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REPRESENTING N-PLAYER TRAGEDY OF THE COMMONS PROBLEM IN CHEMICAL GAME THEORY

MIRAS KATENOV
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Reviewed and approved* by the following:

Darrell Velegol
Distinguished Professor of Chemical Engineering
Thesis Supervisor and Honors Adviser

Themistoklis Matsoukas
Professor of Chemical Engineering
Faculty Reader

* Signatures are on file in the Schreyer Honors College.

ABSTRACT

The primary purpose of this thesis is to represent N-player Tragedy of the Commons problem in the Chemical Game Theory framework of analysis. Chemical game theory attempts to provide a new framework of analysis by introducing Gibbsian Thermodynamics and the concept of entropy to explain the decision making processes. These concepts are applied to Tragedy of the Commons problem, which first was introduced by Garrett Hardin in 1968, and results in the common resource being shared by individuals, where the cost of utilizing the resource is public and profits are individually private. This research attempts to answer fundamental question such as how to combine Ostrom's Institutional Design Principles with traditional classical game theory approach in the new field of Chemical Game Theory. The end product of this thesis is the method of generating N-player Tragedy of the Commons (n-ToC) games in the field of fishing.

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Chapter 1

Introduction

The objective of this thesis is to fundamentally represent a dilemma of Tragedy of the Commons in the Chemical Game Theory, new way of looking at decision making process. Taking a dilemma in Tragedy of the Commons and meaningfully analyzing the problem outcomes, requires a construction of the game and determining a solution in the classical game theory. This thesis transforms the Tragedy of the Commons game into the Chemical Game Theory framework and extends the method to N-number of players participating in the game by developing a model of pain matrix generation. This thesis also attempts to answer a fundamental question of how to incorporate ideas from Ostrom's Institutional Design Principles into non-cooperative games from the Chemical Game Theory modeling perspective.

1.1 Classical Game Theory and Nash Equilibrium

Mathematician John von Neumann and economist Oskar Morgenstern developed a framework to analyze competitive decisions in a quantifiable and mathematical way. Their book "Theory of Games and Economic Behavior" published in 1944 was a breakthrough that opened a new field of economics, which is currently known as "game theory" and will be referred in this thesis as Classical Game Theory. Situation or decision that has choices between people that lead to certain outcomes is turned into a game. The game consists of players making a decision; choices that represent possibilities in making a decision; outcomes that are numerically quantified as "payoffs".

One of the common example is the Prisoners Dilemma game. It consists of two prisoners (player A and player B) that together committed a crime, but have been caught and taken to a police station where they are faced with a decision. The policemen interview them separately by isolating the prisoners in two different cells. Each prisoner faces two options:

(1) Testify against a crime partner, here will be referred as TELL

(2) Cooperate with a crime partner by remaining silent, here will be referred as QUIET

The situation has four possible combinations such as QUIET/QUIET, QUIET/TELL, TELL/QUIET and TELL/TELL. These combinations have potential outcomes, which are represented by the number of years served in prison:

- If both players tell on each other (TELL/TELL), each get 2 years in prison.
- If both players cooperate with one another (QUIET/QUIET), each get 1 year in prison.
- If one of them cooperates and the other one testifies against (TELL/QUIET and QUIET/TELL), player that remains silent gets 3 years, whereas player that testifies gets 0 years.

This game could be represented in a visual matrix, **Table 1-1**. shows all the information in a concise form.

Table 1-1. Prisoners Dilemma Game with payoffs/years in prison

	Player B: a_1 - TELL	**Player B: a_2 - QUIET**
Player A: b_1 - TELL	2,2	0,3
Player A: b_2 – QUIET	3,0	1,1

The game can be solved to yield a solution known as "Nash Equilibrium". John Nash, a Nobel prize winner, introduced this concept and defined it as the set of strategies, where no player has an incentive to unilaterally deviate from it. Nash proved that there will always be at least one equilibrium solution for any game finite set action game (Nash, 1950). In our example, the prisoners' dilemma has a Nash Equilibrium solution TELL/TELL. One of the biggest limitations of classical game theory is the assumption of rationality, which implies that each player will act rationally, in other words self-interested. The Chemical

Game Theory approach allows to expand this assumption and suggest a corrective perspective into the solution of the game.

1.2 Common Costs and Private Profits

In 1968 Garrett Hardin introduced a new problem, which he called Tragedy of the Commons in his article. As the population grows and resources are limited, they become scarce. Due to the inherent nature, humans try to maximize self-benefit and utilize as much resources as possible. As every human being tries to maximize its own benefit utilizing the resources that are common, the inevitable result is mutual destruction of resources to unsustainable level.

In his example of herdsman grazing cattle on the common pasture, Hardin illustrates this concept and provides a simple mathematical explanation. (Hardin, 1968) Each herdsman would try to keep as much cattle grazing as he could possible afford. By adding a sheep, he gains approximately a positive one unit of utility (a measure of happiness or satisfaction) since all the proceeds go to that individual herdsman. On the other hand, the costs in case of overgrazing are shared between all the herdsmen, since the pasture is common, and the herdsman get a fraction of -1. As a rational human being, every herdsman would see a net positive gain from adding an extra sheep and would increase his herd without any limits to the pasture that is limited in space and resource. Hence, the "tragedy" follows as the pasture is destroyed with each and every herdsman overgrazing the common ground, leading to overall negative result.

In general, such problem arises with rivalrous and non-excludable goods, which are called common-pool resources. **Table 1-2**. summarizes the possible goods based on rivalrous/non-rivalrous vs. excludable/non-excludable classification. Rivalrous goods prevent simultaneous consumption of the goods by two consumers, whereas non-excludable goods could be accessed even if it has not been paid for. As a result, the tragedy of the commons could be seen as common cost and private profit problem, where the individual profit and common shared negative cost results for the consumers.

Table 1-2. Common-pool resources as a type of good. Adopted from (Ostrom, 2009)

	Excludable	Non-Excludable
Rivalrous	**Private goods:** Food, clothing, automobiles, etc.	**Common-pool resources (CPR):** Forests, fisheries, lakes, etc.
Non-rivalrous	**Club goods:** Cinema, theaters, private clubs, etc.	**Public goods:** Television, national defense, air, etc.

1.3 Tragedy of the Commons in Fishing

The tragedy of the commons problem has long been known to humans. Given domestic or international fisheries as a common-pool abundant resource of fish, fisheries could fish as much fish as technical capabilities allow them. Scott Gordon first touched on mathematical and conceptual analysis of the fishing problem from the perspective of economics, before Hardin introduced the concept of tragedy of the commons. (Gordon, 1954) Nevertheless, Gordon did acknowledge over-exploitation of a common-property resource under individualistic competition in the fishing industry context, introducing important parameters such as “fishing effort” to explain the consumption and production functions within bio-economics of fisheries.

In late 1970s, the game theoretic perspective came into play to analyze tragedy of the commons of fisheries. Particularly, Levhari & Mirman as well as Clark incorporated the dynamic aspect of biological growth of the fish stock into the Nash equilibrium analysis for strategic options. Most notably, these economists studied the optimal outcomes of the competitive game between N-number of players, leading to the conclusion that the equilibrium level of stock would be driven down to the break-even level resulting in over-fishing (Clark, 1980), taking the stance of Hardin.

A completely different perspective on tragedy of the commons problem comes from Nobel Prize winner Elinor Ostrom. In her book, “Governing the Commons” she achieves more optimistic outcomes for common-pool resource exploitation, presenting compelling counter examples, which are empirically supported. (Ostrom, 1999) For example, a small fishery location in Alanya, Turkey has been able to sustainably self-manage the fish stock despite the competitive nature of the fishermen. The new approach to self-governance has been extensively studied by Ostrom and her colleagues, leading her to propose Institutional Design Principles for stable common-pool resources summarized in Table 1-3.

Table 1-3. Ostrom's Institutional Design Principles (Ostrom, 2009)

1	**Clearly defined boundaries**: The boundaries of the resource system and individuals with use rights are clearly defined
2	**Proportional equivalence between benefits and costs**: Rules specifying resource allocations are related to local conditions and to rules concerning inputs.
3	**Collective choice arrangements**: Many of the individuals affected by harvesting and production rules are included in the group that can modify these groups.
4	**Monitoring**: Monitors who actively monitor biophysical conditions and user behavior are at least partially accountable to the user and/or are the users themselves.
5	**Graduated sanctions**: Users who violate rules receive graduated sanctions from other users, from officials accountable to these users, or from both.
6	**Conflict-resolution mechanisms**. Users and their officials have rapid access to low-cost, local action situations to resolve conflict among users or between users and officials.
7	**Minimal recognition of rights to organize**. Users’ rights to devise their own institutions are not challenged by external governmental authorities, and users have long-term tenure rights.
8	**Nested enterprises**. Appropriation, provision, monitoring, enforcement, conflict resolution and governance activities are organized in multiple players of nested enterprises.

This thesis addresses the shortcomings of classical game theory analysis of Tragedy of the Commons problem on the example of fishing problem. Including the Ostrom’s institutional principles into the newly developing Chemical Game Theory to attempt to model the Tragedy of the Commons problem for multiple players.

