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MATHEMATICAL INTERPRETATION OF THE FINANCIAL TERMS IN INTEREST MODELS THEORY AND PRACTICE

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ABSTRACT

The purpose of the thesis is to compare and contrast different interpretation of financial terms in both the mathematical field and financial field. The referred definitions are selected from the book, *Interest rate models: Theory and practice* by Damiano Brigo and Fabio Mercurio. In each section of the thesis, there is one term firstly defined in the financial field and followed by the interpretation in the mathematical field.

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Chapter 1. Introduction

These following introduced terms are associated with a core concept-contract. In real life, when we deposit a certain amount of money into a bank account and given a certain time period, we actually have signed a potential contract with the bank, which indicates the initial time (when the contract starts), maturity time (when the contract ends) and the type of applied interest rate during the contract period. When the currency is ready to be withdrawn, this contract is mature and going to be terminated. What is interest rate? At the time when we withdraw the deposit, there is a change in the amount compared to the currency at the beginning of the contract. Usually, we will have a surplus deposit that we actually gain which is more than the initial value. That additional amount of money is yielded at a velocity which we call the interest rate. The interest rates, which are not limited to apply to personal finance, are actually important financial signals. On the other hand, they are also interesting from the mathematical point of view. As time when we store the money into bank account under our names, there is potentially a contract between depositor and the bank with a fixed initial value. In this thesis, we are going to give a mathematical interpretation of some basic interest rates. The mathematical language brings out the precise meaning of these concepts which the book *Interest rate models: Theory and practice* lacks.

Chapter 2. Bank Account Value (money-market account)

We are always unable to model real life exactly since there might exist more interventions than the simple setup by a model. Thus, we will assume a continuous basis for predicting the actual value of the bank account at any time t. In a later chapter, we will give a short example to illustrate the fact that error between the continuous assumption and the reality is relatively small. We model the value of a bank account at time t ($t \geqslant 0$) by a function B(t). Assume $B(0) = 1$ and a progressive surplus account is indicated by the following differential equation:

$$dB(t) = r_t B(t)dt, B(0) = 1, t > 0, \tag{1.1}$$

where r_t is a positive function of time. As a consequence,

$$B(t) = exp\left(\int_0^t r_s ds\right). \tag{1.2}$$

(Brigo and Mercurio 2006)
The above equation is telling us the fact that if we invest a unit amount at time 0, then we will receive that value of money as B(t) in (1.2). This instantaneous rate is often defined as instantaneous spot rate or short rate.

It is not hard to know that an equal relationship of how the value of the bank account changes by definition and by the progress calculated from the interest rate. Therefore, at each tiny time period, the r(t) can be used to determine the value of the bank account at that spot time.Thus, we are able to claim that the amount of money in the bank account grows at a certain rate r(t) as the time increases.

Example 1. Mr. Willard decides to deposit his one-month salary of 500 dollars into his savings account for 2 years. The fixed interest annually rate is 2%, paid once a year for this contract. How much is he going to have after 2 years?
The reality is: $500(1 + 0.02)^2 = 520.2$

The mode will predict as:

$$\frac{dB(t)}{dt} = 0.02B(t)$$
$$\int_0^2 \frac{1}{B}dB = \int_0^2 2\% dt$$
$$B(t) = 500e^{\int_0^2 0.02dt} = 520.41.$$

The error is just 20 cents in this case.
Notice here there is a scaler as 500 in the last step, why? B(t) actually is a value respect to 1 unit of currency in the Mr. Willard's bank account. Now, there are 500 units of 1 dollar, then the actual value should be whatever we have for B(t) multiple by 500.
Now, let's look at another usual financial question that we might encounter. What is the value that we have to invest at initial time t to yield one unit of currency at maturity time T? It is a way more convenient to "shrink" the calculation to 1 unit as the maturity value in the contract since in reality, the numbers can be very large. It is common for people to have a goal to take out a certain amount of money at maturity time T since the bank will reward each deposit during that time period.
Assume that the interest rate r(t) and bank value B(t) are fixed, claim that if we initially deposit A units of currency at time 0, then, there are $A \times B(t)$ units of currency in the bank account at $t > 0$. If we fix that there is exactly one unit of currency, as a conclusion $A \times B(t) = 1$. Initially, we have to deposit these amount of money:

