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SEMI-ANALYTICAL NODAL EXPANSION METHOD

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Abstract

In current and next generation nuclear power plants, core designs are utilizing mixed oxide fuel (MOX), high burnup loadings, and advanced designs of fuel assemblies which can complicate calculations based on diffusion theory. In order to make core calculations more accurate, the expansion that is used to approximate the neutron flux must be improved. The nodal expansion method code (NEM) currently utilizes a fourth order polynomial expansion to approximate the neutron flux for diffusion calculations. The implementation of semi-analytical terms in this expansion will allow for more accurate calculations to be made on more heterogeneous core designs such as those with MOX fuel.

In this thesis, the semi-analytical solution to the multi-group neutron diffusion equation will be derived and the procedure for implementing the necessary code changes in NEM will be described. Unfortunately, the current version of the semi-analytical code is unable to solve the response matrix equation, which gives the partial currents in each node. Work is currently being performed in order to debug the code. The problem is most likely with the LAPACK matrix solving routines. A recommendation for future work is to use a different LAPACK routine to solve the response matrix equation.

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1.0 Introduction and Background

1.1 Introduction

Reactor core analysis and design are important tasks that a nuclear engineer must perform. In order to perform fast, accurate reactor physics calculations for the purpose of core design, various computer code modules are used. The Nodal Expansion Method (NEM) is one such code package. Nodal methods have been used for quite some time and have been incorporated into various computer codes. The advantages NEM has opposed to other reactor physics calculation methods are that it can adapt to a coarse mesh spacing, the concepts behind it are simple, and it can easily be extended to different geometries (cylindrical, hexagonal) and multiple energy groups [1]. NEM makes use of the diffusion approximation of the neutron transport equation in order to simplify the reactor physics calculations. Furthermore, the reactor core is discretized into multiple pieces called nodes in order to transform the problem into a more homogeneous structure. The time dependent diffusion equation is integrated over these homogeneous nodes. An approximation must be used in order to solve the resulting transverse integrated diffusion equations due to the complexity of arriving at a direct solution.

Diffusion theory is limited in cases where the region of interest is near a source, strong absorber, or where the material properties of the medium change. The last case poses a significant problem in NEM because of the node homogenization. Core designs that are more heterogeneous contain locations where the material properties vary abruptly and cause steep gradients at assembly interfaces. The most pertinent example of this is in mixed-oxide fuel (MOX). In MOX cores, steep thermal fluxes occur at assembly interfaces, and the fourth-order polynomial expansion that is used in the normal NEM code is insufficient [2]. Similar problems arise in some nonstandard, highly heterogeneous core environments, which can occur during some transients and accidents [2]. The problems are especially noticeable when the node size is large. In both cases, the code does not produce very accurate results, especially when a one node per assembly (npa) model is used [2].The node homogenization approximation does not work over the assembly due to distinct difference in material properties. Therefore more nodes must be used to account for increased heterogeneity. Higher number npa models are much more time-consuming, especially in transient simulations [2].

Current practice is to use fractional core loadings of MOX (with typically between 30 and 50% MOX core fraction), with the rest of the core being made up of conventional uranium oxide assemblies [3]. The special feature of MOX fuel is that the fissile atoms are not homogeneously distributed in the pellets on a microscopic scale, especially in MOX fabricated by the MIMAS process (blending of a master-mix of $UO2$ and $PuO2$ with depleted $UO2$ powder) [3].

1.2 Polynomial Solution vs Semi-Analytical Solution

Past versions of the NEM code package have used a polynomial solution to the transverse integrated diffusion equations. The Semi-Analytical Nodal Expansion Method (SA-NEM) utilizes a semi-analytical solution which proves to be more accurate and allows for calculations to be performed on more heterogeneous core designs. The polynomial solution for the transverse integrated diffusion equations has the following form (for the x direction):

$$\phi_{gx}^{l}(x) = \overline{\phi_g^l} + \sum_{n=1}^{N} a_{gxn}^{l}\, f_n(x), -\frac{\Delta x}{2} \leq x \leq \frac{\Delta x}{2}$$

where l is the node number, g is the group number, and $\overline{\phi_g^l}$ is the node volume average flux. The a_{gxn}^{l} terms are the expansion coefficients and the $f_n(x)$ terms are the basis functions. The expansion coefficients are chosen by a weighted residual procedure, and the basis functions of higher order are chosen such that they become zero at the x-directed boundary of the node. The SA solution introduces semi-analytical terms to the polynomial solution. By introducing these terms, the solution takes the following form:

$$\phi_{gx}^{l}(x) = Asinh\left(k_{gl}x\right) + Bcosh\left(k_{gl}x\right) + \sum_{n=1}^{N} a_{gxn}^{l}\, f_n(x)$$

where the sinh and cosh terms are the semi-analytical terms and the expansion coefficients and basis functions remain the same.

2.0 NEM Overview

The Nodal Expansion Method was first developed in West Germany in the late 1970’s by H. FInnemann and his co-workers at Kraftwerk Union (KWU) [1]. NEM is a few group (up to 10 energy groups can be simulated) three dimensional (3-D) transient nodal core model with three geometry-modeling options: Cartesian, Hexagonal-Z and Cylindrical (R-θ-Z). Like most advanced nodal methods used today, the NEM solution of the multi-group neutron diffusion equation uses the transverse integration procedure [2]. It is based on one-dimensional (1D) polynomial flux expansions and the quadratic transverse leakage approximation (QLA) to calculate the coupling coefficients or currents [2]. The nodal coupling relationships are expressed in a partial current formulation. NEM uses the Response Matrix (RM) technique for inner iterations to calculate ongoing partial currents for each spatial node in the framework of each energy group solution. The coarse-mesh rebalance and asymptotic extrapolation methods are used to accelerate convergence of the outer solution process [1].

3.0 Derivation of the Semi-Analytical Solution

3.1 - SA-NEM for Steady-State, Cartesian Geometry

Derivation of the semi-analytic Nodal Expansion Method for three-dimensional Cartesian coordinates begins with the general steady-state multi-group neutron diffusion equation,

$$\nabla^2 D_g \phi_g - \Sigma_{rg}\phi_g + \sum_{g'=1}^{G} \Sigma_{sg'\to g}\phi_{g'} + \frac{\mathcal{X}_g}{k}\sum_{g'=1}^{G} \nu_{g'}\Sigma_{fg'}\phi_{g'} = 0 \qquad (1)$$

Where

$$\phi_g \equiv Group\ g\ neutron\ flux$$

$$D_g \equiv Group\ g\ diffusion\ coefficient$$

$$\Sigma_{rg} \equiv Group\ g\ removal\ cross\ section$$

$$\Sigma_{sg'\to g} \equiv Group\ g'\ to\ g\ scattering\ cross\ section$$

$$k \equiv Multiplicative\ eigenvalue$$

$$\mathcal{X}_g \equiv Fraction\ of\ fission\ neutrons\ entering\ group\ g$$

$$\nu_{g'} \equiv Average\ number\ of\ neutrons\ produced\ by\ a\ group\ g'\ fission$$

$$\Sigma_{fg'} \equiv Group\ g'\ fission\ cross\ section$$

Equation 1 can be written in two energy groups for an arbitrary node with constant neutronic properties and dimensions x, y, and z, as

$$-D_g^l \frac{\partial^2}{\partial x^2}\phi_g(x,y,z) - D_g^l \frac{\partial^2}{\partial y^2}\phi_g(x,y,z) - D_g^l \frac{\partial^2}{\partial z^2}\phi_g(x,y,z) + A_g^l \phi_g(x,y,z) = Q_g^l(x,y,z) \qquad (2)$$

