AN ANALYSIS OF THE DEVELOPMENT OF SLOPE ACROSS A TEXTBOOK SERIES

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ABSTRACT

This thesis explores the development of the mathematical concept of slope across a textbook series. A reform-based textbook series spanning pre-algebra to precalculus was selected for this study. References to slope in the series were coded for type and level of covariational reasoning as well as other characteristics. The results of this coding were compiled and analyzed to more fully understand the way in which a textbook series develops slope. This is part of a larger body of research on the concept of slope that has already studied teachers, students, and educational standards. The collective goal of these studies is to understand how slope can best be taught and understood by students. In this research, it was found that while slope was well-covered in many areas (covariational reasoning, connection to real world applications), there were areas for improvement (lack of visual representations for slope, overwhelmingly procedural practice problems involving slope, little attention given to steepness of a line, limited extension of slope to non-linear functions). The conclusions of this thesis research were presented at the Mathematical Association of America Allegheny Mountain Section student talks.
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Chapter 1

Literature Review

“Every natural phenomenon, from the quantum vibrations of sub-atomic particles, to the universe itself, is a manifestation of change.... It is of great importance that we should understand and control the changing world in which we live.”

-Ian Stewart, 1990

Background

Slope is a critical concept in the mathematics curriculum. Every student learns it, many times in multiple years of schooling. Accordingly, it is referenced across several grades in the Common Core State Standards for Mathematics (CCSSM, 2010). Yet, if you were to ask a student, “What is slope?” you would probably receive an array of completely different answers. Many may at first regurgitate a quick definition such as “rise over run”, the $m$ in $y=mx+b$, or “change in $y$ over change in $x$”. Pressed further students might explain how a positive slope means a line is increasing and a negative slope means a line is decreasing. Maybe they will recall the fact that the bigger the slope, the steeper a line is. Students who have taken calculus might make the point that slope is not always constant.

All of these students would be correct. Slope is a diverse concept. It evolves across the math curriculum. As students first begin to learn the concept, its importance may not seem so clear, but its usefulness is proven time and time again from algebra to calculus and beyond. It
might not always look the same to students. Sometimes they may be thinking about it more as a ratio of differences. Other times it may just be a point of comparison (for example if one line is steeper than another, or if they are parallel). It is important that students learn all of the different ways slope “looks”. It is also just as important that they learn to connect these different forms of slope.

**Previous Research**

*Slope Conceptualizations*

This is not an entirely new area of study. Slope conceptualizations have been researched for nearly two decades. In 1997, Dr. Sheryl Stump published a study of how preservice and inservice teachers conceptualize slope. This may very well be the first explicit analysis of slope conceptualizations. In her research, Dr. Stump formulates a list of “representations” of slope that bare many similarities to the list of conceptualizations used in this study (more detail on this in Chapter 2). She codes each teacher’s responses to survey and interview questions according to these representations. This approach will characterize future research in slope conceptualizations.

Much of the current research on the topic has been undertaken by Dr. Deborah Moore-Russo and Dr. Courtney Nagle¹, who have worked collaboratively to study slope conceptualizations in standards documents (Stanton & Moore-Russo, 2012; Nagle & Moore-Russo, 2014), among students (Nagle, Moore-Russo, et. al, 2013c), and among instructors (Nagle & Moore-Russo, 2013a). In addition to collecting and analyzing valuable data on how we think about slope, they have also continually refined the definitions of the conceptualizations

¹ Dr. Nagle served as advisor for this thesis
they use in their research. The 11 conceptualizations they have primarily used are described in Table 1.1. The conceptualizations used in this thesis are based on these, and will described in Chapter 2.

Table 1.1 (from Table 1 in Nagle, et. al, 2013c)

Definitions for Slope Conceptualizations

<table>
<thead>
<tr>
<th>Conceptualization</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Ratio</td>
<td>Rise over run of a line; ratio of vertical displacement to horizontal displacement of a line’s graph</td>
</tr>
<tr>
<td>Algebraic Ratio</td>
<td>Change in $y$ over change in $x$; ratio with algebraic expressions (often seen as either $\Delta y/\Delta x$ or $(y_2 - y_1)/(x_2 - x_1)$)</td>
</tr>
<tr>
<td>Physical Property</td>
<td>Property of line often described using expressions like grade, incline, pitch, steepness, slant, tilt, and “how high a line goes up”</td>
</tr>
<tr>
<td>Functional Property</td>
<td>(Constant) rate of change between variables; sometimes seen in responses involving related rates</td>
</tr>
<tr>
<td>Parametric Coefficient</td>
<td>The variable $m$ (or its numeric value) found in $y = mx + b$ and $y_2 - y_1 = m(x_2 - x_1)$</td>
</tr>
<tr>
<td>Trigonometric Coefficient</td>
<td>Property related to the angle a line makes with a horizontal line; tangent of a line’s angle of inclination/decline; direction component of a vector</td>
</tr>
<tr>
<td>Calculus Conception</td>
<td>Limit; derivative; a measure of instantaneous rate of change for any (even nonlinear) functions; tangent line to a curve at a point</td>
</tr>
<tr>
<td>Real World Situation</td>
<td>Static, physical or dynamic, functional situation (e.g. wheelchair ramp, distance versus time)</td>
</tr>
<tr>
<td>Determining Property</td>
<td>Property that determines if lines are parallel or perpendicular; property can determine a line if a point on the line is also given</td>
</tr>
<tr>
<td>Behavior Indicator</td>
<td>Property that indicates increasing/decreasing/horizontal trends of line or amount of increase or decrease; if nonzero, indicates intersection with x-axis</td>
</tr>
<tr>
<td>Linear Constant</td>
<td>Constant property independent of representation; unaffected by translation of a line; reference to what makes a line “straight” or the “straightness” of a line</td>
</tr>
</tbody>
</table>
Covariational Reasoning

With slope as diverse a topic as it is, different conceptualizations do not quite capture all of its aspects. It is important to not just to look at the different types of slope, but also the different levels. For this, we consider covariational reasoning. This is defined as “cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other” (Carlson, 2002, p. 354).

In Dr. Carlson’s study she examined students’ understanding of covariational reasoning. An importance consequence of her research was the codification of the different levels of covariation (described in detail in Chapter 2). Dr. Carlson’s exact definitions are used in the coding of expository material in this textbook.

In a study conducted by Eric Weber and Allison Dorko (2014), it was found that students in multivariable calculus had an inadequate understanding of covariational reasoning. If students even in the upper levels of mathematics do not have a strong grasp on the concept, this points to a larger issue in mathematics education. This is concerning because slope is often thought of (inaccurately) as describing the change of one variable. Slope actually describes the corresponding change in two variables (although in higher mathematics students can begin to describe slope among more than two variables). Therefore, it is very important when looking at this textbook series to analyze how covariational reasoning is fostered.

Multiple Representations

Multiple representations are essentially different ways of looking at the same idea. They differ slightly from the conceptualizations defined earlier, and the difference can be seen in the words themselves. Conceptualizations refer to the different concepts of slope while representations refer to the different ways of representing slope. So for example, two different textbook
problems could describe slope as an indicating behavior and as a geometric ratio, but describe these concepts very similarly, perhaps with an illustrated word problem. Conversely, two different problems could both describe slope as indicating behavior, but in very different ways.

Providing multiple representations has proven to be an effective strategy for increasing student understanding. In one study, a computer-based intervention program demonstrated drastic improvements in understanding of the mathematical concept of estimation (Ainsworth, Bibby, Wood, 2002). The researchers believe this improvement was due to students being able to translate across representations for a deeper understanding of the topic.

Multiple representations can be broken down into many categories, such as pictures, words, numbers, formulas, videos, etc. The study of multiple representations in this series is broken down to a simple approach: visual and non-visual. In this way, we can understand roughly how well the series incorporates multiple representations.

**Previous Textbook Studies**

Since this research breaks new ground in the study of slope conceptualizations, it is important to understand how textbooks have been studied in the past from a research perspective. This research is very different from analyzing a standards document, for example. Whereas a standard is a general idea of something that could be taught many different ways, a textbook problem is a very specific question on a topic presented in a particular way. Furthermore, with a textbook problem you must analyze the likely student response in addition to the words written down on the page. On top of all of this, a textbook is a varied document. There are problems, explanations of concepts, and other miscellaneous content that make the study of textbooks all the more challenging.
Previous studies on textbooks have shown a particular focus on general features of problems/explanations. These are broad classifications that could be used to describe almost any textbook content in any subject. One such example is **cognitive requirement** (Li, 2000). This is simply the distinction between a problem/explanation achieving procedural practice or conceptual understanding. Essentially what this shows is whether the content is meant to reinforce knowledge or expand it. Another feature, which we have termed **flexibility**, examines whether a problem is open-ended or closed-ended (Zhu & Fan, 2006). Flexibility determines whether a student has a singular path to an answer, or multiple ways to respond. Given the nature of flexibility, this feature only applied to textbook problems, not explanations. One final feature, **application**, was also analyzed in problems and explanations (Zhu & Fan, 2006). In short, **application** examines whether content demonstrated a real world or non-real world application. All of these features and how they were coded for (with examples) are described in greater depth in the next chapter.