$$A = \frac{1}{B(T)}.$$

Since B(T) is deterministic, A is deterministic as well. We are able to answer that question by multiplying B(t) on both sides:

$$\text{Initial Investment} = AB(t) = \frac{B(t)}{B(T)}.$$

In the above expression, we know that right hand side is always smaller than 1 which can be considered as a discount during a stochastic interest-rate process.

Chapter 3. Stochastic Discount Factor

<u>Definition in the book:</u> Given the value of a bank account B(t), "the (stochastic) discount factor D(t, T) between two time instants t and T is the amount at time t that has to be converted in order to receive one unit of currency payable at time T, and is given by:" (Brigo and Mercurio 2006)

$$D(t,T) = \frac{B(t)}{B(T)} = exp(-\int_t^T r_s ds). \tag{2.1}$$

Discount factor, generally is a mathematical measurement used to predict the future value or more specifically the value in a given contract duration compared to the present. It is a ratio of present value over future value. Let's take an example to understand in a mathematical way: if we invest one US dollar with a compounded interest rate of 10%per year, after 10 years, we will have $1 \times (1 + 10\%)^{10} = 2.59$ dollars. Thus, the present value of 1 dollar after 10 years will have the value of $\frac{1}{2.59} = 0.39$dollar. As for SDF here, B(t) is the initial value at the beginning of the contract, and then B(T) is corresponded maturity value at T, or the future value. How do we get (2.1)? We know B(t) from (1.2) and similarly, we can also write:

$$B(T) = exp\left(\int_0^T r_s ds\right).$$

Thus:

$$\frac{B(t)}{B(T)} = \frac{exp(\int_0^t r_s ds)}{exp(\int_0^T r_s ds)} = exp(\int_0^t r_s ds - \int_0^T r_s ds) = exp(-\int_t^T r_s ds).$$

It is important to know the value of r(t) since it will influence B(t) at any time (we mention this before). First, r(t) could be deterministic in some cases and as a result, B(t) and D(t) are deterministic as well. Second, for real interest-rate products, they are set based on the interest rate itself to earn money. Therefore, in such cases, they are stochastically processed by a stochastic interest rate which means that it will predict the conditions in the different situations. Stochastic processes always involve stochastic variables that are random, such as inflation factor and intervention of central bank. The purpose of a stochastic model is to predict and forecast the probable outcomes by a random variable.

In contrast to a deterministic model which will usually result in a particular answer or solution, a stochastic model will result in different solutions by adding variables into the modeling in order to see how the outcome would be changed.

Chapter 3.1. Zero-coupon bond

Definition in the book: "A T-maturity zero-coupon bond (pure discount bond) is a contract that guarantees its holder the payment of one unit of currency at time T, with no intermediate payments. The contract value at time $t < T$ is denoted by P (t, T). Clearly, P (T, T) = 1 for all T."
(Brigo and Mercurio 2006)

Simply, we can understand the above definition as the following: the contract value at time t is the price to be paid for the zero-coupon bound at time t. It is modeled as a function denoted by P(t,T).

Zero-coupon bond is a contract that promises 1 unit of currency to be received at a maturity time T from an initial time t. It is very similar to stochastic discount factor which also involves a fixed future value. The difference that lies in these two terms is an "equivalent amount of currency" and a "value of a contract". Whether r(t) fixed or not, that is the key to determine D and P. If the interest rate r(t) is deterministic, then $D(t, T) = P(t, T)$. If r(t) is unknown, then D(t,T) is also unknown at each pair (t,T). However, since P(t,T) is a process in a contract that guarantees the maturity payment, then P(t,T) has to be fixed at initial time t.

