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STUDY OF FEASIBILITY OF ASTEROID-CENTRIC GRAVITY ASSISTS

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ABSTRACT

The focus of this thesis is to study the possibility of using asteroids for gravity assist maneuvers. Planetary flyby maneuvers have long been used to change the magnitude and direction of a spacecraft's velocity without expending propellant. However, with only eight planets in the Solar System, the availability of these maneuvers is limited. If asteroids can be used for gravity assists, the time windows will expand simply due to an increase in available bodies. This study is broken down into two phases: determining whether asteroids can in fact have an effect on a passing spacecraft's velocity, and examining known asteroids for their use in future missions. The results for this study were mixed, as the asteroids proved incapable of significantly affecting spacecraft speed under practical circumstances but useful in changing the direction of the spacecraft. Additionally, four known asteroids—1 Ceres, 2 Pallas, 3 Juno, and 4 Vesta—showed possible use in turning a spacecraft's trajectory. Overall, this study created some interesting information regarding turn angles and opened up avenues for further, more detailed studies into the necessary flyby distance as well as possible future missions.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Spencer for advising me through my honors thesis and pointing me in the right direction. His recommended articles, as well as advice on how to examine this problem, proved very valuable and kept me focused on the right parameters. I would also like to thank Dr. Melton for reading my thesis and guiding me in finding the right topic. Together, Dr. Melton and Dr. Spencer have taught me much about orbital mechanics in a very straightforward and usable way, and have helped me decide my post-undergraduate plans. Finally, I would like to thank my honors advisor Dr. Lesieutre for helping me decide what to study over the past two years and reassuring me I was on the right track.

CHAPTER 1: INTRODUCTION

For decades, planetary flyby maneuvers have been utilized to increase the velocity of a spacecraft and shorten time of flight for interplanetary trajectories. In many cases multiple gravity assists have been used on a single mission in what is known as a grand tour. Mission planners have effectively been using these maneuvers to drastically decrease flight time in recent missions, most notably in the case of the New Horizons spacecraft, which will reach Pluto after only nine years in space[1].

However, because of the low quantity of planets, the launch windows for these missions can be few and far between. For the famous Grand Tour of the Voyager missions, the four-year launch window was available only once every 175 years[2]. Though utilizing gravity assists can make a mission much quicker and more efficient, it can also limit the mission's timeline. However, if there were more bodies available for flyby maneuvers, these windows would open up. In the case of asteroids, even if the change in velocity and trajectory is a fraction of that caused by a planetary flyby, they still can prove useful in certain missions. For example, if asteroids prove to be viable flyby central bodies, a mission to Jupiter can possibly use one of the larger asteroids in the Asteroid Belt—such as 1 Ceres or 2 Pallas—to increase the velocity of the spacecraft and shorten transit time. With more available bodies, advantageous alignments of the bodies will occur more often.

Literature Review

Previous Missions

Currently, there are few missions that employed flybys of asteroids, and, in the case of the missions that have, the flybys were mostly secondary objectives. One such mission is the on-going International Rosetta Mission, which recently made impact with the 67P/Churyumov-Gerasimenko comet. Along its trajectory. Rosetta performed a flyby of 2867 Steins, recording color, infrared, and ultraviolet images of the asteroid to study its composition[3]. However, this operation was performed for observational means and not intended to largely affect the trajectory of the spacecraft.

One mission that did employ an asteroid flyby to alter a spacecraft's trajectory was the Near-Earth Asteroid Rendevous (NEAR) mission, which actualy observed the perturbations to the spacecraft's trajectory to determine the mass and rotation pole of 433 Eros[4].

Not only did this mission show that an asteroid can affect a spacecraft's trajectory, but it also presented some of the challenges associated with an asteroid flyby. Ephemeris data for most asteroids—especially newly discovered asteroids--is not perfect, so in order to properly study 433 Eros, ground-based heliocentric velocity measurements had to be combined with optical navigation frames from the spacecraft. This combined data gave the most accurate orbit information on the asteroid in nearly real time, and allowed the flyby trajectory to be determined.

Past Studies: Tethered Gravity Assist

In 1986, Jet Propulsion Laboratory published a study of using tethers to slingshot a spacecraft around an asteroid. Through their analysis, they found that most asteroids do not have enough mass to be used in gravity assists, but that through an artificial gravity assist using tethers, a spacecraft can utilize asteroids as small one kilometer across[5]. In this maneuver, a spacecraft essentially anchors itself to an asteroid with Kevlar tethers and uses the tension in the tether to swing around the asteroid and change its direction, as seen in Figure 1[6]. Additionally, because of the numerous asteroids of such a small scale, a spacecraft could use multiple tethered assist maneuvers to hop from asteroid to asteroid and collect much data. However, later studies showed that this maneuver would be used most for studying near-Earth asteroids and that many technical aspects, such as the strength and functions of the tethers, required more development.

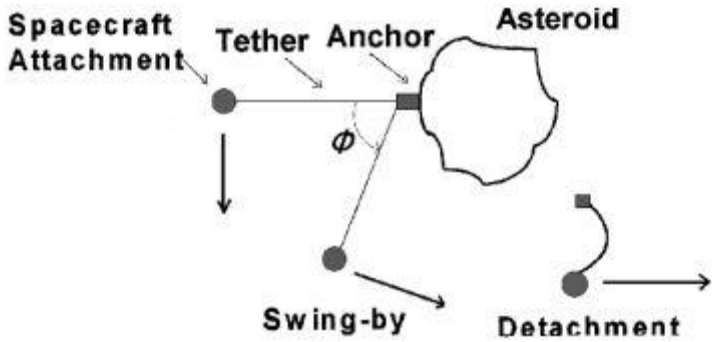

Figure 1. Diagram of Tethered Gravity Assist[6]

CHAPTER 2: PROBLEM STATEMENT

A gravity assist, or flyby, is a maneuver by which a spacecraft uses the gravitational pull and angular momentum of a massive body to passively alter its velocity in magnitude and direction. If a spacecraft enters the sphere of influence (SOI) of a body and has enough velocity to neither orbit nor impact the body, the spacecraft flies by the planet in a hyperbolic trajectory. During this flyby trajectory, the gravitational field of the body accelerates the spacecraft. Finding the heliocentric velocity after the flyby maneuver and when spacecraft exits the body's SOI is a straightforward procedure of finding orbital elements and vector addition. Curtis presents a very algorithmic method of finding the new state vector of the spacecraft and changes undergone by the maneuver[7].

For this study, the parameters studied were the turn angle and change in the magnitude of the spacecraft's heliocentric velocity upon leaving the asteroid's SOI, δ and Δv_∞, respectively. Measured in degrees, the turn angle is defined as the angle between the spacecraft velocity entering and exiting the SOI, $v_{\infty 1}$ and $v_{\infty 2}$, respectively. It can be found from the following equations and is shown in the Figure 2.

$$\delta = 2\sin^{-1}\frac{1}{e}\ (degrees) \tag{1}$$

where

$$e = 1 + \frac{r_p v_{\infty 1} \cdot v_{\infty 1}}{\mu_{Asteroid}} \tag{2}$$

$$r_p = radius\ od\ periapsis\ (km) \tag{3}$$

$$\mu_{Asteroid} = (6.67384 \times 10^{-11}\ \frac{m^3}{kg \cdot s^2}) \times (m_{spacecraft} + m_{Asteroid}) \tag{4}$$

$$\boldsymbol{v}_{\infty 1} = \boldsymbol{V}_{\boldsymbol{1}}^{(v)} - \boldsymbol{V} \tag{5}$$

$$\boldsymbol{V}_{\boldsymbol{1}}^{(v)} = V_1^{(v)}(\cos(\alpha)\,\boldsymbol{u}_V + \sin(\alpha)\,\boldsymbol{u}_s) \tag{6}$$

$$V_1^{(v)} = magnitude\ of\ incoming\ spacecraft\ velocity \tag{7}$$

$$\alpha = angle\ of\ approach \tag{8}$$

$$\boldsymbol{V} = heliocentric\ velocity\ of\ asteroid \tag{9}$$

and the masses of the spacecraft and asteroid are both in kilograms. The unit vector $\mathbf{u}_s$ points toward the sun, while the unit vector $\mathbf{u}_v$ is perpendicular to the $\mathbf{u}_s$ vector and points in the direction of the asteroid's velocity.

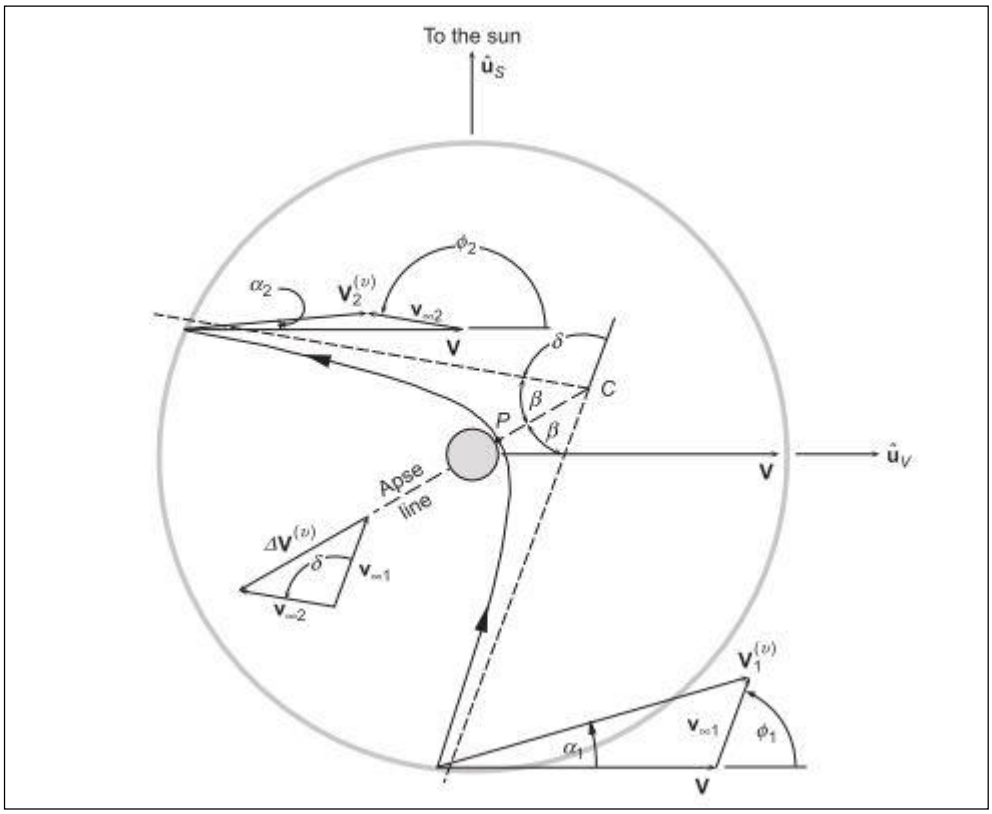

Figure 2. Flyby Maneuver showing incoming and exiting heliocentric velocities, turn angle, and heliocentric velocity of central body. Source: Curtis p.459[7]

Once the turn angle is found, the outgoing heliocentric velocity of the spacecraft can be determined.

$$v_{\infty 2} = \sqrt{v_{r2}^2 + v_{t2}^2} \quad \textit{(km/s)} \tag{10}$$

$$v_{r2} = (v_{\infty 1} \times \sin\frac{\delta \times \pi}{180})\, \boldsymbol{u}_s \quad \textit{(km/s)} \tag{11}$$

$$v_{t2} = (v_{\infty 1} \times \cos\frac{\delta \times \pi}{180})\, \boldsymbol{u}_s \quad \textit{(km/s)} \tag{12}$$

$$\boldsymbol{V}_2^{(v)} = \boldsymbol{V} + \boldsymbol{v}_{\infty 2} \tag{13}$$

and

$$\Delta V^{(v)} = V_1^{(v)} - V_2^{(v)} \quad (km/s) \tag{14}$$

From these equations, it is clearly shown that the key parameters in determining the exit heliocentric velocity are the incoming heliocentric velocity of the spacecraft, angle of approach, mass of the body, radius of periapse, and heliocentric velocity of the central body. By focusing

on these variables, especially mass, the effectiveness of asteroid-centered gravity assists can be examined.

If asteroids do prove capable of altering a spacecraft's trajectory, the next step involves examining known asteroids. For this project, some of the largest known asteroids will be examined for flyby effectiveness. With data on known asteroids, possible missions can be planned in the future. The dimensions of these asteroids ae listed below in Table 1[8].

Table 1. Name, Mass, and Diameter of Asteroids to be examined in this study

Asteroid Number and Name	Diameter (km)	~Mass 10^{15} kg
Ceres	975 x 909	947,000
Pallas	582 x 556 x 500	214,000
Juno	234	20,000
Vesta	569 x 555 x 453	259,000

CHAPTER 3: METHODS

The code developed to examine this problem was designed to evaluate the role of asteroid mass, radius of periapse, and incoming heliocentric velocity on determining the turn angle and change in speed of the spacecraft through the flyby. To focus on these variables, the asteroid was modeled as a point mass. Through this model, it was possible able to obtain data for a wide range of periapse radii, though sacrificing some physical meaning in the process.

The MATLAB code—included in Appendix A— uses nestled for loops to calculate the turn angle and change in velocity for a range of asteroid masses, asteroid orbital radiuses from sun, angles of approach, incoming spacecraft velocities, and flyby periapse radiuses. The innermost loop iterates outputs the turn angle and change in heliocentric speed for an asteroid mass of 1×10^{18} kg at an orbital radius of 300 million kilometers with a spacecraft's angle of approach of 0 degrees, incoming velocity of 1 km/s, and a periapse radius of 10 km to 5000 km. After the periapse radius loop iterates through to 1000 km, it begins again at 10 km and repeats the process for an asteroid of mass 1×10^{18} kg at an orbital radius of 300 million kilometers with a spacecraft's angle of approach of 0 degrees, and incoming velocity of 2 km/s. This continues up to and including an incoming velocity of 30 km/s. The loops continue to compute the turn angle and change in velocity for varying spacecraft angles of attack and asteroid masses and orbital distance. These

Though this script was cumbersome and slow to process, it did create an indexical output of turn angles and changes in spacecraft velocity for a wide range of each of the five parameters. This allows one to quickly look up the results for any possible combination of inputs. For example, entering the command "DELTA (3, 2, 1, 2, 50)" would show the turn angle resulting from an asteroid mass of 10^{20} kg, asteroid orbital radius of 310 million km, 1° angle of approach, incoming heliocentric speed of 2 km/s, and a periapse radius of 500 km.

