THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

DEPARTMENT OF ECONOMICS

A CONSUMPTION BASED ASSET PRICING MODEL OF THE YIELD CURVE

NATHAN BORN
SPRING 2015

A thesis
submitted in partial fulfillment
of the requirements
for a baccalaureate degree
in Economics
with honors in Economics

Reviewed and approved* by the following:

Alessandro Dovis
Assistant Professor of Economics
Thesis Supervisor

Russell Chuderewicz
Senior Lecturer in Economics
Honors Adviser

* Signatures are on file in the Schreyer Honors College.

Abstract

This paper seeks to create a representative model of the yield curve by combining the standard consumption based asset pricing model used by Canzoneri, et al and the equation for the Pure Expectations Hypothesis of the Term Structure of Interest Rates. I begin with reviewing different theories of the yield curve. I then use consumption based asset pricing model in conjunction with the expectations hypothesis to see whether the models can accurately represent the data on yield curve slopes. I conclude with an examination of the forward looking aspects of the yield curve. I find that the forward-looking nature of the yield curve which is implied by the Expectations Hypothesis is not entirely reliable in predicting real GDP.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Introduction

Imagine if policymakers and businesses could know whether a recession is coming. That information could help reduce the policymakers reduce the economic pain felt by consumers during periods of recession. Therein lays the fascination of economists and investors alike in the yield curve. Can expectations of future interest rates allow us to predict recessions based on information available today? This paper seeks to shed some light on these issues.

The first step to understanding the potentially forward-looking nature of the yield curve is to understand the role that nominal interest rates play in the economy. Interest rates contain valuable for information for investors, guiding the flow of capital to projects which create value for the investor, consumers and society. Interest rates also influence how people spend their money. High interest rates may induce some people to save more money, forgoing consumption, while low interest rates may induce others to either consume more or buy different types of assets. In general, interest rates play a key role in financial markets and can contain information about the state of the economy.

One way to extract this information from interest rates is to look at the yield curve. According to the Expectations Theory of the Term Structure of interest rates, the yield curve can be used to dissect the expectations of the market regarding future short term interest rates.

This paper seeks to test the Expectations Theory of the Term Structure of Interest Rates through a consumption based asset pricing model first developed by Robert Lucas in 1978 by comparing model generated yield curve slopes through time to the actual slope of the yield curve. I begin the paper with a general outline of the yield curve, explanations of the theories which seek to explain the yield curve, and empirical evidence in favor or against each theory of the yield curve. I will then outline Lucas's Consumption Based Asset Pricing Model (CBAPM) and explain its connection to the yield curve. Finally I will use the CBAPM and the Pure Expectations Hypothesis to create implied interest rates for treasury securities which will be used to create implied yield curves. The implied yield curves should be consistent with actual yield curves at different points in time if the expectations theory and the CBAPM hold.

General Outline of the Yield Curve

The yield curve is a financial instrument, which charts the path of interest rates for securities which vary only in time to maturity. As seen in Figure 1, the vertical axis of the yield curve represents the interest rate while the horizontal axis tracks the time to maturity. Securities used to model a given yield curve must be identical in every way except for their length to maturity. The most frequently modeled type of yield curve tracks the path of interest rates for risk-free government debt securities which are identical besides the fact that they have different maturity dates. The question for economists is why the interest rates of these securities changes when the time to maturity changes.

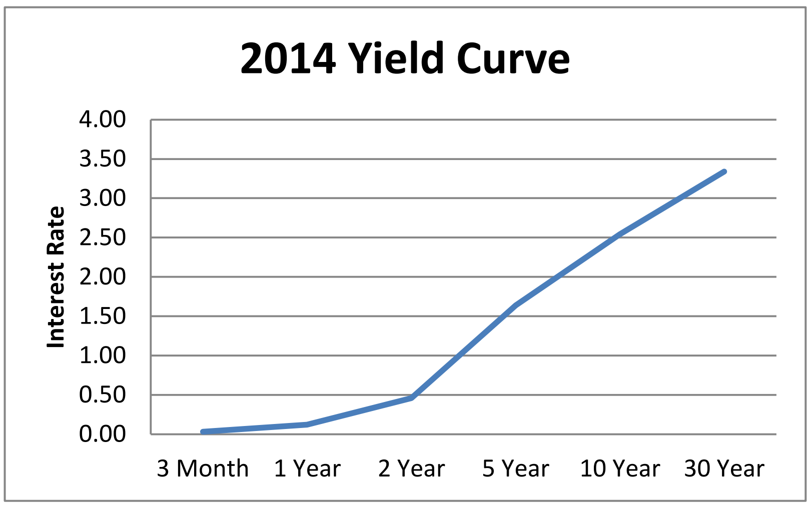

Figure 1 – The ten year yield curve

The relationship between interest rates of different security terms is complex. One must consider the real life decision that the investor is making when he or she chooses to purchase a bond – they are effectively forgoing consumption today in exchange for consumption in the future. The price of assets should therefore reflect the aggregate economy's consumption preferences. The price of the asset should also reflect risks involved. Yet how are these factors mathematically represented in the interest rate?

Economists have proposed three general theories as to why the interest rate changes as length to maturity changes. The first theory and the focus of this paper is the expectations theory of the term structure of interest rates. The liquidity premium theory and the preferred habitat theory are other proposed alternatives (Cox, 1985). The question of which theory holds best in reality is of great concern to economists because of the implications associated with each different theory.

The Pure Expectations Theory of the Term Structure of Interest Rates

The expectations theory of the Term Structure of Interest Rates suggests that the interest rate on a long term security is the geometric mean of a series of expected short term interest rates of interim periods of time. Some models also include a constant risk premium term:

$$(1 + i_{lt})^n = \left(1 + i_{st}^{year\ 1}\right)\left(1 + i_{st}^{year\ 2}\right) \ldots \left(1 + i_{st}^{year\ n}\right) + rP$$

The expectations theory finds its origins in Irving Fisher's *The Theory of Interest*, which first proposed that the yield on a five year debt security was the average of expected one year securities over the course of five years (Fisher, 1930). Friedrich Lutz's 1940 paper built upon Fisher's proposition which challenged the commonly held belief that interest rates of different maturities simply moved together, implying arbitrage opportunities. Lutz on the other hand based his theory on the assumption that there were no arbitrage opportunities for bond investors. In other words, an investor that buys one-year bonds and relends the money every year for ten years will make the same amount of money as someone who buys a ten year bond (Lutz, 1940).

The implied equality between the earnings of the individual who lends once for ten years and the individual who lends for one year every year for ten years implies that the two year interest rate is the expected average of the one year interest rate in period one and the one year interest rate in period two. The yield curve could then theoretically be used to extract the market's expectations of future events, which would be valuable information for policymakers, businesses, and investors.

Some such as Wright posit that the yield curve can be used as a forward looking economic indicator. Theoretically, the yield curve could become inverted when the market believes a recession is approaching (Wright, 2007). The inversion is thought to be caused by the expectation of lower short term interest rates in the future which would occur as a result of anticipated countercyclical monetary easing and the flight to safety – which occurs when investors seek low risk income opportunities, such as bonds, during times of economic uncertainty.

An inversion could be caused by countercyclical monetary easing because investors expect the central bank to lower short term interest rates in the future, causing the geometric mean of expected short term interest rates to become lower than the current short term interest rate. For example suppose we assume that investors expect interest rates to be constant in the future, equal to the current interest rate.

$$(1 + i_{lt})^n = \left(1 + i_{st}^{year\ 1}\right)\left(1 + i_{st}^{year\ 2}\right)$$

$$i_{2t} = i_{st}{}^{year\ 1} = i_{st}{}^{year\ 2}$$

If the market expects a recession to occur, they may expect the central bank to intervene and lower interest rates in the future as a result. Since monetary policy comes with inherent information and implementation lags, the investors will not expect the current one year interest rate to fall to the degree that they expect future short term interest rates to fall. This causes:

$$i_{st}{}^{year\ 1} > i_{st}{}^{year\ 2}$$

This causes the inversion in the yield curve:

$$i_{2t} < i_{st}{}^{year\ 1}$$

Investor expectations as to the course of future short term interest rates inform the forward guidance policies of modern central banks. Lutz observes that if there is a structural relationship between the long and short term interest rate, the central bank can affect long term interest rates by changing the discount rate and committing to keep it low for a considerable period of time (Lutz, 1940). The intuition behind the action of the Bank of Japan following the Japanese stagnation in the late 1990s and the action of the US Federal Reserve following the Financial Crisis of 2007 is the same. The central bank can lower long term interest rates by promising, as the Fed and the Bank of Japan did, to keep short term interest rates and the federal funds rate low for a considerable amount of time.