$$(x,y,z)\epsilon V^l, g = 1,2$$

Where using 2 groups:

$$A_1^l = \Sigma_{a1}^l + \Sigma_{12}^l - \frac{1}{k}\nu\Sigma_{f1}^l$$

$$A_2^l = \Sigma_{a2}^l$$

$$Q_1^l(x,y,z) = \frac{1}{k}\nu\Sigma_{f2}^l\phi_2^l(x,y,z)$$

$$Q_2^l(x,y,z) = \Sigma_{12}^l\phi_1^l(x,y,z)$$

$$V^l = \Delta x \Delta y \Delta z \equiv Volume\ of\ node\ l$$

$$\Sigma_{ag} \equiv Group\ g\ absorption\ cross\ section$$

$$\Sigma_{12} \equiv Group\ 1\ to\ 2\ scattering\ cross\ section$$

Next, by using Fick's Law, which is given for the general u direction by the following equation:

$$J_{gu}^l(x,y,z) = -D_g^l \frac{\partial}{\partial u}\phi_g^l(x,y,z) \quad (3)$$

Where

$$J_{gu}^l(x,y,z) = The\ u\ component\ of\ the\ neutron\ current$$

And u can refer to either x, y, or z. Equation 2 may now be rewritten as:

$$\frac{\partial}{\partial x}J_{gx}^l(x,y,z) + \frac{\partial}{\partial y}J_{gy}^l(x,y,z) + \frac{\partial}{\partial z}J_{gz}^l(x,y,z) + A_g^l\phi_g(x,y,z) = Q_g^l(x,y,z) \quad (4)$$

3.1.1 Nodal Balance Equation

Assuming that the coordinate origin is at the center of cell l, Equation 4 can be integrated over the volume of the cell to obtain a local neutron balance equation. This balance equation is expressed as

$$\frac{1}{\Delta x}\left(J_{gx+}^l - J_{gx-}^l\right) + \frac{1}{\Delta y}\left(J_{gy+}^l - J_{gy-}^l\right) + \frac{1}{\Delta z}\left(J_{gz+}^l - J_{gz-}^l\right) + A_g^l\overline{\phi_g^l} = \overline{Q_g^l} \quad (5)$$

Where:

$$\overline{\phi}_g^l = \frac{1}{V^l}\int_{-\Delta x/2}^{\Delta x/2}\int_{-\Delta y/2}^{\Delta y/2}\int_{-\Delta z/2}^{\Delta z/2}\phi_g^l(x,y,z)\,dxdydz$$
$$\equiv \text{ node volume-average flux}$$

$$\overline{Q}_g^l = \frac{1}{V^l}\int_{-\Delta x/2}^{\Delta x/2}\int_{-\Delta y/2}^{\Delta y/2}\int_{-\Delta z/2}^{\Delta z/2}Q_g^l(x,y,z)\,dxdydz$$
$$\equiv \text{ node volume-average source}$$

$$\frac{1}{\Delta x}\left(J_{gx+}^l - J_{gx-}^l\right) = \frac{1}{V^l}\int_{-\Delta x/2}^{\Delta x/2}\int_{-\Delta y/2}^{\Delta y/2}\int_{-\Delta z/2}^{\Delta z/2}\frac{\partial}{\partial x}j_{gx}^l(x,y,z)\,dxdydz$$
$$J_{gx\pm}^l \equiv \text{ average x-directed net current on node faces } \pm \Delta x/2$$

$$\frac{1}{\Delta y}\left(J^{l}_{gy+}-J^{l}_{gy-}\right)=\frac{1}{V^{l}}\int_{-\Delta x/2}^{\Delta x/2}\int_{-\Delta y/2}^{\Delta y/2}\int_{-\Delta z/2}^{\Delta z/2}\frac{\partial}{\partial y}\,j^{l}_{gy}\left(x,y,z\right)dxdydz$$

$$J^{l}_{gy\pm}\equiv \text{ average y-directed net current on node faces } \pm \Delta y/2$$

$$\frac{1}{\Delta z}\left(J^{l}_{gz+}-J^{l}_{gz-}\right)=\frac{1}{V^{l}}\int_{-\Delta x/2}^{\Delta x/2}\int_{-\Delta y/2}^{\Delta y/2}\int_{-\Delta z/2}^{\Delta z/2}\frac{\partial}{\partial z}\,j^{l}_{gz}\left(x,y,z\right)dxdydz$$

$$J^{l}_{gz\pm}\equiv \text{ average z-directed net current on node faces } \pm \Delta z/2$$

and

$$g=1,G$$
$$\left(x,y,z\right)\in V^{l}$$
$$V^{l}=\Delta x\Delta y\Delta z\equiv \text{ Volume of node l}$$

The equations for the outgoing currents on the left ($-\Delta u/2$) and right ($+\Delta u/2$) surface in arbitrary direction **u** can be solved, via Fick's Law, as functions of the incoming currents if the gradient of the transverse integrated flux $\phi^{l}_{gu}\left(u\right)$ is known. Thus, as with polynomial NEM, the accurate determination of $\phi^{l}_{gu}\left(u\right)$ is a crucial step in the nodal solution process. Considering the x-direction only (u = x), and noting that the results will be analogous for the y- and z-directions, the transverse integration procedure will be now applied

3.1.2 Transverse Integration Procedure

In order to solve for the spatial neutron flux distribution in a medium consisting of neutronically homogeneous nodes, one must derive a relationship between the node-averaged flux and the face-averaged net currents. In both the polynomial and semi-analytic Nodal Expansion Methods, this coupling relationship is provided by a series of three consistently derived one-dimensional polynomial flux expansions. In order to implement these polynomial flux expansions, the transverse integration approximation must be employed. This approximation requires that Equation 4 be spatially integrated over the two dimensions transverse to the particular direction of interest. Such an approximation is motivated by the simple observation that it is generally easier to solve three one-dimensional equations than to solve one three-dimensional equation.

For the x-direction, the transverse integrated diffusion equation within node l takes the form

$$\frac{d}{dx} j^{l}_{gx}(x) + A^{l}_{g}\phi^{l}_{gx}(x) = Q^{l}_{gx}(x) - \frac{1}{\Delta y} L^{l}_{gy}(x) - \frac{1}{\Delta z} L^{l}_{gz}(x) \tag{6}$$

where

$$\phi^{l}_{gx}(x) = \frac{1}{\Delta y \Delta z} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \phi^{l}_{g}(x, y, z)\, dz dy$$

$$\frac{d}{dx} j^{l}_{gx}(x) = \frac{1}{\Delta y \Delta z} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \frac{\partial}{\partial x} j^{l}_{gx}(x, y, z)\, dz dy$$

$$Q^{l}_{gx}(x) = \frac{1}{\Delta y \Delta z} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} Q^{l}_{g}(x, y, z)\, dz dy$$

$$L^{l}_{gy}(x) = \frac{1}{\Delta z} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \frac{\partial}{\partial y} j^{l}_{gy}(x, y, z)\, dz dy$$

$\equiv$ y-direction transverse leakage

$$L^{l}_{gz}(x) = \frac{1}{\Delta y} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \frac{\partial}{\partial z} j^{l}_{gz}(x, y, z)\, dz dy$$

$\equiv$ z-direction transverse leakage

Transverse integration for the y- and z-directions yields analogous results. Additionally, if the current derivative is simplified using Fick's Law, it can be restated in terms of the neutron flux only:

$$-D^{l}_{g} \frac{d^2}{dx^2} \phi^{l}_{gx}(x) + A^{l}_{g}\phi^{l}_{gx}(x) = Q^{l}_{gx}(x) - \frac{1}{\Delta y} L^{l}_{gy}(x) - \frac{1}{\Delta z} L^{l}_{gz}(x) \tag{7}$$

which is the form that will be used for the remainder of this derivation.

