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STATISTICAL PROPERTIES OF THE RISK TRANSFER FORMULA IN THE AFFORDABLE CARE ACT

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ABSTRACT

In March 2010, President Barack Obama signed into law the Affordable Care Act (ACA). This Act created a competitive Marketplace of insurance plans which are required to provide insurance coverage without regard to pre-existing medical conditions. Previously, it was common for insurers to use health information to set premium rates and deny coverage, if appropriate. Because of this major change, the Affordable Care Act introduced several measures to help insurers during a transitional period.

This thesis investigates one such measure, namely the risk transfer formula. This formula transfers funds from insurance plans with healthy members to plans with less healthy ones. By means of these transfer amounts, the formula ameliorates the tendency for insurers to favor healthy members over less healthy ones. By treating these risk transfer amounts as random variables and investigating their means, variances, and covariances, we determine properties of the risk transfer amount for each plan relative to that plan's market share. These results also allow us to quantify this relationship.

The results in this thesis provide mathematical justification for a phenomenon, observed previously by the American Academy of Actuaries, that "Risk adjustment transfers as a percent of premium were more variable and likely to be higher for insurers with a smaller market share" ["Insight on the ACA," 2016]. The results in this thesis determine conditions under which this phenomenon will hold, and also the rates of change of variability of risk transfer amounts as a function of the market shares of competing insurers.

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Chapter 1

Introduction

In March 2010, President Barack Obama signed into law the Affordable Care Act (ACA). This Act created a competitive Marketplace of insurance plans which are required to provide insurance coverage without regard to pre-existing medical conditions. Previously, it was common for insurers to use health information to set premium rates and deny coverage, if appropriate. Because of this major change, the Affordable Care Act introduced several measures to help insurers during a transitional period.

These measures, commonly referred to as the "three R's," are *reinsurance*, *risk corridors*, and *risk adjustment*. Under the *reinsurance* measure, funding will be provided to health insurance plans that incur high claims costs. *Risk corridors* are designed to limit issuers' losses and profits, thereby guarding against inaccurately set health insurance rates. These two measures are only temporary and are planned to end after 2016. The last R, *risk adjustment*, is a permanent measure which will transfer funds from low-risk plans to high-risk plans through the Centers for Medicare and Medicaid Services (CMS). The risk adjustment methodology, and the risk transfer formula in particular, is the focus of this thesis.

The purpose of risk adjustment is "to lessen or eliminate the influence of risk selection on the premiums that plans charge and the incentive for plans to avoid sicker enrollees" [Kauter, Pope, Keenan, 2014]. Ideally, differences in premiums would reflect differences in actuarial value (i.e., generosity of coverage) and induced demand (i.e., the tendency for people with higher levels of coverage to use their insurance more frequently). Moreover, differences in premiums would *not* reflect differences in the pre-existing health conditions of the insured. Risk adjustment amounts are calculated with a risk adjustment model and a risk transfer formula which are designed to meet these ideals. The model utilizes an enrollee's demographics and diagnoses to calculate a plan liability risk score reflecting how relatively costly that person is expected to be to the insurer. All other things being equal, the healthier the person, the lower the risk score. This plan liability risk score is inserted, along with other factors, into the risk transfer formula to calculate the plan's final transfer amount.

The factors used to calculate risk transfer amounts are the following:

- $PLRS_i$: A weighted average of Plan *i*'s plan liability risk scores found in the risk adjustment model, weighted by enrollment months
- IDF_i: Plan *i*'s induced demand factor; this accounts for the tendency for people with higher levels of coverage to use their insurance more frequently
- GCF_i: Plan *i*'s geographic cost factor; this accounts for factors affecting premiums that vary geographically such as input prices and medical care utilization rates
- AVi: Plan *i*'s actuarial value; this term is implicitly included in the PLRS term because it is considered in the risk score model
- $ARFi$: Plan *i*'s allowable rating factor; this is also implicitly included in the PLRS term and reflects the effect of age
- $\bar{P}_s$: State-wide enrollment-weighted market average plan premium
- s_i: Plan *i*'s share of market-wide enrollment

For the *i*th plan, the unweighted risk transfer amount is:

$$T_i = \left[\frac{PLRS_i * IDF_i * GCF_i}{\sum_j \left(s_j * PLRS_j * IDF_j * GCF_j\right)} - \frac{AV_i * ARF_i * IDF_i * GCF_i}{\sum_j \left(s_j * AV_j * ARF_j * IDF_j * GCF_j\right)}\right] \bar{P}_s \qquad (1.1)$$

where the sums over *j* in the denominators are taken over all individual plans in the state in which the plans are offered.