If a business placed a great deal of trust in the Expectations Hypothesis, they could use the yield curve to plan the financing of future projects. If the project was a short term or small investment that could be paid back in a year or two, the business could examine the yield curve to find when short term interest rates would be lowest, minimizing the business's costs (Lutz, 1940).

If the Expectations Hypothesis holds, an individual investor could use the yield curve to find the best times to enter and exit the market. Buying assets while the yield curve is upward sloping with the anticipation of robust economic growth could yield high profits during booms. Selling assets when the yield curve becomes inverted could shield the investor from losses during economic downturn.

So is the Expectations Hypothesis an answered prayer – the full-proof method to beat the market? Unfortunately the only real fortune tellers work at the circus, and they aren't known for giving reliable financial advise. Great deals of empirical and theoretical problems plague the

Expectations Hypothesis. Most studies using data from the United States reject the model but some international studies suggest further investigation. For example, Taylor rejects the Expectations Hypothesis via a Granger-causality test (Taylor, 1992), while a study conducted by MacDonald and Speight using United Kingdom data generates mixed results (MacDonald, 1988).

The assumptions made by the Expectations Hypothesis can also be called into question. The debate over whether the market is able to use all available information at any given time, as the Expectations Hypothesis assumes, is a contested issue. Another theoretical problem is risk variance. As mentioned before most models of the Expectations Hypothesis have either a constant risk premium term or no risk premium term. Macaulay pointed out this issue as early as 1938, while the Expectations Hypothesis was in its infancy (Macaulay, 1938). Variance in risk premium coupled with the repeated empirical shortcomings of the expectations model leads economists to consider the liquidity preference model of the yield curve.

Liquidity Preference Theory

Lutz presents future economists a further opening as he notes the inherent risk in holding bonds. Bonds, an asset which is not convertible into goods and services, are less liquid than money (Lutz, 1940). This alone warrants a risk premium regardless of how easily a bond could be converted into cash. There is always a chance that the seller cannot find a buyer if they need immediate cash. The risk premium compensates the investor for this risk. The key difference between the Expectations Hypothesis and the Liquidity Preference Theory is that the Liquidity Preference Theory allows risk to vary through time. An investor that buys an asset during a time of perceived economic downturn may demand a higher risk premium than they would during a stable time of economic growth. Formally expressed, the Liquidity Preference Theory implies the possibility of a changing risk premium:

$$(1 + i_{lt})^n = \left[\left(1 + i_{st}^{year\ 1}\right)\left(1 + i_{st}^{year\ 2}\right) \ldots \left(1 + i_{st}^{year\ n}\right)\right] + rP_n$$

The lack of perfect foresight also demands consideration for a risk premium. There is always an opportunity cost in any decision and purchasing a bond is no exception. For example, an investor that buys bonds instead of stocks risks the opportunity cost of the appreciation and dividends of stocks they could have bought with the money that they used to purchase a bond. Another risk that a bond investor assumes is the assumption of the Expectations Hypothesis – that the investor is no better off purchasing a five year security than purchasing a one year security every year for five years. If market expectations as to the course of future short term interest rates prove false an investor holding long term securities may incur a capital loss.

According to the liquidity preference theory, the market could compensate the investor of a long term security with a risk premium in addition to the yield received as a result of the expected course of short term interest rates. Specifically, the empirical work of Fama and Bliss find through long regressions that the term premium is increasingly negative as the term to maturity rises (Bliss, 1987).

Preferred-Habitat Theory

The Preferred-Habitat theory is different from Pure Expectations Hypothesis and the Liquidity Premium Theory because it assumes that securities of different term lengths are not perfect substitutes. According to the Preferred-Habitat Theory, each individual term length has a unique market of investors which is influenced only by supply and demand shocks to that market. The preferred-habitat model implies that central bankers cannot influence long term interest rates through short term interest rates. Instead, the Preferred-Habitat Theory argues that the central bank should buy or sell securities of the interest rates that they seek to affect.

Therefore, communication policies which seek to influence long term interest rates by altering the expected path of future short term interest rates is useless (Doh, 2010). The empirical work of Vayanos and Vila examines the US Treasury's long term bond buyback program of 2000-2002. Three weeks following the announcement of the US Treasury to buyback long term bonds, only the yields on long term bonds were affected (Vayanos, 2009). In other words, the Federal Reserve was able to influence long term interest rates without influencing the expected path of future short term interest rates, as the Expectations Hypothesis would suggest. Directly targeting the supply of these long term bonds was enough to lower long term interest rates.

The empirical work of Taylor comes to a similar conclusion. He observes that the government debt repurchasing program that occurred in the United Kingdom may have caused inversions of the UK yield curve. Outside of these empirical studies however, there is little research on the model (Taylor, 1992).

Pulling Everything Together

How can we test the Expectations Hypothesis? Many studies use historical data on interest rates and the forward spread to test the equation for the Expectations Hypothesis. I, however, adopt the standard asset pricing model laid out in a paper by Matthew Canzoneri, Robert Cumby and Dehzad Diba of Georgetown University – the standard consumption based asset pricing model (CBAPM) (Canzoneri, 2007). Why use an asset pricing model? The strongest case for using an asset pricing model is that there is no research firm that asks every investor what they expect the one year interest rate to be in one, two, three…or ten years. An asset pricing model allows us to generate expectations conditional on information that would be available to investors at the time in question. The model is based on assumptions which do not conflict with the Expectations Hypothesis.

One advantage of using the CBAPM to price assets is that the model focuses on the microeconomic principles associated with purchasing a bond – the opportunity cost to the representative agent. If I purchased a one year bond today, the money I used to purchase the bond would not be available until the bond reached maturity – I forgo consumption today in exchange for the opportunity to consume tomorrow.

I follow the example of Jeffrey Fuhrer and adopt the log-linearized adaptation of the model (Fuhrer, 2000) which endogenizes a component of negative covariance between consumption and the interest rate. This is an attempt to capture the "flight to safety effect" often seen during periods of economic uncertainty. When investors expect deterioration in growth and profits, they often turn towards government backed bonds, which are effectively riskless (with of course the exception of Greek government bonds). In short, when consumption is expected to fall, the desirability of bonds is expected to rise. This causes the price of bonds to rise and the interest rate to fall.

The closer that the model can come to representing the way an investor evaluates the desirability of an asset, the closer we can model the investor's expectations. If we can model the investor's expectations through time, we can find out whether the Expectations Hypothesis matches reality.

Methodology – CBAPM and the Expectations Theory

Theoretical Outline – the nuts and bolts

The CBAPM, our formal formulation of an economy that prices assets based on consumption, is derived from the Lucas Asset Pricing Model. The Lucas Model, often called the Lucas Tree Model, is based upon the idea that individuals base investment decisions on the marginal utility of consumption.

Lucas imagines an economy with one type of asset, where consumption is equal to aggregate output. The stock of the asset and population is fixed but the yield on the asset is stochastic, or random. Shares of the asset are perfectly divisible, allowing consumers to buy and sell portions of the asset in a stock-market-eque exchange. Shares of the asset can be traded after the payment of the real dividend of the asset to the owner. Therefore consumption is determined by the yield on the asset as well as the proceeds from selling. Lucas also assumes that the price endogenizes all available information and the assumption of rational expectations (Lucas, 1978).

Christopher Carroll attempts to illustrate Lucas's asset pricing model by imagining shares of fruit trees as the asset and the yield on the asset as fruit. Each fruit tree produces an identical yield, but yields are stochastic through time. The consumer in this model is concerned only with maximizing lifetime utility with minimal volatility. The impatience of the consumer is reflected by the subjective discount factor, beta. When beta is equal to one, the consumer is indifferent between consumption now and consumption in the future. The lower the beta term, the higher the impatience is.

$$U(C_t, C_{t+1}) = u(C_t) + \beta E_t[u(C_{t+1})]$$

We assume, as Cochrane does, that consumption and inflation are lognormal.

$$\ln(C) = c$$

The individual will seek to maximize this function subject to income and payoff constraints given below, where y represents income in a given period, p represents the price of an asset, and Q represents the quantity of the asset purchased.

$$c_t = y_t - p_t Q$$

$$c_{t+1} = y_{t+1} + x_{t+1} Q$$

We see from the constraints that purchasing an asset involves a tradeoff between consumption today and consumption in the future. When we substitute these two constraints into the utility function and take the first order condition with respect to the quantity of an asset purchased we find the basic pricing equation seen below.

$$P_t = E_t[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}]$$

We see from the equation that the price of an asset is determined by its expected payoff, the marginal utility of consumption and the subjective discount factor. The expression is consistent with an individual who seeks to smooth consumption subject to their respective degree of impatience. The price of an asset falls when the marginal utility of consumption in the following period falls. Cochrane further breaks down this expression by introducing the stochastic discount factor or pricing kernel, which integrates all of the individual’s preferences of marginal utility and impatience.