3.1.3 Polynomial Approximation of the One-Dimensional Equations

To obtain a solution for Equation 7, the leakage and source terms must be transformed into a form more amenable to analytic solution procedures. In each case, this is done using a polynomial expansion approximation. Thus, the total leakage for the x-direction is defined as

$$L_g^l(x) = \frac{1}{\Delta y} L_{gy}^l(x) + \frac{1}{\Delta z} L_{gz}^l(x) \tag{8}$$

and is approximated by a parabolic (second-order) polynomial:

$$L_g^l(x) = \overline{L}_g^{xl} + p_{gx1}^l f_1(x) + p_{gx2}^l f_2(x) = \sum_{n=0}^{2} p_{gxn}^l f_n(x) \tag{9}$$

Similarly, the source term is approximated by a fourth order polynomial:

$$Q_{gx}^l(x) = \sum_{n=0}^{4} s_{gxn}^l f_n(x) \tag{10}$$

where

$$s_{gxn}^l = \overline{Q_{gxn}^l} - p_{gxn}^l \tag{11}$$

$$\overline{Q_{gxn}^l} = \sum_{g'=1}^{G} \Sigma_{g\to g'}^l \overline{\phi_{g'xn}^l} + \frac{\chi_g^l}{k} \sum_{g'=1}^{G} \nu\Sigma_{fg'}^l \overline{\phi_{g'xn}^l} \tag{12}$$

Equations 11 and 12 introduce the parameters $\overline{Q_{gxn}^l}$ and $\overline{\phi_{gxn}^l}$, which are the source moments and flux moments, respectively. The basis functions (f_n) for Equations 9 and 10 are the same as those used in the polynomial NEM expansions, namely

$$f_0(x) = 1$$

$$f_1(x) = \frac{x}{\Delta x}$$

$$f_2(x) = 3\left(\frac{x}{\Delta x}\right)^2 - \frac{1}{4}$$

$$f_3(x) = \left(\frac{x}{\Delta x}\right)^3 - \frac{1}{4}\left(\frac{x}{\Delta x}\right)$$

$$f_4(x) = \left(\frac{x}{\Delta x}\right)^4 - \frac{3}{10}\left(\frac{x}{\Delta x}\right)^2 + \frac{1}{80}$$

Equation 7 can now be manipulated algebraically to yield a simpler differential equation:

$$\frac{d^2\phi_{gx}^l}{dx^2} - k_{gl}^2\phi_{gx}^l(x) = -\frac{1}{D_g^l}\sum_{n=0}^{4} b_{gxn}^l f_n(x) \qquad (13)$$

where

$$k_{gl} = \sqrt{A_g^l / D_g^l}$$

$$-\Delta x/2 \le x \le \Delta x/2$$

The simplified polynomial coefficients are

$$b_{gxn}^l = s_{gxn}^l$$

which can be simplified via Equation 11:

$$b^l_{gxn} = \overline{Q^l_{gxn}} - p^l_{gxn}$$

$$p^l_{gx3} = p^l_{gx4} = 0$$

so that

$$b^l_{gx1} = \overline{Q^l_{gx1}} - p^l_{gx1} \qquad (14)$$

$$b^l_{gx2} = \overline{Q^l_{gx2}} - p^l_{gx2} \qquad (15)$$

$$b^l_{gx3} = \overline{Q^l_{gx3}} \qquad (16)$$

$$b^l_{gx4} = \overline{Q^l_{gx4}} \qquad (17)$$

The general solution for Equation 13 is found to be

$$\phi^l_{gx}(x) = A\sinh(k_{gl}x) + B\cosh(k_{gl}x) + \sum_{n=0}^{4} a^l_{gxn} f_n(x) \qquad (18)$$

where the coefficients a^l_{gxn} (n=1,4) are determined by comparison of coefficients:

$$a^l_{gx4} = \frac{1}{A^l_g} b^l_{gx4} \qquad (19)$$

$$a^l_{gx3} = \frac{1}{A^l_g} b^l_{gx3} \qquad (20)$$

$$a^l_{gx2} = \frac{1}{A^l_g} b^l_{gx2} + \left(\frac{4}{\Delta x^2 k^2_{gl}}\right) a^l_{gx4} \qquad (21)$$

$$a^l_{gx2} = \frac{1}{A^l_g} b^l_{gx1} + \left(\frac{6}{\Delta x^2 k^2_{gl}}\right) a^l_{gx3} \qquad (22)$$

Applying the equivalent forms of Equations 14 through 17 and simplifying:

$$a^l_{gx4} = \frac{1}{A^l_g} \overline{Q^l_{gx4}} \qquad (23)$$

$$a^l_{gx3} = \frac{1}{A^l_g} \overline{Q^l_{gx3}} \qquad (24)$$

$$a_{gx2}^{l} = \frac{1}{A_g^l}\left[\overline{Q_{gx2}^l} - p_{gx2}^l + \left(\frac{4}{\Delta x^2 k_{gl}^2}\right)\overline{Q_{gx4}^l}\right] \quad (25)$$

$$a_{gx1}^{l} = \frac{1}{A_g^l}\left[\overline{Q_{gx1}^l} - p_{gx1}^l + \left(\frac{6}{\Delta x^2 k_{gl}^2}\right)\overline{Q_{gx3}^l}\right] \quad (26)$$

The remaining coefficient, a_{gx0}^{l}, follows from the consistency condition:

$$\frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \phi_{gx}^{l}(x)\,dx = \overline{\phi}_g^l \qquad (27)$$

$$a_{gx0}^{l} = \overline{\phi}_g^l - B\frac{2}{\Delta x k_{gl}}\sinh\left(k_{gl}\frac{\Delta x}{2}\right) \qquad (28)$$

so that the overall solution can be rewritten as

$$\phi_{gx}^{l}(x) = \overline{\phi}_g^l + A\sinh\left(k_{gl}x\right) + B\left[\cosh\left(k_{gl}x\right) - \frac{2}{\Delta x k_{gl}}\sinh\left(k_{gl}\frac{\Delta x}{2}\right)\right] + \sum_{n=1}^{4} a_{gxn}^{l} f_n(x) \quad (29)$$

The coefficients A and B are determined by the continuity (discontinuity) boundary conditions at nodal interfaces $\Delta x/2$ and $-\Delta x/2$. The physical surface average fluxes are used as boundary conditions and related to the partial currents as follows:

$$\frac{2}{d_{gx\pm}}\left(J_{gx\pm}^{out,l} + J_{gx\pm}^{in,l}\right) = \phi_{gx\pm}^{l} \qquad (30)$$

where $d_{gx\pm}^{l}$ are pre-calculated discontinuity factors.