This indexing method did create a very orderly array of turn angle and changes in velocity, it did have some problems. First, it took a very long time to process, with nearly 9 million values outputted for each turn angle and velocity change. Additionally, this fact that turn angle and velocity change were stored in a five-dimensional array made it difficult to focus on individual parameters. To further study the effects of approach angle, asteroid mass, and incoming spacecraft velocity, additional MATLAB scripts were created by replacing some for

loops in the aggregate script with set values. For example, the script to study the approach angle used a set asteroid mass of 10^{22} kg, orbital radius of 450 million km[1], periapse radius of 500 km, and incoming speed of 30 km/s. To study known asteroids, these new scripts were used with the respective masses and orbital distances of each specific asteroid. Each of these scripts is contained in Appendix A.

[1] The orbital radius of 450 million km places the asteroid in the middle of the asteroid belt, and this radius is used for hypothetical asteroids through the study.

CHAPTER 4: RESULTS AND DISCUSSION

After examining the data obtained from the MATLAB code, it was shown that -- under the correct circumstances -- asteroids could be used for gravity assist maneuvers. Both the turn angle and speed change showed to be sensitive to the gravitational pull and angular momentum of an asteroid at certain incoming speeds, periapse radiuses, and approach angle. Moreover, these results were duplicated in studies of known asteroids.

Effects of Approach Angle on Turn Angle and Change in Spacecraft Velocity

The first parameter studied was the approach angle, α. Though independent of incoming velocity and periapse radius, the approach angle helps determine incoming velocity of the spacecraft crossing the SOI. By examining the effects of α on the turn angle and speed change, an optimal approach angle could be found and used for further studies of the other parameters. Figures 3 and 4 below show the turn angle and change in heliocentric spacecraft speed for a flyby with an asteroid mass of 10^{21} kg, incoming spacecraft speed of 20 km/s, and a periapse radius 500 km. These values were chosen to maximize the turn angle and speed change so that the optimal approach angle would be more evident. In many physical cases, it would be difficult to find asteroid with the proper mass and radius for such a flyby.

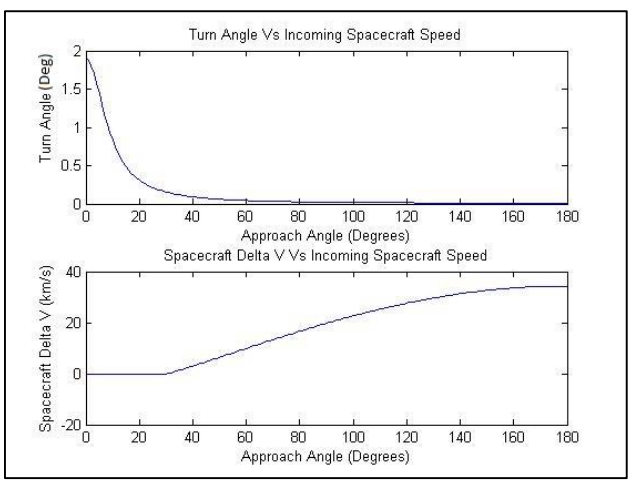

Figure 3. Turn angle and Change in Spacecraft Velocity Magnitude vs Approach Angle for a flyby with an asteroid mass of 10^{22} kg, incoming spacecraft speed of 20 km/s, and a periapse radius 500 km.

Upon viewing these plots, it is evident there is a tradeoff between turn angle and change in velocity. These reasons for these results are fairly intuitive. At an approach angle of zero

degrees, the spacecraft in traveling parallel to the asteroid. In order for the spacecraft to perform a hyperbolic flyby, it will have to change direction towards the asteroid. As the spacecraft is directed more toward the asteroid, it does not need to alter its course as much to have the same hyperbolic trajectory. Additionally, because the approach angle is zero, the difference between the departing angle and approach angle—essentially the turn angle—will be maximized.

In terms of speed change, the opposite results are observed. When the spacecraft is traveling parallel to the asteroid, the asteroid does not need to impart much angular momentum to the spacecraft to keep it constant. However, as the approach angle rises, the velocity of the spacecraft lessens in the $\mathbf{u}_v$ direction, and the asteroid must impart increasing angular momentum to the spacecraft—and thus more velocity in the $\mathbf{u}_v$ direction—to keep the net angular momentum zero. At an approach angle of 180 degrees, the spacecraft is traveling in an opposing direction to the asteroid, so the asteroid must impart a maximum angular momentum to keep net momentum the same, slowing down the spacecraft in its $-\mathbf{u}_v$ velocity. Mathematically, at 180 degrees, $\mathbf{v}_{\infty 1}$ is maximized, and by Equations 1 and 10 through 14, turn angle is minimized and velocity change is maximized. Intuitively, the asteroid carries the spacecraft along its trajectory. When the spacecraft is traveling in the opposing direction, the asteroid cannot carry it with it, but it can slow it down significantly.

To determine the approach angle for the best results for both turn angle and speed change, a practical approach angle with significant results for both parameters must be chosen. As Figure 3 shows, this are few approach angles that can have significant effects on both turn angle and change in velocity, as velocity change stays very close to zero until turn angles in the mid-thirties. At an approach angle of 35 degrees, the turn angle is 0.1167 degrees, and the change in the magnitude of heliocentric spacecraft velocity is 1.332 km/s. However, these results are for a very ideal scenario. In reality, due to the small SOI of an asteroid, it would be too difficult to target the asteroid with such a high approach angle. Practically, only approach angles of 10 degrees or less should be considered. For studies of the other inputted parameters, an approach angle of 5 degrees will be used. Figure 3 shows that there is very little overlap between significant effects on turn angle and velocity change, so the practical approach angle was chosen over the optimizing approach angle. Plots of the effects of the other parameters with approach angles of 0, 45, and 90 degrees can be found in Appendices B and C.

Effects of Asteroid Mass on Turn Angle and Change in Spacecraft Velocity

Once the best approach angle was determine, the analysis focused on the effects of mass. As one purpose of this study is to determine the minimum required mass for a significant gravity assist, this focus would allow the most straight forward evaluation of asteroid-focused gravity assists. Figure 4 below shows these effects of mass for a flyby for an incoming spacecraft velocity of 20 km/s and a periapse radius of 500 km.

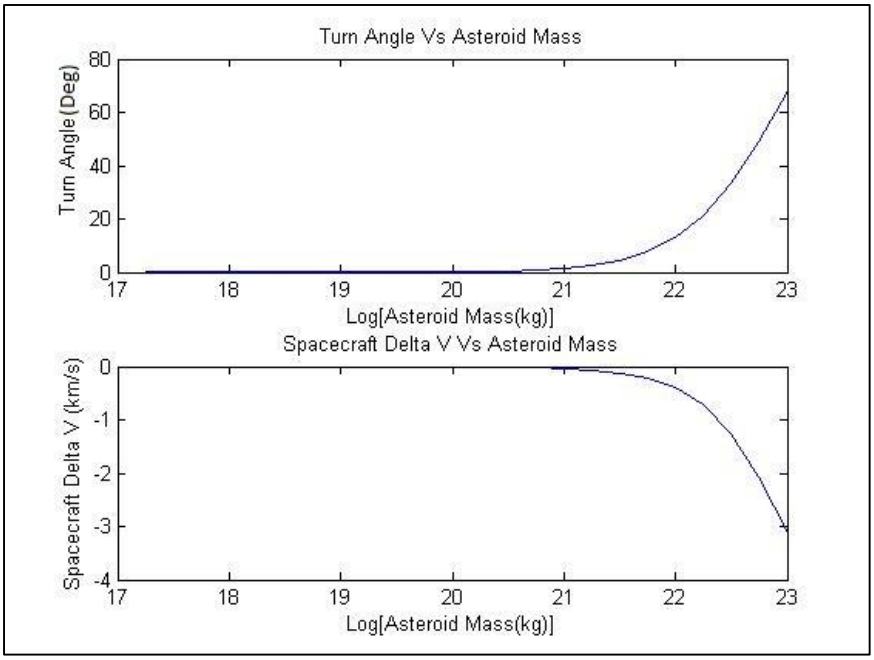

Figure 4. Turn Angle and Change in Spacecraft Speed vs Mass of Asteroid

As is evident in the Figure 4, the effects of a gravity assist increase greatly as body mass reaches 10^{22} kg and higher. Unfortunately, all known asteroids are smaller than 10^{21} kg. Though these results show promising results for dwarf planets, they are also for a periapse radius of 500 km, which would most likely result in a collision with the surface of the bodies of mass 10^{22} kg and higher. Even for a flyby of 1 Ceres, which has a mass of 9.47×10^{20} kg, would have the spacecraft less than 50 km from its surface. These findings do not show much promise for a practical asteroid-based gravity assist.

However, instead of using an approach angle to optimize both turn angle and velocity change, the study can choose angles to optimize turn angle and velocity change individually. These optimal angles are zero and 90 degrees, respectively. Though 180 degrees would result in

the maximum velocity, it is impractical to have a spacecraft traveling in the opposite direct of the other bodies in the solar system. Figures 5 and 6 show these studies.

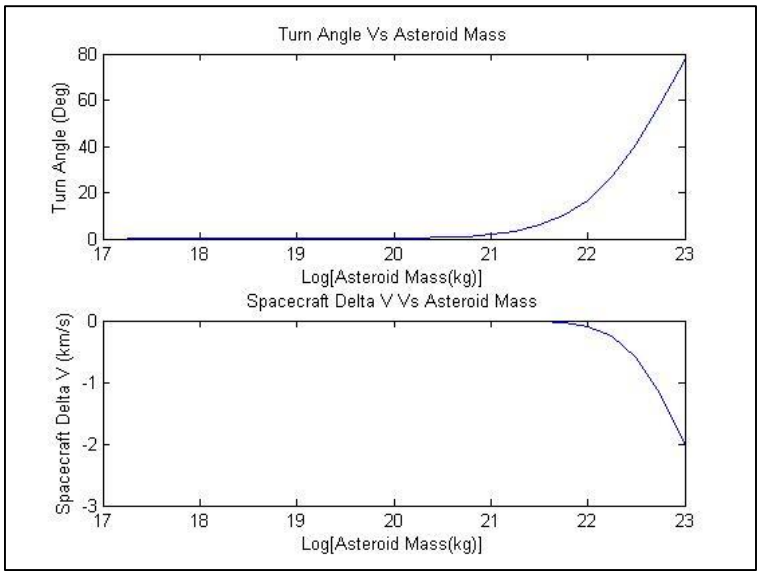

Figure 5. Turn Angle and Change in Spacecraft Speed vs Mass of Asteroid for an Approach Angle of Zero Degrees

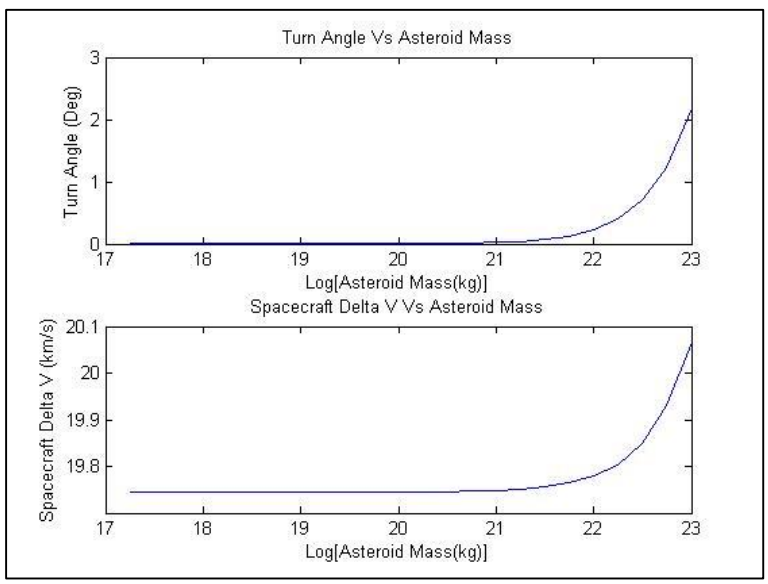

Figure 6. Turn Angle and Change in Spacecraft Speed vs Mass of Asteroid for an Approach Angle of 90 Degrees

Even with trading off turn angle for change in velocity, and vice-versa, the results still only show significant increase at masses at or above 10^{21} kg. In the cases of the largest asteroids, the turn angle would be a maximum of nearly 2 degrees. Though this turn angle is not huge, it

could have use in some missions. Interestingly, the spacecraft actually slows down during a flyby with approach angles of zero and five degrees, hinting that it might actually be captured.

For the optimized velocity change approach angle, the effects predictable don't start increasing until masses of 10^{21} kg. However, even at lower masses, the flatlined change in spacecraft velocity is 19.74 km/s, which a significant increase. This unexpectedly high value is attributed to the fact that at an approach angle of 90 degrees, the spacecraft has no velocity in the $\mathbf{u}_v$ direction, so the asteroid imparts an increase in speed in that direction to keep net angular momentum constant. In other words, when the spacecraft enters the SOI, it is pulled perpendicular to its initial path, causing a significant velocity change. Though these results seem very promising, from a practical viewpoint, it would be incredibly difficult to align the trajectories of the spacecraft and asteroid to meet this criteria, and these results are of only mathematical significance. Once again, for the practical purposes of this study, Figure 4 shows that an asteroid-focused flyby would not cause significant trajectory changes.