Chapter 2

Chemical Game Theory

2.1 Entropy and Gibbsian Thermodynamics

In chemistry and chemical engineering, reactions represent transformation of chemical species into new chemical species. Josiah Willard Gibbs as one of the founders of modern thermodynamics had extensively studied the phenomena and introduced the concept of free-energy, which later was named in his honor as Gibbs free energy, as well as chemical potential. In his monumental paper "On the Equilibrium of Heterogeneous Substances", Gibbs explained thermodynamic reaction tendencies by developing fundamental thermodynamic relations between pressure (P), temperature(T), entropy(S) and chemical potentials of multi-component systems (Gibbs, 1874). As a consequence of Gibbs work, the following relationship results in the **Equation 2-1** (Matsoukas, 2013).

$$dG = -SdT + VdP + \sum_{i=1}^{n} \mu_i dN_i \qquad \text{(Eq. 2-1)}$$

where *G* stands for Gibbs-free energy and μ for chemical potential of component *i* in the system. Using **Equation 2-2**. Rewriting previous expression in terms of extent of the reaction ξ under constant temperature and pressure, where ν_i is the stoichiometric coefficient of component *i* in the reaction, yields **Equation 2-3**.

$$dn_i = \nu_i d\xi_i \qquad \text{(Eq. 2-2)}$$

$$dG = \sum_{i=1}^{n} \mu_i \nu_i d\xi \qquad \text{(Eq. 2-3)}$$

The equilibrium condition further follows as Gibbs-free energy is minimized with respect to the extent of the reaction in **Equation 2-4.**

$$\frac{dG}{d\xi} = 0 \qquad \text{(Eq. 2-4)}$$

Chemical potential of the species could be related to the equilibrium composition of species, y_i of an ideal gas, taking a reference chemical potential μ_i^0, **Equations 2-5, 2-6, 2-7**.

$$\mu_i = \mu_i^0 + RT \ln\left(\frac{y_i P}{P^0}\right) \quad \text{(Eq. 2-5)}$$

$$\sum_{i=1}^{n}\left(\mu_i^0 + RT \ln\left(\frac{y_i P}{P^0}\right)\right)\nu_i = 0 \quad \text{(Eq. 2-6)}$$

$$\sum_{i=1}^{n}\left(\frac{\Delta g^0}{RT} + \nu_i \ln\left(\frac{P}{P^0}\right) + \nu_i \ln(y_i)\right) = 0 \quad \text{(Eq. 2-7)}$$

Equation 2-7. allows calculations of the composition of the ideal mixture at the equilibrium, knowing the Gibbs-free energy change of the reaction of component *i* at the reference temperature, Δg^0.

Previous analysis could be further extended to multiple reactions, *j*, with species *i* in the system and summarized in the **Equation 2-8**. The detailed derivation is explained in Appendix A.

$$\sum_{j=1}^{m}\left(\frac{\Delta g_j^0}{RT} + \ln\left(\frac{P}{P^0}\right)\sum_{i=1}^{n}(\nu_{ij}) + \sum_{i=1}^{n}\nu_{ij}\ln(y_i)\right) = 0 \quad \text{(Eq. 2-8)}$$

Making transition to our analysis of decision making, the same concept of thermodynamic equilibrium is employed in the framework of Chemical Game Theory. Instead of symbolical representation of chemical species, the notion of "knowlecules" is introduced (Velegol, 2015). The knowlecules symbolize the possible choices that individual is faced with in the game. Moreover, the combination of the knowlecules represent decision reactions, that have an associated Gibbs-free energy, equivalently "pain" in Chemical Game Theory. Now, the expected payoffs in the classical game theory games represent the "pains" associated with each decision reaction. The following framework translates normal game theory set up to Chemical Game Theory.

In Chapter 1, Prisoners Dilemma game was analyzed through the lens of classical game theory, yielding a Nash equilibrium solution. Looking at Prisoners Dilemma through Chemical Game Theory perspective decisions are combined in two separate reactors for player A and player B, and then products are further combined in the decider reactor. The concept could be visualized in a Process Flow Diagram with a set of decision reactions generated with corresponding energies.

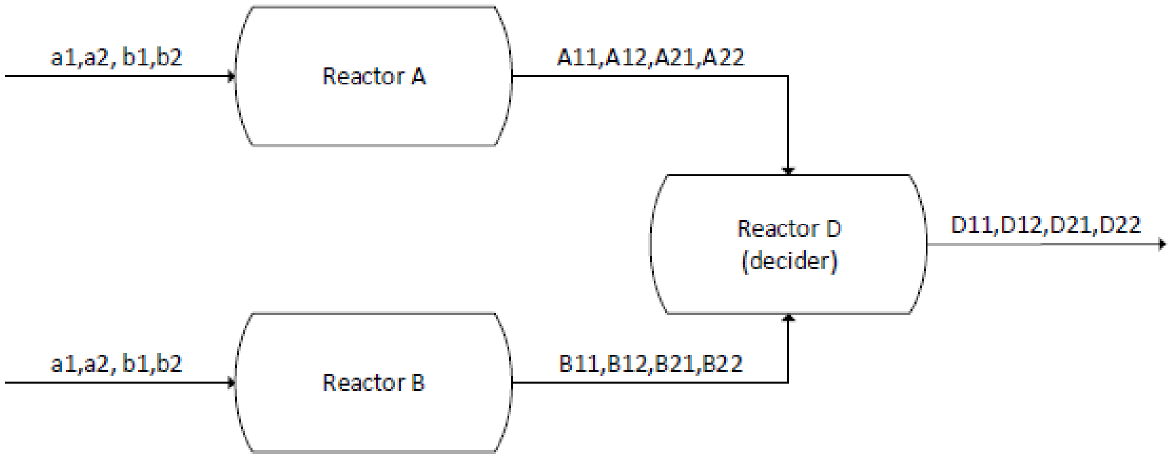

Figure 2-1. Process Flow Diagram for decision reaction process

Multiple reactions are happening in each reactor. R1, R2, R3, R4 designates the reaction order for extents of the reactions, ξ_1, ξ_2, ξ_3, ξ_4 respectively in each reactor. The order of the reactions is important for the reasons explained later in the chapter.

R1: $a1 + b1 + A \rightarrow A11$ $\frac{\Delta g^0}{RT} = P_{11}$ (Eq. 2-9) **R1**: $a1 + b1 + B \rightarrow B11$ $\frac{\Delta g^0}{RT} = P_{11}$ (Eq. 2-10)

R2: $a1 + b2 + A \rightarrow A12$ $\frac{\Delta g^0}{RT} = P_{12}$ (Eq. 2-11) **R2**: $a1 + b2 + B \rightarrow B12$ $\frac{\Delta g^0}{RT} = P_{12}$ (Eq. 2-12)

R3: $a2 + b1 + A \rightarrow A21$ $\frac{\Delta g^0}{RT} = P_{21}$ (Eq. 2-13) **R3**: $a2 + b1 + B \rightarrow B21$ $\frac{\Delta g^0}{RT} = P_{21}$ (Eq. 2-14)

R4: $a2 + b2 + A \rightarrow A22$ $\frac{\Delta g^0}{RT} = P_{22}$ (Eq.2-15) **R4**: $a2 + b2 + B \rightarrow B22$ $\frac{\Delta g^0}{RT} = P_{22}$ (Eq. 2-16)

In the decider reactor, the products from reactors A and B are combined to form new products with reaction energies (c_1,c_2,c_3,c_4), which are set depending whether there is a favorable outcome for each reaction. In general, these energies are set to 0 or -1. **Equations 2-17,18,19,20** show the reactions happening in the decider reactor.

R1: $A11 + B11 \rightarrow D11$ $\frac{\Delta g^0}{RT} = c_1$ (Eq. 2-17)

R2: $A12 + B12 \rightarrow D12$ $\frac{\Delta g^0}{RT} = c_2$ (Eq. 2-18)

$$\textbf{R3}: A21 + B21 \rightarrow D21 \quad \frac{\Delta g^0}{RT} = c_3 \qquad \text{(Eq. 2-19)}$$

$$\textbf{R4}: A22 + B22 \rightarrow D22 \quad \frac{\Delta g^0}{RT} = c_4 \qquad \text{(Eq. 2-20)}$$

In order to calculate the equilibrium concentrations of the species in the reactors, the SPICEY table, or material balance stoichiometric table, is set up reflecting the change from the initial amounts of the reactants in each reactor separately.

Table 2-1. SPICEY table for reactor A (Material Balance Stoichiometric Table)

Species (i)	**Initial ($n_{i,0}$)**	**Change**	**Equilibrium (n_i)**	**Mole fraction (y_i)**
a1	$n_{a1,0}$	$-\xi_1 - \xi_2$	$n_{a1,0} - \xi_1 - \xi_2$	$\frac{n_{a1}}{\Sigma}$
a2	$n_{a2,0}$	$-\xi_3 - \xi_4$	$n_{a2,0} - \xi_3 - \xi_4$	$\frac{n_{a2}}{\Sigma}$
b1	$n_{b1,0}$	$-\xi_1 - \xi_3$	$n_{b1,0} - \xi_1 - \xi_3$	$\frac{n_{b1}}{\Sigma}$
b2	$n_{b2,0}$	$-\xi_2 - \xi_3$	$n_{b2,0} - \xi_2 - \xi_4$	$\frac{n_{b2}}{\Sigma}$
A11	$n_{A11,0}$	$+\xi_1$	$n_{A11,0} + \xi_1$	$\frac{n_{A11}}{\Sigma}$
A12	$n_{A121,0}$	$+\xi_2$	$n_{A11,0} + \xi_2$	$\frac{n_{A12}}{\Sigma}$
A21	$n_{A211,0}$	$+\xi_3$	$n_{A11,0} + \xi_3$	$\frac{n_{A21}}{\Sigma}$
A22	$n_{A22,0}$	$+\xi_4$	$n_{A11,0} + \xi_4$	$\frac{n_{A22}}{\Sigma}$
Total	Σ_0	$\Delta\xi$	Σ	1

Table 2-2. SPICEY table for reactor B (Material Balance Stoichiometric Table)

Species (i)	**Initial ($n_{i,0}$)**	**Change**	**Equilibrium (n_i)**	**Mole fraction (y_i)**
a1	$n_{a1,0}$	$-\xi_1 - \xi_2$	$n_{a1,0} - \xi_1 - \xi_2$	$\frac{n_{a1}}{\Sigma}$
a2	$n_{a2,0}$	$-\xi_3 - \xi_4$	$n_{a2,0} - \xi_3 - \xi_4$	$\frac{n_{a2}}{\Sigma}$
b1	$n_{b1,0}$	$-\xi_1 - \xi_3$	$n_{b1,0} - \xi_1 - \xi_3$	$\frac{n_{b1}}{\Sigma}$
b2	$n_{b2,0}$	$-\xi_2 - \xi_3$	$n_{b2,0} - \xi_2 - \xi_4$	$\frac{n_{b2}}{\Sigma}$
B11	$n_{B11,0}$	$+\xi_1$	$n_{B11,0} + \xi_1$	$\frac{n_{B11}}{\Sigma}$
B12	$n_{B121,0}$	$+\xi_2$	$n_{B11,0} + \xi_2$	$\frac{n_{B12}}{\Sigma}$
B21	$n_{B211,0}$	$+\xi_3$	$n_{B11,0} + \xi_3$	$\frac{n_{B21}}{\Sigma}$
B22	$n_{B22,0}$	$+\xi_4$	$n_{B11,0} + \xi_4$	$\frac{n_{B22}}{\Sigma}$
Total	Σ_0	$\Delta\xi$	Σ	1