The first term inside the square brackets on the right-hand side of the equation includes the *PLRS*, which reflects patient health and actuarial value. By contrast, the second term includes the actuarial value and allowable rating factors in place of the *PLRS*. Thus, the formula subtracts the premium without risk selection from the premium with risk selection. This subtraction is thus designed to achieve the formula's purpose of reducing risk selection.

We remark that both terms on the right-hand side of the formula are calculated as percentages of their corresponding state-wide totals and then are multiplied by $\bar{P}_s$. The outcome is that the formula calculates the actual dollar amount to be transferred, and it also ensures that the weighted sum of all transfers, after being weighted by each plan's market share, sums to zero. Consequently, the risk adjustment procedure is a zero-sum game in that plans receiving payments must be balanced by other plans making payments. This and other mathematical properties of the risk transfer formula will be explored in the following chapter.

Chapter 2

Mathematical Properties of the Risk Transfer Formula

In this chapter, we will treat each plan's ACA risk transfer amount as a random variable and determine some mathematical properties of that random variable.

2.1 Properties of the Sum of Transfers

One of the risk transfer formula's most important characteristics is that the sum of all transfers must equal zero. This ensures that the total amount of money is left unchanged; the money is simply being transferred between plans. Thus, we obtain the following theorem and proof.

Theorem 1: $\sum_i s_i T_i = 0$ where the sums over *i* are taken over all individual plans in the state in which the plans are offered.

Proof: By Equation (1.1),

$$\sum_i s_i T_i = \bar{P}_s \sum_i s_i \left[\frac{PLRS_i * IDF_i * GCF_i}{\sum_j (s_j * PLRS_j * IDF_j * GCF_j)} - \frac{AV_i * ARF_i * IDF_i * GCF_i}{\sum_j (s_j * AV_j * ARF_j * IDF_j * GCF_j)}\right]. \quad (2.1)$$

Let

$$x_i = PLRS_i * IDF_i * GCF_i,$$

$$y_i = AV_i * ARF_i * IDF_i * GCF_i,$$

$$c_1 = \sum_j (s_j * PLRS_j * IDF_j * GCF_j),$$

and

$$c_2 = \sum_j (s_j * AV_j * ARF_j * IDF_j * GCF_j).$$

Then Equation (2.1) reduces to

$$\sum_i s_i T_i = \bar{P}_s \sum_i s_i \left[\frac{x_i}{c_1} - \frac{y_i}{c_2}\right]$$

$$= \bar{P}_s \left(\frac{\sum_i s_i x_i}{c_1} - \frac{\sum_i s_i y_i}{c_2}\right) = \bar{P}_s \left(\frac{c_1}{c_1} - \frac{c_2}{c_2}\right) = 0.$$

The proof is complete.

We also know that $\sum_i s_i = 1$. This is evident because the s_i represent the market shares of the individual plans, and all plans in the state jointly enroll 100% of the state's market share. Furthermore, each $s_i > 0$ because a company cannot have a negative or zero market share. Given this information, the set of possible values for $s_1, \dots, s_N$ form a simplex, i.e., $s_1, \dots, s_N > 0$ and $s_1 + \cdots + s_N = 1$.

2.2 The Set of Possible Values of T_i

Though the sum of transfers are important to the functioning of the risk adjustment program, individual transfers are likely to be of more interest to insurers. We will first consider the set of possible values of the quantity T_i in Equation (1.1), assuming that the market shares are fixed. In fact, the market shares of insurance companies are reported by the National Association of Insurance Commissioners, so we can also assume that the market shares are known. Supposing that an insurance company knows the transfer for its own plan, we wish to know whether the transfer for a rival company can be calculated. To test this, we constructed the following example.

Consider Plan 1 which has 50% of the market share and has a T_1 of \$1 million. If there is only one other plan, Plan 2, then it is trivial to calculate that Plan 2 also has 50% of the market share and a T_2 of -\$1 million. However, if there are three plans, the situation becomes more complicated; in this case, we obtain the following equations:

$$s_2T_2 + s_3T_3 = -s_1T_1 = -500{,}000,$$

equivalently,

$$s_2T_2 + (0.5 - s_2)T_3 = -500{,}000,$$

where $0 < s_2 < 0.5$.

Based on this equation, for any value of s_2 in the interval (0, 0.5) and any value of T_2, T_3 exists. Thus, there is no maximum or minimum transfer amount for other plans that can be calculated based on one plan's transfer amount alone, assuming there are at least three plans in the state.

2.3 The Covariance Between T_i and T_j

In this section, we will consider the covariance between T_i and T_j assuming that there are two plans in the state and that each plan's s_i is fixed for simplicity. There are two ways in which we can express the covariance of a plan's transfer. We will first consider the version based on the transfer amount's variance. We denote $Cov(T_i, T_j)$ by σ_{ij} and $Var(T_i)$ by σ_{ii}.