Effects of Incoming Heliocentric Velocity on Turn Angle and Change in Spacecraft Velocity

For the previous studies, the magnitude of incoming heliocentric velocity, was kept at a constant value of 20 km/s. This parameter plays a large part in determining $v_{\infty 1}$ and thus the turn angle and change in spacecraft velocity. The spacecraft's velocity entering the SOI is the velocity of the spacecraft relative to the velocity of the asteroid. The velocity of the asteroid was held constant in this study, so at low initial spacecraft heliocentric velocities, the asteroid may be moving away from the spacecraft, causing the velocity at the SOI to be negative. As initial spacecraft heliocentric velocity increases, $v_{\infty 1}$ eventually reaches zero and then becomes positive. Examining the turn angle and change in velocity for a range of incoming spacecraft velocities will help highlight the optimal incoming spacecraft velocity. These results are shown in Figure 7.

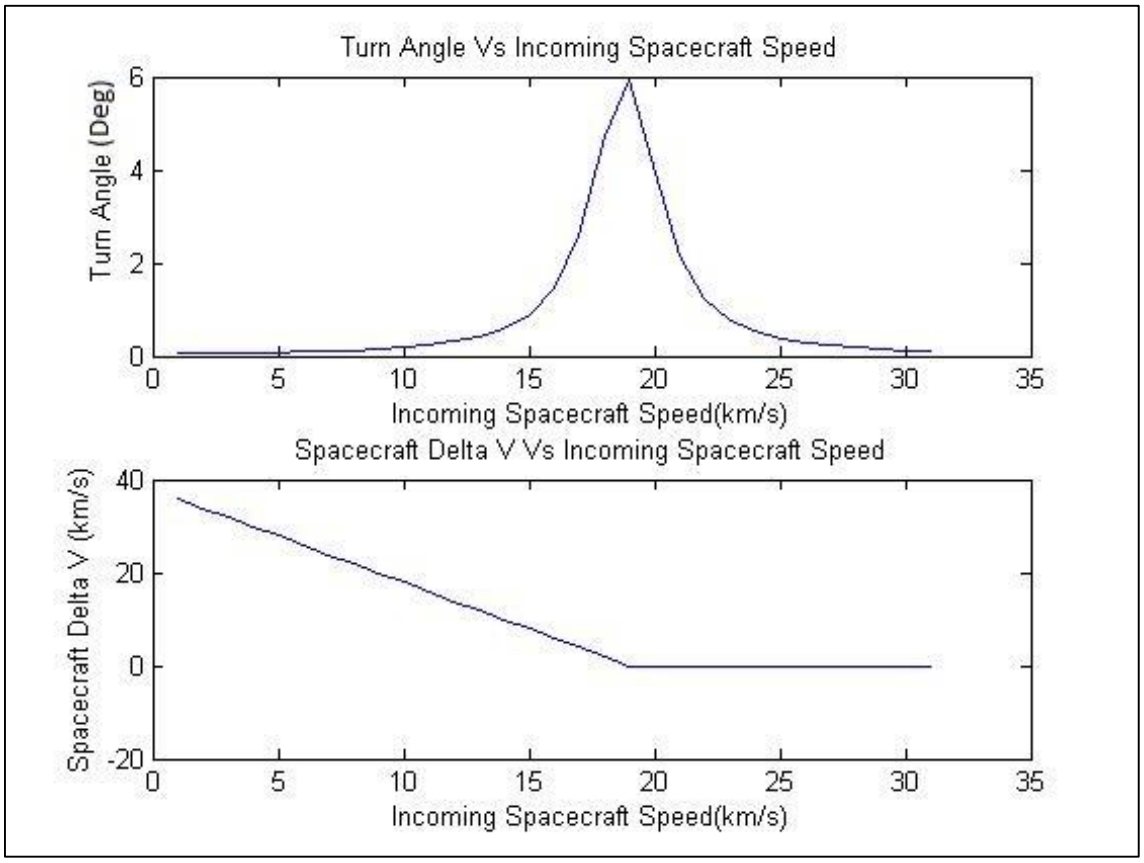

Figure 7. Turn Angle and Change in Spacecraft Speed vs Incoming Spacecraft Heliocentric Velocity

The plots in Figure 7 show that there is a maximum turn angle of 6.0 degrees for an incoming velocity magnitude of 18 km/s. The hypothetical asteroid modeled is traveling at 17.17$\mathbf{u}_v$ km/s. Accounting for the approach angle, the spacecraft is actually moving toward the asteroid at 0.762 km/s for this maximum turn angle value. Additionally, incoming spacecraft speeds in the range of 16 to 22 km/s will result in a turn angle of at least 1 degree. The symmetry of the turn angle plot implies that absolute value of the spacecraft velocity at the SOI determines the effects. This corresponds to $v_{\infty 1}$ being squared in Equation 2.

The plot for change in velocity shows a strictly decreasing trend as incoming spacecraft velocity increases until it zeroes out at incoming velocities of 21 km/s and greater. This is due to both the nature of momentum, and the methods of analysis. At low incoming velocities, the asteroid imparts angular momentum to the spacecraft to keep net angular momentum constant. Since the mass of the asteroid and spacecraft are constant, the final velocities of each body will be constant throughout the various incoming velocities. At lower incoming spacecraft velocities,

the spacecraft must increase velocity by a greater amount to get to the final velocity. Furthermore, at incoming velocities of at least 21 km/s, the spacecraft simply travels past the asteroid unaffected. It may enter the SOI, but it is not captured in a hyperbolic path.

Additionally, the high changes in spacecraft heliocentric velocity corresponding to low initial spacecraft velocities are due in part to the physically simplified model studies. Mathematically, v_∞ is defined by both the spacecraft's heliocentric velocity and the asteroid's heliocentric velocity. Asteroid velocity is from spacecraft heliocentric velocity to get $v_{\infty 1}$, and asteroid velocity is added to $v_{\infty 2}$ to find the post-flyby spacecraft heliocentric velocity. If the spacecraft has little or no heliocentric velocity initially, then the change in velocity is mathematically dependent solely on the asteroid's velocity. Moreover, since $v_{\infty 1}$ is squared, it is only the magnitude of spacecraft velocity relative to the planet that matters, so the asteroid velocities pre- and post- flyby will not cancel out. This causes the change in spacecraft velocity to be equal to asteroid velocity added to itself, and this is evident in Figure 7. Additionally, at low spacecraft velocities and low periapse radiuses, the spacecraft might get captured by the asteroid, in which case the computed post-flyby spacecraft heliocentric velocity is just the velocity of the spacecraft as it orbits the Sun with the asteroid. More sophisticated, graphic models will be needed to further pursue these ideas.

Even if he high changes in velocity for low incoming velocities are not errors in the model, they are once again misleading for practical purposes. In order for a spacecraft to get to the Asteroid Belt, it must be traveling at a much higher velocity than those that correspond to significant changes from the gravity assist. For a spacecraft to be at the necessary low velocities, it would have to slow down by some means and probably burn fuel. As gravity assists are designed to passively alter trajectories without burning fuel, this deceleration would negate the benefits of the gravity assist.

Effects of Periapse Radius on Turn Angle and Change in Spacecraft Velocity

For practical studies, periapse radius and mass are two of the most important parameters in determining asteroid flyby feasibility. Mass studies determine whether the gravity assist is possible, and studies of periapse radiuses show whether the maneuver is possible without impacting the asteroid. Figure 8 below shows how turn angle and velocity change vary with increasing periapse radius. The asteroid is modeled with a mass of 10^{21} kg, and the spacecraft is traveling at 20 km/s and an approach angle of 5 degrees.

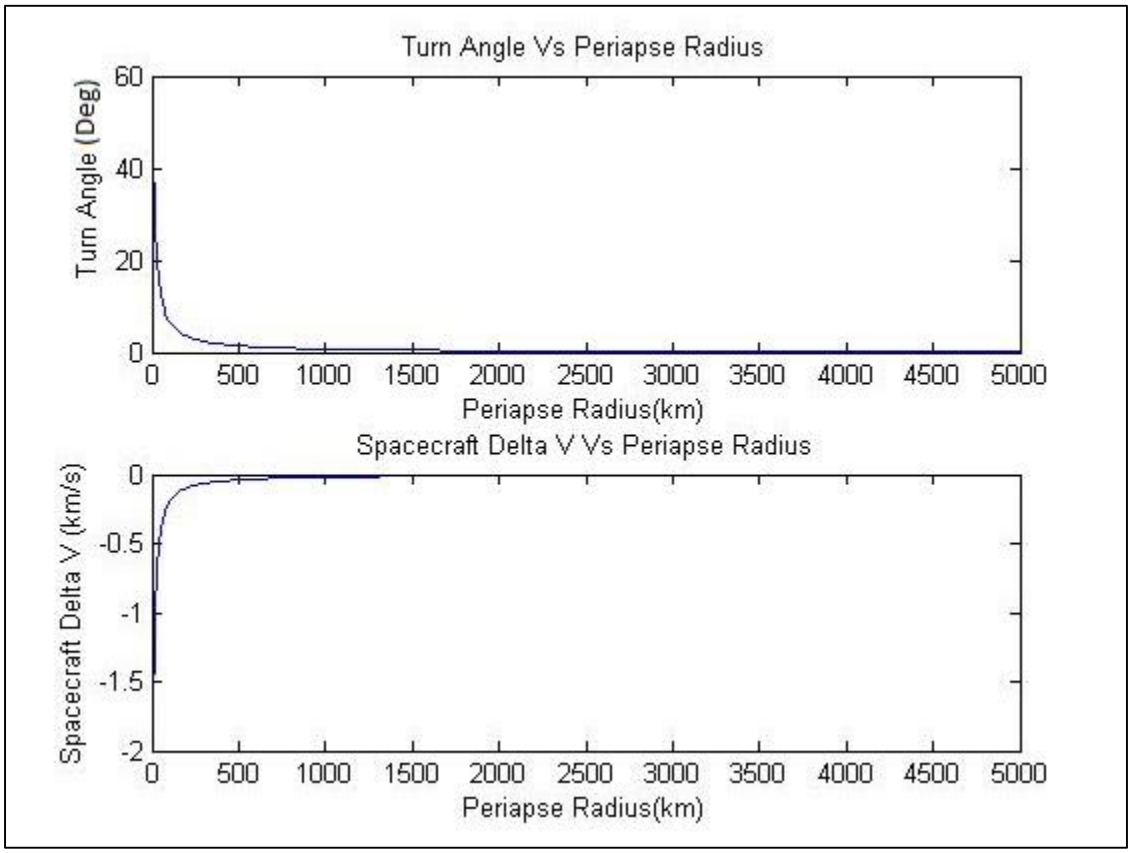

Figure 8. Turn Angle and Change in Spacecraft Speed vs Periapse Radius

This results show that the turn angle decreases rapidly with increasing periapse radius, and is minimal for all practical periapse radii. Even with the asteroid assumed to be incredibly dense, it would require a periapse radius of at least 500 km. The change in velocity shows similar results, with even more rapid decay. Interestingly, the change in velocity corresponding to low periapse radii is negative, indicating that the spacecraft slows down, and is most likely captured. However, these decelerations occur at periapse radiuses that would physically mean traveling through the center of the asteroid. Ultimately, though various approach angles and incoming velocities showed promise for an asteroid-focus gravity assist, the physical realities displayed by studying the mass and periapse radius requirements show there are very few circumstances for a significant asteroid flyby maneuver.

Study of 1 Ceres Flyby

From the above analyses, only 1 Ceres has enough mass to significantly affect the trajectory of a spacecraft through a gravity assist. For this section of the study, incoming

spacecraft velocity, angle of approach, and radius of periapse were studied. The computed results are shown in Figures 9, 10 and 11.