Table 2-3. SPICEY table for decider reactor D (Material Balance Stoichiometric Table)

Species (i)	**Initial ($n_{i,0}$)**	**Change**	**Equilibrium (n_i)**	**Mole fraction (y_i)**
A11	$n_{A11,0}$	$-\xi_1$	$n_{A11,0}-\xi_1$	$\frac{n_{A11}}{\Sigma}$
A12	$n_{A12,0}$	$-\xi_2$	$n_{A21,0}-\xi_2$	$\frac{n_{A12}}{\Sigma}$
A21	$n_{A21,0}$	$-\xi_3$	$n_{A12,0}-\xi_3$	$\frac{n_{A21}}{\Sigma}$
A22	$n_{A22,0}$	$-\xi_4$	$n_{A22,0}-\xi_4$	$\frac{n_{A22}}{\Sigma}$
B11	$n_{B11,0}$	$-\xi_1$	$n_{B11,0}-\xi_1$	$\frac{n_{B11}}{\Sigma}$
B12	$n_{B12,0}$	$-\xi_2$	$n_{B12,0}-\xi_2$	$\frac{n_{B12}}{\Sigma}$
B21	$n_{B21,0}$	$-\xi_3$	$n_{B21,0}-\xi_3$	$\frac{n_{B21}}{\Sigma}$
B22	$n_{B22,0}$	$-\xi_4$	$n_{B22,0}-\xi_4$	$\frac{n_{B22}}{\Sigma}$
D11	$n_{D11,0}$	$+\xi_1$	$n_{D11,0}+\xi_1$	$\frac{n_{D11}}{\Sigma}$
D12	$n_{D12,0}$	$+\xi_2$	$n_{D12,0}+\xi_2$	$\frac{n_{D12}}{\Sigma}$
D21	$n_{D11,0}$	$+\xi_3$	$n_{D21,0}+\xi_3$	$\frac{n_{D21}}{\Sigma}$
D22	$n_{D11,0}$	$+\xi_4$	$n_{D22,0}+\xi_4$	$\frac{n_{D22}}{\Sigma}$
Total	Σ_0	$\Delta\xi$	Σ	1

Combining the results from the tables with Equation 6, the extents of the decision reactions at the equilibrium and subsequent mole fractions could be calculated. Numerical solution to the problem is given in the Appendix B.

Chapter 3

Results and Discussion

3.1 Defining Tragedy of the Commons in Chemical Game Theory

In Tragedy of the Commons problem with multiple players, defining a realistic game for the purpose of inputting into the Chemical Game Theory (CGT), seems problematic given the current methods of solving CGT games is limited to two players faced with two decision choices. Another challenge in incorporating Tragedy of the Commons problem into the CGT framework is the definition of "pains" in the payoff matrix, which in itself should include the stock level reflecting the changes due to consumption or generation of the common stock. The problem should be defined in a way to reflect a competitive decision for Chemical Game Theory. 7P's framework could be used to define the problem, consisting of: *Problem*, *Players*, *Possibilities*, *Pains*, *Priors, Perception, Probabilities* (Velegol, 2015). This framework could be used to formulate the game for fishing tragedy of the commons.

- Problem: The problem of fishing tragedy of the commons could be stated in a question "How to maximize personal gain given the constraints of the common resource and insuring sustainability of the fishery stock?"
- Players: In this game the players are fishers or fishing fleets, depending on the size of the fishery, which could be both domestic or international. Every fisher is considered to be identical have exactly the same technical capabilities and costs associated with fishing.
- Possibilities: The players of the game could choose either to cooperate with the rest of the players to insure sustainable catch and avoid over-fishing or choose not to cooperate and seek personal gain maximization, regardless of other players. Due to the identical nature of the players, the game would be symmetrical having two possibilities with identical pain values.

- Pains: Generation of pains is a more extensive question and will be explained in the next section.
- Priors: Priors represent the initial concentration of the starting species, involved in the reactions in reactors A and B, and show initial biases of the players.
- Perception: Pains in the matrix could be modified depending on the players' perspectives, such as *Loyal/Altruistic, Vengeful, Overall, Rivalrous or Self-interested.* The following table summarizes these equations, where p_A and p_B represent pain of player A and B respectively for a given decision choice.

Table 3-1. Pain changes due to different perspectives

Perspective	**Equation**
Loyal/Altruistic	$0 * p_A + 1 * p_B$
Vengeful	$0 * p_A - 1 * p_B$
Overall	$0.5 * p_A + 0.5 * p_B$
Rivalrous	$0.5 * p_A - 0.5 * p_B$
Self-interested (for player A)	$1 * p_A + 0 * p_B$

- Probabilities: Probabilities are calculated based on the equilibrium concentration of the reactant species and will be explained in the later section.

3.2 Generating Pain Matrix

In most fishery problems that have been analyzed by classical game theory, the players seek to maximize Net Present Value (NPV) of fishing rent. NPV represent the value of the total fishing per person rent that is discounted over time as the level of fish stock is also varying in time. **Equation 3-1** shows NPV constituents as the difference between Revenue and Cost of fishing rent with a discount factor $\delta =$

$\frac{1}{1+r}$, where r is a discount rate constant. Taking the idea that marginal cost is inversely proportional to the stock level, **Equation 3-1** shows the general way to calculate net present value of one fisher with no competition. (Hannesson, 1996)

$$NPV = \sum_{t=0}^{\infty} \delta^t \{p(G(S) - S) - c(\ln(G(S)) - \ln(S))\} \quad \text{(Eq. 3-1)}$$

In the above expression *p* represent unit price, *c* is a cost parameter and *G(S)* is a stock level generation logistic growth function known as Gordon-Schaefer model (Gordon, Schaefer, 1954).

$$G(S) = aS(1 - \frac{S}{K}) \quad \text{(Eq. 3-2)}$$

Equation 3-2. shows the stock level as a function of the stock level that remained in the previous period, where *a* is intrinsic growth rate and *K* is carrying capacity of the given environment and both usually chosen to be constants that could be varied. In the hypothetical example, a pond that could sustain only 100 fishes would have a value of *K=100.* Furthermore, constant *a=0.2* would mean that for every 100 fishes in the pond, there is 20 new generated. An analogy could be drawn to compare Gordon-Schaefer model to the rate expression in the kinetics of the elementary reaction where one reactant is participating: $r_A = k_A C_A$, where k_A is analogous to intrinsic rate of growth a.   The net growth is then expressed as the difference between *G(S)* and *S* and is assumed to be the catch during that period.

Along the cooperative solution, the net gain is equally divided between all the fisherman. Rognvaldur Hannesson derived this result under discounting over infinite time. **Equation 3-4** summarizes this result (Hannesson, 1996).

$$NPV = \frac{\pi^0}{N}\left(\frac{1}{1-\delta}\right) = \frac{\pi^0}{N}\left(\frac{1+r}{r}\right) \quad \text{(Eq. 3-4)}$$

$$\pi^0 = p(G(S^0) - S^0) - c(\ln(G(S^0)) - \ln(S^0)) \quad \text{(Eq. 3-5)}$$

where S^0 is the optimal level of stock under cooperation, which has been calculated for various discounting factors and parameters *a* and *K* in the logistic function by Nils-Arne Ekerhovd (Ekerhovd, 2008).

Non-cooperative path has been modeled in the form of “grim-trigger” strategy, where one player cheats by fishing more stock, equivalently not cooperating, and the rest of the players respond by changing their strategies to punish the cheater upon detection in the first period. The punishment is achieved by fishing down the stock to the break-even point, where no profit is made. The result is illustrated in the **Equation 3-6**, where π^0, π^d and π^* represent profit before cheating, profit during deviation in the second period (when cheater is detected) and profit after punishment in all later periods respectively. In this analysis $S^* = \frac{c}{p}$, which is the level of stock left when fishing is no longer profitable and assumes calculation when marginal cost equals price (Hannesson, 1996).

$$NPV = \frac{\pi^0}{N} + \pi^d + \frac{\pi^*}{N}\frac{\delta}{1-\delta} \qquad \text{(Eq. 3-6)}$$

$$\pi^d = p(S^0 - S^*) - c(\ln(S^0) - \ln(S^*)) \qquad \text{(Eq. 3-7)}$$

$$\pi^* = p(G(S^*) - S^*) - c(\ln\big(G(S^*)\big) - \ln(S^*)) \qquad \text{(Eq. 3-8)}$$

The pain matrix is most feasible for negative pain values, yet with the same proportionality as payoffs in classical games. Consequently, the pain that each fisherman is faced under cooperative decision could be modeled as negative of NPV, **Equation 3-9**.

$$Pain = -NPV \qquad \text{(Eq. 3-9)}$$

Based on the previous derivation and setting constants *a=1* and *K=1,* **Table 3-2** generates the pains for cooperation and no-cooperation strategies for N-number of players. **Figure 3-1** is the graphical representation of Pain values increase as number of players increases in the game, due to less stock becoming available as more people fish. Pain for cooperation is lower for all the numbers of players N, suggesting higher payoff for cooperative strategy regardless of the number of players. This could be explained by an extremely severe punishment by all of the remaining players for the indefinite period after the detection period.

Table 3-2. Pain the certain player gets if cooperates or not for N total number of players.

	Pain $(= -NPV)$	
N	Cooperate (Eq. 3-4)	Not Cooperate (Eq. 3-6)
2	-0.740	-0.515
3	-0.493	-0.346
4	-0.370	-0.262
5	-0.296	-0.211
6	-0.247	-0.177
7	-0.211	-0.153
8	-0.185	-0.135
9	-0.164	-0.120
10	-0.148	-0.109

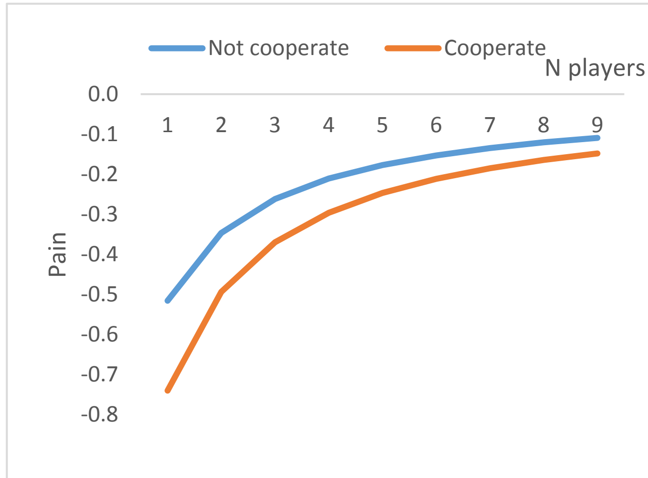

Figure 3-1. Pain values for cooperation and non-cooperation for different N-number of players.