Because we proved that $\sum_i s_iT_i = 0$, it follows that the variance of $Var(\sum_i s_iT_i) = 0$. By Equation (4.1) on page 357 of Ross 2006, we also know that

$$Var\left(\sum_i s_iT_i\right) = \sum_i s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N} s_is_j\sigma_{ij}.$$

Therefore,

$$\sum_i s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N} s_i s_j \sigma_{ij} = 0. \tag{2.2}$$

For simplicity, consider the case in which there are only two plans in the state. Then, Equation (2.2) reduces to

$$s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12} = 0.$$

Substituting $s_2 = 1 - s_1$ and solving this equation for σ_{12}, we obtain

$$\sigma_{12} = -\frac{s_1^2\sigma_{11} + (1-s_1)^2\sigma_{22}}{2s_1(1-s_1)}. \tag{2.3}$$

By graphing σ_{12} as a function of s_1, we obtain an upside-down bathtub function. The graph of this function for $\sigma_{11} = 1$ and $\sigma_{22} = 4$ is provided in Figure 1 below.

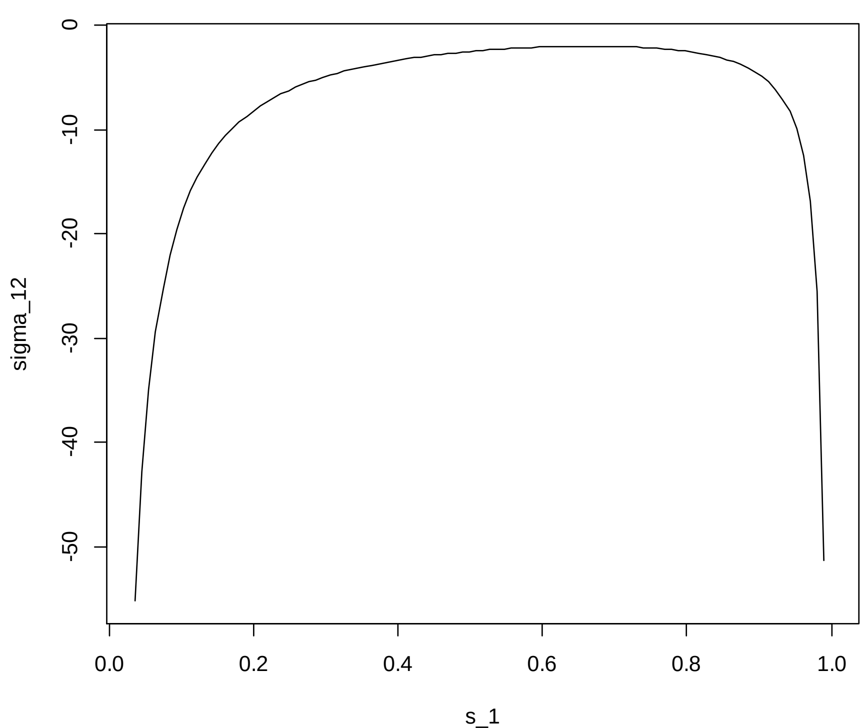

Figure 1: Graph of σ_{12} vs. s_1 where σ_{11}=1 and σ_{22}=4

We observe that this function has no minimum value, but it does have a maximum value that is negative. To find this maximum, we calculate the first derivative of this function and obtain

$$\frac{\partial}{\partial s_1}\sigma_{12} = \frac{\partial}{\partial s_1}\left(-\frac{s_1^2\sigma_{11} + (1-s_1)^2\sigma_{22}}{2s_1(1-s_1)}\right) = \frac{\sigma_{22}(1-s_1)^2 - \sigma_{11}s_1^2}{2(1-s_1)^2 s_1^2},$$

assuming that the variances are constant. Setting this equal to 0, we obtain the roots $\sqrt{\sigma_{22}}/(\sqrt{\sigma_{11}} + \sqrt{\sigma_{22}})$ and $-\sqrt{\sigma_{22}}/(\sqrt{\sigma_{11}} - \sqrt{\sigma_{22}})$. However, the second of these roots is greater than 1 when $\sigma_{22} > \sigma_{11}$, undefined when $\sigma_{22} = \sigma_{11}$, and negative when $\sigma_{22} < \sigma_{11}$. Since the function is defined only between 0 and 1, we can disregard this solution. Thus the only root is $\sqrt{\sigma_{22}}/(\sqrt{\sigma_{11}} + \sqrt{\sigma_{22}})$.

This result implies that, if there are only two plans in the state and assuming that the variances σ_{11} and σ_{22} are fixed, the covariance σ_{12} is maximized as a function of s_1 at $s_1 = \sqrt{\sigma_{22}}/(\sqrt{\sigma_{11}} + \sqrt{\sigma_{22}})$. Moreover, there is no minimum value for the covariance.