Zero-coupon bond P(t,T) is the fundamental concept in our discussion since we use the ratio of present value over maturity value to simplify the process in the case of dealing with too large number.

Chapter 3.2. Time to maturity

The time to maturity $T-t$ is the amount of time (in years) from the present time t to the maturity time $T > t$.

It is not trivial to define this term since in reality there are types of methods to determine the exact days during two dates in the form of day, month, year. In this chapter, we will follow the book to define that:

$$T - t = \frac{Actual}{365}.$$

(Brigo and Mercurio 2006).
Under this consideration, we assume there are 365 days per year and ignore some problems such as holidays where T is not included. For instance, if t= (10th, January, 2009), T= (10th, July, 2009), then:

$$T - t = \frac{182}{365} = 0.4986.$$

Chapter 4. Continuously-compounded Spot Interest Rate

Definition in the book: "The continuously-compounded spot interest rate prevailing at time t for the maturity T is denoted by R(t, T) and is the constant rate at which an investment of P(t, T) units of currency at time t accrues continuously to yield a unit amount of currency at maturity T. In formulas:"
(Brigo and Mercurio 2006)

$$R(t,T) = \frac{\ln P(t,T)}{\tau(t,T)}, \tag{3.1}$$

where $\tau(t,T)$ is the time difference between t and T.
In (3.1), we see that a change of currency over a difference in time, which indicates that R(t, T) is a rate function of interest.
Continuously compounded interest rate is a type of interest yielded constantly compounded, essentially leading to an infinite amount of compounding periods. Compounded interest rate refers to the interest rate that can be applied to the currency yield from the previous compounding period. For instance, if the first time we deposit 100 dollars into the bank account for two years and the compounded interest rate is 2% annually. Then, at the end of the first year, we will receive $500(1 + 0.02) = 510$ dollars. The key point occurs at the second year, when we calculate the new deposit, we apply this interest rate on 510 dollars instead of 500 dollars. Thus, at the end of the second year, we will have $510(1 + 0.02) = 520.2$ dollars
As for the RHS of (3.1), the fraction is the function of investment P(t,T) over a time difference from t to T, so that (3.1) is a rate function. In the following interpretation, the prevailing time t is zero and then the maturity is t which is also the duration of the contract. Thus:

$$R(t,T) = \frac{-\ln P(t)}{\tau(t)} \quad t \in [0, 10], \tag{3.2}$$

where, R(t) is an interest rate function with initial time t and maturity time T. P(t) is an investment function of variable initial time t. P(t) models the amount of investment in a direct proportion of the total amount. $\tau(t)$ models duration of the process.
Let P: $[0, 10] \rightarrow (0, 1)$ and $\tau(t) : [0, T] \rightarrow (0, T)$, then we define the above function R: $[0, T] \rightarrow R_{+}$. Note that we are able to find an equation of investment P in terms of t. This equation helps us to find out how much to invest at initial time t.

$$\begin{aligned} e^{R(t)\tau(t)} &= e^{-\ln P(t)} \\ e^{R(t)\tau(t)} &= P(t)^{-1} \\ P = e^{-R(t)\tau(t)} &= e^{R(t)(T-t)}. \end{aligned} \tag{3.3}$$

In reality, there are two cases categorized by discussing the above interest rate R(t).
<u>Case 1</u>: Assume $-\ln P(t) = ct$,where is a positive constant. Thus:

$$R(t) = \lim_{h \to \infty} \frac{-\ln P(t+h) + \ln P(t)}{h}. \tag{3.4}$$

By the assumption, it is easy to deduce that:

$$R(t) = \lim_{h \to \infty} \frac{-ct + c(t+h)}{h} = c.$$

R(t) is a constant rate in this case. Then we are solving for the initial investment in a given time period. Here is an example:

Example 2: In the year of 2015, Bank A has a continuous compounded interest rate as 1.58% annually. Dr.Smith wants to obtain an amount of money of 10,000 dollars after 10 years. How much money does he have to put into bank account at the beginning?
Solve:

$$\begin{aligned}
1.58\% &= \frac{-\ln P(t)}{10} \\
P = e^{-1.58\%\times 10} &= e^{-0.158} = 0.854 \\
10,000 \times 0.854 &= 8540(dollars).
\end{aligned}$$

Case 2: R(t) is not a constant rate but a variable rate. In order to make $-\ln P(t)$ positive, P(t) has the range (0,1]. Let's assume $P(t) = 1 + \sin(t)$ with the range of (0,1] and the domain is $(\frac{3\pi}{2} + 2\pi h, 2\pi(h+1)]$. In this case, equation (3.4) is the definition of a derivative which gives us:

$$\begin{aligned}
R(t) &= -\frac{1}{P(t)} \times P'(t) \\
R(t) &= -\frac{\cos(t)}{1+\sin(t)},
\end{aligned} \tag{3.5}$$

where h is an integer.

$$\lim_{t\to\frac{3\pi}{2}+2\pi h} -\frac{\cos(t)}{1+\sin(t)} \overset{L'hopital's}{\to} \lim_{t\to\frac{3\pi}{2}+2\pi h} -\frac{\sin(t)}{\cos(t)} = \lim_{t\to\frac{3\pi}{2}+2\pi h} \tan(t) = \infty.$$

Thus the range of R(t) is $(0,\infty)$ and the domain is $R\backslash\{(t = \frac{3\pi}{2} + 2\pi h) \cup (\pi + 2\pi h)\}$. R(t) is no longer a fixed rate and we are looking for the best time t to invest in order to maximum the profit.

Example 3: Give an amount money of $\$10,000$ as an initial investment at the beginning. Bank A has a financial product that with the rate R as a floating interest rate. If Mr. Smith participated in this product for 10 years, does he gain or lose money?

To answer this question, just use the equation (3.5) which gives:

$$R(10) = -\frac{\cos(10)}{1+\sin(10)} = 1.62.$$

It seems that the interest function is positive which gives a positive outcome.

However, R(10) only represents the rate at the 10^{th} year but it cannot reflect the trends during these 10 years. How do we know what was going on during this period? Recall the equation we found in the earlier step calculating the investment P. Use expression (3.3):

$$P(10) = 1 + \sin(10) = 1.46.$$

The ratio of investment and the goal income is greater than 1, thus we know there must be a loss.

Chapter 5. Simply-compounded Spot Interest Rate (SCSIR)

Definition in the book: "The simply compounded spot interest rate prevailing at time t for the maturity T is denoted by L(t, T) and is the constant rate at which an investment has to be made to produce an amount of one unit of currency at maturity, starting from P (t,T) units of currency at time t, when accruing occurs proportionally to the investment time. In formulas:"

$$L(t,T) = \frac{1 - P(t,T)}{\tau(t,T)P(t,T)},$$

where P(t,T) is given.
(Brigo and Mercurio 2006)
Simply compounded interest is the type of interest yield constantly compounded in a finite amount of compounding periods.
Let us explain this type of rate with a basic example. We can define that the SCSIR is used in a contract of financial derivatives, which ensure a fixed amount of money B at the maturity time T, initially growing from a fixed amount of money A. L(t, T) is the currency growth interest rate to reach the guaranteed amount. P(t, T) is the zero-coupon bond that is the ratio of initial investment over the maturity value.
$\tau(t,T)$ is the contract duration and defined as: $\tau(t,T) = T - t$ in unit of year. When people sign the contract with a financial constitution, the contract has a stated goal currency B and initial investment A as well as a stated contract duration $\tau(t,T)$. If the currency growth rate L(t,T) is also fixed, then it is a simple case. If L(t,T) is changeable depend on the variable time t and T, then it will be more complicated.