**Motivation**

It was noted earlier that this is not a new area of study. So why conduct this research? The first and simplest reason is that while slope conceptualizations have been analyzed several times, no such research exists pertaining to textbooks. Taken altogether, the combined work of Stump, Moore-Russo, and Nagle is really an analysis of slope in all aspects of the math curriculum. But even the least textbook-dependent teacher would be hard-pressed to admit that textbooks are not a major influence on the mathematics curriculum. Therefore, it is important that within the extensive study of slope conceptualizations, textbooks are given their day in court.
Chapter 2
Methodology

In this chapter, I describe the process that was followed in conducting my research, as well as the various categories that were used to describe problems, explanations, and conceptualizations of slope. There are 3 sections. The first section details the specific process that was used to collect and analyze data. The second section explains the categorizations that were used to code each problem and explanation. The third section clarifies the different ways in which problems and explanations were coded, as well as what constituted a problem or explanation.

Process

For this project, I wanted to research how the concept of slope is developed across the curriculum. In particular, I wanted to determine how connections are made between different conceptualizations of slope and which representations, contexts, and problem types are associated with a particular conceptualization. I also studied the development of covariational reasoning as it relates to slope.

The following are the specific questions addressed in this thesis:

- What conceptualizations are emphasized within the series and how are they represented (visually or non-Visually)?
- How is covariational reasoning developed throughout the series?
- How do the books present slope in a general sense? Are real world applications often used? Are practice problems procedural or conceptual?

Given the goals of this project, a natural starting point was a review of the previous literature (described in the previous chapter). Earlier studies regarding the development of slope conceptualizations focused on students, teachers, and standards, but a serious analysis of textbooks had not yet been undertaken.

I worked closely with my advisor, Dr. Courtney Nagle on this project. She secured a textbook series written through the University of Chicago School Mathematics Project (UCSMP). This series was written specifically to correlate with Common Core State Standards (UCSMP, 2010a). It is part of a curriculum developed by UCSMP that emphasizes real world application, technology as an instructional tool, multi-dimensional understanding, and mastery learning (UCSMP, 2010b).

The seven textbooks from this series that were analyzed were *Pre-Transition Mathematics (PTM)*, *Transition Mathematics (TM)*, *Algebra (A)*, *Geometry (G)*, *Advanced Algebra (AA)*, *Functions, Statistics, and Trigonometry (FST)*, and *Precalculus and Discrete Mathematics (PC)*. The letters in parentheses denote how the textbook title will be abbreviated in tables and graphs. It should be noted that since this study focused on linear slope, the coding of *Precalculus* excluded examples of variable or instantaneous slope, unless connections were made to linear slope as well.

Once the textbook series was selected, a coding scheme was developed. This involved review of previous research on slope conceptualizations as well as studies of other textbooks. My advisor and I also spoke with Dr. Deborah Moore-Russo by teleconference. Dr. Moore-Russo has conducted much of the available research on slope conceptualizations and is a frequent
collaborator with Dr. Nagle. The sources from which the categorizations came about will be described in more detail in the last section of this chapter.

After developing a coding scheme, I began coding. All data was recorded in Excel. This process took place for three months in the Fall of 2015. Each textbook took roughly one to three weeks to code in full, with the duration depending on the subject of the textbook. For example, Algebra was much denser with references to slope than Geometry, and as a result, took longer to code.

I performed the coding independently and met each week with my advisor to clarify any codings for which I was not confident. We would discuss these cases and come to a conclusion, often referring back to how we specifically defined each categorization. Early on as I was becoming familiar with the categorizations, we discussed most references to slope. As I progressed and became better acquainted with the categorizations, we only discussed a few uncertain codings each week.

After all the texts had been fully coded, data analysis began. From mid-December 2015 to mid-January 2016, I sorted the data to get a comprehensive view of what had been coded. The data was analyzed to consider longitudinal trends over the whole series. I also conducted individual analysis of each textbook. Frequency of categorizations, coincidence of conceptualizations, and more was collected. After review of data with my advisor, I conducted additional analysis based on her suggestions. Primarily this involved re-analyzing data without conceptualizations broken up as visual and non-visual. All of this data will be presented in more detail in Chapter 3: Results. After data collection was finished, I interpreted what I had found for patterns. Further detail on this interpretation will be given in Chapter 4: Discussion and Interpretation.
Categorizations

*Slope Conceptualizations*

Slope conceptualizations included *constant parameter* (CP), *ratio* (R), *indicating behavior* (IB), *steepness* (S), and *determining property* (DP). These conceptualizations were developed over years of research by Dr. Deborah Moore-Russo and Dr. Sheryl Stump. Table 2.1 provides definitions for each conceptualization subdivided into visual and non-visual categorizations. Examples from the textbooks are provided to demonstrate what these conceptualizations may look like in a problem or explanation. Readers may note that Table 1.1 contained 11 conceptualizations, and this table only contains five. This revised list of conceptualizations uses more general definitions (taken directly from a recent update made by Moore-Russo in an unpublished manuscript (2015)) that encompass all of the original 11 conceptualizations. These conceptualizations are used because they have in some form or another been utilized in much of the research in this particular subject. (Stanton & Moore-Russo, 2012; Nagle & Moore-Russo, 2013; Nagle, Moore-Russo, Viglietti, & Martin, 2013; Nagle & Moore-Russo, 2014)

Each conceptualization is subdivided into their visual and non-visual forms. Just as it is important to see both real and non-real world applications of mathematics, it is also important to see concepts through various representations to gain a full understanding.

Moore-Russo defines visual and non-visual as follows:

**Visual:** “Includes a diagram or graph; may involve either numbers or words; could include verbal descriptions meant to promote visualization or mental imagery.”

**Non-Visual:** “Usually an equation, table, set, or, algebraic expression; may involve algebraic symbols, numbers, or words; could include symbols that are not typically considered mathematical” (Moore-Russo, 2015, pg. 1)
Table 2.1 (from Moore-Russo, 2015)

**Definitions for Slope Conceptualizations**

<table>
<thead>
<tr>
<th>Conceptualization</th>
<th>Representation</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Constant Parameter** | **Visual** | ● Emphasis on the uniform “straightness” of the line’s entire graph (i.e., no curvature)  
No matter which segment of the line is considered, the slope remains the same between any two points | Consider the points (3,1), (-4, 4.5), and (5,0). Determine if they lie on the same line; if they do not, explain why not. (Advanced Algebra, p. 162, #14) |
|                     | **Non-Visual** | ● Emphasis that a single constant holds a property for the line’s equation/table (not dependent on input)  
No matter which interval on the line is considered, the slope remains the same between any two points | Write and expression for the amount of money each person has or owes after w weeks:  
a) Eddie is given $100 and spends $4 per week. (Algebra, p. 84. #25) |
| **Ratio** | **Visual** | ● Rise over run or vertical change divided by horizontal change | A slope of \( \frac{-3}{2} \) means that for every change of 3 units to the right there is a change of 2 units. (Advanced Algebra, p. 97, #4) |
|                     | **Non-Visual** | ● \( \frac{b_2-b_1}{a_2-a_1} \)  
A rate between two covarying quantities | Find an equation for the line passing through (5,3) and (0,8). (Student calculates slope using ratio formula) (Functions, Statistics, and Trigonometry, p. 60, #15) |
| **Indicating Behavior** | **Visual** | ● Lines increases (i.e., looks like \( / \)) for positive slope  
Lines decreases (i.e., looks like \( \) ) for negative slope  
Lines is horizontal (i.e., looks like \( \) ) for zero slope | Juana leaves school, 3 miles away from her house, at 2 P.M., to go to drum and bugle practice. She arrives at practice at 2:15, 2 miles away from her house. Halfway through the 2-hour-long practice, she gets sick and goes home for some medicine. She gets home at 3:45. Draw a time-distance graph representing this situation. (Transition Mathematics, p. 665, #16) |
|                     | **Non-Visual** | ● An increasing rate implies a positive slope  
A decreasing rate implies a negative slope  
A flat rate implies a zero slope | According to the Census Bureau, in 2005, Delaware had a population of approximately 840,000, which was increasing at a rate of about 12,000 people a year. Montana had a population of approximately 935,000, increasing at a rate of 6,700 per year. If these rates continue in the future, in how many years after 2005 will the population be equal. (Algebra, p. 215, #18) |
| **Steepness** | **Visual** | ● Relates to how inclined, tilted, slanted, or pitched a line is seen as being  
The greater the absolute value of slope, the more steep the line (i.e., closer to a vertical line)  
The closer to zero the absolute value of slope, the less steep the line (i.e., closer to a horizontal line)  
Since horizontal lines have no tilt, they have zero slope | What property of the graph indicates that Bick biked faster on his way home than on his way to the store? (time-distance graph provided) (Transition Mathematics, p. 656, #7) |
|                     | **Non-Visual** | ● Relates to how extreme a line is calculated as being  
The greater the absolute value of slope, the more steep the line over an interval (i.e., \( \frac{|b_2-b_1|}{|a_2-a_1|} \) is closer to infinity)  
The closer to zero the absolute value of slope, the less steep the line over an interval (i.e., \( \frac{|b_2-b_1|}{|a_2-a_1|} \) is closer to zero)  
Since horizontal lines have \( \frac{|b_2-b_1|}{|a_2-a_1|} = 0 \) for all values, they have zero slope | Between which two years was there the greatest increase in employment rate for men? Women? (numerical chart provided) (Algebra, p. 347, #23) |
| **Determining Property** | **Visual** | ● Two unique lines have the same slope if and only if they never intersect (i.e., they are parallel)  
Two unique lines have different slopes if and only if they intersect at a common point  
Two unique non-vertical lines have negative reciprocal slopes if and only if their intersection is at a right angle (i.e. they are perpendicular) | Find the slope of a line parallel to the line with the given equation:  
\( 4y = -2x + 7 \)  
(Geometry, p. 149, #11) |
|                     | **Non-Visual** | ● Two unique lines have the same slope if and only if a system of these two lines has no solution (i.e., system is inconsistent since lines are parallel)  
Two unique lines have different slopes if and only if a system of these two lines has one solution (i.e., system is consistent; there is a shared ordered pair)  
Two unique non-vertical lines have negative reciprocal slopes if and only if the product of their slopes is -1 (i.e., they are perpendicular) | When solving the equation \( ax + b = cx + d \) for \( x \), there may be no solution, exactly one solution, or infinitely many solutions. What must be true about \( a, b, c, \) and \( d \) to guarantee each of the following?  
a. There is exactly one solution.  
b. There are no solutions.  
c. There are infinitely many solutions. (Algebra, p. 220, #19) |
Cognitive Requirement (Procedural/Conceptual)

