast is the same as the actual series.

The second term of the decomposition, variance, shows whether a model's forecasts are more or less volatile than the underlying series. Like bias, this term is a squared number, so one cannot draw directional conclusions, however, the variance term of a more accurate model is closer to 0 than a less accurate model. A value further from 0 indicates a model that is either over or under-predicting the volatility of the actual observations.

The final noise term of this decomposition accounts for unpredictable, random errors. An accurate model has a high noise term relative to the total MSE, as it shows a model that is unbiased with similar variance to the underlying series and with small random errors.

The MSE decomposition formulas are below:

$$\mathrm{MSE} = (\mathrm{mean(x)} - \mathrm{mean}(\hat{\mathrm{x}}_i))^2 + (\sigma_x - \sigma_{\hat{\mathrm{x}}})^2 + 2 * (1 - \rho) * \sigma_x * \sigma_{\hat{\mathrm{x}}}$$

$$= \text{bias}^2 + \text{variance} + \text{noise}$$

Where:

x = contract market values

$\hat{x}$ = model forecasts

After breaking the MSE of each model for each contract into its three parts, I calculate each term as a percentage of MSE. The absolute value of these terms is not useful in comparing across contracts, so scaling the MSE components makes it possible to evaluate model performance across contracts. The contracts that performed best in these comparisons are the ones that have low absolute MSEs and high noise terms relative to the MSE.

**Diebold-Mariano Statistic**

The Diebold-Mariano statistic quantitatively tests whether the MSE of different forecasting models is the same. Performing this statistical test to compare models is critical since it allows one to conclude whether model forecast differences are due to chance or due to actual underlying differences in the efficacy of the models. The null hypothesis of the test is that the MSE are equivalent and the alternative hypothesis is that they are not equivalent. The test statistic used in this test is constructed below:

$$S = \frac{\text{mean}(d_t)}{\sqrt{2\pi f_d(0)/T}}$$

Where:

$$d_t = L(\varepsilon^1_{t+h|t}) - L(\varepsilon^2_{t+h|t})$$

$$L(\varepsilon^1_{t+h|t}) = \text{MSE loss of first model}$$

$L(\varepsilon^{1}_{t+h|t})$ = MSE loss of second model

$h$ = lag

$f_d(0)$ = spectral density of $d_t$ at frequency 0

$S \sim N(0, 1)$

I perform this test at a 95% significance level, rejecting the null hypothesis with a p-value absolute value below 0.05. I use a lag of 15 to replicate Honig's methodology which found that the Diebold-Mariano statistics stabilized with a lag of 15. Additionally, the p-values that I produce from these calculations have a directional component. A positive p-value less than 0.05 indicates a forecast with a statistically significant higher (worse) MSE values and a negative p-value indicates a statistically significant lower (better) MSE.

# Chapter 6

# Empirical Results

## Mean Squared Error

The results of the mean squared error rankings are inconclusive. I analyze the MSEs with the following process: I first calculate the MSE of each model for each contract. I then rank the MSEs from lowest (rank 1) to highest (rank 3) for each contract. The cells in Table 2 contain the total number of outcomes by model for each rank (e.g. the cell at the intersection of Rank 1 and Binomial Tree has a value of 10, meaning that 10/32 of the binomial tree calculations that I performed had the lowest MSE of the three models for those 10 contracts).

*Table 2: Mean Squared Error Rankings (Number of Contracts)*

| | **Binomial Tree** | **Black-Scholes** | **Monte Carlo** | **Total** |
|---|---|---|---|---|
| **Rank 1** | 10 | 15 | 7 | **32** |
| **Rank 2** | 18 | 10 | 4 | **32** |
| **Rank 3** | 4 | 7 | 21 | **32** |
| **Total** | **32** | **32** | **32** | **96** |

While the rankings of the model's MSEs do not indicate a clear best model, it would appear that the Monte Carlo method performs the worst of all models when comparing them solely on MSE. Of the 32 contracts that I analyze, the Monte Carlo pricing method produced forecasts with the highest MSE 21 times (66% of the contracts). This assertion is not statistically significant, however, as this methodology only analyzes the data at a quick glance. The Diebold-Mariano tests later in the paper do not support this statistical significance.

Table 3 includes the summary statistics for the MSEs of the models. The models all show similar results, but there are some slight differences between the Monte Carlo model and the other two. The Monte Carlo MSEs are higher than and they have a tighter standard deviation than the other two models. The higher MSEs of the Monte Carlo model is consistent with the above results, as the higher error accounts for the model performing worst in the MSE rankings.

*Table 3: MSE Summary Statistics*

| | **Binomial Tree** | **Black-Scholes** | **Monte Carlo** |
|---|---|---|---|
| **Mean** | 19.77 | 19.73 | 20.12 |
| **Standard Deviation** | 58.81 | 58.73 | 58.36 |
| **Minimum** | 0.04 | 0.04 | 0.04 |
| **Q1** | 0.23 | 0.23 | 0.24 |
| **Median** | 1.64 | 1.64 | 1.71 |
| **Q3** | 9.57 | 9.49 | 12.03 |
| **Maximum** | 316.69 | 316.18 | 314.23 |

In Table 4, I take the average proportions of the bias, variance, and noise terms as a percentage of the total MSE. While the decompositions of the MSEs are all very similar, the Monte Carlo has a more positive skew towards the noise term than do the other two models. Though the decomposition of the Monte Carlo MSEs may be more favorable than the other models, albeit by a small margin, the MSEs of the Monte Carlo forecasts tend to be higher on average as found above.