Another way to express the covariance arises as follows. Because $s_1T_1 = -s_2T_2$, then $T_2 = -(s_1/s_2)T_1$. Therefore,

$$\sigma_{12} = Cov(T_1, T_2) = Cov\left(T_1, -\frac{s_1}{s_2}T_1\right) = -\frac{s_1}{s_2}\sigma_{11} = -\frac{s_1}{1-s_1}\sigma_{11}. \qquad (2.4)$$

By equating the expressions in Equations (2.3) and (2.4) for the covariance between the transfer amounts, we shall obtain in Section 2.4 some interesting properties of the variance of the transfer amounts.

2.4 The Variance of *T_i*

In this section, we will consider the variance of T_i. Equating the two expressions obtained in Equations (2.3) and (2.4), we obtain

$$-\frac{s_1^2\sigma_{11} + (1-s_1)^2\sigma_{22}}{2s_1(1-s_1)} = -\frac{s_1}{1-s_1}\sigma_{11}.$$

Simplifying this equation, we obtain

$$\frac{(1-s_1)^2}{s_1^2} = \frac{s_2^2}{s_1^2} = \frac{\sigma_{11}}{\sigma_{22}}.$$

This equation shows that, if $s_2 > s_1$, then $\sigma_{11} > \sigma_{22}$. That is, companies with larger market shares will have smaller variances in their transfer amounts. This phenomenon is to be expected because the transfer amounts are calculated relative to the weighted average, and companies with a larger market share will influence the weighted average more greatly. However, this relationship will then be problematic for companies with smaller market share, because their transfer amount will have higher variance and therefore will be more difficult to predict.

Next, we derive similar results in the case in which there are three plans in a state. Since $s_3 = 1 - (s_1 + s_2)$,

$$\sigma_{13} = -Cov(T_1, T_3) = -Cov\left(T_1, \frac{s_1T_1 + s_2T_2}{s_3}\right) = -\frac{s_1\sigma_{11} + s_2\sigma_{12}}{s_3}, \tag{2.5}$$

and

$$\sigma_{23} = -Cov(T_2, T_3) = -Cov\left(T_2, \frac{s_1T_1 + s_2T_2}{s_3}\right) = -\frac{s_2\sigma_{22} + s_1\sigma_{12}}{s_3}. \tag{2.6}$$

Then, again applying Equation (2.2), we obtain

$$s_1^2\sigma_{11} + s_2^2\sigma_{22} + s_3^2\sigma_{33} + 2s_1s_2\sigma_{12} + 2s_1s_3\sigma_{13} + 2s_2s_3\sigma_{23} = 0.$$

Substituting σ_{13} and σ_{23} from Equations (2.5) and (2.6) respectively, we obtain

$$-s_1^2\sigma_{11} - s_2^2\sigma_{22} + s_3^2\sigma_{33} - 2s_1s_2\sigma_{12} = 0.$$

Solving for Plan 3’s variance, we obtain

$$\sigma_{33} = \frac{s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12}}{s_3^2}.$$

Now we consider the case in which there are N plans in the state. In the cases $N = 2$ and 3, we were able to express σ_{NN} in terms of $\{\sigma_{ij} : 1 \leq i \leq j < N\}$. That is, we expressed the variance of the risk transfer amount for Plan N in terms of the risk transfer variances and covariances for all other plans. For arbitrary N, we obtain the following generalization of the above result.

Theorem 2: For general N, the variance of the risk transfer amount for the Nth plan is given by

$$\sigma_{NN} = \frac{1}{s_N^2}\left(\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\leq i<j\leq N-1}\sum s_is_j\sigma_{ij}\right). \tag{2.7}$$

Proof: From Theorem 1, we know that

$$\sum_i s_iT_i = 0. \tag{2.8}$$

Therefore,

$$0 = Var\left(\sum_i s_iT_i\right) = \sum_{i=1}^{N} s_i^2\sigma_{ii} + 2\sum_{1\leq i<j\leq N}\sum s_is_j\sigma_{ij}$$

$$= s_N^2\sigma_{NN} + \sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{i=1}^{N-1} s_is_N\sigma_{iN} + 2\sum_{1\leq i<j\leq N-1}\sum s_is_j\sigma_{ij}. \tag{2.9}$$

From (2.9), we obtain

$$\sigma_{NN} = -\frac{1}{s_N^2}\left(\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\leq i<j\leq N-1}\sum s_is_j\sigma_{ij} + 2\sum_{i=1}^{N-1} s_is_N\sigma_{iN}\right). \tag{2.10}$$