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

The first step to creating our consumption based model is to define the utility of consumption. I use the power utility function given below:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The shape of the utility function meets some standard economic assumption – utility rises with consumption, yet returns to consumption are diminishing. Finding *u'(c)* we are left with the expression for the relative marginal utility of consumption:

$$u'(c) = c^{-\gamma}$$

Therefore we can rewrite the pricing kernel:

$$m_{t+1} = \beta (\frac{c_{t+1}}{c_t})^{-\gamma}$$

We can take the expression above and substitute it back into the original pricing equation to find the subjective discount factor, which we assume to be constant through time.

$$P_t = E_t[\beta (\frac{c_{t+1}}{c_t})^{-\gamma} x_{t+1}]$$

To find the subjective discount factor, we analyze the average risk-free real interest rate, providing us with the following information:

$$x_{t+1} = P_t(1 + i_t)$$

Substituting the condition we can solve for beta:

$$\frac{P_t}{\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} P_t(1+i_t)} = \beta$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\gamma} \frac{1}{(1+i_t)} = \beta$$

Before we calculate beta, there are more assumptions that we can make. The first and most important assumption is that a unit of consumption today is preferable to a unit of consumption tomorrow. Even if the marginal benefit of consuming now and consuming a year from now were equal, we assume that consumption today is preferred. Therefore we can assume that beta is greater than zero but less than one. Another assumption we are making is that beta is representative of a consistent behavior demonstrated by the representative agent. Therefore beta will be based upon long term averages of consumption growth (calibrated by gamma) and the short term interest rate.

The next part of the pricing kernel that we look at deals with ratios of marginal utilities of consumption in the future and current periods. Since the representative agent is basing the price of the asset on information that he knows at time, t, the model must have a forward looking component that uses only information available at or prior to time, t, to forecast for the next period.

To create these expectations, I use vector autoregressions – abbreviated VAR(p) – with p lags. A vector autoregression is an econometric model that treats lagged values of two or more variables as independent variables. For example a VAR (2) of GDP would be a regression of GDP on GDP in the period before and GDP two periods ago. This is expressed below:

$$GDP = A_0 + A_{11}GDP_{t-1} + A_{12}GDP_{t-2} + \varepsilon$$

The individual equations for our specific symmetric VAR are given below. We naturally need the expectation of future real consumption and values for expected inflation to convert our expectations into nominal terms:

$$C = c_{11} + a_{11}C_{t-1} + a_{21}\pi_{t-1} \ldots a_{p-1,1}C_{t-p} + a_p\pi_{t-k}$$

$$\pi = c_{21} + a_{12}C_{t-1} + a_{22}\pi_{t-1} \ldots a_{p-1,2}C_{t-k} + a_{p,p}\pi_{t-k}$$

Bringing it all together…

I use the real consumption expenditures price index, one year constant maturity interest rate and consumption, all of which are available from FRED. I convert real consumption levels and price levels into natural logs to make it easier to find growth since the change in natural logs is roughly the percent change. This makes values much easier to interpret.

Next I calibrate beta, the subjective discount value using averages of log consumption growth and average real interest rates. I also pick the constant relative risk aversion (gamma). In this case I select 2, consistent with the work done by Canzoneri et al. This yields a beta of .996. The expression for beta is given below:

$$\left(\frac{\overline{c_{t+1}}}{c_t}\right)^{\gamma} \frac{1}{(1 + \overline{\iota_t})} = \beta$$

The next set of information that I need is future consumption and inflation which is based upon information available to the represented agent at the time. To do this I use a vector autoregression to forecast inflation and consumption. First though I must use the augmented Dickey-Fuller test to check for a unit root. Using sets of data which contain unit roots, or variables whose mean changes through time, would make the process far more complicated than anyone would want it to be. I find that inflation does not have a unit root, but natural log level of consumption does. To remove the unit root I take first differences, which yields percent change in consumption. Running the augmented Dickey-Fuller Test on percent change of consumption allows us to reject the presence of a unit root.

Because vector autoregressions are autoregressive models that treat lagged values of the dependent variables as independent variables, selecting the optimal amount of lags is an important factor that can affect the conclusion of the model. I rely upon information criteria (abbreviated IC) tests to determine the optimal amount of lags. I use multiple lag information criteria tests to choose the amount of lags to use, since the quality of performance of the tests is ambiguous. Typically the test which minimizes the information criteria is selected (McMillin, 2010). These tests will help me to reduce information that maybe lost in excluding lags but prevent a loss in precision that could occur from including too many lags. Both the Akaike Information Criterion test and Schwarz-Bayesian Information Criterion test are minimized at p=1, confirming that the optimal amount of lags is one.

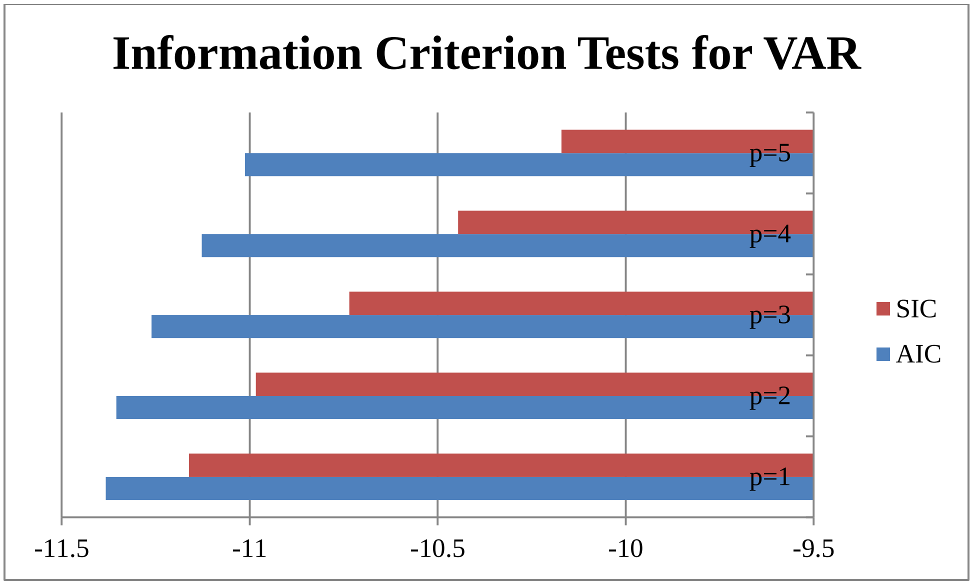

Figure 2 - Information Criterion tests for CBAPM

To generate a yield curve that extends ten years into the future, I must create forecasts for inflation and consumption for ten years into the future. From 1970 to 2014 I create a VAR in each year that is based on information only available to the representative agent in that year to create forecasts.

Vector Y_t will contain the elements of real consumption growth and inflation, and vector V_t contains the elements of the variance of consumption growth and variance of inflation respectively. Our VAR will take the form below:

$$Y_t = A_0 + Y_{t-1}A_1 + V_t$$

The variance covariance matrix is given below:

$$V_t * V_t{}' = \Sigma$$

$$V_{t+1} = A_1 \Sigma A_1' + \Sigma$$

$$\begin{bmatrix} Variance\ of\ Consumption\ Growth & Covariance\ of\ Consumption\ Growth, Inflation \\ Covariance\ of\ Consumption\ Growth, Inflation & Variance\ of\ Inflation \end{bmatrix}$$

Our VAR generates projections for consumption growth, which we must reconvert to natural log levels. This is found by dissecting the consumption growth equation from the VAR using the expression below:

$$\Delta c_t = c_{11} + a_{11}\Delta c_{t-1} + v_t$$

$$c_t = c_{11} + a_{11}(c_{t-1} - c_{t-2}) + c_{t-1} + v_t$$

We now apply our lognormal framework to our original pricing equation (e, Euler's Number, is expressed as 'EXP'), given below.