All problems were coded for whether they promote procedural practice or conceptual understanding. It is important to note that these categories were not mutually exclusive. It is possible for a problem to be coded as both. This often occurred when a problem had multiple parts that built from simple procedure to more conceptual questions based on those procedural observations. Cognitive requirement is important to observe as it gives us a general characterization of the types of problems we are analyzing in this series.

Procedural practice is defined much as it sounds as following a procedure to answer a question. Often the question is presented in the same section or chapter where the procedure is explained. Although procedural practice often still requires some amount of mathematical thinking, it could easily be performed by closely following earlier examples (Li, 2000). The following sample problem from the Algebra textbook is provided as an example of a procedural task:

*Graph the line with the given condition: passes through (0,1) with a slope of 0.4.*

*(Algebra, p. 395, #54)*

This problem simply asks students to perform a procedural task. At no point in the process is it absolutely necessary for the student to understand what they are doing or why they are doing it.

Conceptual understanding refers to problems that encourage a higher level of thinking. Often these problems call for students to perform tasks that do not have a readily available or already known procedure. Additionally, conceptual understanding problems may promote connections across topics and lead to new discoveries. Finally, the generalization of a previously
learned procedure to include a wider problem set was considered highly conceptual (Li, 2000). An example of a conceptual problem is provided below:

*How can you use slope to show that three points do not all lie on the same line?*

*(Algebra, p. 393, #31)*

In this problem, students are not just applying a procedure, but developing the procedure themselves. To do this, they must use what they know about slopes of lines (constant over all intervals).

**Flexibility (Open-Ended/Closed-Ended)**

Another important characteristic of these problems is the flexibility students are given in their answers. The work of Zhu and Fan (2006) inspired this conceptualization although our definitions of open-ended/closed-ended differ slightly.

An open-ended problem is one where more than one correct answer is possible. This correlated well with conceptual understanding, as will be seen further on. Open-ended also includes problems which may be solved in various ways. For many problems, it was clear implicitly (by what had just been described in the previous section in the text) which method a student would use to solve a problem. However, in several instances, students were openly given a choice (stated in problem) or two valid and likely methods were possible, even if unstated. This second type of open-ended problem is where my definition differs from Zhu and Fan. I chose to include these problems as open-ended because for the problems students do in math class, the process is often more important than the final result. So since the solution method is an integral part of the student’s response, it is reflected in the flexibility of the problem. The following illustrates an example coded as open-ended:
Give a strategy for finding an equation for a line when you know two points on the line

(Advanced Algebra, p. 173, #5)

In this example, we can see that students are given a lot of freedom in their answer and have several plausible strategies. For example, one student may use a graphical approach by finding the slope with rise over run. Another student may elect to simply evaluate the points in the slope formula. For these reasons, I coded the problem as open-ended.

A closed-ended problem was simply one where a single procedure was expected with one single possible answer as the result. These problems were rarely conceptual in nature. The next example demonstrates what constituted a closed-ended problem.

Use \( \left( \frac{1}{3}, \frac{2}{5} \right) \) and the slope to determine an equation of the line through \( \left( \frac{1}{3}, \frac{2}{5} \right) \) and \( \left( \frac{7}{3}, \frac{9}{10} \right) \)

(Advanced Algebra, p. 173, #6)

This problem, while similar in content to the previous example has much more direct instructions that could lead to only one possible solution. As a result, it was coded as closed-ended. It should also be noted that like Cognitive Requirement, the two categories of Flexibility were not mutually exclusive, with coincidence of the two often occurring in multi-part questions.

Application (Real World/Non-Real World)

Once again inspired by Zhu and Fan (2006), we have another characteristic, this one being analyzed for both problems and explanations. This text series is described by its publisher as being one that values real world application, so it is an interesting aspect to analyze. It is important that connections are made between real world and non-real world ideas of slope to form a complete concept of slope for students.

The definition for real world versus non-real world is straight-forward. Problems which included real life events (often word problems) were coded as real world. Problems that were
purely mathematical with no other context (equations, $x$-$y$ graphs, etc.) were coded as non-real world.

Real world and non-real world were not *necessarily* mutually exclusive. There were a small amount of codings which featured real world and non-real world components, but for the most part, problems/explanations were coded as one or the other. An example of each is provided:

*Real World Example:* A truck weights 2000 kg when empty. It is loaded with crates of oranges weighing 17 kg each. Write and equation relating the total weight and the number of crates (*Advanced Algebra*, p. 216, #32).

*Non-Real World Example:* Find an equation of the line satisfying the given conditions: The line contains the point (8,2) and goes through the origin (*Advanced Algebra*, p. 215, #8).

**Covariational Reasoning**

After discussion with my advisor, I decided it was important to code for covariational reasoning as well. Since it is not the main focus of this project and it was difficult to predict how students would use it in response to exercises, I only coded for it in explanations. This has been an area of research studied in particular by Marilyn Carlson. Simply put, covariational reasoning is the ability to think of two variables changing in relation to one another. Carlson defines five levels of covariational reasoning, described in Table 2.2 (*Carlson et al.*, 2002).
Table 2.2 (from Table 2 in Carlson et al., 2002)

*Levels of Covariational Reasoning*

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 – Coordination</td>
<td>Images of covariation can support the mental action of coordinating the change of one variable with changes in the other variable</td>
</tr>
<tr>
<td>L2 – Direction</td>
<td>Images of covariation can support the mental actions of coordinating the direction of change of one variable with changes in the other variable</td>
</tr>
<tr>
<td>L3 – Quantitative</td>
<td>Images of covariation can support the mental actions of coordinating the amount of change in one variable with changes in the other variable</td>
</tr>
<tr>
<td>L4 – Average Rate</td>
<td>Images of covariation can support the mental actions of coordinating the average rate of change of the function with uniform changes in the input variable. The average rate of change can be unpacked to coordinate the amount of change of the output variable with changes in the input variable</td>
</tr>
<tr>
<td>L5 – Instantaneous Rate</td>
<td>Images of covariation can support the mental actions of coordinating the instantaneous rate of change of the function with continuous changes in the input variable. This level includes an awareness that the instantaneous rate of change resulted from smaller and smaller refinements of the average rate of change. It also includes awareness that the inflection point is where the rate of change changes from increasing to decreasing, or decreasing to increasing.</td>
</tr>
</tbody>
</table>

It is important to note two things. First, these levels are hierarchical. For an explanation to be coded as L3 for example, it must meet the criteria for L1, L2, and L3. Second, since the scope of this project particularly focused on linear slope, L5 was not coded.

A common misconception among students is that slope is the changing of one variable. Students assume slope to be a unit rate (i.e., $\Delta x = 1$) (Carlson et. al, 2002). Because of this, they
fail to see slope as the rate at which two variable change with one another, but instead think of slope as a single changing quantity. Therefore, they do not exercise covariational reasoning. Although it may only seem tangentially related, strong covariational reasoning is crucial to a good understanding of slope.

It is important to understand the inherent difference between slope conceptualizations and levels of covariational reasoning. Both are classifications, but not on the same scale. Covariational reasoning has several degrees of sophistication. The mental actions required to reason at level 1 are far less advanced than those required for level 5. For slope conceptualizations, there is no definitive difference in sophistication. They are simply different ways of looking at the same thing; different conceptually, but relatively the same cognitively.