3.3 Representing 3-player Tragedy of the Commons Game

In order to extend the representation of the game to more than 2 players, but with two choices, the introduction of new knowlecules is necessary. For instance, in 3 player game with 2 choices (cooperate or not), player C is added with two decision possibilities c1 and c2 (1-cooperate, 2-not). Now the decision reactions should account for all the possible combinations of species a1,a2,b1,b2,c1,c2 in three separate reactors A, B, C, and a decider reactor. Equations below show decision reactions and associated energies in reactor j=A,B or C

R1: $a1 + b1 + c1 \rightarrow j111 \quad \frac{\Delta g^0}{RT} = P^j_{111}$ (Eq. 3-10)

R2: $a1 + b1 + c2 \rightarrow j112 \quad \frac{\Delta g^0}{RT} = P^j{}_{112}$ (Eq. 3-11)

R3: $a1 + b2 + c1 \rightarrow j121 \;\; \frac{\Delta g^0}{RT} = P_{121}^{j}$ (Eq. 3-12)

R4: $a1 + b2 + c2 \rightarrow j122 \;\; \frac{\Delta g^0}{RT} = P_{122}^{j}$ (Eq. 3-13)

R5: $a2 + b1 + c1 \rightarrow j211 \;\; \frac{\Delta g^0}{RT} = P_{211}^{j}$ (Eq. 3-14)

R6: $a2 + b1 + c2 \rightarrow j212 \;\; \frac{\Delta g^0}{RT} = P_{212}^{j}$ (Eq. 3-15)

R7: $a2 + b2 + c1 \rightarrow j221 \;\; \frac{\Delta g^0}{RT} = P_{221}^{j}$ (Eq. 3-16)

R8: $a2 + b1 + c2 \rightarrow j212 \;\; \frac{\Delta g^0}{RT} = P_{212}^{j}$ (Eq. 3-17)

The decider reactor combines products from reactors A, B and C in the same way as in 2x2 case.

R1: $A111 + B111 + C111 \rightarrow D111 \;\; \frac{\Delta g^0}{RT} = P_{111}^{D}$ (Eq. 3-18)

R2: $A112 + B112 + C112 \rightarrow D112 \;\; \frac{\Delta g^0}{RT} = P^{D}{}_{112}$ (Eq. 3-19)

R3: $A121 + B121 + C121 \rightarrow D121 \;\; \frac{\Delta g^0}{RT} = P_{121}^{D}$ (Eq. 3-20)

R4: $A122 + B122 + C122 \rightarrow D122 \;\; \frac{\Delta g^0}{RT} = P_{122}^{D}$ (Eq. 3-21)

R5: $A211 + B211 + C211 \rightarrow D211 \;\; \frac{\Delta g^0}{RT} = P_{211}^{D}$ (Eq. 3-22)

R6: $A212 + B212 + C212 \rightarrow D212 \;\; \frac{\Delta g^0}{RT} = P_{212}^{D}$ (Eq. 3-23)

R7: $A221 + B221 + C221 \rightarrow D221 \;\; \frac{\Delta g^0}{RT} = P_{221}^{D}$ (Eq. 3-24)

R8: $A212 + B212 + C212 \rightarrow D212 \;\; \frac{\Delta g^0}{RT} = P_{212}^{D}$ (Eq. 3-25)

Representing the pain matrix in the case of 3 players, it is no longer possible to view all the possible combinations in one two dimensional matrix, but rather 3D representation such as on **Figure 3-2**, where each quadrant cube within an overall cube represent all the possible outcomes.

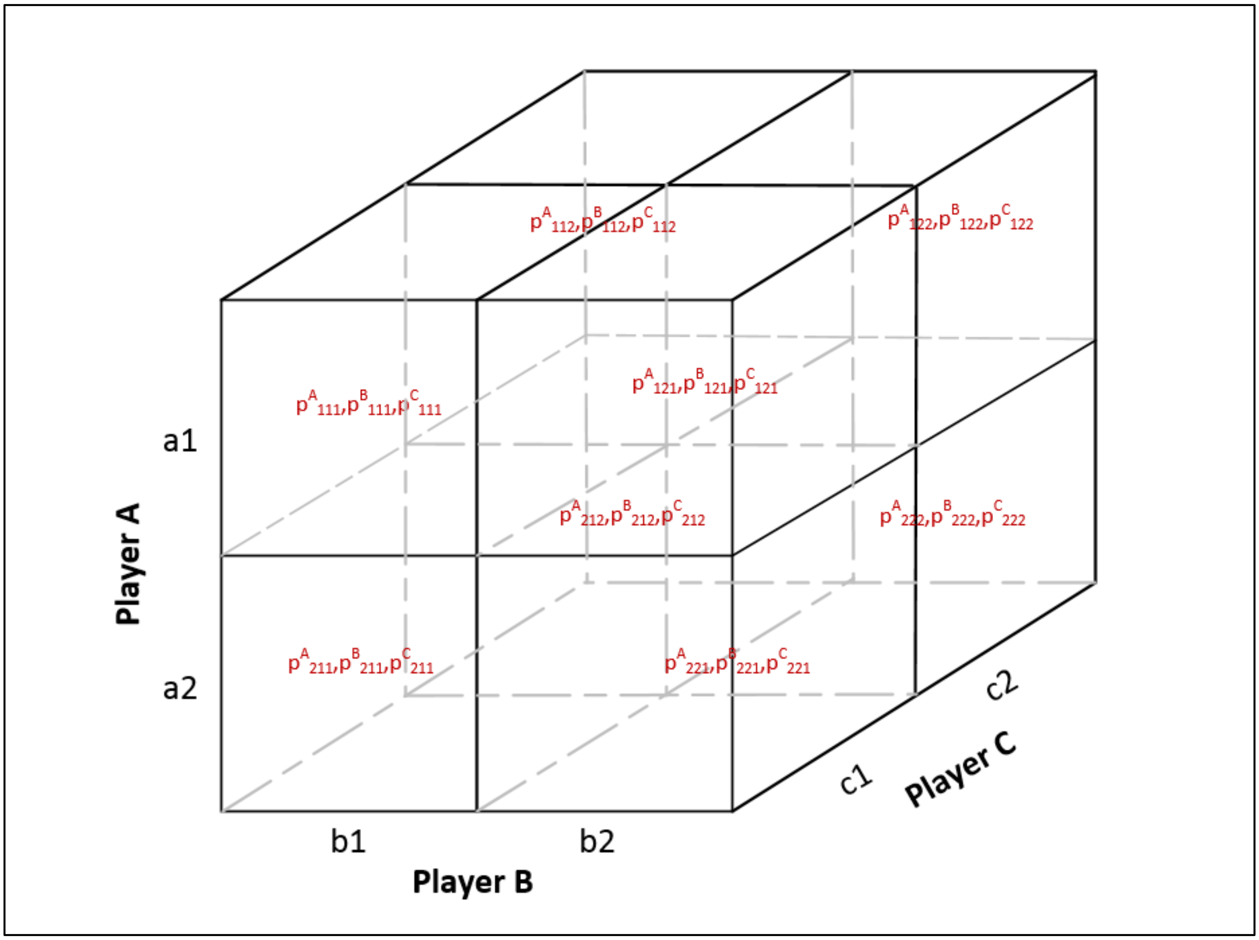

Figure 3-2. 3D matrix for N-player representation

This representation of the pain matrix including all the possible pains could also be decomposed into 2 playing "fields" of each player, keeping one of the player's decision constant.

Table 3-3. Playing "field" of player B and C, when player A chooses a1

Player A: a1	**Player C: c1**	**Player C: c2**
Player B: b1	$p^A_{111}, p^B_{111}, p^C_{111}, p^D_{111}$	$p^A_{112}, p^B_{112}, p^C_{112}, p^D_{112}$
Player B: b2	$p^A_{121}, p^B_{121}, p^C_{121}, p^D_{121}$	$p^A_{122}, p^B_{122}, p^C_{122}, p^D_{122}$

Table 3-4. Playing field of player B and C when player A chooses a2

Player A: a2	**Player C: c1**	**Player C: c2**
Player B: b1	$p^A_{211}, p^B_{211}, p^C_{211}, p^D_{211}$	$p^A_{212}, p^B_{212}, p^C_{212}, p^D_{212}$
Player B: b2	$p^A_{221}, p^B_{221}, p^C_{221}, p^D_{221}$	$p^A_{222}, p^B_{222}, p^C_{222}, p^D_{222}$

A three player game would have 8 decision reactions in reactors A, B, C, D with 12 species in each reactor, totaling to 32 decision reactions and 48 species.

Taking the values from **Table 3-2**, N=3 and inserting into **Table 3-3** and **Table 3-4**, the necessary pain matrix for 3 player game with neutral decider is generated, such that:

Table 3-5. Playing "field" when A chooses a1-cooperate

Player A: a1	**Player C: c1**	**Player C: c2**
Player B: b1	-0.493, -0.493, -0.493, 0	-0.493, -0.493, -0.346, 0
Player B: b2	-0.493, -0.346, -0.493, 0	-0.493, -0.346, -0.346, 0

Table 3-6. Playing "field" when A chooses a2-not cooperate

Player A: a2	**Player C: c1**	**Player C: c2**
Player B: b1	-0.346, -0.493, -0.493, 0	-0.346, -0.493, -0.346, 0
Player B: b2	-0.346, -0.346, -0.493, 0	-0.346, -0.346, -0.346, 0

In general, the number of reactions in the reactor will increase as $\mathbf{2^N}$ and the total number of the reactions will increase as $\mathbf{2^N*(N+1)}$. The number of species in each reactor will grow as $\mathbf{(2^N+2N)}$ and the total number of species would increase as $\mathbf{(2^N+2N)*(N+1)}$. From the representation perspective, the number of "fields" will grow as $\mathbf{2^{N-2}}$ having **N+1** pain values in each corresponding cell.