Since

$$C_t = e^{\ln(C_t)}$$

We can rewrite the pricing equation

$$\frac{1}{(1+i_t)} = \beta\ EXP\left[E_t\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\pi_{t+1}\right]$$

Because consumption at time t is known to the representative agent, we can move the expectation operator inward.

$$\frac{1}{(1+i_t)} = \beta\ EXP[\alpha(E_t c_{t+1} - c_t) - E_t\pi_{t+1}]$$

The expected value of an independent variable in the joint lognormal distribution is given below and is then applied to the pricing equation.

$$E(X) = e^{\mu + \sigma^2 \frac{1}{2}}$$

$$\frac{1}{(1+i_t)} = \beta\ EXP\left[-\alpha(c_t - E_t c_{t+1}) - E_t \pi_{t+1} + \frac{\alpha^2}{2} Variance\ c_{t+1} + \frac{1}{2} Variance\ \pi_{t+1} + \alpha\ cov(c_{t+1}\pi_{t+1})\right]$$

We derive the respective variances and covariance from the variance covariance matrix generated by the vector autoregression. The above equation provides us with the nominal interest rate of a security with a one year maturity. We then extend our forecasting model to project the interest rate of a security with a one year maturity two years into the future by replacing the term c_t with the term c_{t+1} and the term c_{t+1} with the term c_{t+2}. We do the same for inflation and repeat the process every year for ten years into the future. We then have a series of expected one year interest rates based off of information available to the representative agent at the given time, consistent with the assumptions of the Expectations Hypothesis. We then take the projected one year nominal interest rates and substitute them into the Pure Expectations equation given below to find the nominal interest rate for the two year, three year, five year, seven year, and ten year nominal interest rate.

$$(1 + i_{lt})^n = \left(1 + i_{st}^{year\ 1}\right)\left(1 + i_{st}^{year\ 2}\right) \dots \left(1 + i_{st}^{year\ n}\right)$$

I then subtract the one year interest rate from the three, five, seven and ten year interest rates to find the three year, five year, seven year, and ten year yield curves respectively to generate series of yield curve slopes. After generating the series of yield curve slopes from 1970 to 2014 I apply an OLS regression to the actual yield curve slopes of the respective maturities.

The coefficient and constant terms will provide insight on how strongly the Expectations hypothesis holds in reality.

The final product of this methodology will be a series of interest rate securities with maturities from one to ten. I will then analyze the implied and actual interest rates and the implied and actual yield curves. Finally I will analyze the role of the forward looking nature of the yield curve which is implied by the expectations hypothesis. Using Vector Autoregressions I will address this question and its implications for the Expectations Hypothesis.

Results

I use the CBAPM to generate series of interest rates from a maturity of one year a maturity of ten years for every year from 1970 to 2013. I then create yield curves based off of the differences in the one year and the three year, five year, seven year and ten year using a program that I develop in MATLAB. In this section I will present a statistical analysis of implied interest rates and actual interest rates, a statistical analysis of implied and actual yield curves, and an analysis on the forward looking aspects of implied and actual yield curves.

Interest rates

The consumption based asset pricing model was not able to replicate the variance, median, or average values for any specific type of interest rate which was forecasted by the model. Seen in Table 1, the model forecasted average values which were well-above actual values with variances far lower than actual variances. The higher variances of actual interest rates might be caused by an omitted variable bias. Although the model uses consumption, which is largely pro-cyclical, our model is not able to capture volatility driven by uncertainty or other variables that influence large fluctuations in financial markets. The median values for implied interest rates are also very different from the median values for actual interest rates. The bolded values in the following table indicate the result for the CBAPM and non-bolded values represent values of the actual type of asset. These results are similar to those obtained by Canzoneri's standard CRRA model of the money market, which had a mean of 11.56%, which is only .03% away from our one year mean.

Table 1- Statistics for actual interest rates for actual data and model generated data

	1 Year	3 Year	5 Year	7 Year	10 Year
Average	**0.1159** 0.0573	**0.1193** 0.0620	**0.1209** 0.0649	**0.1219** 0.0672	**0.1227** 0.0687
Variance	**0.0007** 0.0012	**0.0005** 0.0011	**0.0004** 0.0010	**0.0003** 0.0009	**0.0003** 0.0008
Median	**0.1190** 0.0575	**0.1208** 0.0624	**0.1223** 0.0630	**0.1230** 0.0657	**0.1238** 0.0671

The statistical results however for the implied and actual yield curves are more similar. In this sample variances are relatively low. This stands in contrast to the results for individual interest rates, where we see low volitilies for implied interest rates and high volitilities for actual interest rates. So is our model able to capture elements of volitility in interest rates or not?

Table 2 – Statistics for actual yield curves for actual data and model generated data

	3 Year Yield Curve	**5 Year Yield Curve**	**7 Year Yield Curve**	**10 Year Yield Curve**
Average	**0.0034** 0.0047	**0.0050** 0.0077	**0.0060** 0.0099	**0.0068** 0.0115
Variance	**0.00003** 0.00003	**0.0001** 0.0001	**0.0001** 0.0001	**0.0002** 0.0001
Median	**0.0031** 0.0059	**0.0043** 0.0088	**0.0050** 0.0111	**0.0058** 0.0124

Comparing variances of interest rates to that of yield curves may not help us analyze the model's effectiveness in capturing volitility. We are comparing apples to oranges – interest rates and yield curves are not the same thing. Therefore we cannot make a meaningful conclusion as to whether our model fails to capture certain aspects of the economy by comparing averages, variances, and medians.

Yield Curve Slopes

We now begin our formal evaluation of the implied and actual yield curves. To analyze the yield curve we take the three year, five year, seven year, and ten year nominal interest rates and subtract them from the one year nominal interest rate to obtain the slope of the yield curve. Figure 3, Figure 4, Figure 5 and Figure 6 illustrate both the slopes of the implied yield curves and actual yield curves for the three year, five year, seven year and ten year respectively.