From (2.8), we obtain

$$T_N = -\frac{1}{s_N}\sum_{j=1}^{N-1} s_j T_j. \tag{2.11}$$

Then, for i = 1,…,N-1,

$$\begin{aligned}\sigma_{iN} &= Cov(T_i, T_N)\\ &= Cov\left(T_i, -\frac{1}{s_N}\sum_{j=1}^{N-1} s_j T_j\right)\\ &= -\frac{1}{s_N}\left(s_i\sigma_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{N-1} s_j\sigma_{ij}\right).\end{aligned} \tag{2.12}$$

Substituting (2.12) into (2.10), we obtain

$$\begin{aligned}\sigma_{NN} &= -\frac{1}{s_N^2}\left(\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N-1}\sum s_i s_j\sigma_{ij} + 2\sum_{i=1}^{N-1} s_i s_N\sigma_{iN}\right)\\ &= -\frac{1}{s_N^2}\left(\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N-1}\sum s_i s_j\sigma_{ij} - 2\sum_{i=1}^{N-1}\left(s_i s_N\frac{s_i\sigma_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{N-1} s_j\sigma_{ij}}{s_N}\right)\right)\\ &= -\frac{1}{s_N^2}\left(-\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N-1}\sum s_i s_j\sigma_{ij} - 2\sum_{i=1}^{N-1}\sum_{\substack{j=1\\ j\neq i}}^{N-1} s_i s_j\sigma_{ij}\right)\\ &= -\frac{1}{s_N^2}\left(-\sum_{i=1}^{N-1} s_i^2\sigma_{ii} + 2\sum_{1\le i<j\le N-1}\sum s_i s_j\sigma_{ij} - 4\sum_{1\le i<j\le N-1} s_i s_j\sigma_{ij}\right).\end{aligned}$$

This reduces to Equation (2.7). The proof is complete.

Thus, we have proven that Theorem 2 provides a general formula valid for any number of plans in a state. In the next chapter, we consider the implications of this formula.

Chapter 3

Implications of the Formula for the Variance of *T_i*

In the previous chapter, we obtained a formula for the variance of the risk transfer amount for Plan *N*. For simplicity, returning to the case in which *N* = 3, Theorem 2 provides that

$$\sigma_{33} = \frac{s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12}}{s_3^2}.$$

To see how this variance behaves, we calculate the gradient of σ_{33} as a function of s_1 and s_2, assuming that σ_{11}, σ_{22}, and σ_{12} are constants. Substituting $s_3 = 1 - s_1 - s_2$, we obtain

$$\frac{\partial}{\partial s_1}\sigma_{33} = \frac{(1-s_1-s_2)^2\left(2s_1\ \sigma_{11} + 2s_2\sigma_{12}\right) + 2(1-s_1-s_2)(s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12})}{(1-s_1-s_2)^4};$$

therefore,

$$\frac{(1-s_1-s_2)^3}{2}\frac{\partial}{\partial s_1}\sigma_{33} = (1-s_1-s_2)\left(s_1\sigma_{11} + s_2\sigma_{12}\right) + (s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12})$$

$$= s_1\sigma_{11} + s_2\sigma_{12} - s_1s_2\sigma_{11} - s_2^2\sigma_{12} + s_2^2\sigma_{22} + s_1s_2\sigma_{12}$$

$$= s_1\sigma_{11}(1-s_2) + s_2\sigma_{12}\left(1-s_2\ \ +s_1\right) + s_2^2\sigma_{22}.$$

Setting this to be greater than or equal to 0, we find that $\partial\sigma_{33}/\partial s_1 \geq 0$ if and only if

$$\sigma_{12} \geq -\frac{s_1\sigma_{11}(1-s_2) + s_2^2\sigma_{22}}{s_2\left(1-s_2\ \ +s_1\right)}. \tag{3.1}$$

By symmetry, we also find that $\partial\sigma_{33}/\partial s_2 \geq 0$ if and only if

$$\sigma_{12} \geq -\frac{s_2\sigma_{22}(1-s_1) + s_1^2\sigma_{11}}{s_1\left(1-s_1\ \ +s_2\right)}. \tag{3.2}$$

Therefore, when Inequalities (3.1) and (3.2) hold, the variance of the risk adjustment transfer amount for Plan 3 increases as its market share decreases. This places insurers who are losing members in an even more precarious situation.

The R code in Appendix A provides examples of cases in which this occurs. In one example, the positive definite covariance matrix was the following:

```
            [,1]       [,2]      [,3]
[1,]  9.2693308 -0.5911575  8.554591
[2,] -0.5911575  8.1328560  4.467307
[3,]  8.5545911  4.4673075 26.171419
```

Using randomly generated market shares of 0.906, 0.029, and 0.065 for Plans 1, 2, and 3, respectively, Inequality (3.1) reduces to $\sigma_{12} \geq$-147.8937 and Inequality (3.2) reduces to $\sigma_{12} \geq$ -67.95267. Since σ_{12} = -0.5911575, then both Inequalities (3.1) and (3.2) are valid.