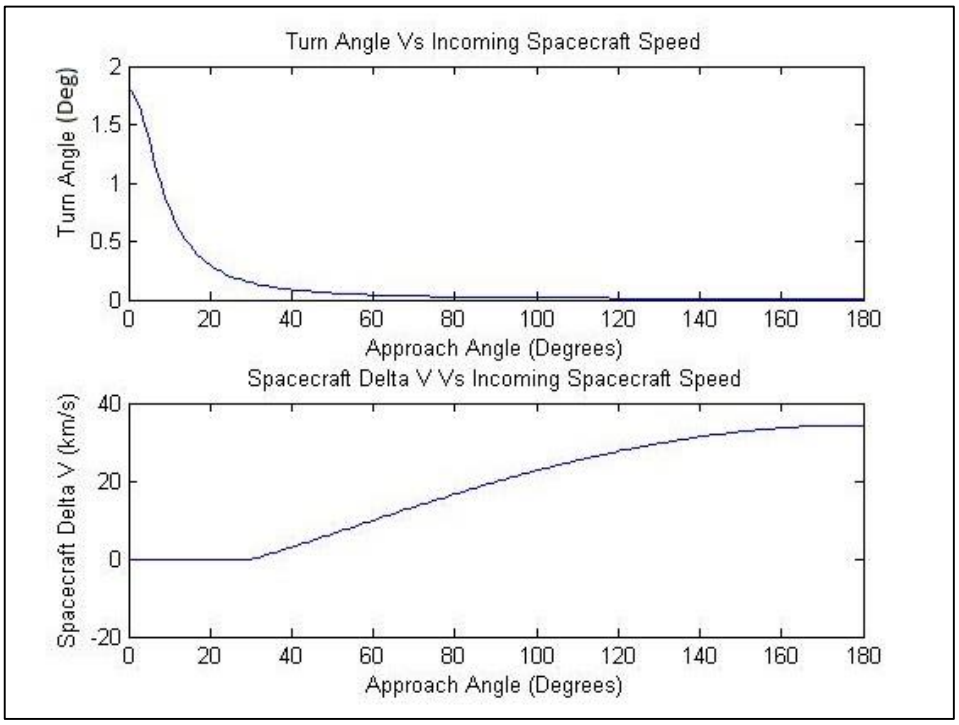

Figure 9. Turn Angle and Change in Spacecraft Speed vs Approach Angle

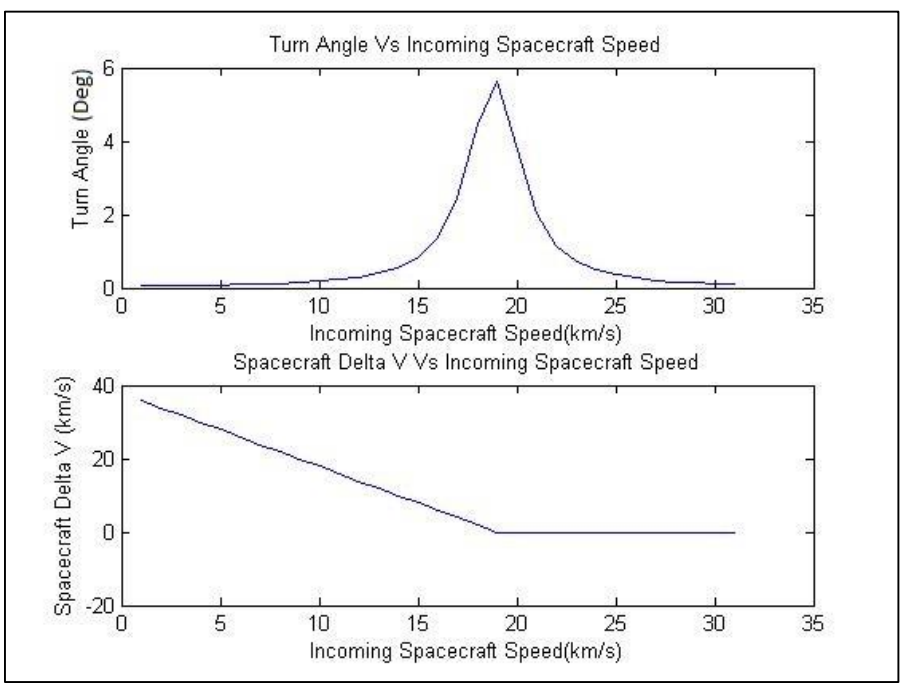

Figure 10. Turn Angle and Change in Spacecraft Speed vs Incoming Heliocentric Spacecraft Speed

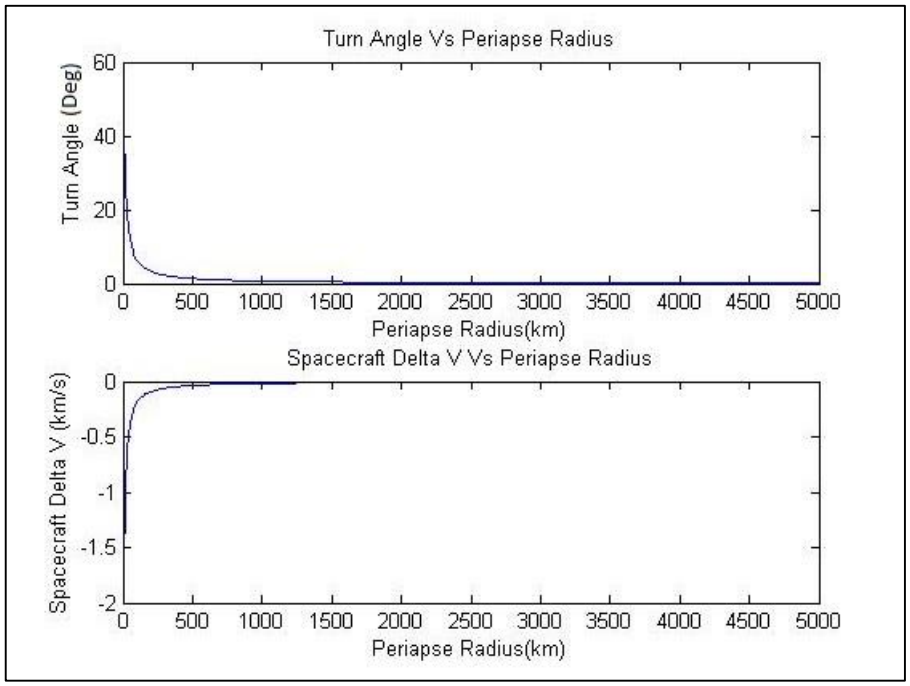

Figure 11. Turn Angle and Change in Spacecraft Speed vs Periapse Radius

These results show that, though 1 Ceres can cause a turn angle and change in velocity, it can only do in a limited set of circumstances. Additionally, the changes it does cause are of little magnitude. As with the hypothetical asteroids studied earlier, choosing the approach angle involves a trade-off between turn angle and velocity change. Once again, an approach angle of 5 degrees was used for the study of the incoming velocity and periapse radius effects, as practicality was chosen over the very limited turn angle and velocity change optimizing approach angle of 35 degrees.

As expected, the turn angle and velocity change relationship to the incoming velocity resemble the calculated results for the hypothetical asteroid. The turn angle plot has a similar peak at in incoming velocity of 18 km/s, and the velocity change plot steadily decreases until it flatlines at zero. However, the values of the plots for 1 Ceres are much lower, with a maximum turn angle of less than 0.15 degrees. The velocity change shows similarly high values for low incoming velocities, but these low incoming velocities are still very impractical, difficult to achieve, and may be due to the errors in the model discussed earlier. Additionally, the turn angle and change in speed decreases rapidly with increasing periapse radius. At a periapse radius of 500 km, for which the spacecraft would be less than 50 km from the surface of Ceres, the turn angle is only 1.363 degrees. Ultimately, these findings suggest that, though 1 Ceres could

marginally effect the trajectory of a passing spacecraft, the conditions necessary are difficult and impractical to obtain.

Comparison to Mars Flyby

To put these findings into context, a study of a flyby of Mars was done. Mars mass was set at 6.42 x 10^{23} kg, and the effects of periapse radius and initial spacecraft heliocentric velocity were chosen. As with the previous studies, the approach angle was set to 5 degrees. Mars' semimajor axis was set to 227.9 million km, and, continuing with the near-impact flybys studied for the asteroids, the periapse radius for the incoming velocity study was set to 4000 km.

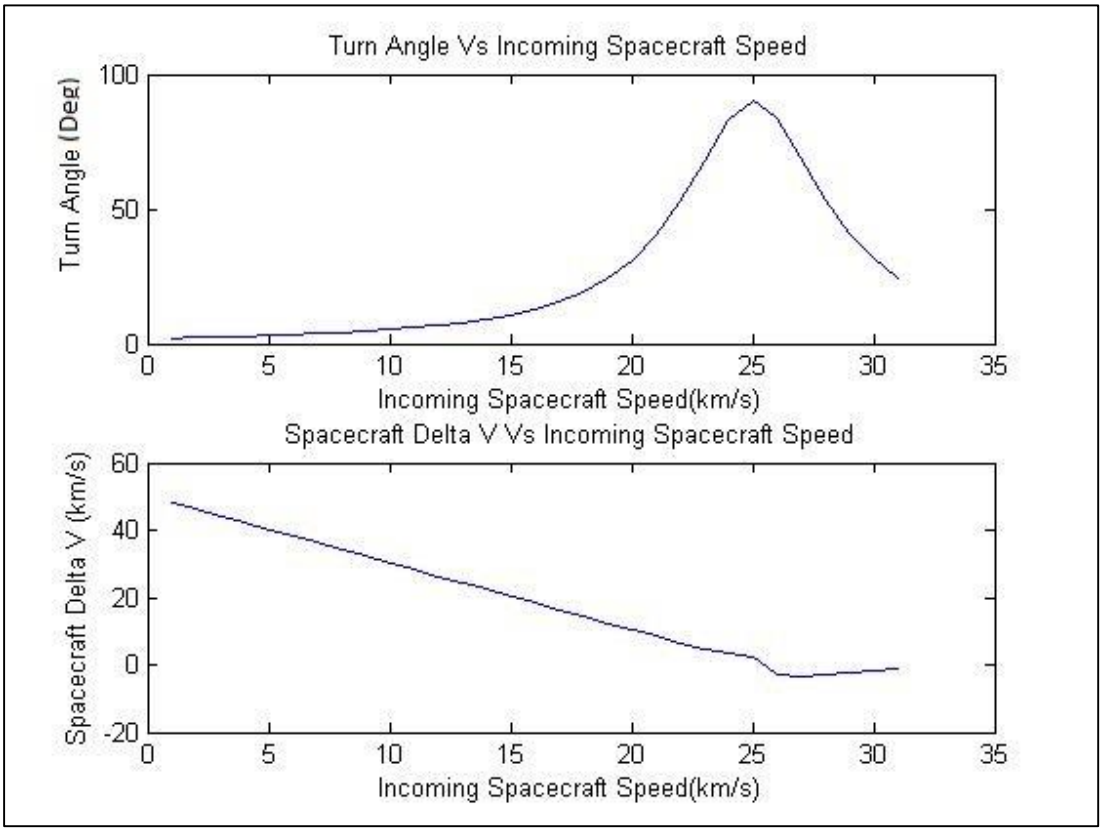

Figure 12. Turn Angle and Velocity Change vs Initial Spacecraft Heliocentric Velocity for Mars Flyby

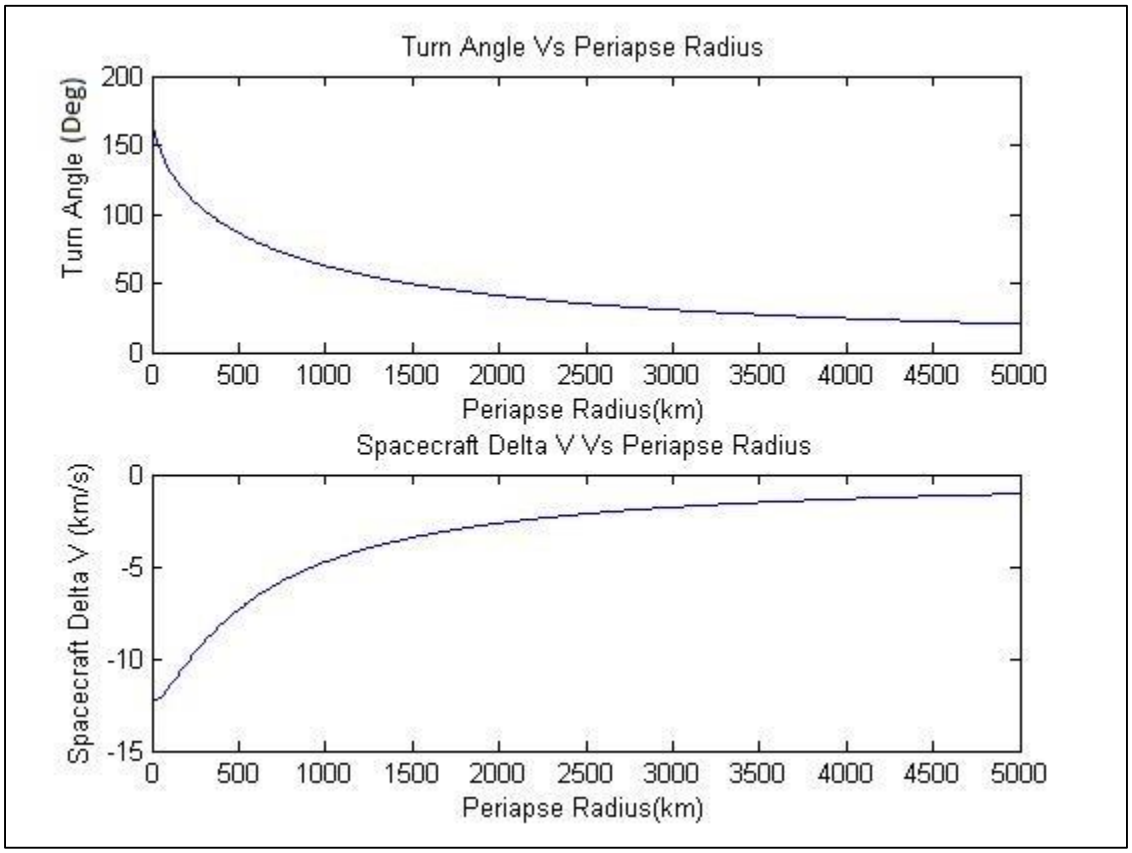

Figure 13. Turn Angle and Velocity Change vs Periapse Radius for Mars Flyby

These figures show the difference between a planetary gravity assist and asteroid-focused gravity assists. Though the graphs largely exhibit the same behavior, the magnitude of the values are much greater for Mars. In the incoming spacecraft velocity study, the turn angle has a maximum value of 90.42 degrees at an initial spacecraft velocity of 25 km/s, showing that the turn angle is maximized when spacecraft velocity in the $\mathbf{u}_v$ direction is close to asteroid velocity in the $\mathbf{u}_v$ direction. Additionally, the Mars flyby shows the same misleadingly high values for velocity change at low initial spacecraft velocities. Furthermore, the change in velocity dips below zero and then increases with increasing velocity. This is most likely to the spacecraft being captured and its heliocentric velocity changing to that of Mars. Ultimately, this underscores how dominant of a factor the mass of the central body is in a gravity assist and how the maneuver cannot simply be geometrically scaled in terms of speed and periapse radius.

The periapse study also follows the same behavior as that for an asteroid, albeit with much higher velocities. The magnitude of turn angle and change in spacecraft speed decreases steadily, but at a gentler slope. As with the asteroid study, these graphs imply physically

impossible behavior at low values, as the spacecraft would impact Mars at any periapse radius value under 3800 km. Taking that physical reality into account shows that, like with the asteroid, for practical periapse radiuses, the effects on turn angle and velocity change are nearly leveled out. Unlike the asteroid study, the magnitudes of these values are usably large. Interestingly, the fact that the physically allowable periapse radiuses are at the low-sloping part of the graph makes them more practical, as an error in the periapse radius will not result in as much of a change in the effects of the periapse.