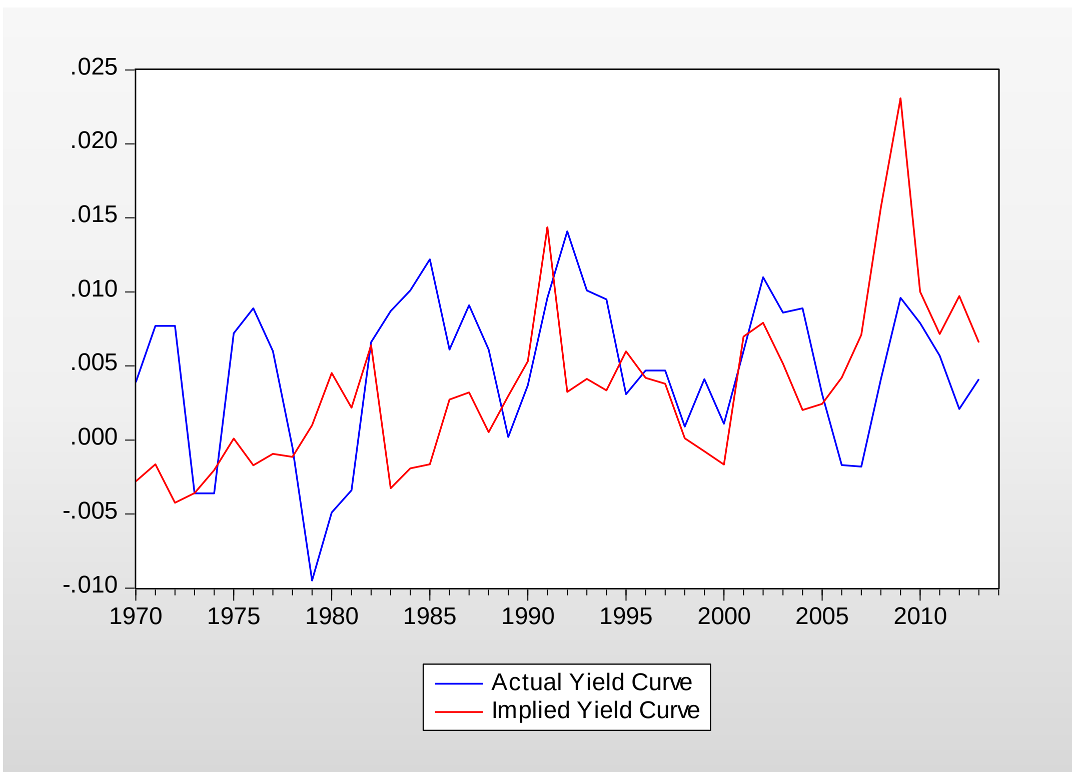

Figure 3 – Three year yield curve slope

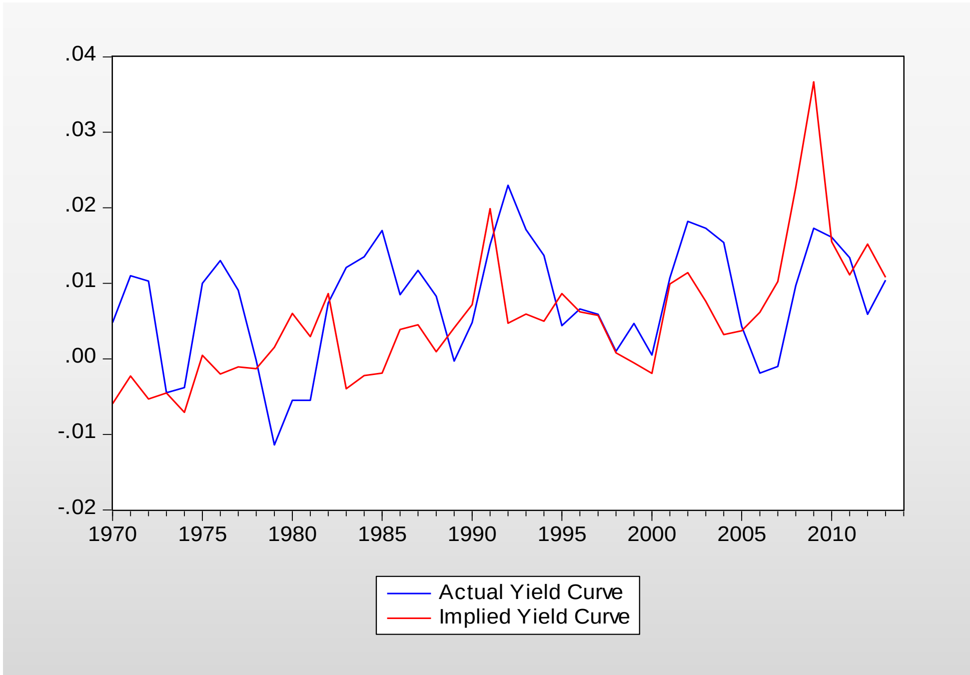

Figure 4 - Five year yield curve slope

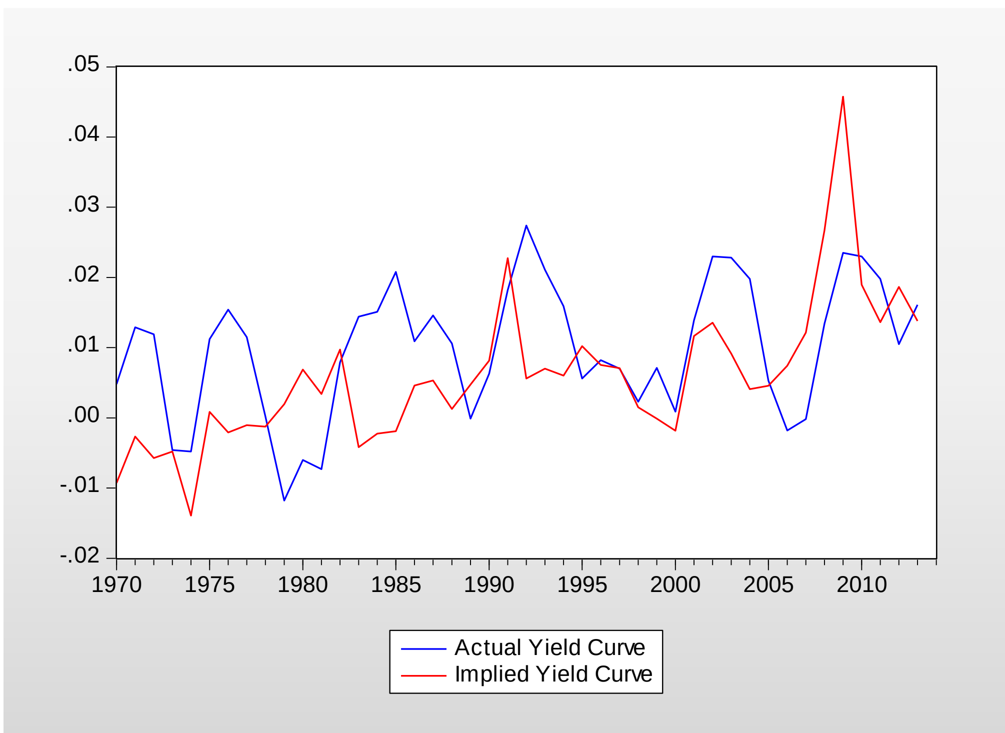

Figure 5 – 7 year yield curve slope

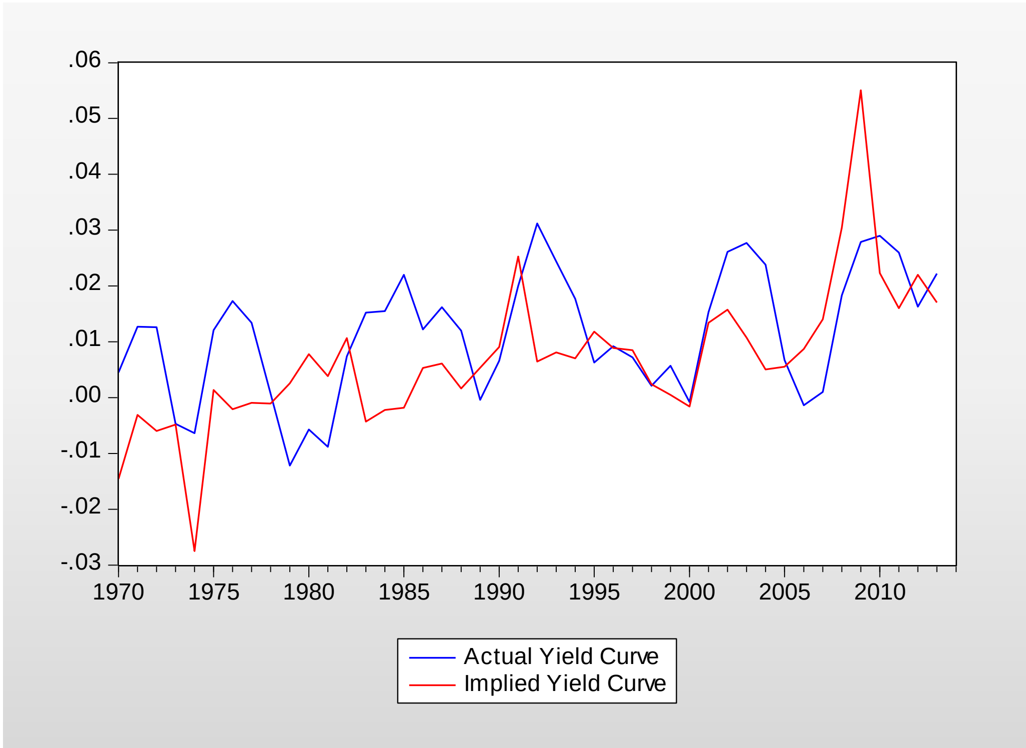

Figure 6 – 10 Year yield curve slope

To formally analyze the ability of the implied model to represent the actual yield curves, I use regressions of implied yield curves on actual yield curves. The results of these tests are mixed. The reason that I perform regressions on the yield curve data and not directly on interest rate data is because of the presence of unit roots in the interest rate data. Yield curves, both implied and actual, have no unit roots. We apply the following form to our regressions:

$$Actual\ Yield\ Curve\ Slope = \beta_0 + \beta_1 Implied\ Yield\ Curve\ Slope + \varepsilon$$

If the Expectations Hypothesis and the quality of our model hold, we should see a constant term of zero and a coefficient term of one. The three and five year yield curve models are unable to demonstrate a statistical relationship with actual three and five year yield curve models. However we do see that as we increase the length to maturity of the model, the results generally improve. We see that the seven year and ten year yield curve models can be said to be statistically significant with *p*-values less than one percent.