*Table 4: Theil MSE Decomposition*

| | MSE | Bias | Variance | Noise |
|---|---|---|---|---|
| **Binomial Tree** | 100.00% | 49.25% | 8.19% | 42.56% |
| **Black-Scholes** | 100.00% | 49.34% | 8.30% | 42.36% |
| **Monte Carlo** | 100.00% | 46.93% | 7.08% | 45.99% |

Table 5 provides the summary statistics for the Theil Decomposition. Similar to the previous results in this paper, there are no glaringly large differences between the models. There are, however, slight differences between the Monte Carlo model and the other two. As indicated above in Table 4, the Monte Carlo method produces a Theil decomposition with a higher noise term on average. In addition, the standard deviation of all terms is lower for the Monte Carlo model than the other two. These results indicate that the Monte Carlo method produces more consistent results than the other models, however, as shown in Table 2, the errors are consistently less accurate than the other two.

*Table 5: Theil Decomposition Summary Statistics*

| | Bias | | | Variance | | | Noise | | |
|---|---|---|---|---|---|---|---|---|---|
| | **BT** | **BS** | **MC** | **BT** | **BS** | **MC** | **BT** | **BS** | **MC** |
| **Mean** | 49.25% | 49.34% | 46.93% | 8.19% | 8.30% | 7.08% | 42.56% | 42.36% | 45.99% |
| **St. Dev.** | 28.05% | 28.05% | 27.75% | 9.73% | 9.72% | 9.11% | 27.60% | 27.60% | 27.22% |
| **Min** | 0.07% | 0.09% | 0.07% | 0.01% | 0.01% | 0.00% | 2.64% | 2.65% | 4.82% |
| **Q1** | 29.01% | 29.46% | 28.24% | 1.71% | 1.72% | 1.63% | 21.75% | 20.93% | 25.27% |
| **Median** | 59.65% | 59.59% | 55.99% | 6.09% | 6.41% | 4.56% | 31.33% | 31.47% | 36.80% |
| **Q3** | 74.57% | 74.49% | 70.54% | 12.38% | 12.05% | 9.32% | 62.46% | 62.37% | 63.77% |
| **Max** | 84.40% | 84.43% | 84.78% | 49.14% | 49.06% | 48.75% | 93.88% | 93.69% | 96.32% |

**Diebold-Mariano Statistics**

The Diebold-Mariano tests are also inconclusive. The results of these tests are summarized in Table 6. Each cell in the table is the directional p-value for the Diebold-Mariano statistic of the first listed model against the second model. Since there are 32 contracts with 3 pairs per contract, there are 96 total p-values.

*Table 6: Diebold-Mariano Statistics*

| Ticker | Maturity | Strike | State | BT vs. BS | BT vs. MC | BS vs. MC |
|---|---|---|---|---|---|---|
| **CELG** | Oct-11 | 60 | ITM | 0.4025 | -0.1933 | -0.2028 |
| **CELG** | Oct-11 | 65 | ITM | -0.1073 | -0.2209 | -0.2577 |
| **CMG** | Oct-11 | 350 | OTM | -0.2546 | -0.2332 | -0.3076 |
| **CMG** | Oct-11 | 300 | ITM | 0.0000 | -0.4962 | 0.4180 |
| **EBAY** | Oct-11 | 30 | ITM | -0.0070 | -0.0586 | -0.0704 |
| **EBAY** | Oct-11 | 35 | OTM | 0.0651 | -0.0482 | -0.0171 |
| **ETFC** | Oct-11 | 10 | OTM | -0.0343 | -0.0989 | -0.1056 |
| **ETFC** | Oct-11 | 15 | OTM | 0.1102 | 0.0201 | 0.0256 |
| **FOSL** | Oct-11 | 95 | OTM | -0.1453 | 0.2959 | 0.2104 |
| **FOSL** | Oct-11 | 105 | OTM | -0.2889 | -0.1337 | -0.1667 |
| **GOOG** | Oct-11 | 600 | OTM | 0.0000 | -0.2200 | -0.0729 |
| **GOOG** | Oct-11 | 500 | ITM | -0.2040 | -0.4858 | 0.4899 |
| **MNST** | Oct-11 | 95 | OTM | 0.3081 | -0.4101 | -0.3414 |
| **MNST** | Oct-11 | 80 | ITM | -0.0022 | 0.3288 | 0.3036 |
| **PCLN** | Oct-11 | 500 | OTM | -0.4290 | -0.0507 | -0.0386 |
| **PCLN** | Oct-11 | 450 | ITM | 0.1489 | 0.3426 | 0.3667 |
| **UA** | Oct-11 | 80 | OTM | -0.3587 | 0.0689 | 0.0719 |
| **UA** | Oct-11 | 70 | ITM | 0.0000 | 0.0525 | 0.0350 |
| **CELG** | Dec-12 | 75 | ITM | 0.3298 | -0.0714 | -0.0621 |
| **CELG** | Dec-12 | 80 | OTM | 0.0068 | 0.4306 | -0.1530 |
| **CMG** | Dec-12 | 250 | ITM | 0.0610 | -0.0714 | -0.0669 |
| **CMG** | Dec-12 | 300 | OTM | -0.4580 | -0.0770 | -0.0780 |
| **EBAY** | Dec-12 | 50 | ITM | 0.0000 | -0.0551 | -0.0216 |
| **EBAY** | Dec-12 | 45 | ITM | 0.3288 | -0.4167 | -0.4131 |
| **FOSL** | Dec-12 | 90 | ITM | 0.0007 | -0.1312 | -0.0916 |
| **FOSL** | Dec-12 | 80 | ITM | 0.0079 | -0.1187 | -0.0890 |
| **GOOG** | Dec-12 | 700 | ITM | 0.0001 | -0.0034 | -0.0022 |
| **GOOG** | Dec-12 | 650 | ITM | 0.0130 | -0.0001 | 0.0000 |
| **MNST** | Dec-12 | 50 | ITM | -0.0050 | 0.2399 | 0.2233 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **MNST** | Dec-12 | 45 | ITM | -0.1844 | -0.0282 | -0.0307 |
| **PCLN** | Dec-12 | 600 | ITM | 0.0676 | -0.2944 | -0.1891 |
| **PCLN** | Dec-12 | 650 | OTM | 0.0052 | 0.2863 | -0.4940 |