Now, define the initial time as 0 and the maturity time T is t, then:

$$\tau(t,T) = T - t = t,$$

and the time t is the only variable in our discussion from now on.
First, rewrite the above formula as:

$$L(t,T) = \frac{1 - P(t)}{\tau(t)P(t)},$$

$L:\mathbb{R}\to\mathbb{R}; P:\mathbb{R}\to[0,1]; \tau:\mathbb{R}\to\mathbb{R}$ Since the range of P(t,T) is [0,1], then we define P(t,T) as following with the same range:

$$p(t) = \sin^2(\ln(t)).$$

Q(t) is the actual amount of currency at time t. $Q(0) = A$,$Q(t) = B$

$$L(t) = \frac{1-P(t)}{tP(t)} = \frac{1}{t}(\frac{1}{P(t)} - 1). \tag{4.1}$$

Here is a reminder of rate function:

$$\frac{Q'}{Q} = L(t), \tag{4.2}$$

Q' is the changed amount of currency at time t and Q is the actual currency at time t. Then $\frac{Q'}{Q}$ is the rate of change of currency, which is the interest rate at time t. Combine (4.1) and (4.2):

$$\begin{aligned}
\frac{\mathrm{d}Q}{\mathrm{d}t}\frac{1}{Q} &= \frac{1}{tP(t)} - \frac{1}{t} \\
\int\frac{dQ}{Q} &= \int(\frac{1}{tP(t)} - \frac{1}{t})dt \\
\ln Q + c &= -\cot(\ln t) + c' - \ln t \\
\ln Q &= -\cot(\ln t) - \ln t + c.
\end{aligned}$$

c and c' are constant. Therefore, in this case we obtain an equation for Q in terms of t.

Now, let us consider the case where L(t) is a constant interest rate function in whole duration of the contract.

$$\begin{aligned}
L(t) = \frac{1-P(t)}{tP(t)} = \frac{1}{t}(\frac{1}{P(t)-1}) &= c \\
t^{-1}P^{-1} - t^{-1} &= c
\end{aligned}$$

Take derivative on both sides:

$$
\begin{aligned}
-t^{-2}P^{-1} + (-t^{-1}P^{-2}P') + t^{-2} &= 0 \\
\frac{1}{t^2} &= \frac{1}{t^2 P} + \frac{P'}{tP^2} \\
tP' &= P^2 - P \\
\frac{\mathrm{d}P}{\mathrm{d}t} t &= P^2 - P
\end{aligned}
$$

Then take integral on both sides:

$$
\begin{aligned}
\int \frac{1}{P^2 - P} dP &= \int \frac{1}{t} dt \\
\ln(\frac{1-P}{P}) &= \ln(t) + c.
\end{aligned}
$$

Chapter 6. Annually-Compounded Spot Interest Rate

Definition in the book: "The annually-compounded spot interest rate prevailing at time t for the maturity T is denoted by Y (t, T) and is the constant rate at which an investment has to be made to produce an amount of one unit of currency at maturity, starting from P (t, T) units of currency at time t, when reinvesting the obtained amounts once a year. In formulas:"

$$Y(t,T) = \frac{1}{P(t,T)^{\frac{1}{\tau(t,T)}}} - 1.$$

(Brigo and Mercurio 2006)

After doing some simple algebra:

$$P(t,T)(1 + Y(t,T)^{\tau(t,T)}) = 1.$$

In this type of interest rate, P(t,T) represents the ratio of $\frac{B(t)}{B(T)}$ and we assume B(T) is 1. As we tackle the previous terms, let t=0 and T=t. The range of P is (0,1). The new formula will be:

$$Y(t) = \frac{1}{P(t)^{\frac{1}{\tau(t)}}} - 1.$$

Let $P(t) = e^{-t}$ where t is positive real number. Since $\tau(t)$ is just t, we have:

$$Y(t) = \frac{1}{(e^{-t})^{\frac{1}{t}}} - 1 = e - 1.$$

This is a constant interest rate regardless of the value of t.
Then, we use a continuous process to model the reality. Recall the rate function (4.2) which we use here again to find Q in this situation:

$$\frac{Q'}{Q} = L(t)$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{1}{e^{-1}} - 1$$