Table 3 – Regression Results for yield curve slopes

Yield Curve Length	Coefficient	Constant	T-statistic	R-squared
Three years	0.144181	0.004244	1.016252	0.024
Five years	0.290743	0.006221	2.132858	0.097727
Seven years	0.367462	0.007723	2.796832	0.157003
Ten years	0.415565	0.008638	3.479835	0.223793

Although we see that the implied yield curve is statistically significan for the regressions of the implied seven and ten year yield curves, we also observe that the coefficient is low (.37 and .42 respectively). The CBAPM, in conjunction with the Expectations equation, is unable to adequately model actual yield curves.

Addressing the Forward Looking Nature of the Yield Curve

To address the forwarding looking nature of the yield curve which is implied by the Expectations Hypothesis I generate VARs featuring lagged values of implied and actual yield curves and real gross domestic product growth. Below, we can visibly see that the ten year yield curve becomes inverted before a recession occurs. Shaded areas indicated offical NBER dates of recessions. We see that the yield curve becomes negative before the recession in the early 1970s and the recessions in the early 1980s.

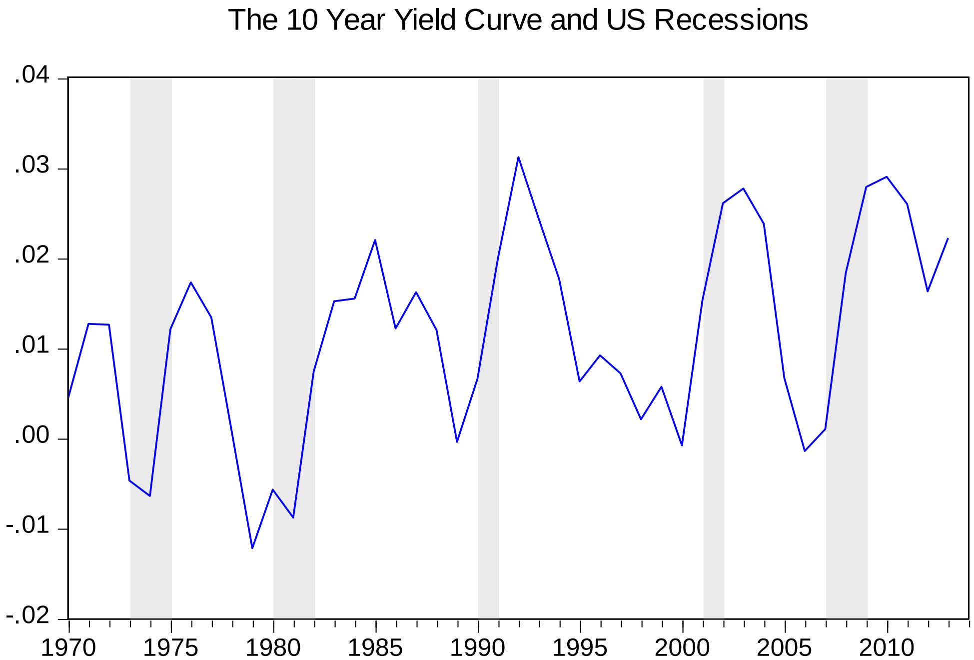

Figure 7 – The slope of the yield curve

Our implied yield curve, below, is not able to replicate this aspect of the actual yield curve.

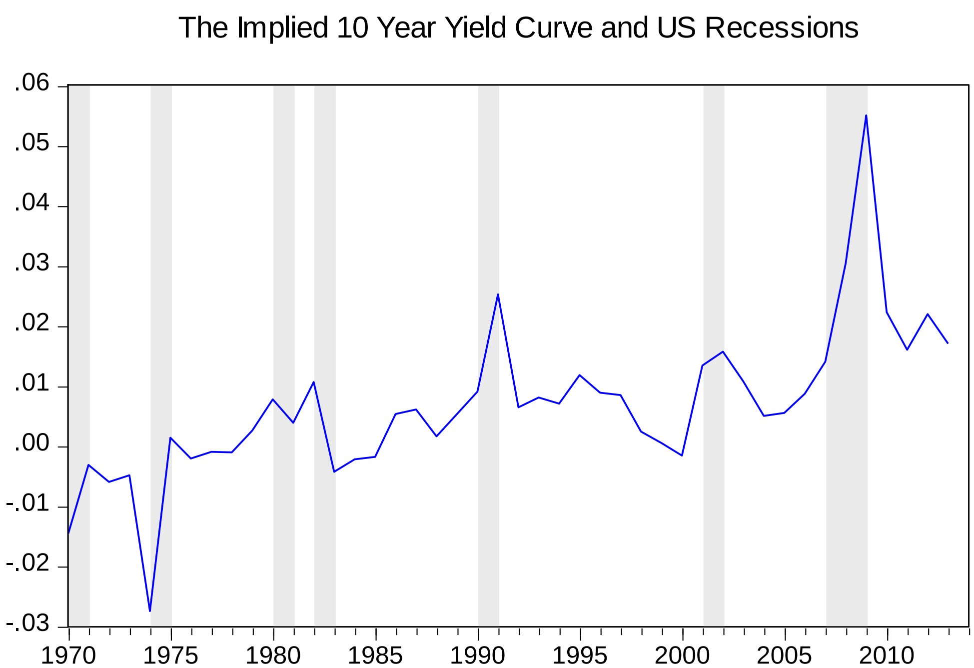

Figure 8 – The model generated yield curve slope

Although the model does not appear to have the same forward looking characteristics of the actual yield curve, we should nonetheless apply a rigorous statistical analysis before drawing any conclusions on either the actual yield curve or the implied yield curve.

Before any VAR estimation I check the Akaike Information Criterion Test and the Schwarz Information Criterion Test for VARs of real log GDP and the actual yield curve and real log GDP and the implied yield curve. Seen below, I select the model which minimizes the value of the criterion test.

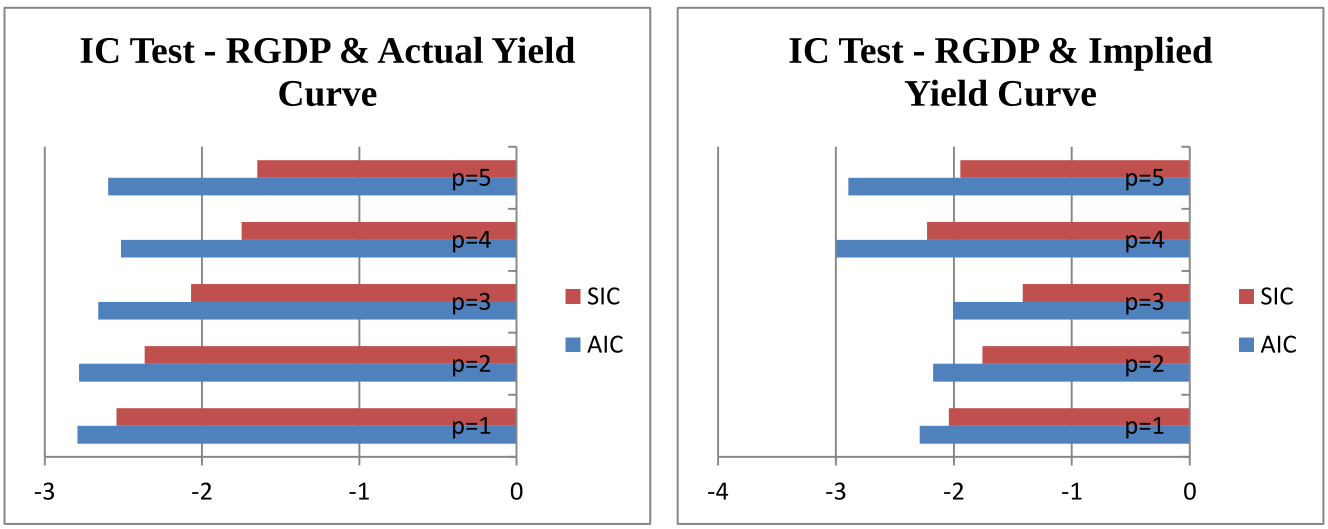

Figure 9 – IC tests for VARs of RGDP and actual/implied yield curves

Values for the IC tests are minimized for the VAR featuring the actual yield curve at p=1. The picture is less clear for the VAR featuring the implied yield curve. Nonetheless, we prefer the IC test which minimizes the value – RGDP and the actual yield curve – and in order to have a consistent analysis, we must be comparing apples to apples. Therefore we select the VAR of one lag for all models.