Problems and Explanations

Problems

Problems refer to all practice exercises from the end of each section, end of chapter self-tests, chapter review, and short in-section problems. These in-section problems were in two categories: Quiz Yourself and Mental Math. They primarily served as a review of previous material and were located within the instructional sections of the textbook. I read all such problems and coded any that included references to slope of linear functions. They were coded for whether they were procedural practice or conceptual understanding, open-ended or closed ended, real world or non-real world application, slope conceptualizations in the problem as posed, and slope conceptualizations required in the problem’s response. An example of such a coding follows:
Table 2.3

*Problem Coding*

<table>
<thead>
<tr>
<th>Problem Location</th>
<th>Problem Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cognitive Requirement</td>
</tr>
<tr>
<td></td>
<td>PP</td>
</tr>
<tr>
<td>Book</td>
<td>Ch.</td>
</tr>
<tr>
<td>PTM</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conceptualizations in Problem as Posed</th>
<th>Conceptualizations in Problem Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>R</td>
</tr>
<tr>
<td>V</td>
<td>NV</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The conceptualizations (*constant parameter, ratio, indicating behavior, steepness, determining property*) are subdivided into visual and non-visual. A 1 indicates that the categorization fits the problem, and a 0 indicates it does not. So looking at this particular coding, one can see that Chapter 1, Section 6 of *Pre-Transition Mathematics*, problem #13 was procedural practice with a closed-ended solution. It involved a real world application of slope. The problem as posed gave a non-visual conceptualization of slope and a non-visual conceptualization of slope was required in response.

In general, it was more straight-forward coding for conceptualizations in the problem as posed. Whatever was given in the problem was coded appropriately. Coding for in-response was more difficult. Some conceptualizations were required in-response, even if not explicitly stated. The following criteria were used to judge whether a conceptualization was present for the in-response portion of the problem:

1. The response calls for the student to perform a procedure which specifically requires the student to use that conceptualization, as in the following example.
Find the slope of a line parallel to the line with the given equation:

\[-3x + 2y = 8 - 4x. \, (Geometry, \, p. \, 149, \, \#12)\]

In this problem, the student needs to use the fact that parallel lines have the same slope, an example of visual *determining property*.

2. The response calls for the student to perform a procedure which will implicitly require the student to use a conceptualization to arrive at the overall answer, even if that conceptualization is not the main focus of the problem.

   Using the same problem from the previous example, one can see that once the equation is converted into slope-intercept form the student will need to find the slope. It is reasonably certain they will do this by identifying the slope from the equation. This is an example of non-visual *constant parameter*. Even though it is not the explicit goal of the problem to use this conceptualization, it is still used in some form.

   By satisfying one of the above criteria, the conceptualization was deemed appropriate to code.

   One other difficult situation was when a problem had two clear and distinct methods in which it could be solved that used different conceptualizations. In this case, neither was coded as it is uncertain how an individual student would interpret the problem. The problem itself may still have been coded if there were clear references to slope in its posing, but the response would be uncoded. The example below is one such problem.
The graph at the right (provided in textbook) shows the four equations \( y = 4x, \)

\[ y = -4x, \quad y = \frac{1}{4}x, \quad \text{and} \quad y = -\frac{1}{4}x. \]  

Match each graph with its equation. (Advanced Algebra, p. 98, #17)

It is unclear from context for this problem how a student would solve it. One may presume a student would use visual steepness and visual indicating behavior to choose the graphs, but this is not necessarily so. Another approach would be to calculate the rise over run (visual ratio) for each line using the graph. The graph provided grid lines so this would not be unreasonable.

This is not meant to be critical of these types of problems. Allowing students multiple paths to the same answer is not necessarily a bad thing. However, for the purposes of this research, it does not provide clear evidence of what slope conceptualizations are being emphasized.

Explanations

Explanations comprised the instructional content of the textbook. This includes chapter introductions, explanations of concepts, examples, guided examples, and activities. Explanations were divided into two categories: Expository Material and Sample Problems. Expository Material included chapter introductions, explanations of concepts, activities, and guided examples. This material typically contained complete information allowing students to understand the concepts without practicing it themselves. Sample Problems only included the regular examples (distinguished from guided examples by the textbook’s formatting, not from the researcher’s judgment). These examples were also fairly complete but did leave some
information out for students to practice the skills themselves and prepare them for the end of section problems.

Explanations were coded for much of the same information as the problems were with a few differences. First, there was no need to break up explanations by as posed and in response. Secondly, explanations were coded in an additional category: level of covariational reasoning. Finally, coding for open-ended/closed-ended and procedural practice/conceptual understanding was not done as these codings related more to problems than explanations. This category will be discussed further in the next section.

An example of an explanation coding is provided in Table 2.4:

Table 2.4

Explanation Coding

<table>
<thead>
<tr>
<th>Explanation Location</th>
<th>Explanation Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book</td>
<td>Ch.</td>
</tr>
<tr>
<td>PC</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Table 2.4](image)

One can see that this is a real world application sample problem in Chapter 1, Section 8 of Precalculus. The problem contained the conceptualizations non-visual constant parameter and non-visual ratio. It also reaches the third level of covariational reasoning.
Chapter 3

Results

In this chapter, I will present the results of my research. These results will first focus on the textbook series as a whole. This will include summative data representing the overall distribution of conceptualizations and other categorizations. Additionally, longitudinal data will be presented to show the development of conceptualizations and other categorizations across the series. To gain a deeper understanding of development through the series, data will also be presented for each book individually.

Analysis of Series Overall

The following data summarizes what was emphasized across all seven books in the series.

The following data represents codings of 960 problems and 201 explanations.

Conceptualizations

Figure 3.1 presents the frequency of references for each conceptualization. The data is also subdivided to show how many references were in problems and how many were in explanations. Ratio and constant parameter were by far the most common and with a surprisingly close total. Steepness was the least common with just 42 references throughout the entire textbook series.

Figure 3.2 presents the relative frequency of conceptualizations. The problem relative frequency is the number of problems that were coded with a particular conceptualization divided by the total number of problems (960). So for example, there are 645 problems that were coded
constant parameter, and \(645/960 \approx 0.67\). Therefore, approximately 67% of problems involving slope used the constant parameter conceptualization. It does not mean that 67% of all the textbook problems involved constant parameter, just that 67% of problems with slope involved constant parameter. The explanation relative frequency is found similarly using the total number of explanations (201). Notice that neither the problem nor the explanation relative frequencies sum to 1.00. This is because problems and explanations could be coded for more than one conceptualization, and they often were.

**Figure 3.1**

*Frequency of Conceptualizations*

<table>
<thead>
<tr>
<th>Conceptualization</th>
<th>Problems</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>645</td>
<td>134</td>
</tr>
<tr>
<td>R</td>
<td>644</td>
<td>132</td>
</tr>
<tr>
<td>IB</td>
<td>256</td>
<td>77</td>
</tr>
<tr>
<td>S</td>
<td>179</td>
<td>42</td>
</tr>
<tr>
<td>DP</td>
<td>124</td>
<td>37</td>
</tr>
</tbody>
</table>

**Figure 3.2**

*Relative Frequency of Conceptualizations*

The data in Fig. 3.2 shows that a conceptualization’s problem and explanation relative frequency were typically similar. The only conceptualization to break this trend is *indicating behavior*, for which the explanation relative frequency is approximately double the problem relative frequency. There is also some difference in *steepness’s* relative frequencies, but there is not enough data on *steepness* (only 42 total codings) to make any strong inferences.
Next, in Figures 3.3 and 3.4 we can see how conceptualizations were distributed throughout the series. This data includes references to conceptualizations in both explanations and problems. Figure 3.4 uses a stacked bar graph to help visualize the change over the series.

Figure 3.3

*Conceptualizations over Series*

<table>
<thead>
<tr>
<th></th>
<th>CP</th>
<th>R</th>
<th>IB</th>
<th>S</th>
<th>DP</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTM</td>
<td>8</td>
<td>73</td>
<td>28</td>
<td>2</td>
<td>2</td>
<td>113</td>
</tr>
<tr>
<td>TM</td>
<td>94</td>
<td>108</td>
<td>32</td>
<td>8</td>
<td>2</td>
<td>244</td>
</tr>
<tr>
<td>A</td>
<td>311</td>
<td>289</td>
<td>99</td>
<td>16</td>
<td>17</td>
<td>732</td>
</tr>
<tr>
<td>G</td>
<td>82</td>
<td>70</td>
<td>17</td>
<td>5</td>
<td>66</td>
<td>240</td>
</tr>
<tr>
<td>AA</td>
<td>198</td>
<td>154</td>
<td>44</td>
<td>10</td>
<td>58</td>
<td>464</td>
</tr>
<tr>
<td>FST</td>
<td>61</td>
<td>24</td>
<td>22</td>
<td>0</td>
<td>2</td>
<td>109</td>
</tr>
<tr>
<td>PC</td>
<td>25</td>
<td>58</td>
<td>14</td>
<td>1</td>
<td>5</td>
<td>103</td>
</tr>
<tr>
<td>Total</td>
<td>779</td>
<td>776</td>
<td>256</td>
<td>42</td>
<td>152</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.4

*Visualization of Conceptualizations over Series*

Analyzing Figure 3.4, it is clear that the number of overall slope conceptualizations coded is
highest in *Algebra* and *Advanced Algebra*. We also see that *constant parameter* and *ratio* were significantly present in most books, while *steepness* consistently not present.