Take the integral on both sides:

$$\int \frac{1}{Q} dQ = \int (e-1) dt$$

$$\ln Q + c = et - t.$$

Chapter 7. K-times-per-year Compounded Spot Interest Rate

<u>**Definition in the book**</u>: “The k-times-per-year compounded spot interest rate prevailing at time t for the maturity T is denoted by $Y^k(t,T)$ and is the constant rate (referred to a one-year period) at which an investment has to be made to produce an amount of one unit of currency at maturity, starting from P(t, T) units of currency at time t, when reinvesting the obtained amounts k times a year. In formulas: ”

$$Y^k(t,T) = \frac{k}{P(t,T)^{\frac{1}{k\tau(t,T)}}} - k. \tag{5.1}$$

(Brigo and Mercurio 2006)

It would be more easily understood if we change the above formula in another form when we do a little change to (5.1)

$$P(t,T)(1+\frac{Y^k(t,T)}{k})^{k\tau(t,T)} = 1. \tag{5.2}$$

This formula is “similar” to the one in the previous section of annually-compounded spot interest rate. As we interpret how we obtain that formula, the only thing needed to deal with is the “k-times”. First, these two interest rate functions are all constant that are fixed during the whole contract time from t to T. The “k-times” divide one year into several compounding periods. And each period has length as $\frac{1}{k}$. Assume P_0 is the initial investment at the beginning of a compounding period so that P_n is the actual balance in the bank account at time t. From one full compounding periods, we have:

$P_{n+1} = P_n+$the interest yield in this one compounded period.
$P_{n+1} = P_n+P_n\times$the length of one compounding period$\times$the interest rate
$P_{n+1} = P_n+P_n \times (\frac{1}{k}) \times R.$
$P_{n+1} = P_n(1+\frac{R}{k})$
Easily we can deduce the general formula as: $P_n = P_0(1+\frac{R}{k})^n$ If we fix the initial deposit and the duration of the contract $k\tau(t)$, we have earned more interest as more compounding periods per year. The number of compounding

periods is then $k\tau(t)$ which is equivalent to n in formula above. In (5.1), P(t,T) is a ratio of the present balance at time t over the maturity deposit at T. Then, $P(t,T) = \frac{P_t}{P_T}$.

In financial field, the "compounded interest" is the yielded interest that would be added into the balance as a new deposit to yield interest. Therefore, the obtained interest would also be able to earn the interest during the next time interval which is where the name "compounded" originated. The k-times-per-year compounded interest rate is where we repeat calculating the interests yielded in each time interval for k times totally.

When k is largely enough, let's say $k \to \infty$, then we find:

$$\lim_{k\to+\infty} \frac{k}{P(t,T)^{\frac{1}{k\tau(t,T)}}} - k = \frac{-\ln P(t,T)}{\tau(t,T)},$$

which is the formula corresponding to continuously-compounded interest rate.

Chapter 8. Zero-coupon Curve

Definition in the book: "The zero-coupon curve (sometimes also referred to as 'yield curve') at time t is the graph of the function:"

$$T \to \begin{cases} L(t,T) & t < T < t+1 \\ Y(t,T) & T > t+1, \end{cases}$$

where t and T are in the unit of year. It is a plot of time t of L(t, T) simply-compounded interest rates when the maturity duration is less than one year and of annually compounded interest rate when the maturity duration is larger than one year. (Brigo and Mercurio 2006)

Zero-coupon curve is usually obtained from the market data of interest rate at given time t. It is a graph reflecting the function mapping maturity duration into rates at time t.