Similarly, the net currents are given by

$$J_{gx+}^{out,l} - J_{gx+}^{in,l} = J_{gx+}^{l} \qquad (31)$$

$$J_{gx-}^{out,l} - J_{gx-}^{in,l} = -J_{gx-}^{l} \tag{32}$$

Eliminating the outgoing currents from Equations 30, 31 and 32, using Fick's law to replace the $J_{gx\pm}^{l}$ terms with the flux solution of Equation 29, and performing some simplifying algebra yields a matrix equation for the determination of A and B:

$$4\begin{bmatrix} J_{gx^+}^{in,l} \\ J_{gx^-}^{in,l} \end{bmatrix} = \begin{bmatrix} r_{11}^x & r_{12}^x \\ r_{21}^x & r_{22}^x \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} + \begin{bmatrix} t_1^x \\ t_2^x \end{bmatrix}$$

$$\begin{bmatrix} r_{11}^x & r_{12}^x \\ r_{21}^x & r_{22}^x \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 4\begin{bmatrix} J_{gx^+}^{in,l} \\ J_{gx^-}^{in,l} \end{bmatrix} - \begin{bmatrix} t_1^x \\ t_2^x \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = 4\begin{bmatrix} r_{11}^x & r_{12}^x \\ r_{21}^x & r_{22}^x \end{bmatrix}^{-1}\begin{bmatrix} J_{gx^+}^{in,l} \\ J_{gx^-}^{in,l} \end{bmatrix} - \begin{bmatrix} r_{11}^x & r_{12}^x \\ r_{21}^x & r_{22}^x \end{bmatrix}^{-1}\begin{bmatrix} t_1^x \\ t_2^x \end{bmatrix}$$

$$A = \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x}\left[r_{22}^x\left(4J_{gx^+}^{in,l} - t_1^x\right) + r_{12}^x\left(t_2^x - 4J_{gx^-}^{in,l}\right)\right]$$

$$B = \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x}\left[r_{11}^x\left(4J_{gx^-}^{in,l} - t_2^x\right) + r_{21}^x\left(t_1^x - 4J_{gx^+}^{in,l}\right)\right]$$

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l} - D_g^l \frac{d}{dx}\phi_{gx}^l(x)\big|_{x=\Delta x/2}$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l} + D_g^l \frac{d}{dx}\phi_{gx}^l(x)\big|_{x=-\Delta x/2}$$

Substituting the flux expression evaluated at the boundary:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l} - D_g^l\left\{k_{gl}A\cosh\left(\frac{k_{gl}\Delta x}{2}\right) + k_{gl}B\sinh\left(\frac{k_{gl}\Delta x}{2}\right) + \frac{a_{gx1}^l}{\Delta x} + \frac{3a_{gx2}^l}{\Delta x} + \frac{a_{gx3}^l}{2\Delta x} + \frac{a_{gx4}^l}{5\Delta x}\right\}$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l} + D_g^l\left\{k_{gl}A\cosh\left(\frac{k_{gl}\Delta x}{2}\right) - k_{gl}B\sinh\left(\frac{k_{gl}\Delta x}{2}\right) - \frac{a_{gx1}^l}{\Delta x} + \frac{3a_{gx2}^l}{\Delta x} - \frac{a_{gx3}^l}{2\Delta x} + \frac{a_{gx4}^l}{5\Delta x}\right\}$$

Substituting the A and B coefficients:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l} - D_g^l \left\{ k_{gl} \left[\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} \left[r_{22}^x \left(4J_{gx^+}^{in,l} - t_1^x \right) + r_{12}^x \left(t_2^x - 4J_{gx^-}^{in,l} \right) \right] \right] \cosh\left(\frac{k_{gl}\Delta x}{2} \right) \right.$$
$$+ k_{gl} \left[\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} \left[r_{11}^x \left(4J_{gx^-}^{in,l} - t_2^x \right) + r_{21}^x \left(t_1^x - 4J_{gx^+}^{in,l} \right) \right] \right] \sinh\left(\frac{k_{gl}\Delta x}{2} \right) + \frac{a_{gx1}^l}{\Delta x}$$
$$\left. + \frac{3a_{gx2}^l}{\Delta x} + \frac{a_{gx3}^l}{2\Delta x} + \frac{a_{gx4}^l}{5\Delta x} \right\}$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l} + D_g^l \left\{ k_{gl} \left[\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} \left[r_{22}^x \left(4J_{gx^+}^{in,l} - t_1^x \right) + r_{12}^x \left(t_2^x - 4J_{gx^-}^{in,l} \right) \right] \right] \cosh\left(\frac{k_{gl}\Delta x}{2} \right) \right.$$
$$- k_{gl} \left[\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} \left[r_{11}^x \left(4J_{gx^-}^{in,l} - t_2^x \right) + r_{21}^x \left(t_1^x - 4J_{gx^+}^{in,l} \right) \right] \right] \sinh\left(\frac{k_{gl}\Delta x}{2} \right) - \frac{a_{gx1}^l}{\Delta x}$$
$$\left. + \frac{3a_{gx2}^l}{\Delta x} - \frac{a_{gx3}^l}{2\Delta x} + \frac{a_{gx4}^l}{5\Delta x} \right\}$$

Simplifying the expression above:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l} \left[1 - 4D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2} \right) + 4D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2} \right) \right]$$
$$+ J_{gx^-}^{in,l} \left[4D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2} \right) \right.$$
$$\left. - 4D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2} \right) \right]$$
$$+ t_1^x \left[D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2} \right) - D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2} \right) \right]$$
$$+ t_2^x \left[D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2} \right) - D_g^l k_{gl} \frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2} \right) \right]$$
$$- \frac{D_g^l}{\Delta x} \left[a_{gx1}^l + 3a_{gx2}^l + \frac{a_{gx3}^l}{2} + \frac{a_{gx4}^l}{5} \right]$$

$$J_{gx^-}^{out,l} = J_{gx^+}^{in,l}\left[1 - 4D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) - 4D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)\right]$$
$$+ J_{gx^-}^{in,l}\left[4D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right)\right.$$
$$\left. - 4D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)\right]$$
$$- t_1^x\left[D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) + D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)\right]$$
$$+ t_2^x\left[D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right) + D_g^l k_{gl}\frac{1}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right)\right]$$
$$+ \frac{D_g^l}{\Delta x}\left[a_{gx1}^l \mp 3a_{gx2}^l + \frac{a_{gx3}^l}{2} - \frac{a_{gx4}^l}{5}\right]$$

The following substitutions are made for simplification:

$$\Psi = \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) - \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)$$
$$\Theta = \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) - \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)$$

$$\Omega = \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{12}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) + \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{11}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)$$

$$\Lambda = \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{22}^x \cosh\left(\frac{k_{gl}\Delta x}{2}\right) + \frac{D_g^l k_{gl}}{r_{11}^x r_{22}^x - r_{12}^x r_{21}^x} r_{21}^x \sinh\left(\frac{k_{gl}\Delta x}{2}\right)$$

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1 - 4\Psi] + J_{gx^-}^{in,l}[4\Theta] + t_1^x[\Psi] + t_2^x[-\Theta] - \frac{D_g^l}{\Delta x}\left[a_{gx1}^l + 3a_{gx2}^l + \frac{a_{gx3}^l}{2} + \frac{a_{gx4}^l}{5}\right]$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l}[1 - 4\Omega] + J_{gx^-}^{in,l}[4\Lambda] - t_1^x[\Lambda] + t_2^x[\Omega] + \frac{D_g^l}{\Delta x}\left[a_{gx1}^l - 3a_{gx2}^l + \frac{a_{gx3}^l}{2} - \frac{a_{gx4}^l}{5}\right]$$