**Covariational Reasoning**

In Figure 3.5, we see the levels of covariational reasoning most emphasized in explanations throughout the series. Recall that L5 (Instantaneous Rate), was not coded. L3 (Quantitative Coordination) was the most commonly coded. Over a third of the explanations (73) did not display any levels of covariation.

**Figure 3.5**

**Levels of Covariational Reasoning**

![Bar Chart]

Similar to Figures 3.4, Figures 3.6 show development across the series, this time for covariational reasoning.
Notice in the above chart that there is much less consistency than in Figure 3.4. For example, Level 1 represents nearly all covariational reasoning in *Pre-Transition Mathematics*, but is scarcely coded for the remainder of the series. Similarly, Levels 2 and 4 are prevalent in some books, but completely absent from others. This can be explained in part by the hierarchical nature of the level of covariational reasoning. Recall that if an explanation was coded as a certain level of covariation, it included all levels below it. So for example, a coding of L3 meant that the explanation also met the requirements of being coded L1 and L2. So lower codings, such as L1, L2, and L3 begin to gradually fade out as more sophisticated levels of coariational reasoning are emphasized in later texts.
Attributes

Figure 3.7

Problem Attributes

In problem attributes, we can see problems were overwhelmingly more procedural than conceptual, as well as more closed-ended than open-ended. Interestingly, real world and non-real world problems were almost equally present. In explanations, expository material and sample problems were almost evenly split, and real world and non-real world explanations were roughly even as well.

Analysis of Series by Book

The following section provides a book-by-book analysis with a similar format to the overall analysis (without longitudinal analysis). Particular attention will be given to how each book
differed from the series as a whole. Examples will be provided to give a sample of a typical problem or explanation for the book.

**Pre-Transition Mathematics**

The *Pre-Transition Mathematics* text was coded for 79 problems and 11 explanations involving slope. This accounts for 7.8% of all codings across the series.

*Pre-Transition Mathematics (PTM)* has several notable characteristics. First, as can be seen in Figures 3.8 and 3.9, the book has very few references to *constant parameter*. This is mainly due to the fact that slope was not formally introduced at all in this book and functions were being taught at an introductory level. So there were very few times where a connection was made between a rate of change and a functional property, which is the very essence of *constant parameter*. The problems and examples in this textbook often had to rely on real world scenarios to introduce the concept without stating it outright, as seen in the following example.

**Example 1**

*Problem from Pre-Transition Mathematics, pg. 496, #12*

If Alfred runs 100 meters in 18 seconds, how long will it take him to run 1 kilometer?

This problem does indeed involve rates comparing the change in two variables, which at a very foundational level involves slope. However, there is no need to put that slope in the context of an algebraic function or a graph emphasizing the straightness of the line. So while *ratio* is an appropriate coding for this problem, it does not meet the criteria for a *constant parameter* coding.
Second, *PTM* has more explanations coded as L1 covariational reasoning than the rest of the series combined (Figure 3.10). This again stems from the book’s very elementary level of mathematics. L1 only requires students to know that there is a change occurring between two variables, with no regard for direction or quantitative value. At this point in the series, this is all that is required of students.

Finally, *PTM* also has a strong tendency towards real world problems and examples as opposed to non-real word (Figures 3.11 and 3.12). However, this is not so surprising considering that slope was not formally introduced. It is easy to “hide” the foundational skills of slope in familiar contexts. This is apparent in Example 1. Many casual observers would not even recognize this as a problem involving slope, because it is a very natural everyday example that does not involve all of the traditionally associated slope concepts (e.g. “rise over run”, “steepness of a line”).

*Conceptualizations*

**Figure 3.8**

*Frequency of Conceptualizations*

![Frequency of Conceptualizations](chart1)

**Figure 3.9**

*Relative Frequency of Conceptualizations*

![Relative Frequency of Conceptualizations](chart2)
Covariational Reasoning

Figure 3.10
Levels of Covariational Reasoning

Attributes

Figure 3.11
Problem Attributes

Figure 3.12
Explanation Attributes
Transition Mathematics

The Transition Mathematics text was coded for 110 problems and 28 explanations involving slope. This accounts for 11.9% of all codings across the series.

In Transition Mathematics, there is an increase in constant parameter codings (Figures 3.13 and 3.14), which is more consistent with the rest of the series.

Example 2

*Problem from Algebra, pg. 783, #22*

Debbie has borrowed $2,700, and will pay $75 every month until she has repaid that amount. Paul has borrowed $3,000 and will pay $100 every month. Write and solve an inequality showing how many months it will be until Paul owes less than Debbie.

As we can see in this example, now students are not only required to understand a ratio concept of slope (“$75 every month”), but also how to interpret this in the context of an algebraic expression.

Transition Mathematics is very consistent with the overall series. The only significant aberration is the proportion of real world problems (Figures 3.16 and 3.17), which are popular in this book (one example is highlighted above) for the same reason that they were in PTM. Since slope is not yet clearly defined, it is easier to “hide” slope in a real world context.
Conceptualizations

Figure 3.13  
*Frequency of Conceptualizations*

![Graph showing frequency of conceptualizations across different levels.]

Figure 3.14  
*Relative Frequency of Conceptualizations*

![Graph showing relative frequency of conceptualizations across different levels.]

Covariational Reasoning

Figure 3.15  
*Levels of Covariational Reasoning*

![Graph showing levels of covariational reasoning.]

Levels: L1, L2, L3, L4, None
**Attributes**

Figure 3.16

**Problem Attributes**

![Bar chart showing problem attributes](image)

Figure 3.17

**Explanation Attributes**

![Bar chart showing explanation attributes](image)

**Algebra**

The *Algebra* text was coded for 330 problems and 70 explanations involving slope. This accounts for 34.5% of all codings across the series.

The codings in *Algebra* are fairly consistent with the series overall. This makes sense given that the book contains over a third of the series’ references to slope. It differs from the previous books in its increased emphasis on *constant parameter* (Figures 3.18 and 3.19) and an increasing number of non-real world problems and explanations (Figures 3.21 and 3.22). The latter is due to the book explicitly defining slope. As can be seen in the example below, this allows for examples of slope that do not need to be disguised in a real world context.
Example 3

Problem from Algebra, pg. 436, #10

Suppose \( L(x) = 12x - 18 \).

a) Calculate \( L(5) \).

b) Calculate \( L(3) \).

c) Calculate \( \frac{L(5) - L(3)}{5 - 3} \).

d) What is the meaning of your calculation in Part c?

It is also notable in this textbook that L1 covariational reasoning becomes more prevalent. At this point in the series, slope has been explicitly defined, allowing for more specific exploration of slope in terms of direction and quantitative value.

Conceptualizations

Figure 3.18

Frequency of Conceptualizations

Figure 3.19

Relative Frequency of Conceptualizations
Covariational Reasoning

Figure 3.20

Levels of Covariational Reasoning

Attributes

Figure 3.21

Problem Attributes

Geometry

The Geometry text was coded for 95 problems and 21 explanations involving slope. This accounts for 10.0% of all codings across the series.

Geometry deviates from the series as a whole in many ways. Since the other six books in the series focus on pre-algebra, algebra, and algebra-based subjects, this is not so surprising. The
most striking difference is the prevalence of **determining property** (Figures 3.23 and 3.24). This is due to connections being made between the slope and parallel lines. There is also an unexpected and extreme shift towards non-real world problems and explanations (Figures 3.26 and 3.27). This is because the book had more rigorous and purely mathematical content (proofs, simple geometric figures, etc.) than the other texts. There were more real world applications than the data may suggest, but these applications did not typically involve slope.

One final way that *Geometry* differs from the rest of the series is its low covariational reasoning (Figure 3.25). This is because covariational reasoning at its basis involves coordinating change between two variables. This very algebraic reasoning is not necessary since *Geometry* often (but not always) presented synthetic geometry rather than analytic geometry.

Example 4

*Problem from Geometry, pg. 181, #54*

Line $l$ has slope 7.4. Maxine says the line with equation $5y + 37x = -15$ is parallel to $l$.

Is she correct? Explain your answer.