CHAPTER 5: CONCLUSIONS AND FUTURE WORK

From the results of this study, it is concluded that asteroids lack the mass to affect the trajectory of a passing spacecraft. Though studying the approach angle and incoming heliocentric velocity of the spacecraft highlighted the optimal trajectory of the spacecraft for a gravity assist, the results of the mass and periapse radius studies proves the necessary circumstances to be very unlikely. Planning for such a maneuver would require an incredible amount of precision and might not be practical or efficient. Essentially, this study showed that though an asteroid-focused gravity assist could cause some changes to trajectory, they are not of much practical use. Rather, the effects of an asteroid on a passing spacecraft are variables to be aware of in mission planning to avoid trajectory disturbances.

Future work on this project will serve two purposes: verifying the findings of this study and developing possible scenarios in which an asteroid flyby could be useful. To verify the findings of this study, the relationship between parameters must be studied. The approach angle, incoming spacecraft velocity, and asteroid mass each contribute to the periapse radius of the hyperbolic flyby trajectory. Possibly, the required mass and incoming velocity for a significant gravity assist might not allow the necessary periapse radius. This study assumed each parameter was largely independent of the others, and that is simply not the case. In the future, a more interconnected analysis must be done to examine the relations between the inputted variables.

Additionally, more robust and professionally accepted software will be utilized to verify the findings of this study. As the data was found through a student-created MATLAB code, it needs to be verified by industry experts. On way to do this would be to utilize a widely-used program such as Systems Tool Kit (STK) or General Missions Analysis Tool (GMAT) to simulate flybys of asteroids. With GMAT, it is possible to add asteroids by importing known ephemeris data. With this analysis, not only would the simulations show the feasibility of asteroid-centric flyby, but it could also be used to plan entire missions. These simulations could then help determine launch dates and trajectories of any missions that plan on utilizing these flybys. Ultimately, though the results of this study are very interesting, there is much work do before this information is practically usable.

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APPENDIX A: MATLAB SCRIPTS

1.aggregate.m

```
      %This program uses a series of nestled for loops to find the change in
      %speed and turn angle of a spacecraft undergoing a flyby of an asteroid
      %with variable mass, asteroid orbital radius, angle of approach,
incoming velocity, and radius of %periapse. Values for each parameter are
saved on each iteration as the variable name in all caps


      G=6.7384*10^(-20);
%km^3/s^2

      for l=[1:5]
%Start of mass loop

          M(l)=10^(17+l)
%Saving mass value
          m=10^(17+l);
          mu=G*m;
%Calculating mu

      for k=[1:29]
%Start of asteroid orbital radius loop

          R_AST(l,k)=10000000*(29+k);
%Orbital radius has range of 300-600 million km
          r_ast=10000000*(29+k);
          V_ast=[sqrt(132712000000/r_ast);0];

      for h=[1:19]
%Iterating approach angle, in degrees
          ALPHA(l,k,h)=10*(h-1);
          alpha=10*(h-1);

      for i=[1:1:30]
%Iterating Heliocentric speed of spacecraft in km/s

          SC_Helio_Speed(l,k,h,i)=i;
          v_sc1=i;
%Heliocentric speed of spacecraft
          V_sc1=[v_sc1*cos(alpha*pi/180);v_sc1*sin(alpha*pi/180)];


          V_inf1=V_sc1-V_ast;
%Spacecraft velocity relative to asteroid when entering SOI
          v_inf1=sqrt(dot(V_inf1,V_inf1));
%Magnitude of V_inf
          PHI1(l,k,h,i)=atan(V_inf1(2)/V_inf1(1))*180/pi;
%Spacecraft flight path angle relative to asteroid
          phi1=atan(V_inf1(2)/V_inf1(1))*180/pi;
```

```
      for j=[1:1:100]
%Start of Periapse Radius Loop. Covers range of 0-1000km

          RP(l,k,h,i,j)=10*j;
          rp=10*j;
%radius of periapsis, in km

          ECC(l,k,h,i,j)= 1 + rp*v_inf1^2/mu;
%calculating eccentricity
          ecc= 1 + rp*v_inf1^2/mu;
          DELTA(l,k,h,i,j) = 2*asin(1/ecc)*180/pi;
%turn angle, in degrees
          delta = 2*asin(1/ecc)*180/pi;
          theta_inf(l,k,h,i,j) = acos(-1/ecc)*180/pi;
%true anomaly
          PHI2(l,k,h,i,j)=phi1+delta;
%new spacecraft flight path angle relative to asteroid
          phi2=phi1+delta;


          V_inf2=[v_inf1*(cos((phi2)*pi/180)); v_inf1*(sin((phi2)*pi/180))];
%Spacecraft velocity relative to asteroid when exiting SOI

          V_sc2=V_ast+V_inf2;
%Heliocentic spacecraft velocity after flyby
          v_sc2=sqrt(dot(V_sc2,V_sc2));
          Change_v_sc(l,k,h,i,j)=(v_sc2-v_sc1);
%Change in speed

      end
      end
      end
      end
      end
```

2.alpha_study.m

```
      %This script outputs turn angle and change in spacecraft heliocentric
speed
      %for a range of angles of attack. For brevity, comments that are used
in
      %aggregate.m are excluded
      G=6.7384*10^(-20);
      M=10^22
%Set asteroid mass
      mu=G*M

      V_ast=[sqrt(132712000000/450000000);0];
%Set asteroid orbital radius
```

```
      for i=[1:181]
%Alpha iterated by 1 degree from 0 to 180

          ALPHA(i)=(i-1);
          alpha=(i-1);

          v_sc1=20;
%Heliocentric speed of spacecraft
          V_sc1=[v_sc1*cos(alpha*pi/180);v_sc1*sin(alpha*pi/180)];


          V_inf1=V_sc1-V_ast;
          v_inf1=sqrt(dot(V_inf1,V_inf1));
          PHI1(i)=atan(V_inf1(2)/V_inf1(1))*180/pi;
          phi1=atan(V_inf1(2)/V_inf1(1))*180/pi;


          rp=500;
%radius of periapsis, in km

          ECC(i)= 1 + rp*v_inf1^2/mu;
          ecc= 1 + rp*v_inf1^2/mu;
%eccentricity
          DELTA(i) = 2*asin(1/ecc)*180/pi;
%turn angle, in degrees
          delta = 2*asin(1/ecc)*180/pi;
          theta_inf(i) = acos(-1/ecc)*180/pi;
%true anomaly
          PHI2(i)=phi1+delta;
          phi2=phi1+delta;
          V_inf2=[v_inf1*(cos((phi2)*pi/180)); v_inf1*(sin((phi2)*pi/180))];
%New heliocentric transverse velocity

          V_sc2=V_ast+V_inf2;
          v_sc2=sqrt(dot(V_sc2,V_sc2));
          CHANGE_V_SC(i)=(v_sc2-v_sc1);
%Change in speed

      end
```

3.RP_study.m

```
      %This script outputs turn angle and change in spacecraft heliocentric
speed
      %for a range of periapse radiuses. For brevity, comments that are used
in
      %aggregate.m are excluded
      G=6.7384*10^(-20);
      M=10^22
%set asteroid mass
      mu=G*M
%km^3/s^2
```

```
      V_ast=[sqrt(132712000000/450000000);0];
%set asteroid velocity

      alpha=35;
%set alpha

      for i=[1:1:500]

          v_sc1=20;
%Heliocentric speed of spacecraft
          V_sc1=[v_sc1*cos(alpha*pi/180);v_sc1*sin(alpha*pi/180)];


          V_inf1=V_sc1-V_ast;
          v_inf1=sqrt(dot(V_inf1,V_inf1));
          PHI1(i)=atan(V_inf1(2)/V_inf1(1))*180/pi;
          phi1=atan(V_inf1(2)/V_inf1(1))*180/pi;

          RP(i)=10*i;
          rp=10*i;
%radius of periapsis, in km

          ECC(i)= 1 + rp*v_inf1^2/mu;
          ecc= 1 + rp*v_inf1^2/mu;
%eccentricity
          DELTA(i) = 2*asin(1/ecc)*180/pi;
%turn angle, in degrees
          delta = 2*asin(1/ecc)*180/pi;
          theta_inf(i) = acos(-1/ecc)*180/pi;
%true anomaly
          PHI2(i)=phi1+delta;
          phi2=phi1+delta;


          V_inf2=[v_inf1*(cos((phi2)*pi/180)); v_inf1*(sin((phi2)*pi/180))];
%New heliocentric transverse velocity

          V_sc2=V_ast+V_inf2;
          v_sc2=sqrt(dot(V_sc2,V_sc2));
          CHANGE_V_SC(i)=1.36-(v_sc2-v_sc1);
%Change in speed

      end
```

4.mass_study.m

```
      %This script outputs turn angle and change in spacecraft heliocentric
speed
      %for a range of periapse radiuses. For brevity, comments that are used
in
      %previous scripts are excluded
      G=6.7384*10^(-20);

      V_ast=[sqrt(132712000000/450000000);0];
```

```
      alpha=90;

      for i=[1:24]

          M(i)=10^(17+0.25*i);
%Storing asteroid mass
          m=10^(17+0.25*i);
%setting asteroid mass for future calculations
          mu=G*m;

          v_sc1=20;
%Heliocentric speed of spacecraft
          V_sc1=[v_sc1*cos(alpha*pi/180);v_sc1*sin(alpha*pi/180)];


          V_inf1=V_sc1-V_ast;
          v_inf1=sqrt(dot(V_inf1,V_inf1));
          PHI1(i)=atan(V_inf1(2)/V_inf1(1))*180/pi;
          phi1=atan(V_inf1(2)/V_inf1(1))*180/pi;

          rp=500;

          ECC(i)= 1 + rp*v_inf1^2/mu;
          ecc= 1 + rp*v_inf1^2/mu;
          DELTA(i) = 2*asin(1/ecc)*180/pi;
          delta = 2*asin(1/ecc)*180/pi;
          theta_inf(i) = acos(-1/ecc)*180/pi;
          PHI2(i)=phi1+delta;
          phi2=phi1+delta;


          V_inf2=[v_inf1*(cos((phi2)*pi/180)); v_inf1*(sin((phi2)*pi/180))];

          V_sc2=V_ast+V_inf2;
          v_sc2=sqrt(dot(V_sc2,V_sc2));
          CHANGE_V_SC(i)=(v_sc2-v_sc1);
%Change in speed

      end
```

5.incoming_v.m

```
      %This script outputs turn angle and change in spacecraft heliocentric
speed
      %for a range of periapse radiuses. For brevity, comments that are used
in
      %previous scripts are excluded
      G=6.7384*10^(-20);
      M=10^22
      mu=G*M

      V_ast=[sqrt(132712000000/450000000);0];
```

```
      alpha=35;

      for i=[1:1:30]

          SC_Helio_Speed(i)=i;
%Storing heliocentric spacecraft speed
          v_sc1=i;
%Heliocentric speed of spacecraft for future calculations
          V_sc1=[v_sc1*cos(alpha*pi/180);v_sc1*sin(alpha*pi/180)];


          V_inf1=V_sc1-V_ast;
          v_inf1=sqrt(dot(V_inf1,V_inf1));
          PHI1(i)=atan(V_inf1(2)/V_inf1(1))*180/pi;
          phi1=atan(V_inf1(2)/V_inf1(1))*180/pi;

          rp=500;

          ECC(i)= 1 + rp*v_inf1^2/mu;
          ecc= 1 + rp*v_inf1^2/mu;
          DELTA(i) = 2*asin(1/ecc)*180/pi;
          delta = 2*asin(1/ecc)*180/pi;
          theta_inf(i) = acos(-1/ecc)*180/pi;
          PHI2(i)=phi1+delta;
          phi2=phi1+delta;


          V_inf2=[v_inf1*(cos((phi2)*pi/180)); v_inf1*(sin((phi2)*pi/180))];

          V_sc2=V_ast+V_inf2;
          v_sc2=sqrt(dot(V_sc2,V_sc2));
          CHANGE_V_SC(i)=(v_sc2-v_sc1);
%Change in speed

      end
```

APPENDIX B: TURN ANGLE AND CHANGE IN VELOCITY VS ASTEROID MASS FOR VARIOUS APPROACH ANGLES

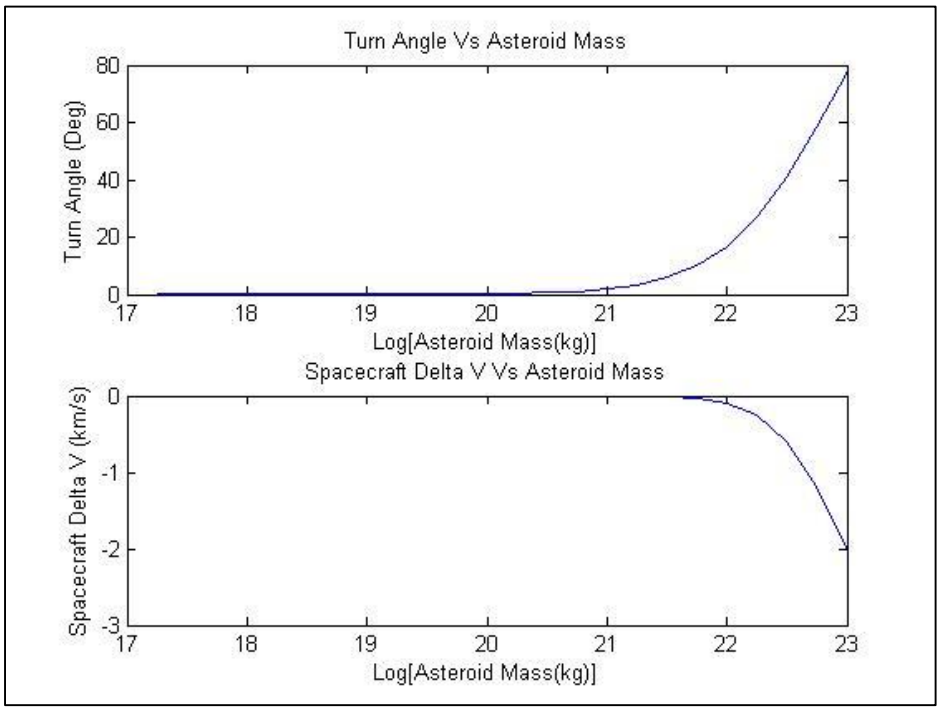

Figure B-1. Turn Angle and Change in Velocity vs Asteroid Mass at 0º Approach Angle