The expressions for t_1 and t_2 are substituted in:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1-4\Psi] + J_{gx^-}^{in,l}[4\Theta] + \left[d_{gx^+}^l\overline{\phi_g^l} + \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)a_{gx1}^l + \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)a_{gx2}^l + \frac{D_g^l}{\Delta x}a_{gx3}^l + \frac{2D_g^l}{5\Delta x}a_{gx4}^l\right][\Psi] + \left[d_{gx^-}^l\overline{\phi_g^l} - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)a_{gx1}^l + \left(\frac{d_{g-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)a_{gx2}^l - \frac{D_g^l}{\Delta x}a_{gx3}^l + \frac{2D_g^l}{5\Delta x}a_{gx4}^l\right][-\Theta] - \frac{D_g^l}{\Delta x}\left[a_{gx1}^l + 3a_{gx2}^l + \frac{a_{gx3}^l}{2} + \frac{a_{gx4}^l}{5}\right]$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l}[1-4\Omega] + J_{gx^+}^{in,l}[4\Lambda] - \left[d_{gx^+}^l\overline{\phi_g^l} + \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)a_{gx1}^l + \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)a_{gx2}^l + \frac{D_g^l}{\Delta x}a_{gx3}^l + \frac{2D_g^l}{5\Delta x}a_{gx4}^l\right][\Lambda] + \left[d_{gx^-}^l\overline{\phi_g^l} - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)a_{gx1}^l + \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)a_{gx2}^l - \frac{D_g^l}{\Delta x}a_{gx3}^l + \frac{2D_g^l}{5\Delta x}a_{gx4}^l\right][\Omega] + \frac{D_g^l}{\Delta x}\left[a_{gx1}^l - 3a_{gx2}^l + \frac{a_{gx3}^l}{2} - \frac{a_{gx4}^l}{5}\right]$$

Simplifying the above expressions:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1-4\Psi] + J_{gx^-}^{in,l}[4\Theta] + \overline{\phi_g^l}\left[d_{gx^+}^l\Psi - d_{gx^-}^l\Theta\right] + a_{gx1}^l\left[\left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right] + a_{gx2}^l\left[\left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right] + a_{gx3}^l\left[\frac{D_g^l}{\Delta x}\left(\Psi + \Theta - \frac{1}{2}\right)\right] + a_{gx4}^l\left[\frac{D_g^l}{5\Delta x}(2\Psi - 2\Theta - 1)\right]$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l}[1-4\Omega] + J_{gx^+}^{in,l}[4\Lambda] + \overline{\phi_g^l}\left[d_{gx^-}^l\Omega - d_{gx^+}^l\Lambda\right] + a_{gx1}^l\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right] + a_{gx2}^l\left[\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right] + a_{gx3}^l\left[\frac{D_g^l}{\Delta x}\left(\frac{1}{2} - \Lambda - \Omega\right)\right] + a_{gx4}^l\left[\frac{D_g^l}{5\Delta x}(2\Omega - 2\Lambda - 1)\right]$$

Substituting the 'a' coefficients:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1-4\Psi] + J_{gx^-}^{in,l}[4\Theta] + \overline{\phi_g^l}\left[d_{gx^+}^l\Psi - d_{gx^-}^l\Theta\right]$$
$$+\frac{1}{A_g^l}\left[\overline{Q_{gx1}^l} - p_{gx1}^l + \frac{6}{\Delta x^2 k_{gl}^2}\overline{Q_{gx3}^l}\right]\left[\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right]$$
$$+\frac{1}{A_g^l}\left[\overline{Q_{gx2}^l} - p_{gx2}^l + \frac{4}{\Delta x^2 k_{gl}^2}\overline{Q_{gx4}^l}\right]\left[\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right]$$
$$+\left[\frac{1}{A_g^l}\overline{Q_{gx3}^l}\right]\left[\frac{D_g^l}{\Delta x}\left(\Psi+\Theta-\frac{1}{2}\right)\right] + \left[\frac{1}{A_g^l}\overline{Q_{gx4}^l}\right]\left[\frac{D_g^l}{5\Delta x}(2\Psi-2\Theta-1)\right]$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l}[1-4\Omega] + J_{gx^+}^{in,l}[4\Lambda] + \overline{\phi_g^l}\left[d_{gx^-}^l\Omega - d_{gx^+}^l\Lambda\right]$$
$$+\frac{1}{A_g^l}\left[\overline{Q_{gx1}^l} - p_{gx1}^l + \frac{6}{\Delta x^2 k_{gl}^2}\overline{Q_{gx3}^l}\right]\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Omega\right]$$
$$+\frac{1}{A_g^l}\left[\overline{Q_{gx2}^l} - p_{gx2}^l + \frac{4}{\Delta x^2 k_{gl}^2}\overline{Q_{gx4}^l}\right]\left[\left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right]$$
$$+\left[\frac{1}{A_g^l}\overline{Q_{gx3}^l}\right]\left[\frac{D_g^l}{\Delta x}\left(\frac{1}{2}-\Lambda-\Omega\right)\right] + \left[\frac{1}{A_g^l}\overline{Q_{gx4}^l}\right]\left[\frac{D_g^l}{5\Delta x}(2\Omega-2\Lambda-1)\right]$$

Grouping the 'Q' and 'p' terms together:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1-4\Psi] + J_{gx^-}^{in,l}[4\Theta] + \overline{\phi_g^l}\left[d_{gx^+}^l\Psi - d_{gx^-}^l\Theta\right]$$
$$+\frac{\overline{Q_{gx1}^l}}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right]$$
$$+\frac{\overline{Q_{gx2}^l}}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right]$$
$$+\frac{\overline{Q_{gx3}^l}}{A_g^l}\left[\frac{6}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right) + \frac{D_g^l}{\Delta x}\left(\Psi+\Theta-\frac{1}{2}\right)\right]$$
$$+\frac{\overline{Q_{gx4}^l}}{A_g^l}\left[\frac{4}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right) + \frac{D_g^l}{5\Delta x}(2\Psi-2\Theta-1)\right]$$
$$-\frac{p_{gx1}^l}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right]$$
$$-\frac{p_{gx2}^l}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right]$$

$$J_{gx^-}^{out,l} = J_{gx^-}^{in,l}[1-4\Omega] + J_{gx^+}^{in,l}[4\Lambda] + \overline{\phi_g^l}\left[d_{gx^-}^l\Omega - d_{gx^+}^l\Lambda\right]$$
$$+\frac{\overline{Q_{gx1}^l}}{A_g^l}\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right]$$
$$+\frac{\overline{Q_{gx2}^l}}{A_g^l}\left[\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right]$$
$$+\frac{\overline{Q_{gx3}^l}}{A_g^l}\left[\frac{6}{\Delta x^2 k_{gl}^2}\left(\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right) + \frac{D_g^l}{\Delta x}\left(\frac{1}{2} - \Lambda - \Omega\right)\right]$$
$$+\frac{\overline{Q_{gx4}^l}}{A_g^l}\left[\frac{4}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right) + \frac{D_g^l}{5\Delta x}(2\Omega - 2\Lambda - 1)\right]$$
$$-\frac{p_{gx1}^l}{A_g^l}\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right]$$
$$-\frac{p_{gx2}^l}{A_g^l}\left[\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right]$$