The Diebold-Mariano statistics in Table 6 are unusual in that they do not show any systematic outperformers or underperformers of the models. Of the 96 comparisons in the above table, 27 (28%) show significance in MSE differences, as indicated by gray cells. This number is far higher than the roughly 5 significant values (5% of 96) that one would expect if the entire process were random, but the pattern of significance, if it exists, is not clear. Although the objective of this study is not to distinguish between contracts of varying moneyness, I include the ending state of each option contract to examine whether a pattern exists in the significant pairs based on the contract's ending moneyness.

The Binomial Tree versus Black-Scholes statistics have the most number of significant pairs (14 out of 32), but they are not all directionally consistent. Of these 14 pairs, 4 contracts lead to a better forecast from the Binomial Tree method, and the remaining 10 lead to a worse forecast. Though this number is skewed towards a worse forecast for the Binomial Tree model, 15 p-values of the total pairs indicate a better forecast, and the other 17 indicate a worse forecast—essentially split in half. There is no relationship between whether the contracts finished in-the-money or out-of-the-money.

The Binomial Tree versus Monte Carlo statistics have fewer significant pairs than the previously mentioned pairing, however the significant pairs are more directionally consistent. Of the 5 significant pairs, 4 indicate better predictions from the Binomial Tree model. Additionally, 23 of the 32 overall contracts show a favorable result towards the Binomial Tree method. Again, the results of these Diebold-Mariano tests are not strong enough to make a definitive statement on the efficacy of the Binomial Tree method versus the Monte Carlo method, however, based on the above numbers, the Binomial Tree method seems to be slightly better.

The final pairing of the Diebold-Mariano test, the Black-Scholes model versus Monte Carlo model does not provide much more insight into the effectiveness of these models. Eight of the 32 contracts show statistically significant results. Five are in favor of the Black-Scholes model and the other 3 are in favor of the Monte Carlo model. Since roughly half of the significant results are in favor of each, we cannot draw any meaningful conclusions. Overall, 23 of the 32 contracts are in favor of the Black-Scholes model, but since the majority are not statistically significant, we cannot conclude that either the Black-Scholes model or Monte Carlo model forecasts with more accuracy than the other.

Compared to Honig (2009), this research finds a higher proportion of statistically significant contracts (28% versus 5%), and while it would be tempting to conclude that the heightened volatility of the time period of this study was responsible for the results, the discrepancy could be due to several factors. These factors include a different profile of the types of companies whose stocks were analyzed, the fact that none of the stocks analyzed in this paper pay dividends, differing moneyness states, and the difference in volatility characteristics of the markets. Similar to Honig, this paper finds no statistically significant result in the comparison of the models to each other through either the decomposition of MSE or the Diebold-Mariano analysis.

# Chapter 7

## Conclusion

This paper attempts to quantify which, if any, of the Binomial Tree, Black-Scholes, or Monte Carlo methods of valuing American call options on non-dividend paying equities during the fourth quarters of 2011 and 2012 where volatility rapidly increased would have been most effective. An analysis of each model for each of 32 contracts is performed on the models' MSE, its decomposition, and a Diebold-Mariano test.

The analyses performed in this study are inconclusive. A comparison of MSEs suggests that the Monte Carlo method is the least effective model, with 21 out of 32 of its MSEs being the highest (worst) of the models used. A relative Theil decomposition of the MSEs yields very similar results across models, though the Monte Carlo method does produce a slightly better decomposition than the other models with lower bias and variance terms. Though the decomposition is more favorable for the Monte Carlo method, as found in the MSE rankings, the Monte Carlo ranks worse in the absolute value of its MSEs. The above findings are empirically unsupported, however, as the Diebold-Marinao test does not indicate significance of any of the relationships. There is an abnormally high number of statistically significant Diebold-Mariano statistics found (greater than 5% of the total), however there are no patterns to the significance that would indicate superiority of one model compared to the others.