Next, we test whether a similar situation exists for the case of four plans in the state. Starting with

$$\sigma_{44} = \frac{s_1^2\sigma_{11} + s_2^2\sigma_{22} + s_3^2\sigma_{33} + 2s_1s_2\sigma_{12} + 2s_1s_3\sigma_{13} + 2s_2s_3\sigma_{23}}{(1 - s_1 - s_2 - s_3)^2},$$

we obtain

$$\frac{(1 - s_1 - s_2 - s_3)^3}{2}\frac{\partial}{\partial s_1}\sigma_{44} = (1 - s_1 - s_2 - s_3)\big(s_1\ \sigma_{11} + s_2\sigma_{12} + s_3\sigma_{13}\big)$$

$$+ s_1^2\sigma_{11} + s_2^2\sigma_{22} + s_3^2\sigma_{33} + 2s_1s_2\sigma_{12} + 2s_1s_3\sigma_{13} + 2s_2s_3\sigma_{23}$$

$$= s_1\sigma_{11} + s_2\sigma_{12} + s_3\sigma_{13} + s_1s_2\sigma_{12} + s_1s_3\sigma_{13} - s_1s_2\sigma_{11} - s_2s_3\sigma_{13}$$

$$-s_2^2\sigma_{12} - s_1s_3\sigma_{11} - s_2s_3\sigma_{12} - s_3^2\sigma_{13} + s_2^2\sigma_{22} + s_3^2\sigma_{33} + 2s_2s_3\sigma_{23}.$$

Therefore, $\partial\sigma_{44}/\partial s_1 \geq 0$ if and only if

$$0 \leq s_1\sigma_{11} + s_2\sigma_{12} + s_3\sigma_{13} + s_1s_2\sigma_{12} + s_1s_3\sigma_{13} - s_1s_2\sigma_{11} - s_2^2\sigma_{12} - s_2s_3\sigma_{13} - s_1s_3\sigma_{11} - s_2s_3\sigma_{12} - s_3^2\sigma_{13} + s_2^2\sigma_{22} + s_3^2\sigma_{33} + 2s_2s_3\sigma_{23}.$$

Similarly, by differentiating with respect to s_2 we find that $\partial\sigma_{44}/\partial s_2 \geq 0$ if and only if

$$0 \leq s_2\sigma_{22} + s_1\sigma_{12} + s_3\sigma_{23} - s_1s_2\sigma_{22} - s_1s_3\sigma_{12} - s_1^2\sigma_{12} - s_2s_3\sigma_{22} - s_1s_3\sigma_{23} + s_1s_2\sigma_{12} - s_3^2\sigma_{23} + s_1^2\sigma_{11} + s_3^2\sigma_{33} + 2s_1s_3\sigma_{13}.$$

Finally, by differentiating with respect to s_3 we find that $\partial\sigma_{44}/\partial s_3 \geq 0$ if and only if

$$0 \leq s_3\sigma_{33} + s_1\sigma_{13} + s_2\sigma_{23} - s_1s_3\sigma_{33} - s_1s_2\sigma_{23} - s_2s_3\sigma_{33} - s_1s_2\sigma_{13} - s_1^2\sigma_{13} + s_1s_3\sigma_{13} + s_2s_3\sigma_{23} - s_2^2\sigma_{23} + s_1^2\sigma_{11} + s_2^2\sigma_{22} + 2s_1s_2\sigma_{12}.$$

Using the R code from Appendix B, we obtain the following example of a covariance matrix which satisfies these inequalities:

```
            [,1]      [,2]      [,3]       [,4]
[1,]  42.751597 25.970697 -4.138777 -16.202942
[2,]  25.970697 52.989897 41.949209   2.056357
[3,]  -4.138777 41.949209 55.207588  20.205847
[4,] -16.202942  2.056357 20.205847  23.600923
```

With market shares of 0.862, 0.100, 0.017, and 0.021 for Plans 1, 2, 3, and 4, respectively, the derivatives of σ_{44} with respect to s_1, s_2, and s_3 are 37.65271, 37.42307, and 36.862, respectively. Because these values all are nonnegative, we conclude that the variance of the risk transfer amount for Plan 4 increases as any of the market shares for Plans 1, 2, or 3 increase in this example.