This an example of a typical problem in *Geometry*. The problem focuses on the *determining property* conceptualization. The *constant parameter* conceptualization is also present as students have to use their understanding of this conceptualization to “pull” the slope from the equation once they have presumably written it in $y = mx+b$ form. This type of problem typified almost every slope coding the textbook.
Conceptualizations

Figure 3.23

Frequency of Conceptualizations

![Graph showing frequency of conceptualizations]

Figure 3.24

Relative Frequency of Conceptualizations

![Graph showing relative frequency of conceptualizations]

Covariational Reasoning

Figure 3.25

Levels of Covariational Reasoning

![Graph showing levels of covariational reasoning]

L1  L2  L3  L4  None

- L1: 4
- L2: 0
- L3: 17
- L4: 0
- None: 0
Advanced Algebra

The *Advance Algebra* text was coded for 231 problems and 40 explanations involving slope. This accounts for 23.3% of all codings across the series.

Much like *Algebra*, *Advanced Algebra* is very much aligned with the overall trends of the series due to its high proportion of codings. In fact, if we ignore *Geometry*, this book simply picks up where *Algebra* left off. The one major difference is an emphasis on non-real world problems (Figure 3.31). In the following example, we see a typical non-real world problem. This type of purely calculation-based question was common.
Example 5

*Problem from Advanced Algebra, pg. 173, #6*

A line passes through the points \((\frac{1}{3}, \frac{2}{5})\) and \((\frac{7}{3}, \frac{9}{10})\).

a) Compute the slope of the line.

b) Use \(\left(\frac{1}{3}, \frac{2}{5}\right)\) and the slope to determine the equation of the line.

c) Check that \(\left(\frac{7}{3}, \frac{9}{10}\right)\) satisfies this equation.

It seems as the series progresses, the focus begins to shift slightly away from real world application and towards pure mathematical non-real world problems. Interestingly, the explanations are roughly even among real world and non-real world (Figure 3.32).

*Conceptualizations*

**Figure 3.28**

*Frequency of Conceptualizations*

**Figure 3.29**

*Relative Frequency of Conceptualizations*
**Covariational Reasoning**

Figure 3.30

*Levels of Covariational Reasoning*

<table>
<thead>
<tr>
<th>Level</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>2</td>
</tr>
<tr>
<td>L2</td>
<td>1</td>
</tr>
<tr>
<td>L3</td>
<td>16</td>
</tr>
<tr>
<td>L4</td>
<td>3</td>
</tr>
<tr>
<td>None</td>
<td>18</td>
</tr>
</tbody>
</table>

**Attributes**

Figure 3.31

*Problem Attributes*

Figure 3.32

*Explanation Attributes*
Functions, Statistics, and Trigonometry

The *Functions, Statistics, and Trigonometry* text was coded for 61 problems and 14 explanations involving slope. This accounts for 6.5% of all codings across the series.

References to slope become scarce in the final two books of the series. Therefore, data should be considered somewhat volatile. That being said, there are a few interesting things that can be gleaned from the data. First, there is a total absence of *steepness* in explanations and problems (Figures 3.33 and 3.34). This is surprising in a book that features “Trigonometry” in the title. *Steepness* can be defined as the tangent of the angle of inclination from a horizontal line. This connection is made nowhere in the book.

A typical problem or explanation in *FST* was coded for *constant parameter* and was non-real world.

Example 6

*Problem from Functions, Statistics, and Trigonometry, pg. 554, #3*

The first term of an arithmetic sequence is -3 and the constant difference is \(d\). Find the 12\(^{th}\) term.

Here, the constant difference can be thought of as the slope of the arithmetic equation. Disappointingly, this is no more sophisticated a conception of slope than anything found in the previous texts, yet it is the most slope-heavy topic in this text.
**Conceptualizations**

Figure 3.33

*Frequency of Conceptualizations*

![Frequency of Conceptualizations](image)

Figure 3.34

*Relative Frequency of Conceptualizations*

![Relative Frequency of Conceptualizations](image)

**Covariational Reasoning**

Figure 3.35

*Levels of Covariational Reasoning*

![Levels of Covariational Reasoning](image)
Precalculus

The *Precalculus* text was coded for 54 problems and 17 explanations involving slope. This accounts for 6.1% of all codings across the series.

In *FST* the proportion of *ratio* is roughly half of the proportion for the whole series, but *constant parameter* is the same. The opposite occurs in *Precalculus*. In this book, *ratio’s* representation is consistent with the series as a whole, while the proportion of *constant parameter* is about half of the proportion for the series as a whole (Figure 3.39). Perhaps the most significant difference between *Precalculus* and the rest of the series is that slope is extended to include average slope. This accounts for the prevalence of L4 covariational reasoning (Figure 3.40).
Example 7

*Problem for Precalculus, pg. 422, #13*

Determine the average rate of change of the function \( f \) defined by \( f(x) = x^2 \) over the interval \([1,4]\).

This is an example of the extension of slope to the average slope. This is a great lead in to calculus where average slope will be taken to a limit giving the exact slope at a point on a non-linear graph.

*Conceptualizations*

Figure 3.38

*Frequency of Conceptualizations*

Figure 3.39

*Relative Frequency of Conceptualizations*
Covariational Reasoning

Figure 3.40

Levels of Covariational Reasoning

![Levels of Covariational Reasoning Chart]

Attributes

Figure 3.41

Problem Attributes

![Problem Attributes Graph]

Figure 3.42

Explanation Attributes

![Explanation Attributes Graph]
Analysis by Conceptualization

In this section, each slope conceptualization will be analyzed. This gives a specific judgment on how these conceptualizations presence (or lack of presence) impacted the series. Examples will be provided to give a sample of a typical problem/explanation for the conceptualization.

Constant Parameter

The following data represents codings across 779 problems and explanations. This means that all of these 779 codings contained the constant parameter conceptualization.

In Figure 3.43, it is clear that constant parameter is represented the most in Algebra and Advanced Algebra. This will be true for most conceptualizations since those two texts contained the most references to slope. Constant parameter is also coded much more often as procedural than conceptual in problems (Figure 3.44). This is another property that will be fairly common among all conceptualizations. One property that is fairly unique to constant parameter is the overwhelming dominance of the non-visual representation over the visual one (Figure 3.44).

Example 8

Problem Coded for Constant Parameter in Algebra, pg. 364, #2

Suppose a 15-minute call costs $5.26, and a 30-minute call costs $10.24. Find a formula relating time (in minutes) and cost (in dollars).

This was coded for constant parameter in the response because the student must understand that the rate translates to a constant slope in the formula, which models the situation. It was also coded as non-visual. Some might consider a real-world situation, such as the one presented in this problems, as visual by default, but this is not the case. Because there is no need
to interpret the situation graphically or in some other visual manner, this is an example of non-visual constant parameter.

Figure 3.43

*Representation of Constant Parameter by Book*

![Pie chart showing representation of constant parameter by book.

2 The first number is the number of conceptualizations coded as constant parameter in each book. The second number is the percentage of constant parameter codings in the book compared to constant parameter codings across the whole series.

Figure 3.44

*Attributes for Constant Parameter*
Ratio

The following data represents codings across 718 problems and explanations. This means that all of these 718 codings contained the *ratio* conceptualization.

Just like *constant parameter*, *ratio* is most prevalent in *Algebra* and *Advanced Algebra*, although there is more representation of *ratio* in *PTM* than there is for *constant parameter* (Figure 3.45). Like *constant parameter* it was also usually coded as heavily procedural and non-visual (Figure 3.46).

Example 9

*Problem Coded for Ratio in Advanced Algebra, pg. 97, #4*

**Fill in the Blanks** A slope of $-\frac{2}{5}$ means that for every change of 5 units to the right there is a change of ___ units ____. It also means that for every changes of 1 horizontal unit there is a vertical change of ____ unit.

This was coded as a conceptual problem with visual *ratio* as well as visual *indicating behavior*.

Figure 3.45

*Representation of Ratio by Book*
Indicating Behavior

The following data represents codings across 256 problems and explanations. This means that all of these 256 codings contained the indicating behavior conceptualization.

*Indicating behavior* is distributed fairly similarly to *ratio* and *constant parameter* (Figure 3.47). What differs is that *indicating behavior* has a much higher proportion of visual representations.

Example 10

*Problem Coded for Indicating Behavior in Advanced Algebra, pg. 97, #7*

**Fill in the Blanks** The graph of \( y = kx \) slants up as you read from left to right if \( k \) is _____.

It slants down as you read from left to right if \( k \) is _____.

This problem was coded as conceptual with visual *indicating behavior*.  

![Bar chart showing data for indicating behavior, ratio, constant parameter, and non-visual]

<table>
<thead>
<tr>
<th>Category</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural</td>
<td>627</td>
</tr>
<tr>
<td>Conceptual</td>
<td>43</td>
</tr>
<tr>
<td>Visual</td>
<td>63</td>
</tr>
<tr>
<td>Non-Visual</td>
<td>744</td>
</tr>
</tbody>
</table>
Figure 3.47

*Representation of Indicating Behavior by Book*

![Pie chart showing distribution of indicating behavior by book]

Figure 3.48

*Attributes for Indicating Behavior*

![Bar chart showing procedural, conceptual, visual, and non-visual attributes]

**Steepness**

The following data represents codings across 42 problems and explanations. This means that all of these 42 codings contained the *steepness* conceptualization.