Additionally, while the outcomes all of the analyses performed in the paper are slightly different than Honig (2009), the predominant results are the same in that none of the models can consistently forecast options prices with more accuracy than the others. The difference in the statistical outcome cannot be attributed solely to the differing market environment, as there are several different factors in the way these studies were conducted. These factors include different stocks analyzed, the fact that none of the stocks in this paper paid dividends, differing moneyness profiles, and the overall volatility characteristics of the market.

These results are not entirely unexpected, as the models were implemented with the same underlying assumptions. In fact, these results could be taken as a testament to the robustness of the different models and fact that there are many options pricing models that come to similar conclusions in different ways.

# Appendix A

## Selected Case Studies

### GOOG 700 December 2012 Call

The first contract that I would like to draw further attention to is the $700 strike December 2012 call for GOOG since it is one of two contracts that I analyzed that had 3 significant pairings under the Diebold-Mariano test. Based on the Diebold-Mariano tests, the ranking of the three models from best to worst for this contract are Black-Scholes, Binomial Tree, and Monte Carlo.

*Figure 3: GOOG Stock Price and 700 December 2012 Option Prices*

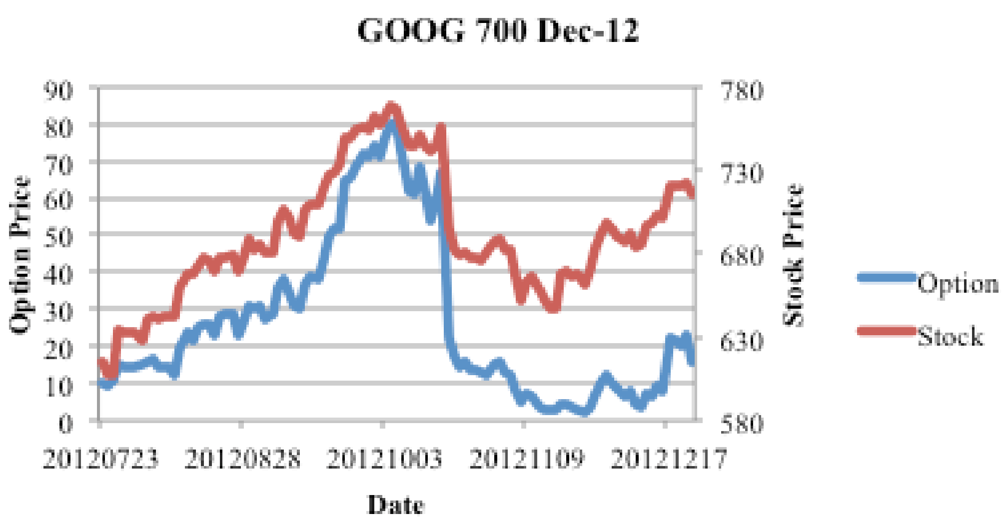


Figure 3 shows the GOOG stock price over the same period as the lifetime of this contract. The sharp drop in both the stock and option price were due to a weak earnings report for the quarter. This sudden change in price is likely the reason for the relatively large discrepancies between the pricing models.