Substituting the transverse leakage terms and the average flux in:

$$J_{gx^+}^{out,l} = J_{gx^+}^{in,l}[1-4\Psi] + J_{gx^-}^{in,l}[4\Theta]$$
$$+\left\{\frac{1}{A_g^l}\left[\overline{Q_g^l} - \frac{1}{\Delta x}\left(J_{gx^+}^{out,l} - J_{gx^+}^{in,l} + J_{gx^-}^{out,l} - J_{gx^-}^{in,l}\right) - \frac{1}{\Delta y}\left(J_{gy^+}^{out,l} - J_{gy^+}^{in,l} + J_{gy^-}^{out,l} - J_{gy^-}^{in,l}\right)\right.\right.$$
$$\left.\left. - \frac{1}{\Delta z}\left(J_{gz^+}^{out,l} - J_{gz^+}^{in,l} + J_{gz^-}^{out,l} - J_{gz^-}^{in,l}\right)\right]\right\}\left[d_{gx^+}^l\Psi - d_{gx^-}^l\Theta\right]$$
$$+\frac{\overline{Q_{gx1}^l}}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right]$$
$$+\frac{\overline{Q_{gx2}^l}}{A_g^l}\left[\left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right]$$
$$+\frac{\overline{Q_{gx3}^l}}{A_g^l}\left[\frac{6}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right) + \frac{D_g^l}{\Delta x}\left(\Psi + \Theta - \frac{1}{2}\right)\right]$$
$$+\frac{\overline{Q_{gx4}^l}}{A_g^l}\left[\frac{4}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right) + \frac{D_g^l}{5\Delta x}(2\Psi - 2\Theta - 1)\right]$$
$$-\frac{12}{A_g^l}\left(\frac{L_{gyx1}^l}{\Delta y} + \frac{L_{gzx1}^l}{\Delta z}\right)\left[\left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Psi + \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Theta - \frac{D_g^l}{\Delta x}\right]$$
$$-\frac{60}{A_g^l}\left(\frac{L_{gyx2}^l}{\Delta y} + \frac{L_{gzx2}^l}{\Delta z}\right)\left[\left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Psi - \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Theta - \frac{3D_g^l}{\Delta x}\right]$$

$$
\begin{aligned}
J_{gx^-}^{out,l} = {} & J_{gx^-}^{in,l}[1-4\Omega] + J_{gx^+}^{in,l}[4\Lambda] \\
& + \left\{\frac{1}{A_g^l}\left[\overline{Q_g^l} - \frac{1}{\Delta x}\left(J_{gx^+}^{out,l} - J_{gx^+}^{in,l} + J_{gx^-}^{out,l} - J_{gx^-}^{in,l}\right) - \frac{1}{\Delta y}\left(J_{gy^+}^{out,l} - J_{gy^+}^{in,l} + J_{gy^-}^{out,l} - J_{gy^-}^{in,l}\right)\right.\right. \\
& \left.\left. - \frac{1}{\Delta z}\left(J_{gz^+}^{out,l} - J_{gz^+}^{in,l} + J_{gz^-}^{out,l} - J_{gz^-}^{in,l}\right)\right]\right\}\left[d_{gx^-}^l\Omega - d_{gx^+}^l\Lambda\right] \\
& + \frac{\overline{Q_{gx1}^l}}{A_g^l}\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right] \\
& + \frac{\overline{Q_{gx2}^l}}{A_g^l}\left[\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right] \\
& + \frac{\overline{Q_{gx3}^l}}{A_g^l}\left[\frac{6}{\Delta x^2 k_{gl}^2}\left(\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right) + \frac{D_g^l}{\Delta x}\left(\frac{1}{2} - \Lambda - \Omega\right)\right] \\
& + \frac{\overline{Q_{gx4}^l}}{A_g^l}\left[\frac{4}{\Delta x^2 k_{gl}^2}\left(\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right) + \frac{D_g^l}{5\Delta x}(2\Omega - 2\Lambda - 1)\right] \\
& - \frac{12}{A_g^l}\left(\frac{L_{gyx1}^l}{\Delta y} + \frac{L_{gzx1}^l}{\Delta z}\right)\left[\frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega\right] \\
& - \frac{60}{A_g^l}\left(\frac{L_{gyx2}^l}{\Delta y} + \frac{L_{gzx2}^l}{\Delta z}\right)\left[\left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda - \frac{3D_g^l}{\Delta x}\right]
\end{aligned}
$$

Finally, grouping all like terms together yields the final partial current equations:

$$
\begin{aligned}
J_{gx^+}^{out,l}&\left[1+\frac{1}{A_g^l\Delta x}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\\
&= J_{gx^+}^{in,l}\left[1-4\Psi+\frac{1}{A_g^l\Delta x}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]+J_{gx^-}^{in,l}\left[4\Theta+\frac{1}{A_g^l\Delta x}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\\
&+J_{gx^-}^{out,l}\left[\frac{-1}{A_g^l\Delta x}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\\
&+\left[\frac{-1}{A_g^l\Delta y}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\left(J_{gy^+}^{out,l}-J_{gy^+}^{in,l}+J_{gy^-}^{out,l}-J_{gy^-}^{in,l}\right)\\
&+\left[\frac{-1}{A_g^l\Delta z}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\left(J_{gz^+}^{out,l}-J_{gz^+}^{in,l}+J_{gz^-}^{out,l}-J_{gz^-}^{in,l}\right)\\
&+\overline{Q_g^l}\left[\frac{1}{A_g^l}\left(d_{gx^+}^l\Psi-d_{gx^-}^l\Theta\right)\right]\\
&+\left[\overline{Q_{gx1}^l}-12\left(\frac{L_{gyx1}^l}{\Delta y}+\frac{L_{gzx1}^l}{\Delta z}\right)\right]\left(\frac{1}{A_g^l}\right)\left[\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi+\left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta-\frac{D_g^l}{\Delta x}\right]\\
&+\left[\overline{Q_{gx2}^l}-60\left(\frac{L_{gyx2}^l}{\Delta y}+\frac{L_{gzx2}^l}{\Delta z}\right)\right]\left(\frac{1}{A_g^l}\right)\left[\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi-\left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta-\frac{3D_g^l}{\Delta x}\right]\\
&+\overline{Q_{gx3}^l}\left(\frac{1}{A_g^l}\right)\left[\frac{6}{\Delta x^2k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Psi+\left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Theta-\frac{D_g^l}{\Delta x}\right)\right.\\
&\left.+\frac{D_g^l}{\Delta x}\left(\Psi+\Theta-\frac{1}{2}\right)\right]\\
&+\overline{Q_{gx4}^l}\left(\frac{1}{A_g^l}\right)\left[\frac{4}{\Delta x^2k_{gl}^2}\left(\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Psi-\left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Theta-\frac{3D_g^l}{\Delta x}\right)\right.\\
&\left.+\frac{D_g^l}{5\Delta x}(2\Psi-2\Theta-1)\right]
\end{aligned}
$$