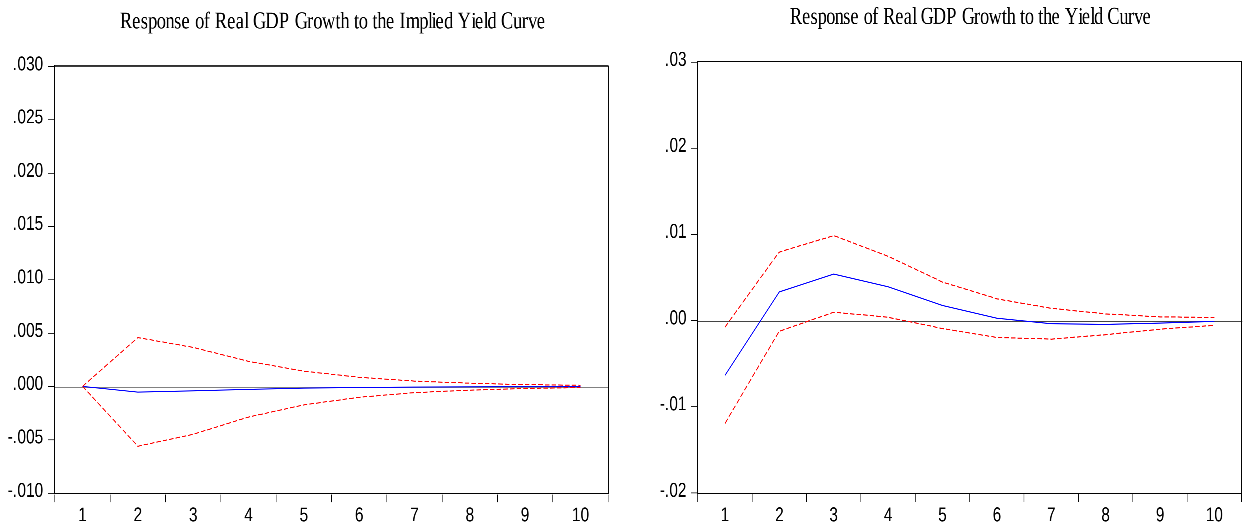

Figure 10 – Impulse response functions for implied/actual slopes and real GDP

The results do not bode well for our model. Our VAR analysis shows us that the implied yield curve has no forward looking ability. The actual yield currve, however, may have a limited forwarding-looking ability.

When we combine the information available in both the actual ten year yield curve and the implied ten year yield curve we see that there maybe some information about the future contained in the spread. Subtracting the implied yield curve from the actual yield curve creates a series that represents fluctuations between the two series.

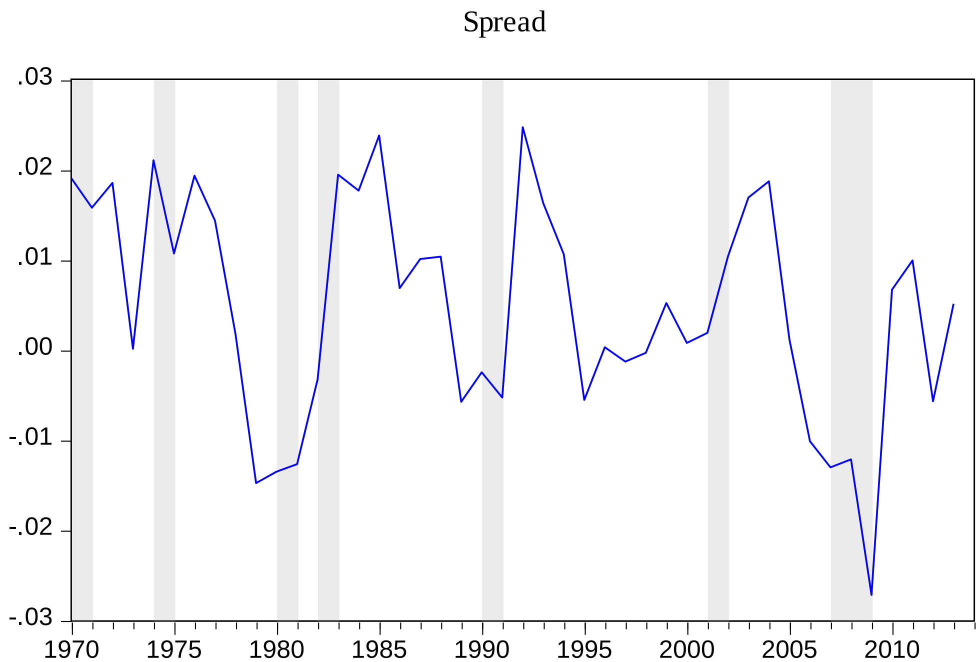

Figure 11 – Spread between the actual ten year yield curve slope and model slope

We see in Figure 11 that the spread falls before most of the recessions between 1970 and 2015. We find more evidence for a relationship through a vector autoregression (Figure 12) of growth and the spread. We find that a lagged shock in the spread causes a shock to growth. This technically serves as qualification of the spread as a forward looking indicator of real GDP growth. However compelling the statistical results may be the lack of a theoretical explanation as to the cause of the connection warrant further investigation before drawing a conclusion.

If we return to Figure 11 we can see that there are many points in time where the spread implies a recession that in reality never happen. For example we see that the spread falls from 1992 to 1995 but no recession follows. The spread fell again in 2012, and there was no recession. If an investor used the spread as an economic indicator they would have lost a lot of money betting against the economy in 1996 and 2013.

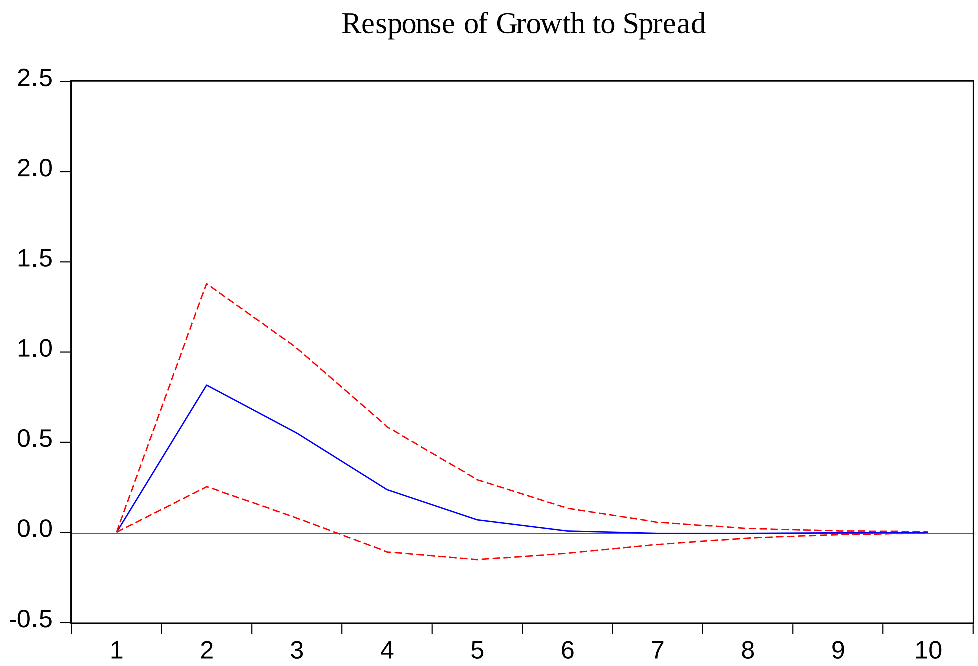

Figure 12 – VAR of the spread and real GDP growth

The combination of information available in our CBAPM model and actual yield curve data warrants further investigation. These models can help us gain some important insight on the ability of market expectations to forecast growth.

Pitfalls of the Model

One problem that is present in our model is the nature of the implied slope through time. The CBAPM model is unable to replicate the consistent upward sloping nature of the yield curve. Every version of the implied model has negative slopes ocurring more frequently than actual yield curves (Figure 13).