*Figure 4: GOOG 700 December 2012 Pricing Model Forecasts vs. Market Price*

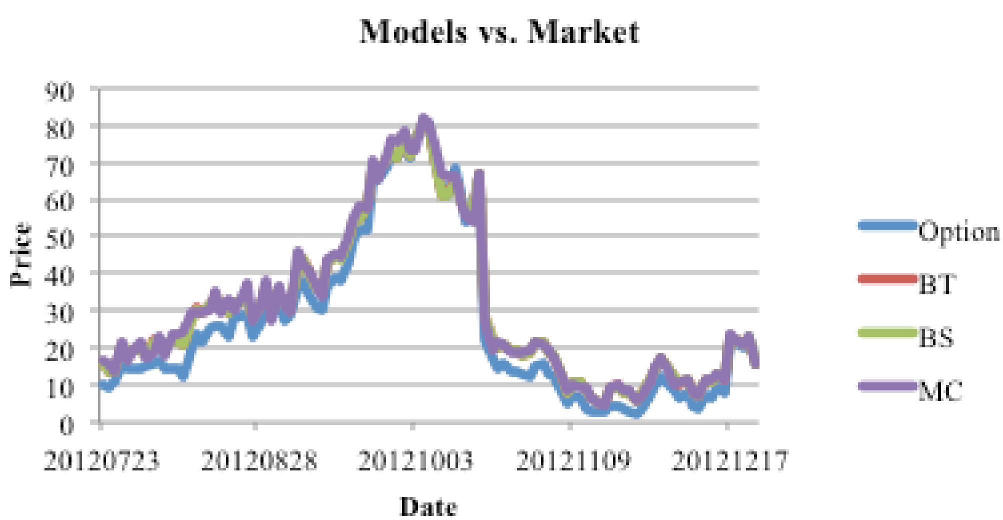



*Figure 5: GOOG 700 December 2012 Model Errors*

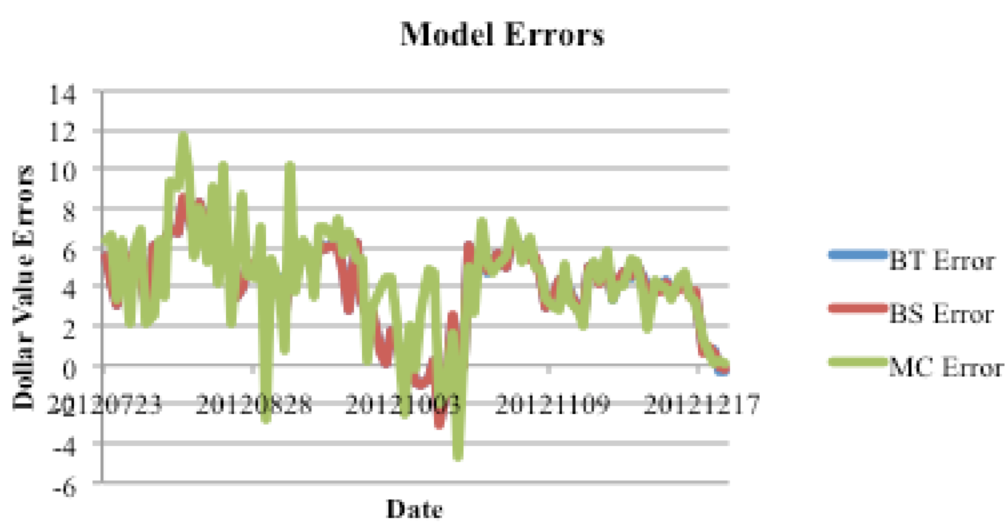


Figure 4 shows the models' forecasted option price for a particular day against the actual options price. There is a positive bias from the models (as seen by the forecast tendencies to be higher than the actual price), but overall they track the actual price relatively closely. As seen in Figure 5, the model errors remain on a downward trend, excluding the period of several days where the stock price rapidly dropped.

*Table 7: GOOG December 2012 700 Theil MSE Decomposition*

| | MSE | Bias | Variance | Noise |
|---|---|---|---|---|
| **Binomial Tree** | 100.00% | 75.00% | 6.72% | 18.29% |
| **Black-Scholes** | 100.00% | 75.01% | 6.73% | 18.27% |
| **Monte Carlo** | 100.00% | 69.96% | 1.64% | 28.41% |

Table 7 includes the relative MSE decomposition for each of the models. While the Monte Carlo model performs worst in the Diebold-Mariano test, its relative MSE decomposition is the most favorable of the models, as the noise component was significantly higher than the other two.

Overall, the increased stock specific volatility is the likely cause for the different performance from each of the models. This result supports the original hypothesis that volatility changes the effectiveness of the pricing models, but in this case, company specific volatility has a much larger effect than did market volatility on the forecasts.

**GOOG 650 December 2012 Call**

The second contract to discuss further in depth is the GOOG $650 strike December 2012 call. This contract is interesting for two reasons: 1) all 3 model pairs are significant in the

Diebold-Mariano test, and 2) despite having the exact same inputs as the $700 strike, except for strike price, the Diebold-Mariano and MSE results are different than the other contract.

*Figure 6: GOOG Stock Price and 650 December 2012 Option Prices*

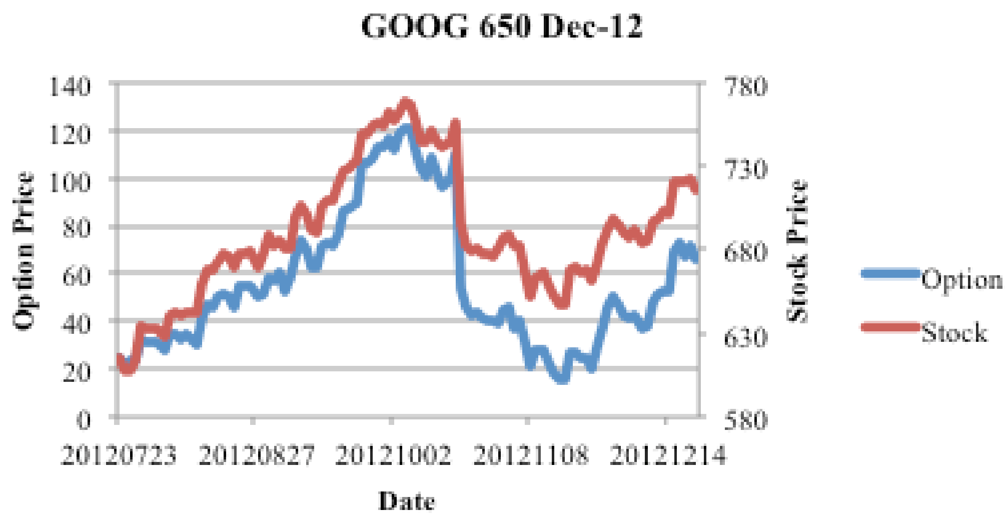



Figure 6 demonstrates essentially the same pattern as Figure 1, with the exception of the option price and its sensitivity to price changes in the underlying stock. Since the strike price of this contract is lower than the previous contract analyzed, the contract's delta (price sensitivity) is higher.