$$
\begin{aligned}
&J_{gx^-}^{out,l}\left[1+\frac{1}{A_g^l\Delta x}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\\
&\qquad= J_{gx^-}^{in,l}\left[1-4\Omega+\frac{1}{A_g^l\Delta x}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]+J_{gx^+}^{in,l}\left[4\Lambda+\frac{1}{A_g^l\Delta x}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\\
&\qquad+J_{gx^+}^{out,l}\left[\frac{-1}{A_g^l\Delta x}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\\
&\qquad+\left[\frac{-1}{A_g^l\Delta y}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\left(J_{gy^+}^{out,l}-J_{gy^+}^{in,l}+J_{gy^-}^{out,l}-J_{gy^-}^{in,l}\right)\\
&\qquad+\left[\frac{-1}{A_g^l\Delta z}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\left(J_{gz^+}^{out,l}-J_{gz^+}^{in,l}+J_{gz^-}^{out,l}-J_{gz^-}^{in,l}\right)+\overline{Q_g^l}\left[\frac{1}{A_g^l}\left(d_{gx^-}^l\Omega-d_{gx^+}^l\Lambda\right)\right]\\
&\qquad+\left[\overline{Q_{gx1}^l}-12\left(\frac{L_{gyx1}^l}{\Delta y}+\frac{L_{gzx1}^l}{\Delta z}\right)\right]\left(\frac{1}{A_g^l}\right)\left[\frac{D_g^l}{\Delta x}-\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Lambda-\left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Omega\right]\\
&\qquad+\left[\overline{Q_{gx2}^l}-60\left(\frac{L_{gyx2}^l}{\Delta y}+\frac{L_{gzx2}^l}{\Delta z}\right)\right]\left(\frac{1}{A_g^l}\right)\left[\left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Omega-\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Lambda-\frac{3D_g^l}{\Delta x}\right]\\
&\qquad+\overline{Q_{gx3}^l}\left(\frac{1}{A_g^l}\right)\left[\frac{6}{\Delta x^2k_{gl}^2}\left(\frac{D_g^l}{\Delta x}-\left(\frac{d_{gx^+}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Lambda-\left(\frac{d_{gx^-}^l}{2}+\frac{2D_g^l}{\Delta x}\right)\Omega\right)+\frac{D_g^l}{\Delta x}\left(\frac{1}{2}-\Lambda-\Omega\right)\right]\\
&\qquad+\overline{Q_{gx4}^l}\left(\frac{1}{A_g^l}\right)\left[\frac{4}{\Delta x^2k_{gl}^2}\left(\left(\frac{d_{gx-}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Omega-\left(\frac{d_{gx^+}^l}{2}+\frac{6D_g^l}{\Delta x}\right)\Lambda-\frac{3D_g^l}{\Delta x}\right)\right.\\
&\qquad\left.+\frac{D_g^l}{5\Delta x}(2\Omega-2\Lambda-1)\right]
\end{aligned}
$$

Similar equations are derived in the y and z directions (the discontinuity factors are replaced with 1's in the z direction)

The six partial current equations can be combined into what is called the response matrix equation:

$$[\mathbf{A}] \cdot \mathbf{J}_{\mathbf{g}}^{\mathbf{out,l}} = [\mathbf{C}] \cdot \mathbf{J}_{\mathbf{g}}^{\mathbf{in,l}} + [\mathbf{B_1}] \cdot \mathbf{Q}_{\mathbf{g}}^{\mathbf{l}} + [\mathbf{B_2}] \cdot \mathbf{L}_{\mathbf{g}}^{\mathbf{l}}$$

Where the matrices and vectors are as follows:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & a_3 & a_4 & a_4 \\ a_5 & a_6 & a_7 & a_7 & a_8 & a_8 \\ a_9 & a_9 & a_{10} & a_{11} & a_{12} & a_{12} \\ a_{13} & a_{13} & a_{14} & a_{15} & a_{16} & a_{16} \\ a_{17} & a_{17} & a_{18} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{21} & a_{22} & a_{22} & a_{23} & a_{24} \end{bmatrix}$$

$$\mathbf{J}_{\mathbf{g}}^{\mathbf{out,l}} = \left(J_{gx+}^{out,l} \quad J_{gx-}^{out,l} \quad J_{gy+}^{out,l} \quad J_{gy-}^{out,l} \quad J_{gz+}^{out,l} \quad J_{gz-}^{out,l} \right)^T$$

$$C = \begin{bmatrix} c_1 & c_2 & c_3 & c_3 & c_4 & c_4 \\ c_5 & c_6 & c_7 & c_7 & c_8 & c_8 \\ c_9 & c_9 & c_{10} & c_{11} & c_{12} & c_{12} \\ c_{13} & c_{13} & c_{14} & c_{15} & c_{16} & c_{16} \\ c_{17} & c_{17} & c_{18} & c_{18} & c_{19} & c_{20} \\ c_{21} & c_{21} & c_{22} & c_{22} & c_{23} & c_{24} \end{bmatrix}$$

$$\mathbf{J}_{\mathbf{g}}^{\mathbf{in,l}} = \left(J_{gx+}^{in,l} \quad J_{gx-}^{in,l} \quad J_{gy+}^{in,l} \quad J_{gy-}^{in,l} \quad J_{gz+}^{in,l} \quad J_{gz-}^{in,l} \right)^T$$

$$B_1 = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_6 & b_7 & b_8 & b_9 & b_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{11} & 0 & 0 & 0 & 0 & b_{12} & b_{13} & b_{14} & b_{15} & 0 & 0 & 0 & 0 \\ b_{16} & 0 & 0 & 0 & 0 & b_{17} & b_{18} & b_{19} & b_{20} & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{26} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{27} & b_{28} & b_{29} & b_{30} \end{bmatrix}$$

$$\mathbf{Q}_{\mathbf{g}}^{\mathbf{l}} = \left(\overline{Q}_g^l \quad \overline{Q}_{gx1}^l \quad \overline{Q}_{gx2}^l \quad \overline{Q}_{gx3}^l \quad \overline{Q}_{gx4}^l \quad \overline{Q}_{gy1}^l \quad \overline{Q}_{gy2}^l \quad \overline{Q}_{gy3}^l \quad \overline{Q}_{gy4}^l \quad \overline{Q}_{gz1}^l \quad \overline{Q}_{gz2}^l \quad \overline{Q}_{gz3}^l \quad \overline{Q}_{gz4}^l \right)^T$$

$$B_2 = \begin{bmatrix} b_{31} & b_{32} & b_{33} & b_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{35} & b_{36} & b_{37} & b_{38} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{39} & b_{40} & b_{41} & b_{42} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{43} & b_{44} & b_{45} & b_{46} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{47} & b_{48} & b_{49} & b_{50} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{51} & b_{52} & b_{53} & b_{54} \end{bmatrix}$$

$$\mathbf{L}_{\mathbf{g}}^{\mathbf{l}} = \left(L_{gyx1}^l \quad L_{gyx2}^l \quad L_{gzx1}^l \quad L_{gzx2}^l \quad L_{gxy1}^l \quad L_{gxy2}^l \quad L_{gzy1}^l \quad L_{gzy2}^l \quad L_{gxz1}^l \quad L_{gxz2}^l \quad L_{gyz1}^l \quad L_{gyz2}^l \right)^T$$

The elements of each matrix are defined in Appendix A.

4.0 Code Changes

The matrix elements were coded into the current polynomial version of NEM which is written in FORTRAN 77. The same matrix equation solving routines were thought to be sufficient for solving the new matrices, but unfortunately the code cannot handle the new matrices. Investigation and debugging is currently being performed in order to remedy the problem.

The Semi-Analytical solution uses 4 source moments in each direction instead of the 2 that the polynomial version uses. Therefore, there must also be 2 addition flux moments in each direction. The flux moments are calculated after the partial currents are found by solving the response matrix equation. The first two flux moments [4] are:

$$\bar{\phi}_{gx1}^{l} = -\frac{1}{A_g^l}\left(\frac{1}{2\Delta x}\left(J_{gx+}^l + J_{gx-}^l\right) + \frac{D_g^l}{\Delta x^2}\left(\frac{\phi_{gx+}^l}{d_{gx^+}^l} - \frac{\phi_{gx-}^l}{d_{gx-}^l}\right) - \overline{Q_{gx1}^l} + \frac{L_{gyx1}^l}{\Delta y} + \frac{L_{gzx1}^l}{\Delta z}\right)$$

$$\bar{\phi}_{gx2}^{l} = -\frac{1}{A_g^l}\left(\frac{1}{2\Delta x}\left(J_{gx+}^l - J_{gx-}^l\right) + \frac{3D_g^l}{\Delta x^2}\left(\frac{\phi_{gx+}^l}{d_{gx^+}^l} + \frac{\phi_{gx-}^l}{d_{gx-}^l} - 2\bar{\phi}_g^l\right) - \overline{Q_{gx2}^l} + \frac{L_{gyx2}^l}{\Delta y} + \frac{L_{gzx2}^l}{\Delta z}\right)$$