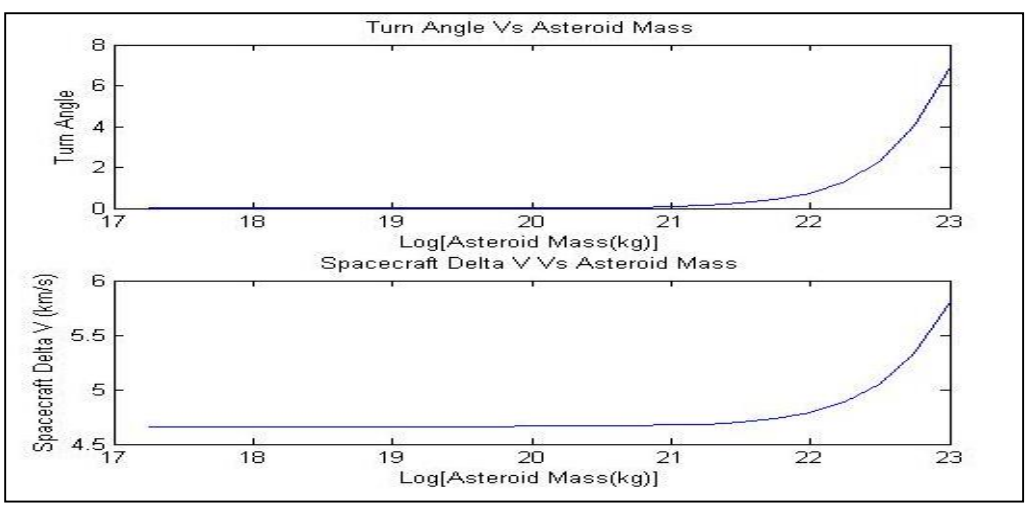

Figure B-2. Turn Angle and Change in Velocity vs Asteroid Mass at 45º Approach Angle

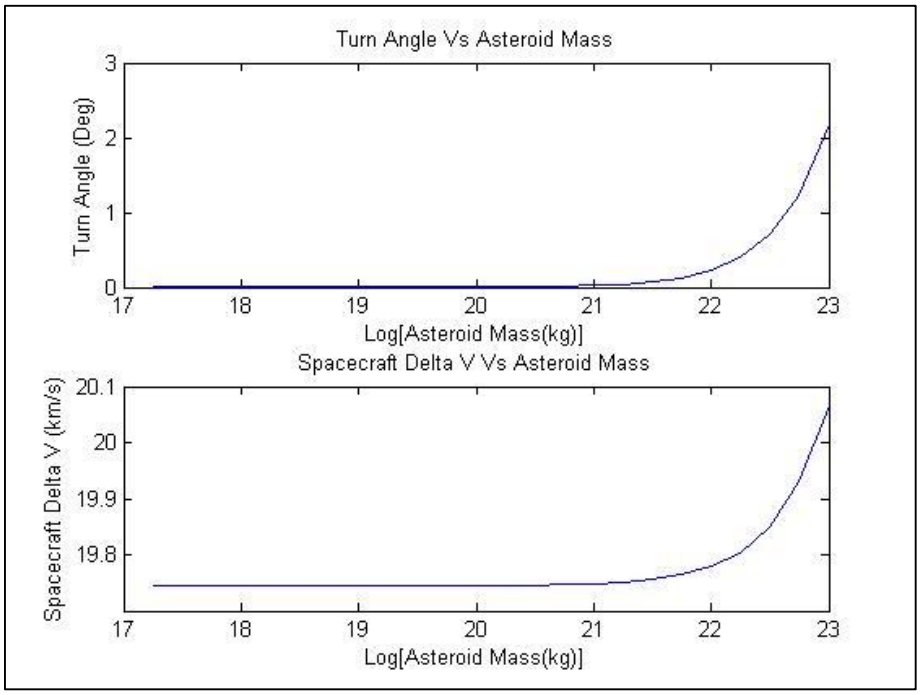

Figure B-3. Turn Angle and Change in Velocity vs Asteroid Mass at 90º Approach Angle

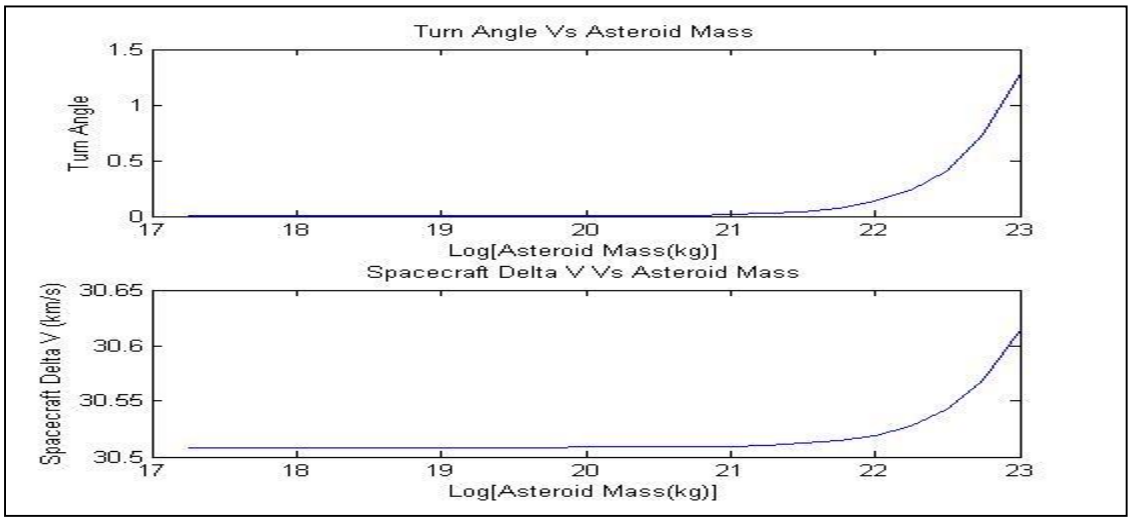

Figure B-4. Turn Angle and Change in Velocity vs Asteroid Mass at 135º Approach Angle

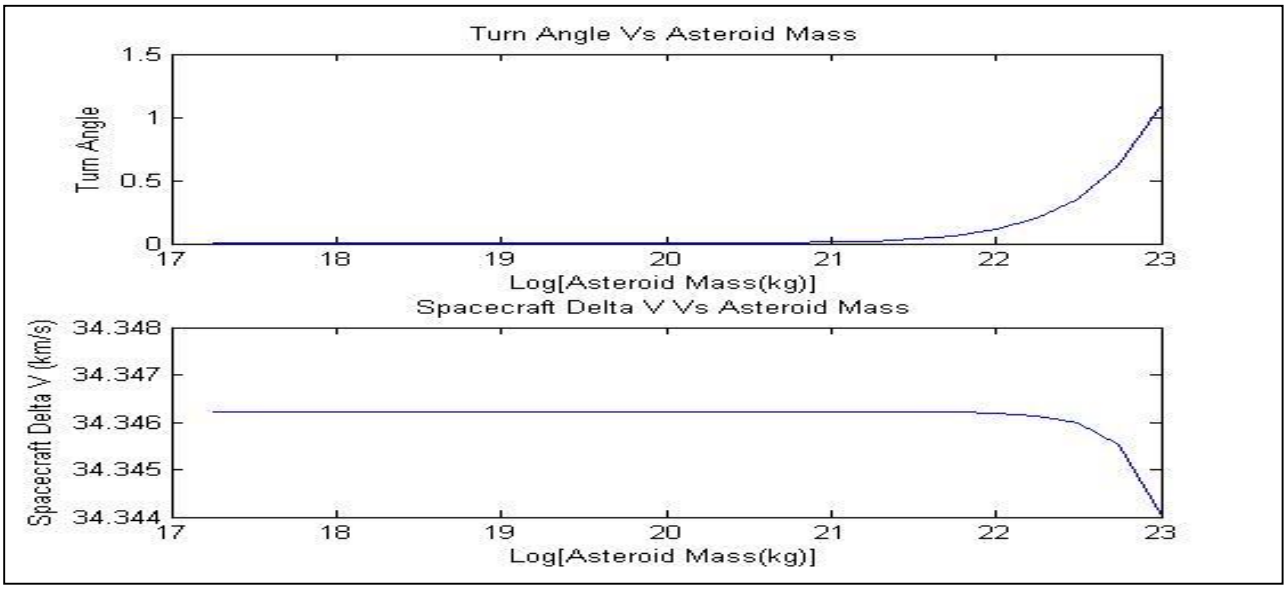

Figure B-5. Turn Angle and Change in Velocity vs Asteroid Mass at 180º Approach Angle

APPENDIX C: TURN ANGLE AND CHANGE IN VELOCITY VS Periapse Radius FOR VARIOUS APPROACH ANGLES

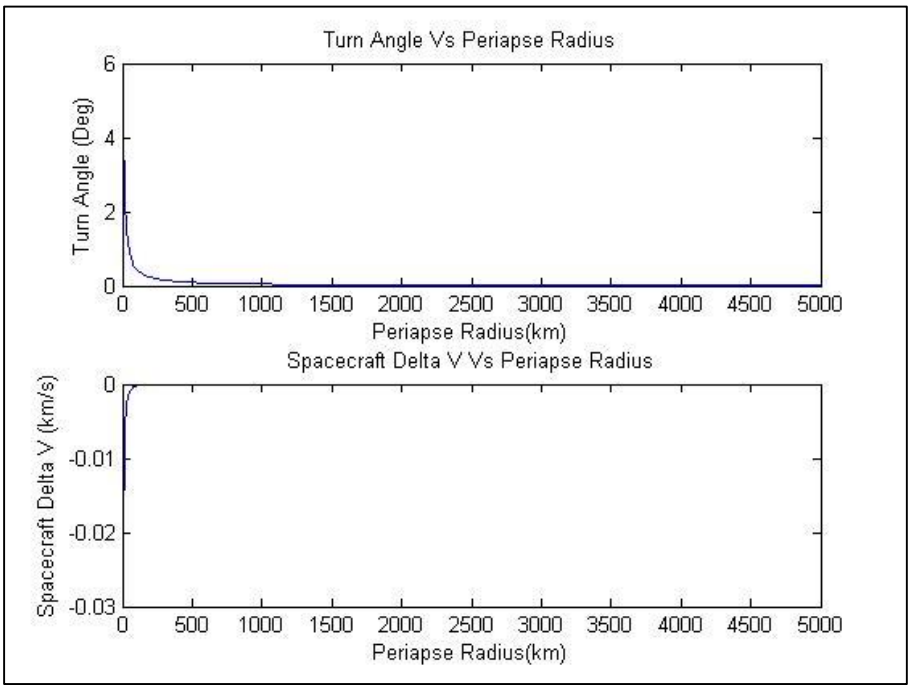

Figure C-1. Turn Angle and Change in Velocity vs Periapse Radius at 0º Approach Angle

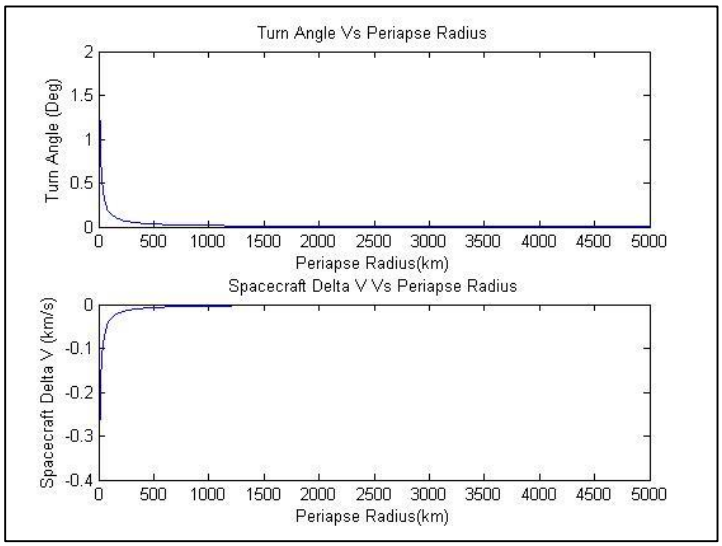

Figure C-2. Turn Angle and Change in Velocity vs Periapse Radius at 45º Approach Angle

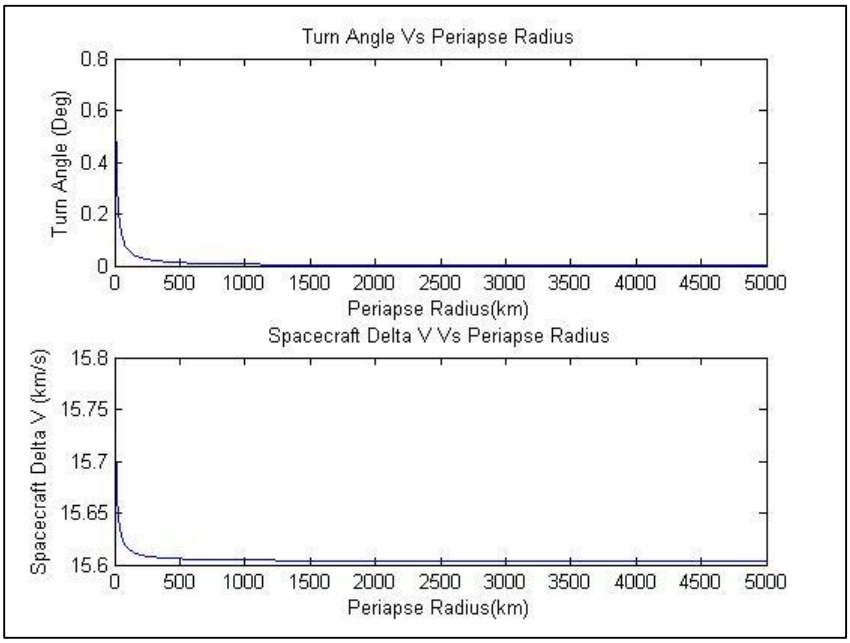

Figure C-3. Turn Angle and Change in Velocity vs Periapse Radius at 90º Approach Angle