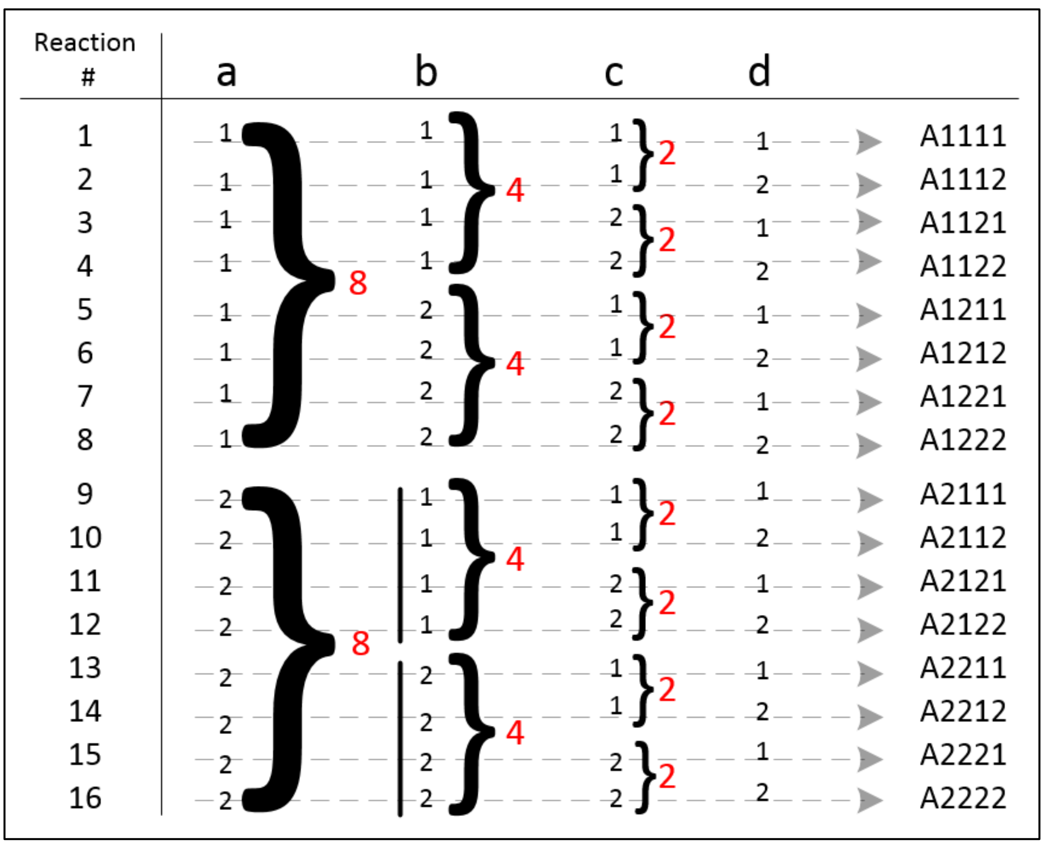

Figure 3-3. Reaction generation for N=4 players

For the purpose of consistency, the reaction generation in each reactor should follow the following scheme, as the order of the reactions is important. This example generation is for 4x2 case with four players and two choices each. However, the pattern follows for any other combination of N players.

3.4 Deriving Algorithm for Solving N-player Games

Starting back from 2x2 case the SPICEY Table could further be extended using matrix algebra if the reactions and species are arranged in the correct way. After the reactions have been generated and number assigned for each reaction in all of the reactors reactor (e.g. R1: a1+b1->A11, with ξ_1), SPICEY table is condensed into the following vector equation, where *j* for reactor A,B,C,D… etc. (Velegol, 2015)

$$\vec{n}^j = \vec{n}_0^j + \vec{\nu}^j \cdot \vec{\xi}^j \qquad \text{(Eq. 3-26)}$$

$\vec{n}_0^j$ represent a column vector of species initial amounts, $\vec{\nu}^j$- stoichiometric coefficient matrix, and $\vec{\xi}^j$ - extents of the reactions (either 0,-1 or 1) in reactor *j*. For instance, 2x2 case results in reactor A such that:

$$\begin{bmatrix} n_{a1,0} \\ n_{a2,0} \\ n_{b1,0} \\ n_{b2,0} \\ n_{A11,0} \\ n_{A12,0} \\ n_{A21,0} \\ n_{A22,0} \end{bmatrix} + \begin{bmatrix} \nu_{a1}^1 & \nu_{a1}^2 & \nu_{a1}^3 & \nu_{a1}^4 \\ \nu_{a2}^1 & \nu_{a2}^2 & \nu_{a2}^3 & \nu_{a2}^4 \\ \nu_{b1}^1 & \nu_{b1}^2 & \nu_{b1}^3 & \nu_{b1}^4 \\ \nu_{b2}^1 & \nu_{b2}^2 & \nu_{b2}^3 & \nu_{b2}^4 \\ \nu_{A11}^1 & \nu_{A11}^2 & \nu_{A11}^3 & \nu_{A11}^4 \\ \nu_{A12}^1 & \nu_{A12}^2 & \nu_{A12}^3 & \nu_{A12}^4 \\ \nu_{A21}^1 & \nu_{A21}^2 & \nu_{A21}^3 & \nu_{A21}^4 \\ \nu_{A22}^1 & \nu_{A22}^2 & \nu_{A22}^3 & \nu_{A22}^4 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \begin{bmatrix} n_{a1} \\ n_{a2} \\ n_{b1} \\ n_{b2} \\ n_{A11} \\ n_{A12} \\ n_{A21} \\ n_{A22} \end{bmatrix}$$

Furthermore, the mole fractions of species *i* in reactor *j*, y_i^j, calculated:

$$\vec{y}^j = \frac{\vec{n}^j}{n_T^j} \qquad \text{(Eq. 3-27)}$$

where n_T^j is a scalar quantity representing total moles in reactor *j* and calculated by multiplying column vector $\vec{n}^j$ by vector $\vec{I}$, defined to have the same number of species elements as $\vec{n}^j$ all of them equaling to one. Arranging pains into column vectors and minimizing the square difference of two terms from the equilibrium condition Eq. 6:

$$\Delta\vec{g}^{j} = (p_{11}^{j}, p_{12}^{j}, p_{21}^{j}, p_{22}^{j}) \quad \text{(Eq. 3-28)}$$

$$\left[\Delta\vec{g}^{j} - \left(-\vec{v}^{j} * \overrightarrow{\ln(y^{J})}\right)\right]^{2} = obj(J) \quad \text{(Eq. 3-29)}$$

Lastly minimizing the total objective functions to determine extents of the reactions and subsequent mole fractions by minimizing the total objective function $obj^{tot} = \sum obj(j)$.

Current efforts in Dr. Velegol's group are directed to developing a computational method of solving N-player games using computer software called GAMS or Mathematica.

3.5 Linear Approximation of 2x2 and 3x2 case

Another linear approach may be taken to estimate the decision reaction extents in the reactors. Taking the results from the SPICEY table, we convert the reaction energies to the equilibrium constant condition, from taking an exponential of negative energy values associated with each reaction.

$$K = \exp(-\frac{\Delta g^{0}}{RT}) \quad \text{(Eq. 3-30)}$$

Since the values of the reaction extents are very small, the product of two different reaction extents is even smaller and could be approximated to equal to zero, $\xi_{n} * \xi_{m} \approx 0$, *m* and *n* are integers. For a 2x2 case (2 players & 2 decisions), the equations are as follows, where K_{A11}, K_{A12}, K_{A21}, K_{A22} are equilibrium constants for each decision reaction in reactor A and n_{i} is the initial concentration of specie *j*.

$$K_{A11} = \frac{\Sigma_{0} n_{A11} - n_{A11}(\xi_{1}+\xi_{2}+\xi_{3}+\xi_{4}) + \Sigma_{0}\xi_{1}}{n_{a1}n_{b1} - n_{a1}(\xi_{1}+\xi_{3}) - n_{b1}(\xi_{1}+\xi_{2})} \quad \text{(Eq. 3-31)}$$

$$K_{A12} = \frac{\Sigma_{0} n_{A12} - n_{A12}(\xi_{1}+\xi_{2}+\xi_{3}+\xi_{4}) + \Sigma_{0}\xi_{2}}{n_{a1}n_{b2} - n_{a1}(\xi_{2}+\xi_{4}) - n_{b2}(\xi_{1}+\xi_{2})} \quad \text{(Eq. 3-32)}$$

$$K_{A21} = \frac{\Sigma_{0} n_{A21} - n_{A21}(\xi_{1}+\xi_{2}+\xi_{3}+\xi_{4}) + \Sigma_{0}\xi_{3}}{n_{a2}n_{b1} - n_{a1}(\xi_{1}+\xi_{3}) - n_{b1}(\xi_{3}+\xi_{4})} \quad \text{(Eq. 3-33)}$$

$$K_{A22} = \frac{\Sigma_0 n_{A22} - n_{A22}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_4}{n_{a2} n_{b2} - n_{a2}(\xi_2+\xi_4) - n_{b1}(\xi_3+\xi_4)} \quad \text{(Eq. 3-34)}$$

Similar results are derived for reactor B and reactor D.

$$K_{B11} = \frac{\Sigma_0 n_{B11} - n_{B11}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_1}{n_{a1} n_{b1} - n_{a1}(\xi_1+\xi_3) - n_{b1}(\xi_1+\xi_2)} \quad \text{(Eq. 3-35)}$$

$$K_{B12} = \frac{\Sigma_0 n_{B12} - n_{B12}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_2}{n_{a1} n_{b2} - n_{a1}(\xi_2+\xi_4) - n_{b2}(\xi_1+\xi_2)} \quad \text{(Eq. 3-36)}$$

$$K_{B21} = \frac{\Sigma_0 n_{B21} - n_{B21}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_3}{n_{a2} n_{b1} - n_{a1}(\xi_1+\xi_3) - n_{b1}(\xi_3+\xi_4)} \quad \text{(Eq. 3-37)}$$

$$K_{B22} = \frac{\Sigma_0 n_{B22} - n_{B22}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_4}{n_{a2} n_{b2} - n_{a2}(\xi_2+\xi_4) - n_{b1}(\xi_3+\xi_4)} \quad \text{(Eq. 3-38)}$$

$$K_{D11} = \frac{\Sigma_0 n_{D11} - n_{D11}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_1}{n_{A11} n_{B11} - n_{A11}(\xi_1+\xi_3) - n_{B11}(\xi_1+\xi_2)} \quad \text{(Eq. 3-39)}$$

$$K_{D12} = \frac{\Sigma_0 n_{D12} - n_{D12}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_2}{n_{A12} n_{B12} - n_{A12}(\xi_2+\xi_4) - n_{B12}(\xi_1+\xi_2)} \quad \text{(Eq. 3-40)}$$

$$K_{D21} = \frac{\Sigma_0 n_{D21} - n_{D21}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_3}{n_{A21} n_{B21} - n_{A21}(\xi_1+\xi_3) - n_{B21}(\xi_3+\xi_4)} \quad \text{(Eq. 3-41)}$$

$$K_{D22} = \frac{\Sigma_0 n_{D22} - n_{D22}(\xi_1+\xi_2+\xi_3+\xi_4) + \Sigma_0 \xi_4}{n_{A22} n_{B22} - n_{A22}(\xi_2+\xi_4) - n_{B22}(\xi_3+\xi_4)} \quad \text{(Eq. 3-42)}$$

Taking these equations, the extents of the reactions could be estimated upon construction of the matrix equations and solving linear systems of equations, Appendix C.