Chapter 4

Conclusions

In this thesis, we investigated some of the statistical properties of the risk transfer payments insurers must bear under the Affordable Care Act. We investigated insurers' risk transfer amounts, the covariances between the risk transfer amounts of all pairs of plans, and the variances of the risk transfer amounts for each plan. In particular, we derived some interesting properties of the variance of the risk transfer amounts relative to each plan's market share. Specifically, assuming that the covariances between plans' risk transfer amounts are within certain bounds we proved that, as the market share of any competing plan increases, the variance of the risk transfer amount of a given plan increases. By simulating examples in the cases of three and four plans in a state, we demonstrated that those assumptions can be satisfied in the real world.

This result provides mathematical justification for a phenomenon which has been observed previously. According to an article by The American Academy of Actuaries, "Risk adjustment transfers as a percent of premium were more variable and likely to be higher for insurers with a smaller market share. Insurers with a larger market share were by definition closer to the market average while small-market-share insurers were more likely to be skewed toward either low-risk or high-risk individuals" ["Insight on the ACA," 2016]. Figure 2 below further illustrates this point; we observe that plans with small market share have a much higher variance in their risk transfer amount as a percent of premium.

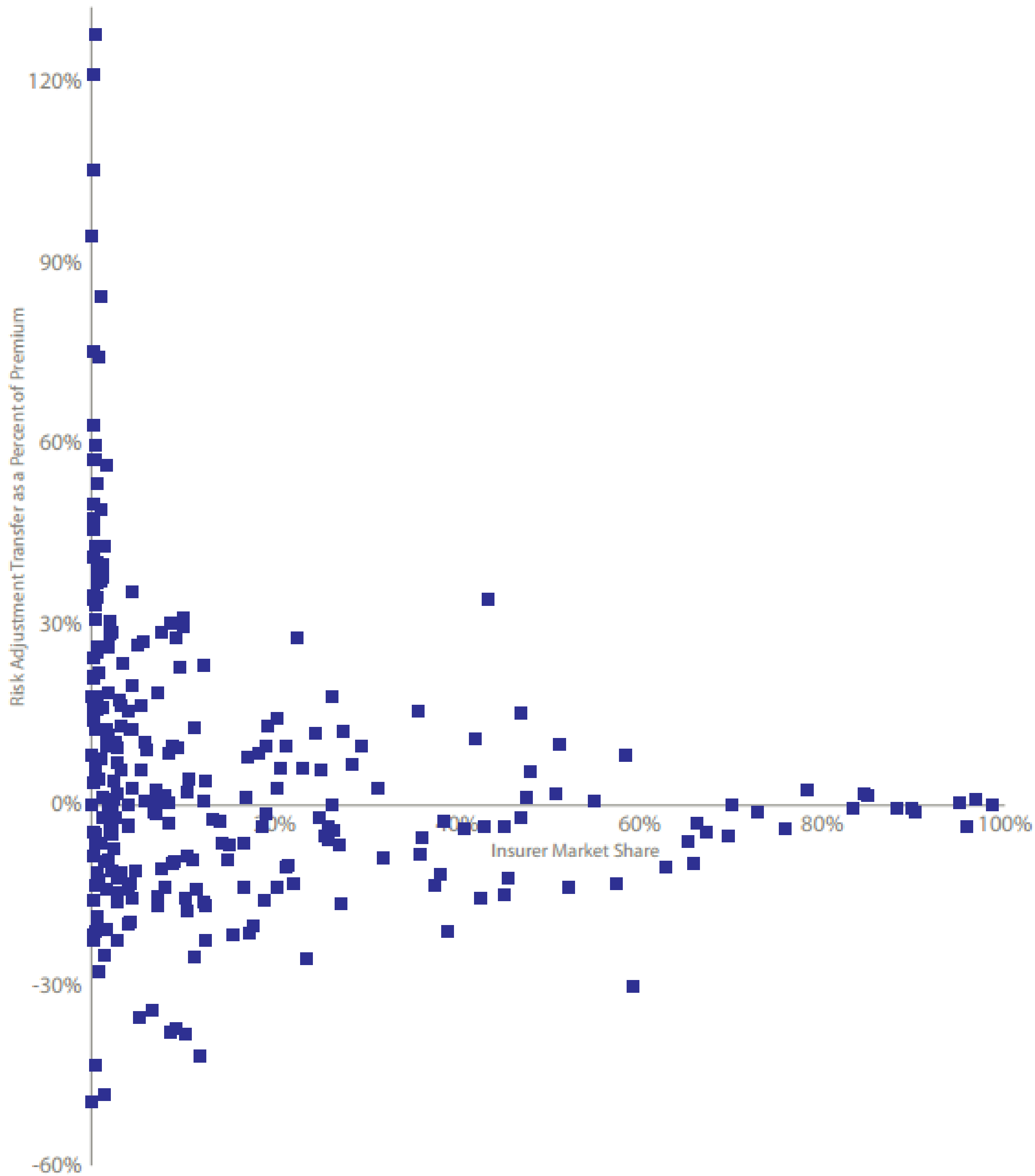

Figure 2: Risk Adjustment Transfer as a Percent of Premium vs. Insurer Market Share

Source: “Insight on the ACA Risk Adjustment Program.” *American Academy of Actuaries*, 2016, http://actuary.org/files/imce/Insights_on_the_ACA_Risk_Adjustment_Program.pdf. Accessed 14 Nov. 2016.