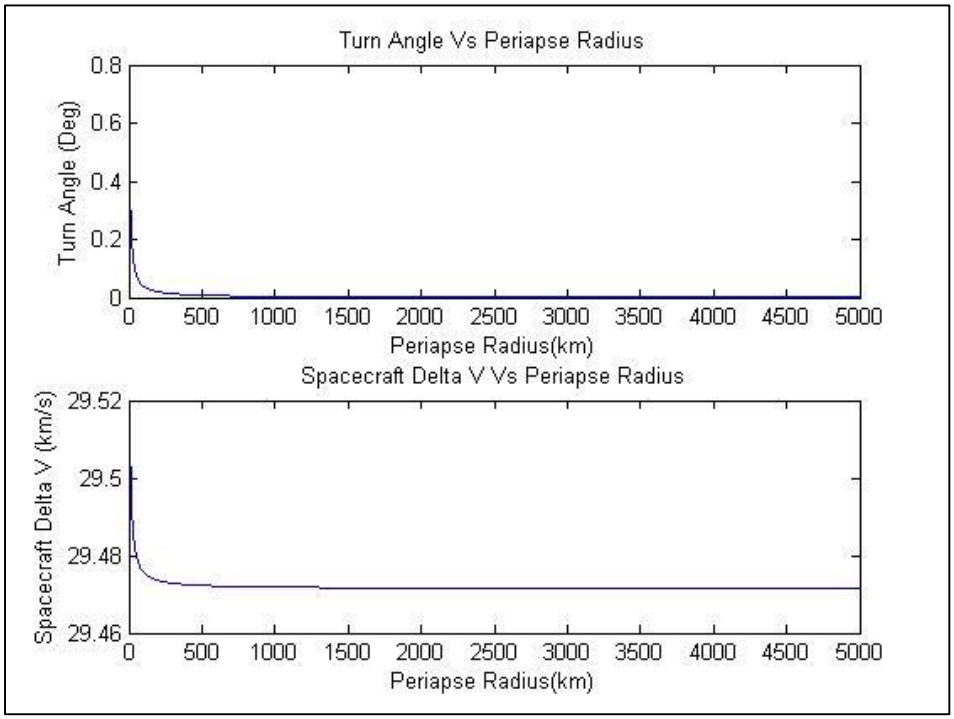

Figure C-4. Turn Angle and Change in Velocity vs Periapse Radius at 135º Approach Angle

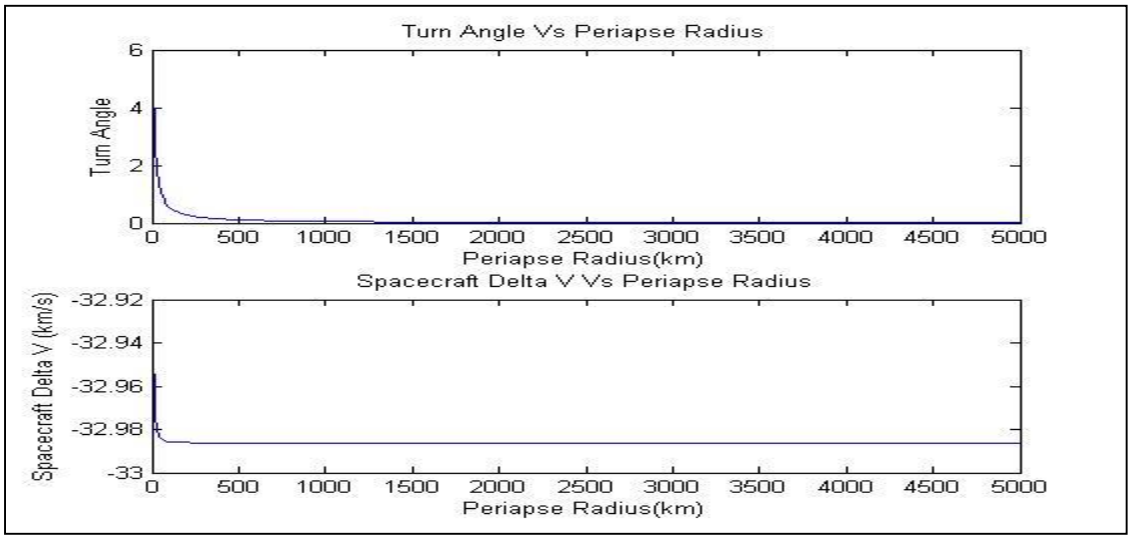

Figure C-5. Turn Angle and Change in Velocity vs Periapse Radius at 180º Approach Angle

APPENDIX D: COMPUTED RESULTS OF GRAVITY ASSISTS WITH KNOWN ASTEROIDS

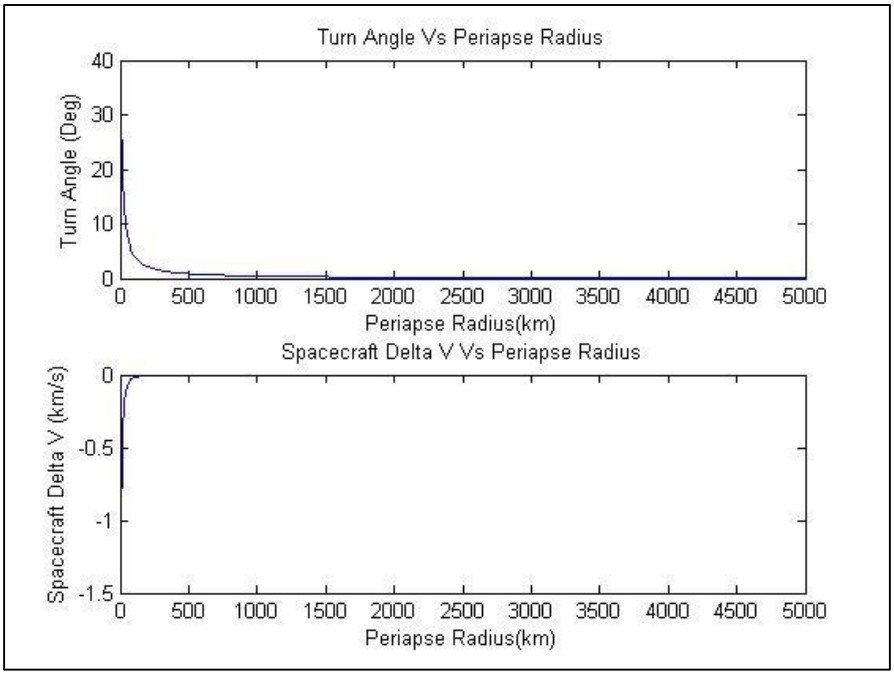

Figure D-2. Turn Angle vs Periapse Radius for Ceres Flyby with a 0 Degree Approach Angle

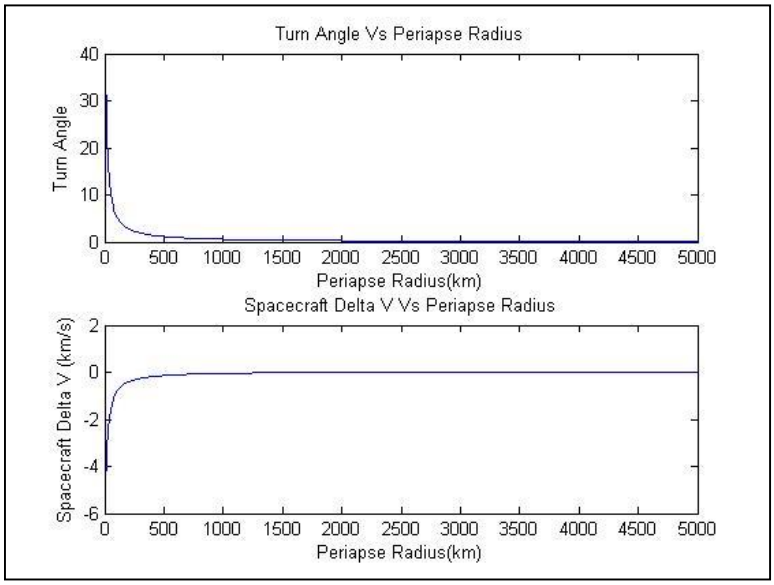

Figure D-2. Turn Angle vs Periapse Radius for Ceres Flyby with a 35 Degree Approach Angle

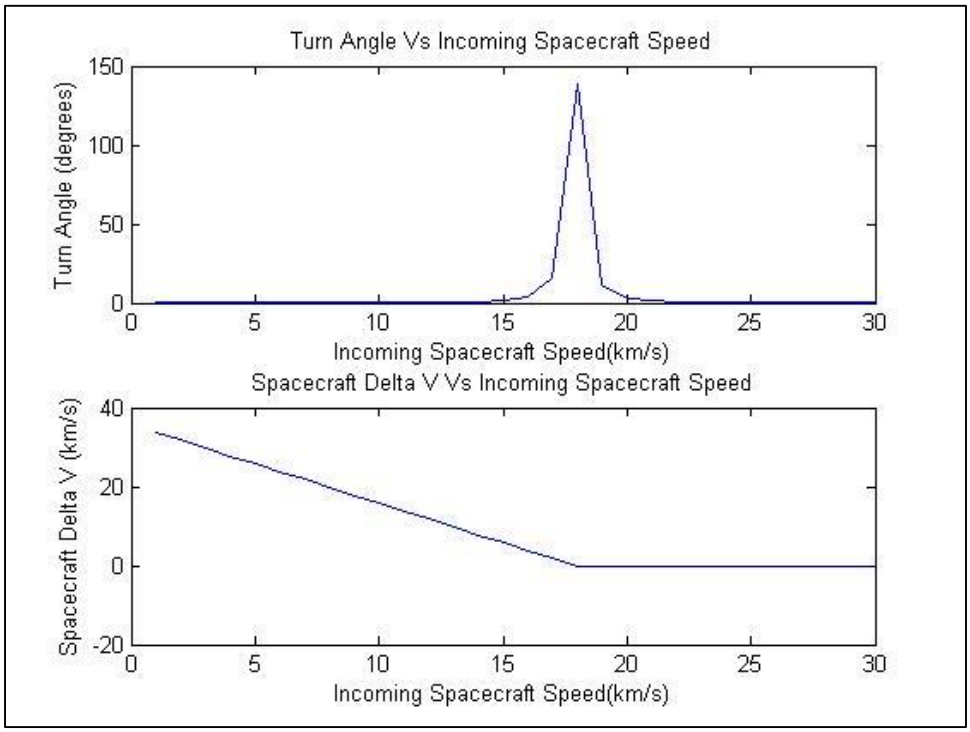

Figure D-3. Turn Angle vs Initial Spacecraft Heliocentric Velocity for Ceres Flyby with a 0 Degree Approach Angle

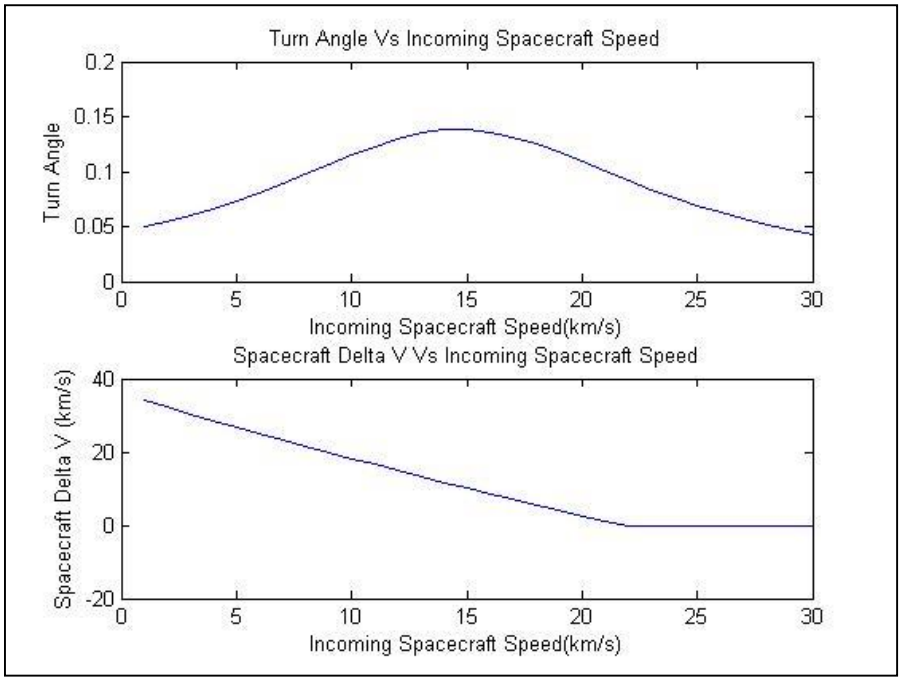

Figure D-4. Turn Angle vs Initial Spacecraft Heliocentric Velocity for Ceres Flyby with a 35 Degree Approach Angle