Similar analysis could be extended to derive initial estimates of 3x2 (3 players & 2 decisions) game as a precursor to N-player estimations and could be seen in more detail in Appendix D. Here one more assumption is made to be able to linearize the result such that $\xi_n * \xi_m * n_j \approx 0$ in addition to the previous assumption.

$$K_{A111} = \frac{(\Sigma_0)^2 n_{A111} - 4 n_{A111}\xi_t + (\Sigma_0)^2 \xi_1}{n_{a1} n_{b1} n_{c1} - n_{a1} n_{b1}(\xi_1+\xi_3+\xi_5+\xi_7) - n_{a1} n_{c1}(\xi_1+\xi_2+\xi_5+\xi_6) - n_{b1} n_{c1}(\xi_1+\xi_2+\xi_3+\xi_4)}$$

$$K_{A112} = \frac{(\Sigma_0)^2 n_{A111} - 4 n_{A111}\xi_t + (\Sigma_0)^2 \xi_2}{n_{a1} n_{b1} n_{c2} - n_{a1} n_{b1}(\xi_2+\xi_4+\xi_6+\xi_8) - n_{a1} n_{c2}(\xi_1+\xi_2+\xi_5+\xi_6) - n_{b1} n_{c2}(\xi_1+\xi_2+\xi_3+\xi_4)}$$

$$K_{A121} = \frac{(\Sigma_0)^2 n_{A111} - 4 n_{A111}\xi_t + (\Sigma_0)^2 \xi_3}{n_{a1} n_{b2} n_{c1} - n_{a1} n_{b2}(\xi_1+\xi_3+\xi_5+\xi_7) - n_{a1} n_{c1}(\xi_3+\xi_4+\xi_7+\xi_8) - n_{b2} n_{c1}(\xi_1+\xi_2+\xi_3+\xi_4)}$$

$$K_{A122}=\frac{(\Sigma_0)^2 n_{A111}-4n_{A111}\xi_t+(\Sigma_0)^2\xi_4}{n_{a1}n_{b2}n_{c2}-n_{a1}n_{b2}(\xi_2+\xi_4+6+\xi_8)-n_{a1}n_{c2}(\xi_3+\xi_4+\xi_7+\xi_8)-n_{b2}n_{c2}(\xi_1+\xi_2+\xi_3+\xi_4)}$$

$$K_{A211}=\frac{(\Sigma_0)^2 n_{A111}-4n_{A111}\xi_t+(\Sigma_0)^2\xi_5}{n_{a2}n_{b1}n_{c1}-n_{a2}n_{b2}(\xi_1+\xi_3+\xi_5+\xi_7)-n_{a2}n_{c1}(\xi_1+\xi_2+\xi_5+\xi_6)-n_{b2}n_{c1}(\xi_5+\xi_6+\xi_7+\xi_8)}$$

$$K_{A212}=\frac{(\Sigma_0)^2 n_{A111}-4n_{A111}\xi_t+(\Sigma_0)^2\xi_6}{n_{a2}n_{b1}n_{c2}-n_{a2}n_{b1}(\xi_2+\xi_4+\xi_6+\xi_8)-n_{a2}n_{c2}(\xi_1+\xi_2+\xi_5+\xi_6)-n_{b2}n_{c2}(\xi_5+\xi_6+\xi_7+\xi_8)}$$

$$K_{A221}=\frac{(\Sigma_0)^2 n_{A111}-4n_{A111}\xi_t+(\Sigma_0)^2\xi_7}{n_{a2}n_{b2}n_{c1}-n_{a2}n_{b2}(\xi_1+\xi_3+\xi_5+\xi_7)-n_{a2}n_{c1}(\xi_3+\xi_4+\xi_7+\xi_8)-n_{b2}n_{c1}(\xi_5+\xi_6+\xi_7+\xi_8)}$$

$$K_{A222}=\frac{(\Sigma_0)^2 n_{A111}-4n_{A111}\xi_t+(\Sigma_0)^2\xi_8}{n_{a2}n_{b2}n_{c2}-n_{a2}n_{b2}(\xi_2+\xi_4+\xi_6+\xi_8)-n_{a2}n_{c2}(\xi_3+\xi_4+\xi_7+\xi_8)-n_{b2}n_{c2}(\xi_5+\xi_6+\xi_7+\xi_8)}$$

Similar eight linear equations are constructed for reactor B, C and D. These equations could be deconstructed in the same way as 2x2 case to construct 4 matrix equations to estimate the extents of the reactions and subsequent mole fractions in order to calculate the expected pain.

3.6 Institutional Design Principles Inclusion

An important aspect in our analysis of tragedy of the commons is the derivation of pains for cooperative and non-cooperative strategy. In case of "grim-trigger" strategy, the deflector is punished severely, which minimizes the payoffs of that player's strategy. In Ostrom's Institutional Design principles she talks about enforcing rules-in-use as in collective action, as opposed to external body, such as government, monitoring the rules. The result of the pain functions may be affected by graduating sanctions. For example, in the non-cooperative strategy, instead of fishing down the stock by all other players, a sanction fee may be in place. This fee could be in terms controlling profits gained after detection π^* (profit remained from the punishment) i.e. increasing π^* by 10%, 30%, 60% for high, low, medium sanctions respectively. The table represents the effect of various levels of sanctions on the cheater. The pains for non-cooperative strategy decreases as sanctions becoming lower, which is reasonable to expect.

Table 3-7. Pains at different levels of sanctions

		High	Medium	Low
N	**Cooperate**	**Not cooperate**		
2	-0.740	-0.610	-0.657	-0.799
3	-0.493	-0.409	-0.441	-0.535
4	-0.370	-0.309	-0.332	-0.403
5	-0.296	-0.249	-0.267	-0.324
6	-0.247	-0.208	-0.224	-0.271
7	-0.211	-0.180	-0.193	-0.234
8	-0.185	-0.158	-0.170	-0.205
9	-0.164	-0.141	-0.152	-0.183
10	-0.148	-0.128	-0.137	-0.166

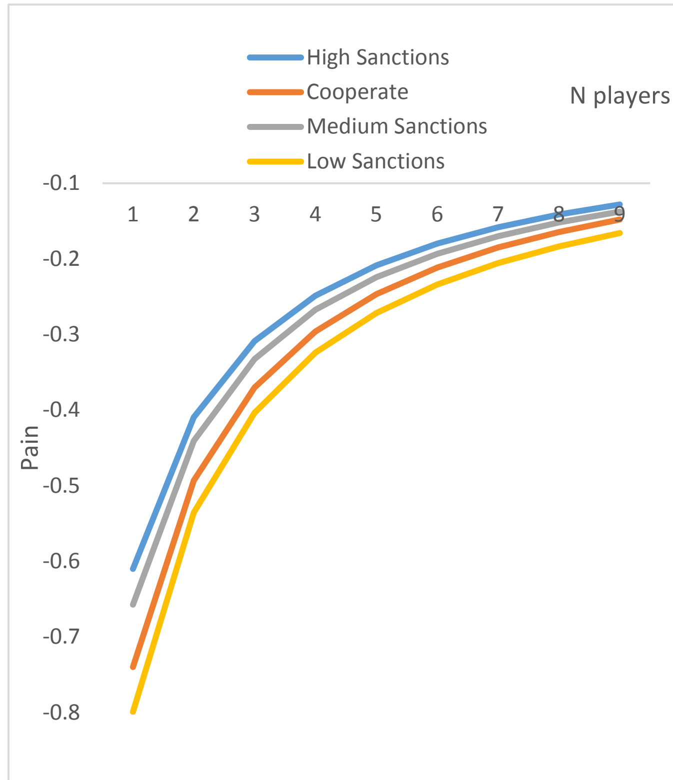

Figure 3-4. Graphical representation of the result

Another way to enforce cooperation is through utilizing the effect of the decision reaction pains in the decider reactor, favoring the cooperation solution. Since the value for those pains is set according to the decider rule (0 for neutral decider), setting negative values for cooperative decisions could act as a monitor of the situation and model fourth principle in the Design Institutional Principles by Ostrom. The monitor would be in charge of preventing the over-exploitation, or biophysical condition of the fishery resource. For example, the pain values for reactions where all of the players cooperate (a1, b1, c1, d1, etc.) would be set to -1, thus strongly favoring that decision to prevent over-exploitation. On the other hand, for the decision reactions where all the players choose not to cooperate (a2, b2, c2, d2, etc.), the value would be set to 1 to strongly disfavor that course of action. In order to account for a few cheaters in the game, the proportional system would be in place to compensate for cheaters, such that$(p^{max} - p^{min})/N$. For example, for 3x2 case, each player carries a weight of$\frac{[1-(-1)]}{3}=\frac{2}{3}$. Thus, the following pain matrix would be constructed.