*Figure 7: GOOG 650 December 2012 Pricing Model Forecasts vs. Market Price*

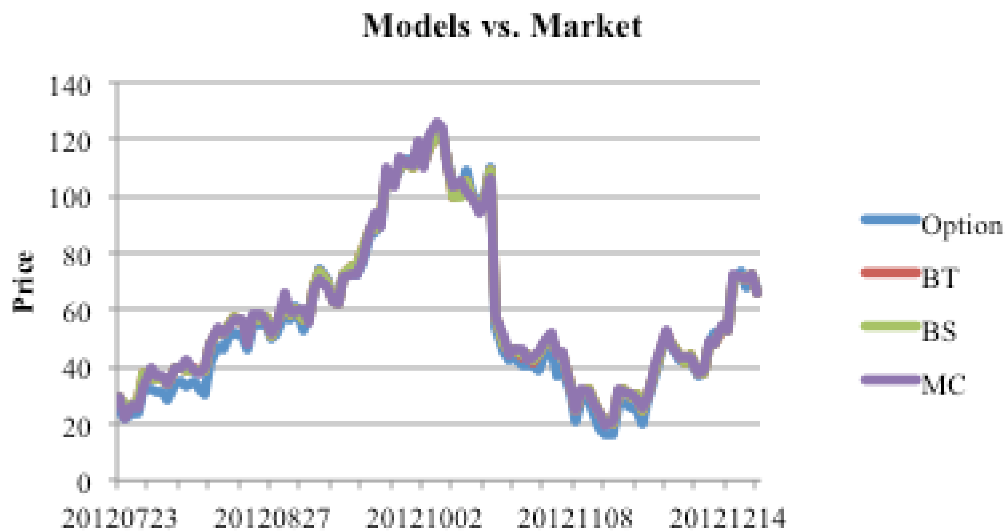
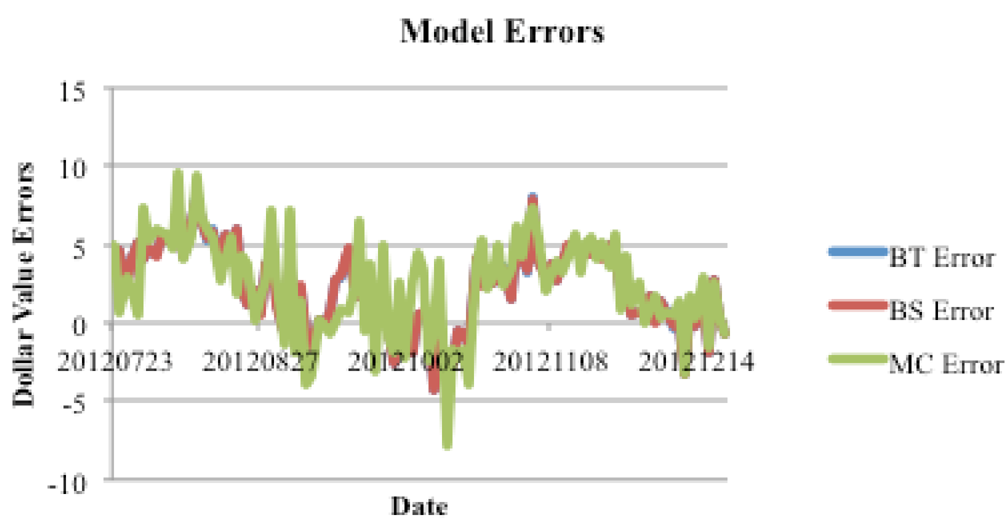



*Figure 8: GOOG December 2012 650 Model Errors*




The patterns in the above graphs nearly mirror the patterns seen in the other GOOG December 2012 contract. Since the underlying stock for both contracts was the exact same, it is reasonable to expect the Diebold-Mariano test to produce similar significant results, but the difference in significance patterns is strange.

*Table 8: GOOG 650 December 2012 Theil MSE Decomposition*

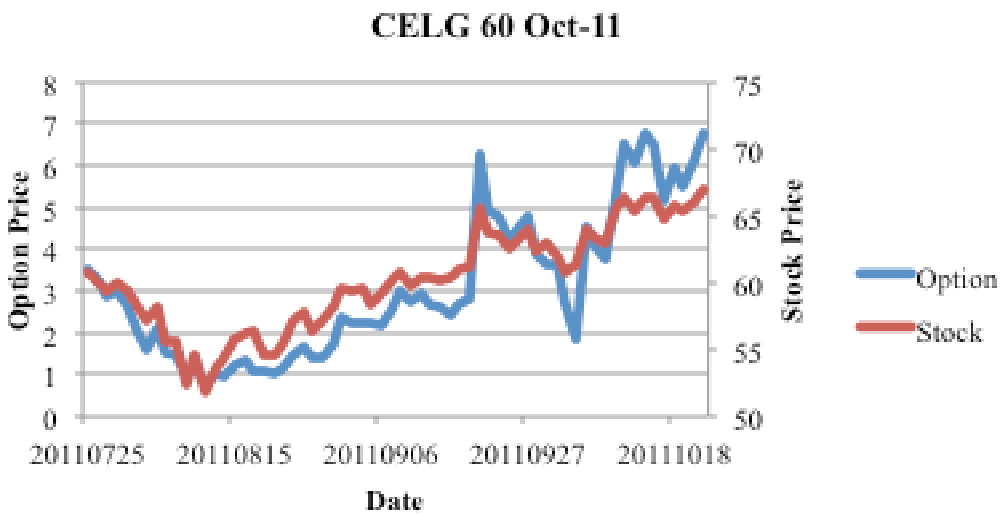
| | MSE | Bias | Variance | Noise |
|---|---|---|---|---|
| **Binomial Tree** | 100.00% | 43.10% | 24.50% | 32.40% |
| **Black-Scholes** | 100.00% | 42.91% | 24.38% | 32.71% |
| **Monte Carlo** | 100.00% | 38.12% | 13.38% | 48.50% |