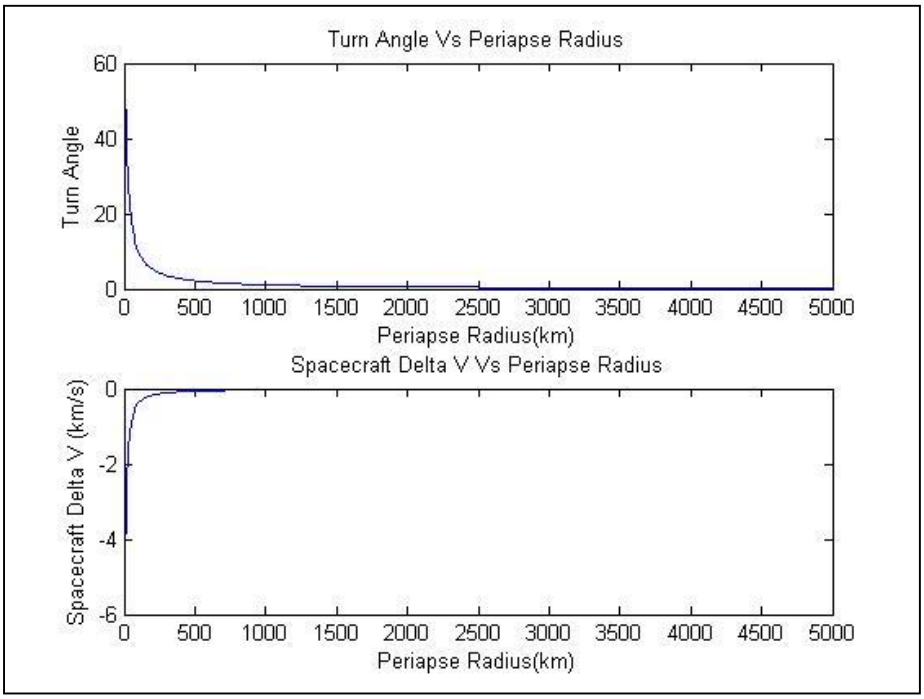

Figure D-5. Turn Angle vs Periapse Radius for Pallos Flyby with a 5 Degree Approach Angle

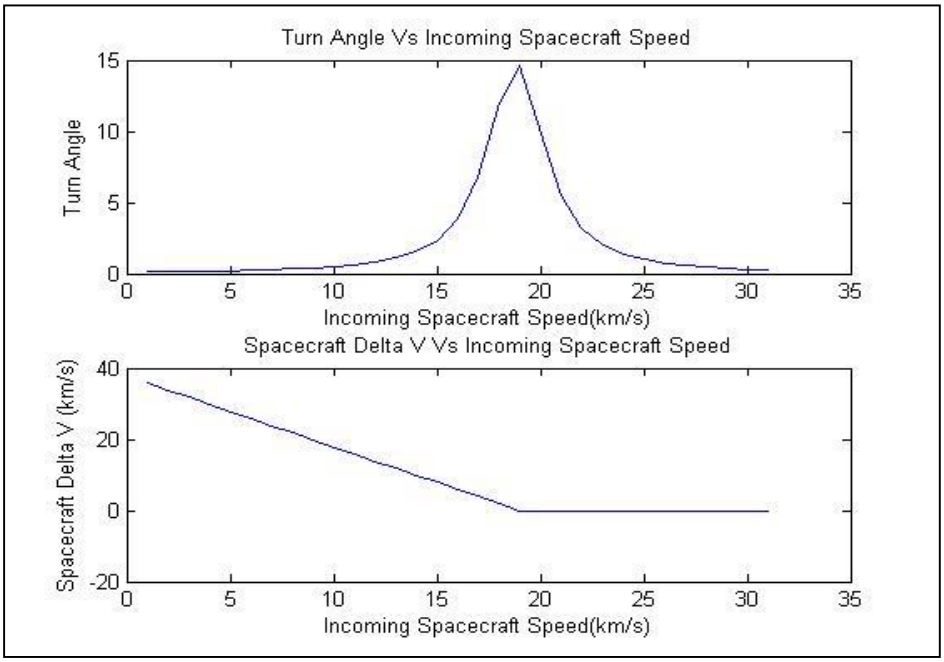

Figure D-6. Turn Angle vs Initial Spacecraft Heliocentric Velocity for Pallos Flyby with a 5 Degree Approach Angle

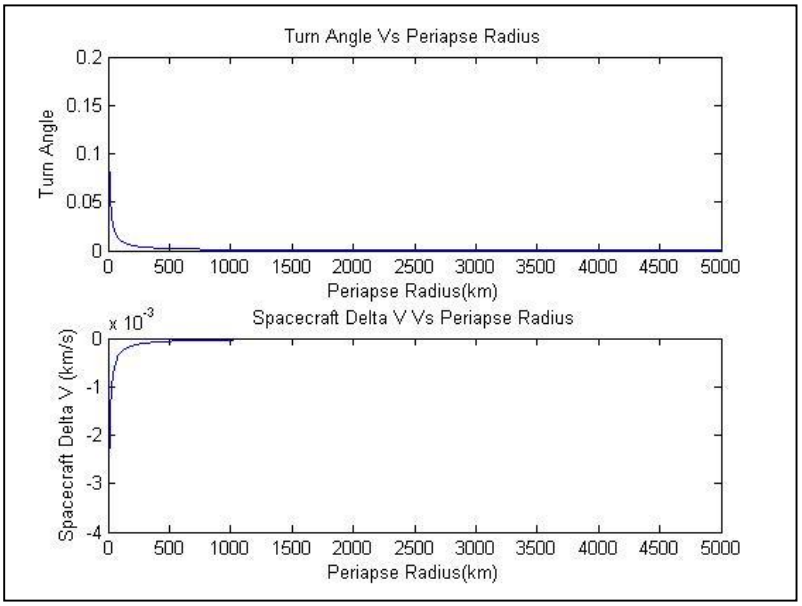

Figure D-7. Turn Angle vs Periapse Radius for Juno Flyby with a 5 Degree Approach Angle

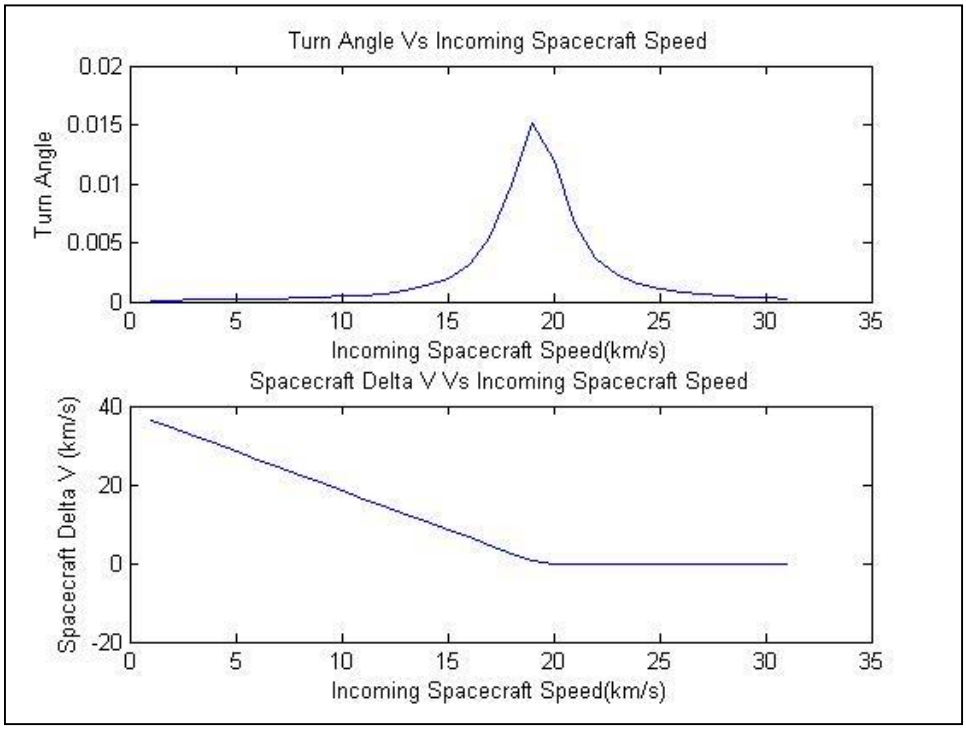

Figure D-8 Turn Angle vs Initial Spacecraft Heliocentric Velocity for Juno Flyby with a 5 Degree Approach Angle

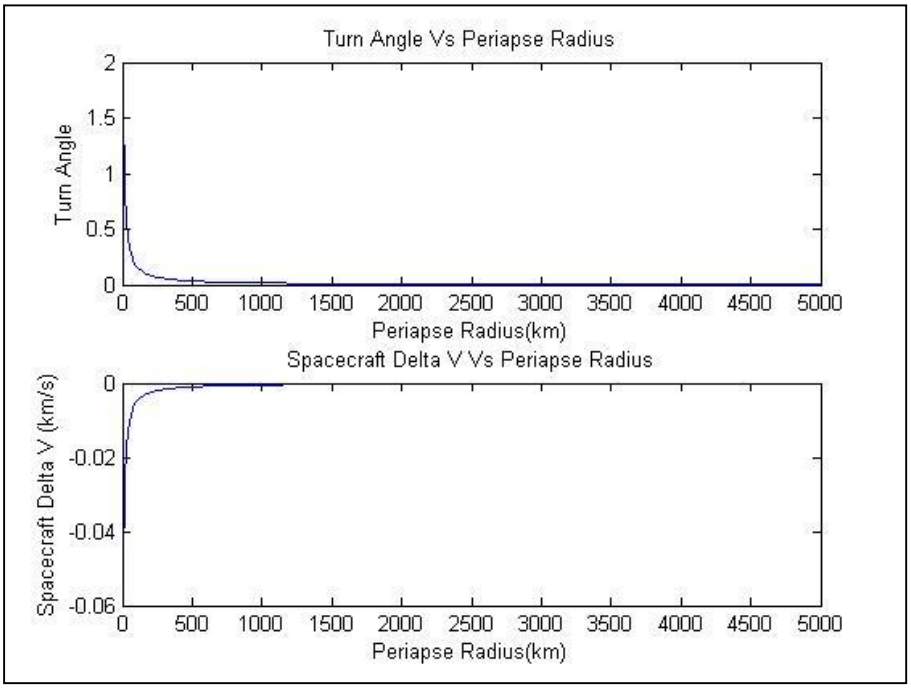

Figure D-9. Turn Angle vs Periapse Radius for Vesta Flyby with a 5 Degree Approach Angle

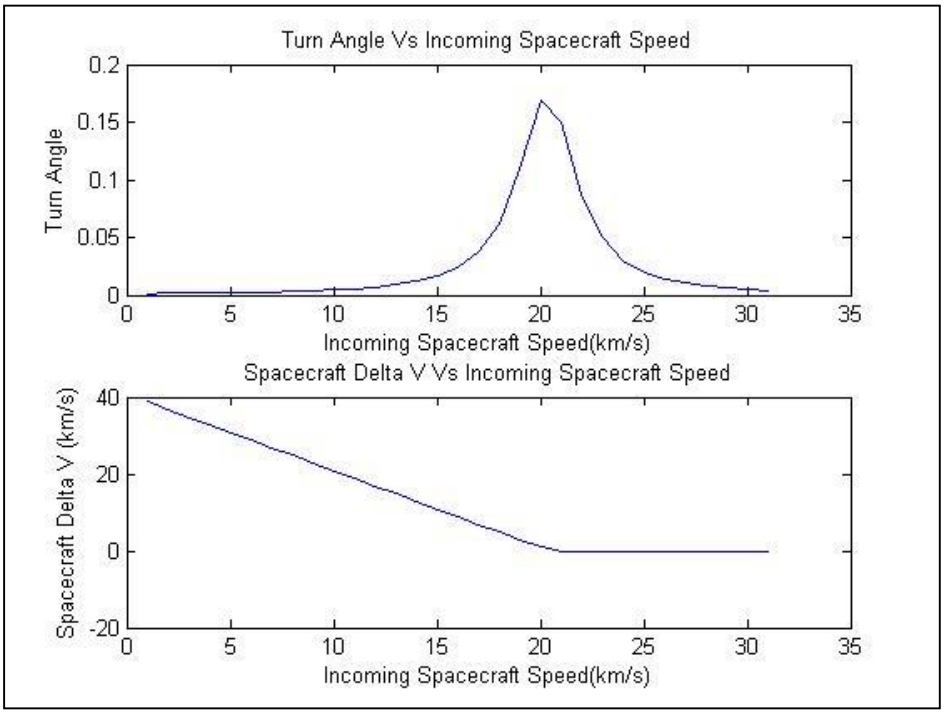

Figure D-10. Turn Angle vs Initial Spacecraft Heliocentric for Vesta Flyby with a 5 Degree Approach Angle

Academic Vita of Christopher P. Tombasco

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Education
Pennsylvania State University (Graduating with Honors in Spring 2015)
B.S. in Aerospace Engineering, Minor in Economics
Thesis: Study of Feasibility of Asteroid-Centric Gravity Assists (Supervised by Dr. David B. Spencer)

Projects:
RAS-CAL EDL Competition
Date: Fall 2014-Spring 2015
Description: Worked as propulsion lead in designing a mission to land twenty metric tons on the surface of Mars.

Honors Thesis
Date: Fall 2014- Spring 2015
Description: Analyzed the effects of a gravity assist maneuver on the velocity of a spacecraft for a wide range of asteroid masses and incoming spacecraft velocities; determined minimum asteroid mass needed for significant change to the spacecraft's velocity.

Honors and Awards
Schreyer Honors Scholar: 2011-2015
Galen E. Dreibelbis Renaissance Scholarship: 2011-2015
Digital Global Scholarship: Fall 2014
Sigma Gamma Tau Member: Spring2014-Spring 2015
Oder of Omega: Spring 2013-Spring 2015

Work Experience
Date: May 2013-August 2013
Title: Orientation Leader
Institution: Pennsylvania State University Office of Student Transition and Orientation Programs
Description: Lead small groups of students through conversations and activities to help them smoothly transition
Supervisor: Daniel Murphy