*Steepness* is noticeably underrepresented in the series. Interestingly enough, it is also one of just two conceptualizations that was coded as visual more than non-visual (Figure 3.50), the other being *determining property*. 
Example 11

*Problem Coded for Steepness in Advanced Algebra, pg. 726, #39*

A wheelchair ramp is built with a slope of \( \frac{1}{12} \). To the nearest tenth of a degree, what angle does the ramp make with the horizontal?

This problem was coded as a procedural problem with non-visual *constant parameter* and non-visual *steepness*.

*Figure 3.49*

*Representation of Steepness by Book*

*Figure 3.50*

*Attributes for Steepness*
Determining Property

The following data represents codings across 152 problems and explanations. This means that all of these 152 codings contained the determining property conceptualization.

All conceptualizations up to this point have followed a trend where Algebra and Advanced Algebra have contained the most references. Determining property instead has Geometry and Advanced Algebra providing the most references (Figure 3.51). This conceptualization also overwhelmingly favors visual over non-visual.

Example 12

Problem Coded for Determining Property in Advanced Algebra, pg. 726, #39

Find an equation of the line satisfying the given conditions. The line is parallel to \( y = 6x - 1 \) and contains the point (7,1).

This problem was coded as procedural with non-visual constant parameter and visual determining property.

Figure 3.51

Representation of Determining Property by Book
All of the tables and graphs in the world could not perfectly describe the treatment of slope in this textbook series. They can provide an adequate summary, but for a fuller picture, one must interpret these results and discuss the relevant outcomes. The next chapter will begin this discussion on some of the major takeaways from this data and begin to address the questions this research set out to answer.
Chapter 4
Discussion

One can draw many observations from the wealth of data provided in Chapter 3. Some observations are encouraging, some worthy of criticism, and others do not fall strictly into either category. The focus of this chapter is to discuss these observations, their significance, and recommendations to improve potential issues.

Comparison with Previous Research

It is important to begin this discussion by observing how this research does or does not correlate with other research done in this area. This will help guide later discussion on the strengths and weaknesses of the series.

Standards

Since this textbook prides itself on its alignment with the Common Core, it seems appropriate to start by analyzing how the development of slope is handled in the textbook series and in the Common Core State Standards for Mathematics (CCSSM). Research on slope in standards shows that ratio and constant parameter are the most frequently referenced conceptualizations (Nagle & Moore-Russo, 2014), similar to what was found in this study. However, the series differs in that determining property is far more emphasized than it is in standards. In Nagle and Moore-Russo’s research, it was found that in 53 CCSSM references coded for slope, only 1 mentioned determining property. This translates to 1.9% of codings. This is much less than the
12.9% of references coded with determining property in the textbook series. This is perhaps expected, however. The concept of determining property is one that can be described briefly, but applied often. So a standards document need only reference the conceptualization once or twice, while a textbook series should have many more problems that involve it.

Students and Teachers
In a study of college calculus students and instructors, it was found that students most often hold conceptions of slope as indicating behavior and constant parameter (Nagle & Moore-Russo, 2013c). Ratio was also popular among students. Instructors also commonly thought of slope in terms of a ratio or constant parameter, although few considered it for indicating behavior. Professors were also more likely than students to consider slope in real world contexts. It was also found (as one might expect) that professors on average had more conceptualizations of slope than students did.

The textbook series was fairly accommodating for both groups. Constant parameter, ratio, and indicating behavior rank as the top three conceptualizations over the series. The series also features a high share of real world problems.

Areas of Strength
In many ways, the USCMP textbooks exemplify what a reform-based textbook series should be. As the results of this research clearly show, there is certainly significant attention given to the core concept of slope. In particular, many of the ways slope is presented line up with current educational trends.
Covariational Reasoning

The series’ most impressive accomplishment is a logically-sequenced development of covariational reasoning. In Figure 3.6 from the previous chapter, one can see that the initial emphasis is on the L1 (Coordination) level. Then in three of the next four books, L3 (Quantitative Coordination) is the most common coding, with L2 (Direction) a distant second. The one book that did not exhibit these results was Geometry, which was an aberration from the rest of the series with regard to slope. It is also important to remember that in coding at the L3 level, L1 and L2 are implicitly included. Finally, at the end of the series, L4 (Average Rate) is introduced. It first appears in Advanced Algebra and is then one of the most prominent covariational reasoning codings in Precalculus.

L4 is conspicuously missing from Functions, Statistics, and Trigonometry, one of the few ways in which the series mishandles their development of covariational reasoning. This is in part due to the overall drop in slope references at this point in the series. This will be expounded upon later in the chapter (see: Decline in Functions, Statistics, and Trigonometry and Precalculus).

Real World Applications

Another strength of this series is the extremely even distribution of real world and non-real world applications. Across all grades, CCSSM states the importance of applying concepts to realistic scenarios. It is important that students go beyond the context-less mathematics, which is not to say that it does not have its place. Context-less mathematics can certainly build logical thinking skills while also giving students the tools to solve problems with context. However, many textbooks fall short in this area.

One study involving several modern, commercial textbooks found that only 12-15% of problems involving slope had a real life context (Arnold & Son, 2011). The series studies in this
these had a 50.7% rate of problems and explanations with a real life context. Additionally, a case study of several preservice teachers found that many did not include real world contexts for slope in their lessons (Stump, 1997). One reason for this may have been that the textbook the preservice teachers were using did not include many real world applications. Textbooks are an important teaching tool and it is important that they reflect what the standards emphasize on real world applications. The fact that this textbook series regularly utilizes real world application speaks to its commitment to the standards.

Areas for Improvement

Despite the strengths in the series and its relative consistency with previous research on the development of slope, there are many areas in which it could improve.

Lacking in Visual Representations

One glaring problem is the over-reliance on non-visual problems and explanations. As was seen in Figures 3.44 and 3.46, constant parameter and ratio (by far the two most prominent conceptualizations in the series) were overwhelmingly conceptualized non-visually (approximately 92.5% of the time combined). Previous research has noted the importance of visual representations of slope in the classroom (Zaslavsky, 2002; Arcavi, 2003). So it is disappointing that the vast majority of representations for slope in this series are non-visual.

Some may not regard this as an issue, since slope is primarily an algebraic concept and visual approaches to mathematics are often associated more with geometry. Therein lies a deeper issue. Many students and teachers alike do not believe in visual mathematics outside of geometry. In a study of undergraduate math students and mathematics professors, it was found
that while both groups recognized visual representations as important, the students restricted the approach to geometry while professors noticed a wide range of applicability (Stylianou & Silver, 2004). It should be the goal of textbook publishers, teachers, and curriculum writers to make students aware of the ubiquity of visual representations in mathematics, and how those representations can help students understand problems better.

**Overly Procedural Problems**

Figure 3.7 shows that 885 problems (92.2% of all problems) were listed as procedural, while just 113 (12.8%) were considered conceptual. Problems were also coded as closed-ended 922 times (96.0%) versus 60 (6.3%) coded as open-ended.

In Figure 4.1, it can be seen that these two pairs of data show some amount of correlation (column frequencies in parentheses). Closed-ended problems were much more likely to be procedural than conceptual. Open-ended problems, while still more likely to be procedural, were much more likely to be conceptual compared to closed-ended problems. Closed-ended problems were inflexible in the how they could be solved, leading to them being largely procedural.

**Figure 4.1**

*Two-Way Table for Procedural/Conceptual vs. Open-Ended/Closed-Ended*

<table>
<thead>
<tr>
<th></th>
<th>Open-Ended</th>
<th>Closed-Ended</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Procedural</strong></td>
<td>42 (55.3%)</td>
<td>864 (90.1%)</td>
</tr>
<tr>
<td><strong>Conceptual</strong></td>
<td>34 (44.7%)</td>
<td>95 (9.9%)</td>
</tr>
</tbody>
</table>

Research by Riley, Greeno, and Heller shows that for students to become better problem solvers they must be able to “represent relationships within problems” (1983). Essentially, students must have a strong conceptual base. It is simply not enough to teach students the basic
skills necessary to finish a problem. This only allows students to “go through the motions” on contrived problems that will likely never relate to problems they will face in practical situations. By teaching conceptual skills, students can link newly taught material with earlier lessons. Then, if they have trouble solving one problem, they can think back to how it relates to previous lessons and try a different approach. This is not just a skill for math class; this is critical thinking for all areas of life.

There are some easy to find conceptual problems in the book. The last problem at the end of each section was typically conceptual. In addition, at the end of each chapter there were several projects, nearly all of which required some level of conceptual knowledge. In a book scarce with conceptual problems, it is important that teachers can find some without digging too much, especially in classrooms based on a discovery approach, where conceptual often precedes procedural. The Common Core calls for rigor, and conceptual problems are crucial in answering this call.