The Theil decomposition of the 650 strike contract is much different than the decomposition of the 750 strike contract. The biases are all lower while the variance terms are higher. Additionally, the noise terms are all higher. The same pattern of the Monte Carlo noise term being higher than that of the other two models remains the same. It is possible that the different relative composition of the MSEs of the models of the 650 strike contract are responsible for the different Diebold-Mariano results. The results could also be driven by the higher delta of the 650 strike contract, skewing the sensitivities of the models to price changes in the underlying stock.

**CELG 60 October 2011 Call**

The final contract that is examined further in depth is the CELG 60 October 2011 as it represents a typical contract that I analyze.

*Figure 9: CELG Stock Price and 60 October 2011 Option Prices*




As seen in Figure 9, this CELG contract finished strongly in the money. There is a clear positive relationship between the stock and the price of the option (as we would expect), and there were no large price swings in the underlying stock.

*Figure 10: CELG 60 October 2011 Pricing Model Forecasts vs. Market Price*

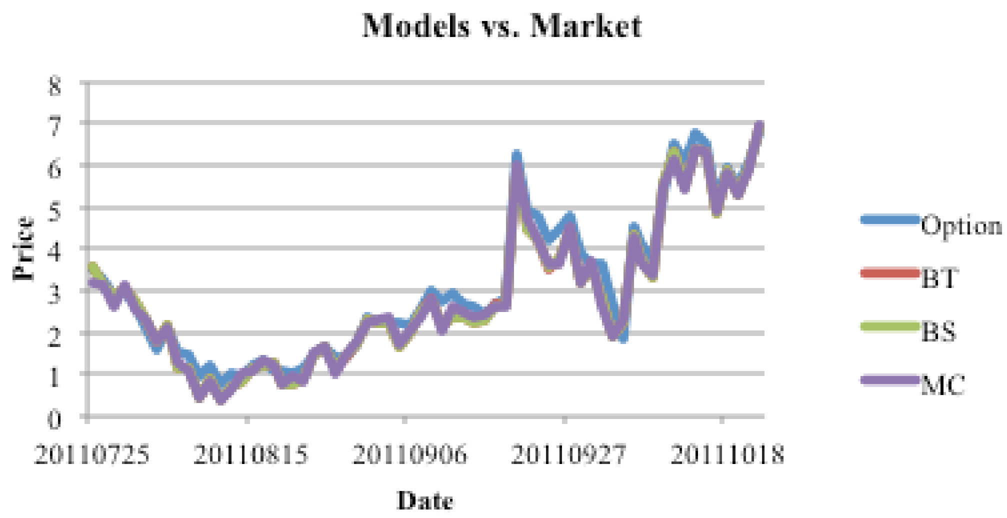
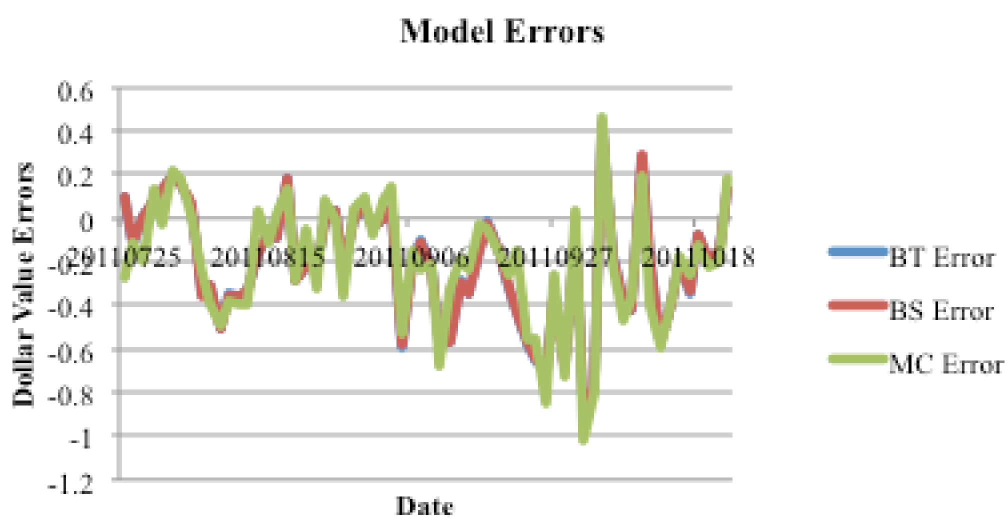