Table 3-8. Pain matrix enforcing cooperation through decider, a1-cooperate

Player A: a1	**Player C: c1**	**Player C: c2**
Player B: b1	-0.493, -0.493, -0.493, -1	-0.493, -0.493, -0.346, -0.33
Player B: b2	-0.493, -0.346, -0.493, -0.33	-0.493, -0.346, -0.346, 0.33

Table 3-9.Pain matrix enforcing cooperation through decider, a2-not cooperate

Player A: a2	**Player C: c1**	**Player C: c2**
Player B: b1	-0.346, -0.493, -0.493, -0.33	-0.346, -0.493, -0.346, 0.33
Player B: b2	-0.346, -0.346, -0.493, 0.33	-0.346, -0.346, -0.346, 1

Chapter 4

Conclusion and Future Work

Ultimately, the main contribution of this research to the field of Chemical Game Theory is pain generation functions (**Eq. 3-4, 3-6**) yielding the result in Table 3-5, 3-6 that could readily be taken in nToC calculations. **Chapter 3.3** goes in detail as to how to generate the framework of the game with N-number of players, and subsequently could be extended in the similar fashion to NxN games, where each player is faced with N number of decision options. Lastly, Institutional Design Principles inclusion in **Chapter 3.6** adds a few options to the nToC game and results from **Table 3-8, 3-9** could readily be taken for more calculations.

The next steps of this research lie in improving the method of pain generation to more realistically depict the Pain evolution from one strategy to another. This research should further be extended in terms of improving the assumptions made in pain generation and incorporation of Ostrom's ideas into the model. A few main points such as normalization of costs, coalition stability and heterogeneity of players could potentially make the pain generation more accurate.

4.1 Normalizing Costs

An important aspect that has not been addressing in this thesis is normalization of costs in the derivation of the Net Present Values of the fishing rent. The usual cost structure consists of the fixed and the variable costs. Fixed costs may have high implications on the profitability and, thus, ideally should be considered along with all of the operating costs. One way to account for various types costs has been discussed in the GAMIFESTO model (Merino, 2007) and follows a four-component structure:

- Trade costs that are dependent on the revenue level. Those costs may include various taxes, such as fisherman association taxes, commercialization taxes or sales taxes.
- Daily costs are associated with day to day running of the fishing boat or fleet, and may include fuel, transportation, net mending, food expenses, etc.
- Labor costs account for the payments to the crewmembers and are paid for after trade and daily costs have been paid.
- Compulsory costs such as license, insurance costs as well as any other fixed costs not accounted for in the previous categories.

4.2 Coalition Stability

The next big step in the analysis of multi-player games in Tragedy of the Commons is developing computational methods to present actual solutions of the games. The primary goal of this thesis was to lay the foundation in this question, mainly constructing and defining a game in the form so that Chemical Game Theory could be used in the analysis.

Current efforts in Tragedy of the Commons problem development in classical game theory are in the direction of looking at the formation of coalitions between players in the game. Primarily investigating stability of the coalitions of various sizes between the players (Kaitala, 1998, Duarte, 2000). For example, (Kennedy, 2003) had looked at the mackerel fishery in the north-eastern Atlantic, where he identified three international players Norway, European Union and Russia with the possibility of different coalitions. Arnason looked at Norwegian fishery with five international players and concluded that the grand coalition is the most stable one. On the other hand, Brasao showed that the coalition of two players is most stable for distant water fishing nations (Brasao, 2000). In order to check that hypothesis from Chemical Game Theory perspective, development of the pain matrix would be required to incorporate coalition formations.

How coalitions would be represented in the game and how the payoffs would be calculated are the next big questions for the research to be extended upon.

4.3 Heterogeneity of Players

In constructing Tragedy of the Commons game in this thesis, one important assumption has been made that dictated the results of payoff function in the pain matrix. All the players in the game were assumed to be identical. In other words, each player had exactly the same technical abilities and costs associated with fishing. Obviously, in a realistic case, not all of the players are equals. For example, on the international arena, where multiple countries fish in the same ocean or see, smaller countries in terms of capital will obviously have less resources and consequently will have a smaller catch. A lot of previous authors considered the heterogeneity of players and how it results in a different outcome. Chemical Game Theory could be used to either support or disprove the hypothesis made by the results and conclusions in classical game theory.

Appendix A

Multiple reactions derivation

Let *i* represent species and *j* represent reactions. Then each reaction would have an extent of the reaction defined as ξ_j and expressed in the form:

$$d\xi_j = \frac{dn_{ij}}{\nu_{ij}}$$

Where ν_{ij} represents stoichiometric coefficient of species *i* in reactor *j*. Then the total change in moles for reactant *i* is:

$$dn_i = \sum_{j=1}^{r} d\xi_j$$

The total change in Gibbs free energy is then:

$$dG = \sum_{i=1}^{m} \mu_i \sum_{j=1}^{r} \nu_{ij} d\xi_j$$

Dividing by the reaction extents, the equilibrium condition yields:

$$\sum_{i=1}^{m} \sum_{j=1}^{r} \mu_i \nu_{ij} = 0$$

Using the reference chemical potential similarly as in chapter 2 derivation, equilibrium condition transforms into the following expression upon division by RT, which is assumed to be constant:

$$\sum_{j=1}^{m} \left(\frac{\Delta g_j^0}{RT} + \ln(\frac{P}{P^0}) \sum_{i=1}^{n} (\nu_{ij}) + \sum_{i=1}^{n} \nu_{ij} \ln(y_i) \right) = 0$$

Appendix B

2x2 case solution using Excel

Pain Matrix			
dg0/RT	A	B	D
11	1	-1	0
12	0	-3	0
21	3	0	0
22	2	-2	0

* Fill in GREEN fields

species	initial	change	end	y mole frxn
a1	0.5	-0.1175	0.382501	0.20536471
a2	0.5	-0.01996	0.480043	0.25773517
b1	0.5	-0.0406	0.459398	0.24665074
b2	0.5	-0.09685	0.403146	0.21644915
A11	0	0.034707	0.034707	0.01863433
A12	0	0.082792	0.082792	0.04445102
A21	0	0.005895	0.005895	0.00316499
A22	0	0.014062	0.014062	0.00754989
inert	0	0	0	0
SUM	2	-0.13746	1.862544	1

species	initial	change	end	y mole frxn
a1	0.5	-0.33982	0.160176	0.111156388
a2	0.5	-0.21918	0.280823	0.194880965
b1	0.5	-0.16601	0.333993	0.231778978
b2	0.5	-0.39299	0.107006	0.074258375
B11	0	0.100918	0.100918	0.070033035
B12	0	0.238906	0.238906	0.165791901
B21	0	0.065089	0.065089	0.045169311
B22	0	0.154088	0.154088	0.106931048
inert	0	0	0	0
SUM	2	-0.559	1.441	1

species	initial	change	end	y mole frxn
A11	0.034707	-0.00438	0.030328	0.04536275
A12	0.082792	-0.02039	0.062398	0.09333106
A21	0.005895	-0.00052	0.005376	0.00804071
A22	0.014062	-0.0026	0.011464	0.01714761
B11	0.100918	-0.00438	0.096538	0.14439606
B12	0.238906	-0.02039	0.218512	0.32683686
B21	0.065089	-0.00052	0.06457	0.09657947
B22	0.154088	-0.0026	0.15149	0.22658922
D11	0	0.004379	0.004379	0.0065502
D12	0	0.020394	0.020394	0.03050403
D21	0	0.000519	0.000519	0.00077657
D22	0	0.002598	0.002598	0.00388546
inert	0	0	0	0
SUM	0.696456	-0.02789	0.668566	1

Player A	extent	guess		
eps1	0.034707	0.032532		0.18738078
eps2	0.082792	0.0757		0.10663416
eps3	0.005895	0.005516		0.25335928
eps4	0.014062	0.013271		0.14418104

Player B	extent	guess		
eps1	0.100918	0.094591		0.187380776
eps2	0.238906	0.125592		0.253359278
eps3	0.065089	0.046033		0.106634163
eps4	0.154088	0.129214		0.144181036

Player D	extent	guess		
eps1	0.004379	0.004209		0.04464908
eps2	0.020394	0.019427		0.03456114
eps3	0.000519	0.0005		0.03456114
eps4	0.002598	0.002506		0.02905046

eq #	Dg0/RT	-SUM(nui ln yi)	dif^2
1	1	1	3.2397E-18
2	0	1.85E-09	3.4321E-18
3	3	3	2.898E-19
4	2	2	2.1617E-19
Total			7.1777E-18

eq	Dg0/RT	-SUM(nui ln yi)	dif^2
5	-1	-1	7.24333E-18
6	-3	-3	2.22696E-19
7	0	7.54E-10	5.69213E-19
8	-2	-2	3.17087E-19
Total			8.35232E-18

eq	Dg0/RT	-SUM(nui ln yi)		dif^2
9	0	1.02E-10		1.041E-20
10	0	8.8E-10		7.7455E-19
11	0	6.05E-10		3.6609E-19
12	0	-1.5E-09		2.2015E-18
Total			SUM	3.3526E-18

	SOLVER	
	A	7.18E-18
	B	8.35E-18
	D	3.35E-18
Minimize	TOTAL	1.9E-17

			through D's	
	actual	fraction	actual	fraction
a1	0.4573229	0.65664252	0.024773	0.888244
a2	0.23913352	0.34335748	0.003117	0.111756
b1	0.20660874	0.2966571	0.004898	0.175633
b2	0.48984767	0.7033429	0.022992	0.824367
D11	0.00437924	0.15701797	0.004379	0.157018
D12	0.02039397	0.73122633	0.020394	0.731226
D21	0.00051919	0.01861547	0.000519	0.018615
D22	0.00259769	0.09314024	0.002598	0.09314

DH / RT = energy/decision										
fa1	fa2	fb1	fb2	EA	EB	E tot	fD11	fD12	fD21	fD22
0.888244	0.111756	0.175633435	0.824367	0.399145	-2.53698	-2.13783	0.157018	0.731226	0.018615	0.09314024

Appendix C

Estimating initial guesses

Converting to equilibrium constants

K = exp(-dg0/RT)

	A	B	D
11	0.367879	2.718282	1
12	1	20.08554	1
21	0.049787	1	1
22	0.135335	7.389056	1

INITIAL GUESS — Reactor A

Ax = B, so x = A^-1 B

A				x =	B
2.367879	0.18394	0.18394	0		0.09196986
0.5	3	0	0.5		0.25
0.024894	0	2.049787	0.024894		0.01244677
0	0.067668	0.067668	2.135335		0.03383382

x =	A^-1				B
0.032532	0.428302	-0.02641	-0.03865	0.00663471	0.09196986
0.0757	-0.07179	0.339531	0.00907	-0.0796086	0.25
0.005516	-0.00523	0.000452	0.488516	-0.0058008	0.01244677
0.013271	0.002441	-0.01077	-0.01577	0.47101711	0.03383382