Absence of Steepness

As seen in Figure 3.3, steepness is coded just 42 times in the 6000+ pages of the series, by far the least of any of the conceptualizations. That is simply too low. The steepness of a line and how it relates to the slope is a concept that can help link concepts in algebra, trigonometry, and calculus (Nagle Moore-Russo, 2013b; Teuscher & Reys, 2010). Even in the problems and explanations that were coded for steepness, the concept was mostly self-contained to algebra, as the following example problem demonstrates.

Which section(s) of the graph below shows the a) fastest increase? b) slowest decrease?

(graph provided with several connected linear segments (Algebra, p. 360, #22)
This is not to say problems like these are “bad” problems, but they were typical for problems coded for steepness, and the series could have gone further by connecting slope across subject areas.

**Extension of Slope to Non-Linear Functions**

Although the series generally aligned with the standards (as is expected from a series based upon those standards), there is one crucial area in which it falls short. In the high school curriculum, the CCSSM encourages teachers to extend the concept of slope to non-linear functions (Nagle & Moore-Russo, 2014). In many ways this extension is a precursor to calculus, but it also serves as a way to help students expand their concept of slope so that they may more easily apply it to things they see in real life.

In the series, this is hardly seen. A glimpse is given at the end of the series in Precalculus as problems begin requiring students to calculate the average slope of a function. However, this is insufficiently covered, with only a passing reference here and there. Additionally, there were very few problems in which students directly compared linear functions with non-linear functions from a slope perspective. Once again, an opportunity is missed to use slope as a way of connecting concepts across multiple subjects.

**Decline in Functions, Statistics, and Trigonometry and Precalculus**

The final two books of the series represent roughly 28.7% of the entire series (by page count), yet only combine for 10.6% of the series’ references to slope. Additionally, individually they are ranked last and second to last in references to slope among all books. This means there were more references to slope concepts in Pre-Transition Mathematics than there were in either of Functions, Statistics, and Trigonometry or Precalculus. Recall that slope was never even formally defined in Pre-Transition Mathematics. The final two books in this series serve as a
very poor lead-in to calculus, a subject for which slope serves as the basis for an entire half of the course. The end of this series should have been a time to increase the emphasis on slope, not dramatically decrease it.

In Nagle & Moore-Russo’s 2013 study of undergraduate calculus students’ conceptions of slope, responses were analyzed and categorized using conceptualizations very similar to the ones used in this research. Their analysis led to the following conclusion: “In order to grasp the concept of the derivative, students need a conceptual understanding of slope beyond what was evidenced in the majority of their responses” (Nagle, Moore-Russo, et al. 2013c, p. 1508). If students do not receive adequate instruction in slope leading up to calculus, they will have the exact same issue.
Chapter 5

Conclusions

The issues presented in the previous chapter are not uncommon ones and are certainly correctable. In this chapter, recommendations will be made to improve the quality of the textbook series. Recommendations are also provided for teachers on how this series might best be used.

Lacking in Visual Representations

This is a broad problem, so of course there are more ways to increase the proportion of visual representations than could reasonably be described here. However, there is one change that could drastically remedy the issue. Many times throughout the series, a problem will ask a student to construct a graph of a line using a very procedural method that does not quite activate any visual representation of slope. It may be helpful to replace some of these problems with ones where students deconstruct graphs. Below is an example problem from the book.

Example 1 (Original)

*Problem from Algebra, pg. 395, #56*

Graph the line satisfying the given conditions: slope 8 and y-intercept -8.

This problem does not require student to engage in a very sophisticated visual conception of slope, even though the problem requires them to create something visual. Below is a revised version of this problem so that students deconstruct the graph instead.
Example 1 (Revised)

What is the slope of the graph? What is the y-intercept?

In the revised version, students can calculate the slope and deduce the relationships between the slope and the visual representation of the line (which could be helpful in strengthening conceptualizations of steepness and indicating behavior). Problems like these were scarce throughout, although not totally missing.

Problems in the text similar to this revised version often provide two sets of coordinates, which allow students to easily calculate the slope using a formula instead of using what they see from the graph. This does very little to strengthen students visual concept of slope.

Overly Procedural Problems

As was noted in the previous chapter, there was a strong correlation between closed-ended and open-ended problems with procedural and conceptual problems respectively. The National Research Council stresses both procedural and conceptual knowledge in their Five Strands of Mathematics Proficiency (NRC, 2001). To increase the amount of conceptual problems in the series, it may be helpful to simply have problems be more open-ended. Allowing students to think through a problem in their own unique way opens up a world of opportunities. While procedural skills are certainly important, it is also important that students can demonstrate that
they can use those skills in an unfamiliar context. Open-ended problems tend to be unfamiliar because they cannot usually be solved simply by parroting another example.

It is important to be careful when making problems open-ended. An easy way to do it is to simply add on a question asking students about their thought process and/or justifications for their answer (an example of this is provided below). This is not a bad strategy, but if done too much, it can just turn into a tedious exercise. As always, balance is key.

For an example of such a problem that could be improved with follow-up questions, look to Example 1 (Revised) on the previous page. As stated, the problem is completely procedural, but short follow-up questions could engage the student in much deeper thinking, such as “How did you find the slope? Explain.” or “Can you think of a real world situation this line describes? How are the slope and y-intercept represented in this situation?”

**Absence of Steepness**

*Steepness* is a conceptualization that can connect across multiple subjects. Drs. Courtney Nagle & Deborah Moore-Russo, published a lesson in NCTM’s *Mathematics Teacher* detailing one such connection (Nagle & Moore-Russo, 2013b). In this lesson, students discover how the angle of inclination is related to the slope of a line. This intimately links geometry and algebra to give students a fuller understanding of slope. Even a single section of a chapter demonstrating this relationship would go a long way in strengthening students’ conceptualizations of *steepness*.

*Steepness* is also an important skill leading into calculus. As such, attending to this conceptualization in the final two books of the series would also have the dual effect of raising the low amount of slope conceptualizations in those books. Problems could focus on how the
average slope of a curve changes over several intervals. Paying special attention to whether the curve is becoming “more steep” or “less steep” this may serve as a good lead-in to concavity.

By emphasizing steepness in the manners described above, slope can more easily be extended from linear to non-linear functions. This can help ensure that the concept of slope does not drop out of the curriculum as it does in the final two books of this textbook series.

**Recommendations for Teachers**

The textbook is a guide, not a manual. It may seem obvious to teachers who have likely been taught something similar in their education courses or in-services, but it is something that is often taken for granted. Hopefully, this thesis shows that even in good textbooks, it is important to carefully interpret what the book is emphasizing. The chances that it aligns perfectly with how a certain teacher would like to teach are incredibly low. There are always differences, big or small, that will contrast how the teacher teaches and how the textbook recommends teaching. It is important that teachers first stay true to the practices they believe to be most beneficial.

It is also important that teachers teach in a way that is true to their students. In the last chapter, this textbook was compared with research on how students conceptualized slope, and it was found that there was some level of disconnect. Teachers must take care to realize the specific needs of their own classroom. If students identify strongly with the visual ratio conceptualization, but they need to learn a concept that heavily involves non-visual indicating behavior, teachers should build from the conceptualization students are comfortable with using already. Once again, it may seem like an obvious platitude, but teachers must not forget to always consider how to build form their students’ strengths.
Going Forward

This thesis was the first investigation specifically analyzing the development of slope across a textbook series. It takes its place within a larger body of work on the subject, led by Dr. Courtney Nagle and Dr. Deborah Moore-Russo, that has studied slope conceptualizations in students, teachers (pre-service and in-service), and standards documents. The limitation of this study is that it only analyzes one major textbook series. To gain a fuller understanding of the way textbooks develop slope, it is necessary to study multiple series and compare. This is a much larger task and an opportunity for future research.

It is my hope that textbook publishers consider the suggested recommendations in the future when writing their textbooks. Slope is an extremely important topic in the math curriculum; one that we should be extremely vigilant in making sure is taught well. Publishers and teachers alike owe it to their students to make sure they are providing them with the best instruction possible.
References


Moore-Russo. (2015) Star coding. *Unpublished manuscript, Department of Learning and Instruction, University at Buffalo, Buffalo, NY.*


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Sept 2015 – Dec 2015

Introductory Field
Completed at Iroquois Junior High School (Erie, PA)
Jan 2015 – Apr 2015

WORK & ACTIVITIES

Penn State Behrend
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Aug 2013 – present

Penn State Abington
Math Tutor
Jan 2013 – April 2013

Penn State Behrend Math Club President
Aug 2015 – May 2016

HONORS/AWARDS

Honors Programs
Schreyer Honors College
2012 – present
Penn State Behrend Honors
2013 – 2014
Penn State Abington Honors
2012 – 2013

Awards
Dean’s List
Each semester
Evan Pugh Scholar Award
2015 & 2016
Outstanding Tutor in Math
2014
President Sparks Award
2014
President’s Freshman Award
2013
AP Honors with Distinction

Scholarships/Grants
Penn State Behrend Math Scholarship
2016
Bucks County Penn State Alumni Association Scholarship
2015 – 2016
Penn State Behrend School of Science Scholarship
2014

Other
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2013 – present