*Figure 11: CELG 60 October 2011 Pricing Model Errors*




Similar to the GOOG contracts, the CELG contract model forecasts track the observed market price over the life of the contract. Unlike the GOOG contracts, however, the CELG models tend to have a negative bias. This negative bias can potentially be attributed to the heightened volatility seen in the markets—as expected future volatility was higher than the trailing 2 year volatility, the market price of the option increased faster than the models which used historical data. This explanation for the negative bias may not be entirely correct, however, as the GOOG contracts both experienced significant stock-specific volatility.

*Table 9: CELG 60 October 2011 Theil MSE Decomposition*

| | **MSE** | **Bias** | **Variance** | **Noise** |
|---|---|---|---|---|
| **Binomial Tree** | 100.00% | 37.99% | 0.70% | 61.32% |
| **Black-Scholes** | 100.00% | 38.18% | 0.61% | 61.21% |
| **Monte Carlo** | 100.00% | 35.53% | 1.03% | 63.44% |

Unlike the GOOG contracts, the Theil MSE Decomposition of the CELG models is more consistent. All three models produced relatively similar breakdowns. Additionally, the noise term of the models is higher than for the GOOG contracts, a better indicator for the models. The variance terms are extremely small as well.

There are several conclusions that we can draw from these three case studies. First, volatility driven by internal company events can have a much stronger influence over the pricing of the stock's options than does a general market increase in volatility. Second, while there are some patterns exhibited by the options models that are consistent across contracts, there are many factors that influence volatility and options pricing in general. And finally, though models based on historical data are not entirely accurate, they do provide a quick way to price options that is simple and relatively close to market prices.

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# Academic Vita
# Dale Peterson

## EDUCATION

**The Pennsylvania State University | Schreyer Honors College** — **University Park, PA**
*Bachelor of Science in Finance, Minors in Mathematics and Statistics* — *Class of May 2015*

## RELEVANT EXPERIENCE

**Lincoln International, LLC** — **New York, NY**
*Investment Banking Summer Analyst* — *May 2014 – July 2014*
- Formulated DCF, LBO, and accretion/dilution models through reference of management and external information
- Served on several live transactions in the chemicals, business services, and automotive industries with responsibilities including drafting client materials, performing industry research, and processing company information
- Performed buyer diligence to evaluate firms' fit as potential strategic and financial buyers of businesses

**Nittany Lion Fund, LLC** — **University Park, PA**
*Vice President* — *January 2014 – December 2014*
- Maintained relationships with investors through presentations and distribution of fund materials
- Led weekly organizational meetings of 35 members on topics including current events, investing, and finance

*Lead/Associate Fund Manager, Financials Sector* — *May 2012 – December 2013*
- Managed a $1.1 million portfolio of Financials equities within the $6.8 million fund, resulting in outperformance of the Financials Sector of the S&P 500 by 1.5% during the 2013 calendar year
- Analyzed companies quantitatively and qualitatively through study of discounted cash flow valuations, excess equity valuations, financial statements, comparable ratios, and news to present buy and sell recommendations
- Constructed a self-updating portfolio tracker using Excel and FactSet to analyze performance of the fund

**JL Squared Group, LLC** — **Philadelphia, PA**
*Summer Analyst* — *June 2013 – August 2013*
- Conducted due diligence on hedge funds across a variety of strategies through in-person meetings, conference calls, scenario analysis of disclosed holdings, and reviewing of legal documents to analyze potential investments
- Produced statistical analyses on hedge fund returns through metrics including correlation, R-squared, Sharpe ratio, and volatility to assist portfolio managers in allocating capital

**Penn State Trading Room** — **University Park, PA**
*FactSet and Morningstar Analyst* — *September 2013 – Present*
- Created and presented seminars with teams covering FactSet and Morningstar software and their application to analysis of historical economic events

## LEADERSHIP EXPERIENCE

**Penn State IFC/Panhellenic Dance Marathon (THON)** — **University Park, PA**
*Merchandise and OPPerations Committee Member* — *October 2013 – February 2015*
- Ensured logistical effectiveness of THON, the largest student-run philanthropy in the world, and related events

**Student Support Services Program** — **University Park, PA**
*Tutor* — *September 2012 – May 2014*
- Guided students in "Introduction to Probability Theory," "Elementary Statistics," and "Basic Management Concepts," leading to improved exam and graded assignment performance

**Penn State Fencing Club** — **University Park, PA**
*Team Member* — *August 2011 – Present*
- Performed inventories and assisted with setup of tournaments to support budgeting process and operational efficiency
- Trained at tri-weekly practices and competed in club and regional tournaments

## SKILLS & INTERESTS

**Skills:** Proficient in Bloomberg, FactSet, S&P Capital IQ, and Microsoft Office (Excel, PowerPoint, and Word)
**Interests:** Tang-Soo-Do Karate (Fourth Degree Black Belt), Fencing, Running, and Reading