INITIAL GUESS — Reactor B

Ax = B, so x = A^-1 B

A				x =	B
4.718282	1.359141	1.359141	0		0.67957
10.04277	22.08554	0	10.04277		5.021384
0.5	0	3	0.5		0.25
0	3.694528	3.694528	9.389056		1.847264

x =	A^-1				B
0.094591	0.277748	-0.02262	-0.16656	0.033069557	0.67957
0.125592	-0.16717	0.06962	0.179197	-0.08400986	5.021384
0.046033	-0.06127	0.008922	0.399013	-0.03079169	0.25
0.129214	0.089892	-0.03091	-0.22752	0.15168059	1.847264

INITIAL GUESS — Reactor D

Ax = B, so x = A^-1 B

A				x =	B
0.832081	0	0	0		0.003503
0	1.018154	0	0		0.01978
0	0	0.76744	0		0.000384
0	0	0	0.864606		0.002167

x =	A^-1				B
0.004209	1.201806	0	0	0	0.003503
0.019427	0	0.982169	0	0	0.01978
0.0005	0	0	1.303033	0	0.000384
0.002506	0	0	0	1.15659615	0.002167

Appendix D

3x2 case derivation

Species (i)	**Initial ($n_{i,0}$)**	**Change**	**Equilibrium (n_i)**	**Mole fraction (y_i)**
a1	$n_{a1,0}$	$-\xi_1 - \xi_2 - \xi_3 - \xi_4$	$n_{a1,0} - \xi_1 - \xi_2 - \xi_3 - \xi_4$	n_{a1}/Σ
a2	$n_{a2,0}$	$-\xi_5 - \xi_6 - \xi_7 - \xi_8$	$n_{a2,0} - \xi_5 - \xi_6 - \xi_7 - \xi_8$	n_{a2}/Σ
b1	$n_{b1,0}$	$-\xi_1 - \xi_2 - \xi_5 - \xi_6$	$n_{b1,0} - \xi_1 - \xi_2 - \xi_5 - \xi_6$	n_{b1}/Σ
b2	$n_{b2,0}$	$-\xi_3 - \xi_4 - \xi_7 - \xi_8$	$n_{b2,0} - \xi_3 - \xi_4 - \xi_7 - \xi_8$	n_{b2}/Σ
c1	$n_{c1,0}$	$-\xi_1 - \xi_3 - \xi_5 - \xi_7$	$n_{c1,0} - \xi_1 - \xi_3 - \xi_5 - \xi_7$	n_{c1}/Σ
c2	$n_{c2,0}$	$-\xi_2 - \xi_4 - \xi_6 - \xi_8$	$n_{c2,0} - \xi_2 - \xi_4 - \xi_6 - \xi_8$	n_{c2}/Σ
A111	$n_{A111,0}$	$+\xi_1$	$n_{A111,0} + \xi_1$	n_{A111}/Σ
A112	$n_{A112,0}$	$+\xi_2$	$n_{A112,0} + \xi_2$	n_{A112}/Σ
A121	$n_{A121,0}$	$+\xi_3$	$n_{A121,0} + \xi_3$	n_{A121}/Σ
A122	$n_{A122,0}$	$+\xi_4$	$n_{A122,0} + \xi_4$	n_{A122}/Σ
A211	$n_{A211,0}$	$+\xi_5$	$n_{A211,0} + \xi_5$	n_{A211}/Σ
A212	$n_{A212,0}$	$+\xi_6$	$n_{A212,0} + \xi_6$	n_{A212}/Σ
A221	$n_{A221,0}$	$+\xi_7$	$n_{A221,0} + \xi_7$	n_{A221}/Σ
A222	$n_{A222,0}$	$+\xi_8$	$n_{A222,0} + \xi_8$	n_{A222}/Σ
Total	Σ_0	$\Delta\xi$	Σ	1

Equilibrium defined as : $K_{A111} = \frac{[A111]}{[a1][b1][c1]}$, define e_t as the sum of all the extents and assumption $\xi_n * \xi_m * n_j = 0$ applied.

$$K_{A111} = \frac{([A111] + \xi_1)(\Sigma_0 - 2e_t)}{\frac{[a1] - \xi_1 - \xi_2 - \xi_3 - \xi_4]}{(\Sigma_0 - 2e_t)} * \frac{[b1 - \xi_1 - \xi_2 - \xi_5 - \xi_6]}{\Sigma_0 - 2e_t} * \frac{[c1 - \xi_1 - \xi_3 - \xi_5 - \xi_7]}{\Sigma_0 - 2e_t}}$$

$$K_{A111} = \frac{([A111] + \xi_1)((\Sigma_0)^2 - 4\Sigma_0 e_t)}{[a1 - \xi_1 - \xi_2 - \xi_3 - \xi_4][b1 - \xi_1 - \xi_2 - \xi_5 - \xi_6][c1 - \xi_1 - \xi_3 - \xi_5 - \xi_7]}$$

Decomposing the result so that ξ_n has constant coefficients:

$$\xi_1\{4[A111]\Sigma_0 - (\Sigma_0)^2 - K_{A111}([a1][b1] + [a1][c1] + [b1][c1])\} +$$

$$\xi_2\{4[A111]\Sigma_0 - K_{A111}([a1][c1] + [b1][c1])\} +$$

$$\xi_3\{4[A111]\Sigma_0 - K_{A111}([a1][b1] + [b1][c1])\} +$$

$$\xi_4\{4[A111]\Sigma_0 - K_{A111}([b1][c1])\} +$$

$$\xi_5\{4[A111]\Sigma_0 - K_{A111}([a1][c1] + [a1][b1])\} +$$

$$\xi_6\{4[A111]\Sigma_0\} +$$

$$\xi_7\{4[A111]\Sigma_0 - K_{A111}([a1][b1] + [a1][c1])\} +$$

$$\xi_8\{4[A111]\Sigma_0\} = (\Sigma_0)^2[A111] - K_{A111}\{[a1][b1][c1]\}$$

$$K_{A111} = \frac{(\Sigma_0)^2 n_{A111} - 4n_{A111}\xi_t + (\Sigma_0)^2\xi_1}{n_{a1}n_{b1}n_{c1} - n_{a1}n_{b1}(\xi_1 + \xi_3 + \xi_5 + \xi_7) - n_{a1}n_{c1}(\xi_1 + \xi_2 + \xi_5 + \xi_6) - n_{b1}n_{c1}(\xi_1 + \xi_2 + \xi_3 + \xi_4)}$$

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ACADEMIC VITA

EDUCATION

The Pennsylvania State University **University Park, PA**
College of Engineering: B.S. Chemical Engineering, Engineering Leadership Development Minor May 2017
College of Liberal Arts: Economics Minor
Schreyer Honors Scholar

WORK EXPERIENCE

ExxonMobil Production Company – Global Operations **Spring, TX**
Reliability Integrity & Optimization – Workflow Automation Group Intern (DTAM) *May – August 2016*

- Developed relevant facilities surveillance requirements for two assets in Sakhalin, Russia.
- Built data visualization screens for power management in Chad production unit.
- Collaborated with engineers in remote locations and studied PFDs, P&IDs, DCS graphics of the Production Units.
- Classified Root Cause Failure Analyses as part of Operations Integrity Management System for global assets.

United States Gypsum Corporation **Jacksonville, FL**
Project Engineer Coop *January – July 2015*

- Independently managed 20 projects valued at up to $300,000 and assigned to outside contractors.
- Collaborated with in-house engineers and plant personnel to implement safety policies and project assignments.
- Trained outside contractors on company safety procedures and supervised work completion schedules.
- Designed and implemented engineering solutions to in-plant operations.

Penn State Department of Chemical Engineering **University Park, PA**
Teaching Assistant CHE 430 – Chemical Reaction Engineering *August 2016 – December 2016*

- Assist professor in designing homework problems and grading in-class quizzes & exams.
- Hold office hours to clarify lectures and guide students to help them complete the assignments.

Penn State Learning **University Park, PA**
Mathematics/Calculus Tutor *July 2014 – May 2016*

- Simplifying and explaining calculus and algebra concepts to students individually in 30 minutes per session.
- Lead group and study sessions of 5 to 50 students to prepare them for the exams.

LEADERSHIP EXPERIENCE

Penn State Global Programs **University Park, PA**
Orientation Student Leader *August 2014, August 2015, August 2016*

- Executed orientation week of activities in a team of 40 for around 1000 incoming international students each year.
- Lead a group of ~20 international students through various orientation activities to familiarize them with campus.

Global Brigades Organization **University Park, PA**
Treasurer Business Brigades *August 2015 – May 2016*

- Lead and organized class room presentations and chapter meetings as part of recruitment program.
- In charge of check requests, pick up checks and receive transaction reports.
- Responsible for managing the Associated Student Activities account in collaboration with campus treasurer.

RESEARCH &VOLUNTEERING EXPERIENCE

Penn State Department of Chemistry **University Park, PA**
Polyphosphazene Research Group *November 2012 – September 2013*

- Synthesized unconventional synthetic polyphosphazene polymers and characterized via NMR and GC-MS.
- Isolated and purified polymers via Rotavapor to produce pure products in the lab.

Global Business Brigades **Embera, Panama**
Business Brigades Volunteer *March 2014*

- Microfinancing consulting for indigenous kiosk businesses on strategies to improve performance and profitability.
- Delivered a series of financial lectures on money management and implemented profit tracking system.

SKILLS ACTIVITIES & INTERESTS

- <u>Language skills</u>: Fluent – Russian, Ukrainian, English; Basic – Spanish, Kazakh (Don't require sponsorship)
- Computer skills: CAD, SolidWorks, HYSYS, OneNote, XHQ, Microsoft Word, Excel, PowerPoint
- Sports: Club Tennis club (ranked #1), Ski club, Tennis and Volleyball High School varsity teams.
- Junior Treasurer Alpha Kappa Lambda, Phi Eta Sigma Honors Society, Hedge Fund Club
- <u>Lived in 5 countries</u> – USA, UK, Ukraine, Turkmenistan, Kazakhstan; Travelled to over 30 countries.