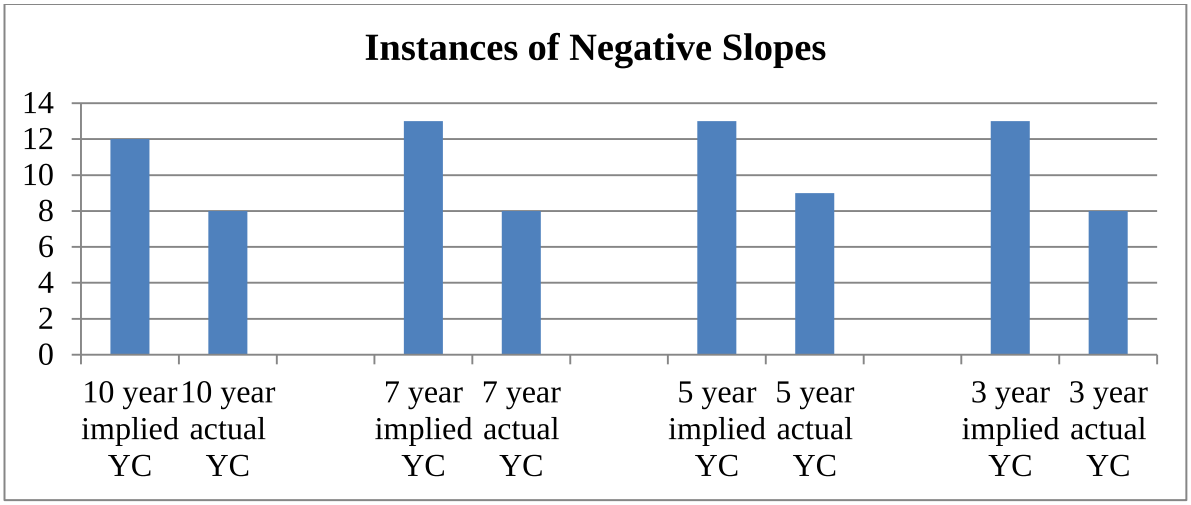

Figure 13 – Instances of negatively sloping yield curves (YC) of actual and implied data

Another potential problem with this model is the clarity of information available to the representative agent at the time that they price the asset. Often macroeconomic data is later revised after being initially published. This means that the representative agent might have been looking at a set of different data or a different forecast model than what we used to price the assets. For example, we know in hindsight that inflation in 1985 was about 4%. However an investor in 1985, not having knowledge of future revisions might have believed inflation to be 5% or 3%. This is a difficulty that Orphanides points out with his study on the Taylor Rule (Orphanides, 1997).

Another inherent problem in the CBAPM model is the difficulty in generating useful forecasts. Is it realistic to base decisions today on forecasts for consumption in ten years when weathermen have difficulties in determining whether it will rain in ten days? Although the yield curve and the model may represent the market's expectations, these expectations may never come to fruition. In other words, just because the weather station believes that rain is in the ten day forecast, it does not mean that it will actually rain. Expectations and reality can be two different things.

Conclusion

Fortune tellers belong in the circus, along with economists who believe that they can predict the future. Economic models, no matter how sophisticated, can help us to understand the economy and market expectations at certain points in time, but they cannot be used as tarot cards.

Although the CBAPM is based upon microeconomic principles that are frequently involved in pricing a given asset, the simplicity of the model is unable to account for the unpredictable and volatile nature of financial markets. The model predicted significantly lower variances and higher average values for individual interest rates than what actually exist. The CBAPM was also unable to replicate consistent upward sloping nature of the yield curve and the theoretical forward looking nature of the yield curve.

The experience was valuable to me however, in that I had the opportunity to learn how to use MATLAB (through plenty of trial and error) and merge two different theoretical models – the CBAPM and the Expectations Hypothesis. I also learned a very important lesson: humility. There are severe limitations to what economic theory can tell us about the future. Economics can teach us a lot about the world around us, but we should never forget that fortune telling and astrology are best left to the carnival.

Work Cited

Canzoneri, M., Cumby, R., Diba, B. (2007). Euler equations and money market interest rates: A challenge for monetary policy models. *Journal of Monetary Economics 54*. 1863-1881.

Carroll, C. (2014). The Lucas Asset Pricing Model. Lecture Notes. http://www.econ2.jhu.edu/people/ccarroll/public/lecturenotes/AssetPricing/LucasAssetPrice.pdf

Cochrane, J. (2000). *Asset Pricing*. Princeton University Press.

Cox, J., Ingersoll, J., & Ross, S. (1985). A Theory of the Term Structure of Interest Rates. *Econometrica 53*(2), 385-385.

Bliss, R., Fama, E. (1987). The Information in Long-Maturity Forward Rates. *The American Economic Review* 77*(4).*

Doh, T. (2010). The Efficacy of Large-Scale Asset Purchases at the Zero Lower Bound. Kansas City Fed.

Fisher, I. (1930). *The theory of interest* (p. 30). New York, New York: Macmillan.

Fuhrer, J. (2000). Habit Formation in Consumption and Its Implications for Monetary-Policy Models. *The American Economic Review. 90(3)*. 367-390.

Lucas, Robert E. (1978). Asset Prices in an Exchange Economy. *Econometrica 46*. 1429–1445.

Lutz F. (1940). “The Structure of Interest Rates.” *The Quarterly Journal of Economics 55(1).* 36-63.

Macaulay, F (1938) “The Concept of Long Term Interest Rates.” *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856*. NBER.

MacDonald, R., Speight A. (1988). The Term Structure of Interest Rates in the UK. *Bulletin of Economic Research 40(4).* 287-299.

McMillin, W. Ozcicek, O. (2010). Lag Length Selection in Vector Autoregressive Models: Symmetric and Asymmetric Lags. *Applied Economics 31(4)*. 517-524.

Orphanides, A.(1997). Monetary Policy Rules Based on Real-Time Data. Board of Governors of the Federal Reserve System.

Vayanos, D., Vila, J. (2009). A Preferred-Habitat Model of the Term Structure of Interest Rates. NBER Working Paper 15487.

Wright, J. (2007). The Yield Curve and Predicting Recessions. *Finance and Economics Discussion Series Divisions of Research & Statistics and Monetary Affairs*. Federal Reserve Board, Washington, D.C.

ACADEMIC VITA

Nathan Born
260 Limekiln Road
Reading, Pennsylvania 19606

Institute of Economic Affairs — **London, United Kingdom**
Research Intern — *Summer 2014*

- Assessed the forward guidance policies of central banks in context of rules versus discretionary monetary policy
- Received best presentation award at the end of the six-week program

Pennsylvania State University Economics Department — **State College, PA**
Research Experience for Undergraduates — *Spring 2014*
(Offered Through Bates White Economic Consulting)

- Produced economic research on yield curves and the Expectations Theory on the Term Structure of Interest Rates using time series data with an academic advisor
- Generated independent research which was then presented at a colloquium at the University of Pennsylvania on the Theory of Purchasing Power Parity

Teaching Assistant. - International Finance and Open Market Economics — *Spring 2015- Present*

Born Investment Advisory — **Shillington, PA**
Summer Intern — *Summer 2012*

- Developed an understanding of ethical and personal financial advising
- Assessed mutual funds and stocks to help create portfolios for clients

Global Microfinance Brigades — **Ghana**
Microfinance Consultant — *Summer 2013*

- Collaborated with a team in rural Ghana to promote active savings for impoverished families
- Taught basic accounting practices, Susu, and loan education
- Invested a pool of $900 into a corn processor to raise income for the community

LEADERSHIP EXPERIENCE

US Naval Academy Leadership Conference — **Annapolis, MD**

- Represented Schreyer Honors College, discussed technological applications of the Servant Leadership model with representatives from military and civilian institutions — *January, 2015*

Penn State Residence Life — **University Park, PA**
Residence Assistant — *Fall 2013*

- Assessed and responded to the needs of 38 undergraduate residents
- Designed and delegated responsibilities for social, developmental, and educational events

Serve State: Students for Philanthropy — **University Park, PA**
Founder and Treasurer — *Spring 2014-Fall 2014*

Alpha Phi Omega, National Service Fraternity — **University Park, PA**
Recruitment Chair — *Spring 2013*

ADDITIONAL EXPERIENCE

Accepted Student Program	*Panelist*	*Spring 2015*
Jugend mit einer Mission	*Maintenance Assistant*	*Summer 2013*
Bertie's Bar and Grill	*Bartender and Server*	*Summers 2011-2013*

SKILLS

Computer: MATLAB, STATA, E-Views 7, Morning Star Advisor Work Station, Excel, C++
Math: Calculus, Regression Analysis, Linear Algebra, Statistics