What we have proven is that, under a wide range of conditions, smaller plans will have more variable and therefore less predictable risk transfer amounts. With this knowledge, perhaps it will be possible to make changes to improve the risk adjustment process in the Affordable Care Act.

Appendix A

R Code for the Example of Three Plans

```
M = matrix(runif(9,-5,5),nrow=3,ncol=3)

det(M)

sigma=M%*%t(M)

sigma

s1=runif(1,0,1)

s2=runif(1,0,1-s1)

s3=1-s1-s2

s1

s2

s3

bound1=-(s1*sigma[1,1]*(1-s2)+s2^2*sigma[2,2])/(s2*(1-s2+s1))

bound2=-(s2*sigma[2,2]*(1-s1)+s1^2*sigma[1,1])/(s1*(1-s1+s2))

bound1

bound2
```

Appendix B

R Code for the Example of Four Plans

```
M = matrix(runif(9,-5,5),nrow=4,ncol=4)
det(M)
sigma=M%*%t(M)
sigma
s1=runif(1,0,1)
s2=runif(1,0,1-s1)
s3=runif(1,0,1-s1-s2)
s4=1-s1-s2-s3
s1
s2
s3
s4
bound1=s1*sigma[1,1]+s2*sigma[1,2]+s3*sigma[1,3]+s1*s2*sigma[1,2]+s1*s3*sigma[1
,3]-s1*s2*sigma[1,1]-s2*s2*sigma[1,2]-s2*s3*sigma[1,3]-s1*s3*sigma[1,1]-s2*s3*sigma[1,2]-
s3*s3*sigma[1,3]+s2*s2*sigma[2,2]+s3*s3*sigma[3,3]+2*s2*s3*sigma[2,3]
bound2=s2*sigma[2,2]+s1*sigma[1,2]+s3*sigma[2,3]-
s1*s2*sigma[2,2]+s2*s3*sigma[2,3]-s1*s3*sigma[1,2]-s1*s1*sigma[1,2]-s2*s3*sigma[2,2]-
s1*s3*sigma[2,3]+s1*s2*sigma[1,2]-
s3*s3*sigma[2,3]+s1*s1*sigma[1,1]+s3*s3*sigma[3,3]+2*s1*s3*sigma[1,3]
```

bound3=s3*sigma[3,3]+s1*sigma[1,3]+s2*sigma[2,3]-s1*s3*sigma[3,3]-s1*s2*sigma[2,3]-s2*s3*sigma[3,3]-s1*s2*sigma[1,3]-s1*s1*sigma[1,3]-s2*s2*sigma[2,3]+s1*s1*sigma[1,1]+s2*s2*sigma[2,2]+s1*s3*sigma[1,3]+s2*s3*sigma[2,3]+2*s1*s2*sigma[1,2]

bound1

bound2

bound3

BIBLIOGRAPHY

"Insight on the ACA Risk Adjustment Program." *American Academy of Actuaries*, 2016, http://actuary.org/files/imce/Insights_on_the_ACA_Risk_Adjustment_Program.pdf. Accessed 14 Nov. 2016.

Pope, G. C., Bachofer, H., Pearlman, A., Kautter, J., Hunter, E., Miller, D., & Keenan, P. "Risk Transfer Formula for Individual and Small Group Markets Under the Affordable Care Act." Medicare and Medicaid Research Review 4.3 (2014): n. pag. Web. 5 Nov 2016.

Ross, Sheldon. *A First Course in Probability*. 7th ed., Prentice Hall, 2006.

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Passed Exam P — May 2014
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Completed Economics, Corporate Finance, and Applied Statistical Methods VEEs

WORK EXPERIENCE

Highmark Health (Pittsburgh, PA) — Summer 2016
Actuarial Intern
- Built and tested a model for Medicare Part D risk adjustment payments in SAS
- Created and ran tests for quality assurance of incremental HCCs
- Presented work to actuaries of all levels at end of internship

Mutual of Omaha (Omaha, NE) — Summer 2015
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- Developed persistency and commission assumptions using SQL and Excel in MedSupp team
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- Presented work to actuaries of all levels at end of internship

LEADERSHIP/ACTIVITIES

Asian American Students In Action — 2013-2016
- Mentored Asian American students in transition to university
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COMPUTER SKILLS

Excel, Access: Experienced with PivotTables, queries, and formulas through coursework and internship
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