Here is an example:

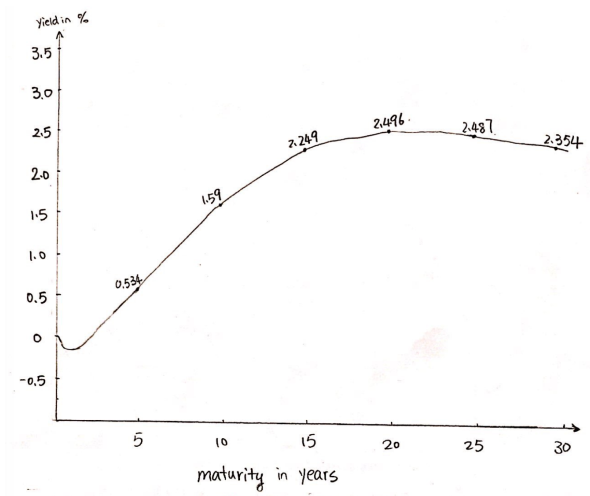

Figure 1: Zero Coupon curve at May 3, 2013

From the Figure above, we know that the graph of euro rate is not monotonic in this 30 years. It goes down in the early period which result as some particular currency of EURO showing reversed behavior before being monotonic. The EURO rate is the average interbank interest rate in the area where EURO dominates.

Chapter 9. Zero-bond curve

Definition in the book: "The zero-bond curve at time t is the graph of the function"

$$T \to P(t,T), \qquad T > t$$

which, because of the positivity of interest rates, is a T-decreasing function from $P(t,T) = 1$. (Brigo and Mercurio 2006)

Remind from the previous content about discount factor D(t, T) and P(t, T). In order to calculate the discount factor, we apply the principle of $P_n = P_0(1+\frac{R}{k})^n$. For instance, on May 3rd, 2013, zero-coupon yield curve spot rate of EURO market is 1.54% for maturity 5 years. Then $\frac{P_0}{P_n} = (\frac{1}{1+\frac{0.0154}{12}})^{5\times 12} = 0.9260$. By repeating this procedure, we can generate a zero-bond curve: (data is from DataMarket)

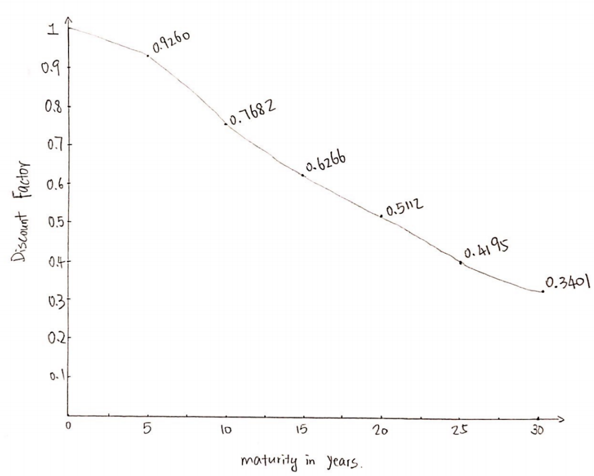

Figure 2: Zero Bond curve at May 3, 2013

By increasing the maturity time, the discount factor decreases since we approach the maturity from $P(t,T) = 1$ initially. The curve is not as diverse as zero-coupon curve and it is monotonic.

Appendix A. Figure1 data

The data of Zero-Coupon curve from market Euro rate at May 3rd, 2013

Maturity	Rate
1year	-0.019
5year	0.534
10year	1.590
15year	2.249
20year	2.496
25year	2.487
30year	2.354

Appendix B. Figure2 data

The data of Term structure of discount factor from market Euro rate at May 3rd, 2013

Maturity	Rate	Discount Factor
5year	1.54	0.9260
10year	2.64	0.7682
15year	3.12	0.6266
20year	3.36	0.5112
25year	3.51	0.4195
30year	3.61	0.3401

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Thesis Supervisor: Manfred Denker

Work Experience

Date: From Fall 2014 to Spring 2017

Title: Penn State Learning math and language tutor Description: Drop-in one-on-one math tutor and one-on-one language tutor by appointment Institution: Penn State Learning

Supervisor's Name: R. Neill Johnson

Language Proficiency: Chinese and English