These moments were determined by the moments weighting method. The transverse integrated flux is weighted with each basis function and integrated from $-\Delta x/2$ to $\Delta x/2$.

$$\bar{\phi}_{gx1}^{l} \equiv < w_1(x), \phi_{gx}^l > = \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \frac{x}{\Delta x}\, \phi_{gx}^l(x)\, dx$$

Where the flux expression is

$$\phi_{gx}^l(x) = -\frac{1}{A_g^l}\left(\frac{d}{dx}J_{gx}^l(x) - Q_{gx}^l(x) + \frac{L_{gy}^l}{\Delta y} + \frac{L_{gz}^l}{\Delta z}\right)$$

The 3rd and 4th flux moments for are then found to be:

$$\bar{\phi}_{gx3}^{l} = -\frac{1}{A_g^l}\left(\frac{D_g^l}{2\Delta x^2}\left(\frac{\phi_{gx+}^l}{d_{gx^+}^l} - \frac{\phi_{gx-}^l}{d_{gx-}^l}\right) - \overline{Q_{gx3}^l}\right)$$

$$\bar{\phi}_{gx4}^{l} = -\frac{1}{A_g^l}\left(\frac{D_g^l}{5\Delta x^2}\left(\frac{\phi_{gx+}^l}{d_{gx^+}^l} + \frac{\phi_{gx-}^l}{d_{gx-}^l}\right) - \overline{Q_{gx4}^l}\right)$$

Other minor code changes include changing the value of nblen (number of elements in B, 54), nclen(number of elements in C, 24), nphi (number of flux moments, 13), and nqou (number of source and leakage moments, 25). These values are used to set up the appropriate arrays and for loop bounds. The index values in the flux, source, and leakage moment arrays were changed in order to reflect the addition of the new moments. Also, new pointer variables were introduced for the temporary variables used to simplify the coding of the matrix elements. Lastly, the ibb and icc arrays which are used to set up the B and C matrices by mapping out the locations of the elements needed to be changed in order to reflect the new matrix setups.

5.0 Test Case Results

The C3 and C5 benchmark problems will be used as a basis to investigate the performance and validity of the SA-NEM solution compared to the P-NEM solution once it is complete. Both problems consist of a small core with MOX and UO_2 fuel assemblies. Results for the P-NEM will be given and compared to the results obtained from PARAGON, a lattice physics code by Westinghouse.

5.1 Test Case Set-up

Figure 5.1 gives a diagram of the composition of each fuel assembly. Each fuel assembly consists of a 17x17 arrangement of fuel-pin cells. The square lattice pitch (length of each side of a fuel-pin) is 1.260082 cm. Tables 5.1 – 5.4 below give additional specifications for the test cases.

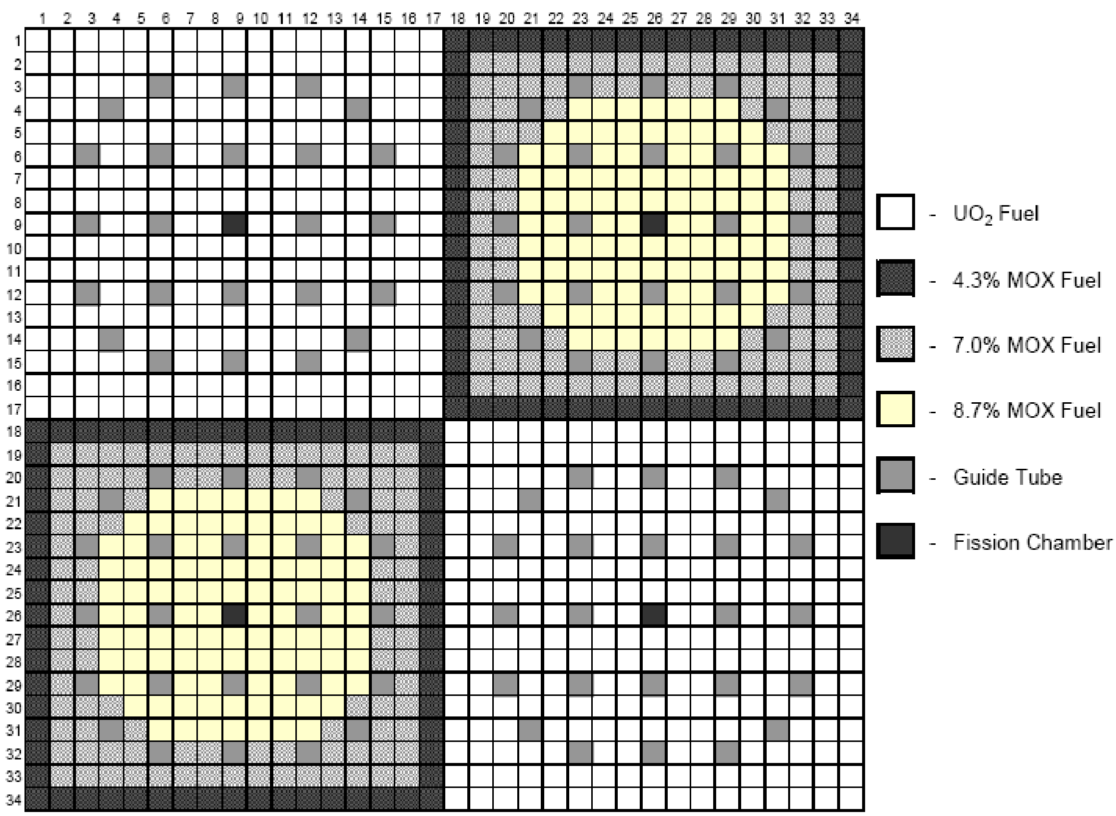

Figure 5.1: Fuel pin compositions and numbering scheme [4]

Table 5.1: Dimensions for a fuel cell in test cases C3 and C5 [4]

Medium	External Radius
Fuel	0.409501 cm
Zirconium Clad	0.540000 cm
Moderator	Square lattice pitch = 1.260082 cm

Table 5.2: Dimensions for a guide tube cell in test cases C3 and C5 [4]

Medium	External Radius
Moderator	0.340000 cm
Aluminum clad	0.540000 cm
Moderator	Square lattice pitch = 1.26 cm

Table 5.3: Isotopic number densities for each medium [4]

	MOX 4.3%	MOX 7.0%	MOX 8.7%	UOX
92235	5.0017595E-05	5.0020146E-05	4.9986268E-05	8.6557939E-04
92238	2.2096719E-02	2.2097846E-02	2.2082880E-02	2.2243897E-02
92234	1.0000000E-09	1.0000000E-09	1.0000000E-09	1.0000000E-09
92236	1.0000000E-09	1.0000000E-09	1.0000000E-09	1.0000000E-09
94238	1.5042693E-05	2.4025892E-05	3.0016944E-05	
94239	5.8165081E-04	9.3100330E-04	1.1606552E-03	
94240	2.4068309E-04	3.9042074E-04	4.9027675E-04	
94241	9.8278930E-05	1.5216398E-04	1.9010731E-04	
94242	5.4153696E-05	8.4090620E-05	1.0505930E-04	
95241	1.3037001E-05	2.0021576E-05	2.5014120E-05	
8016	4.6299170E-02	4.7499189E-02	4.8267995E-02	4.6218958E-02

Table 5.4: Isotopic number densities for the moderating and cladding [4]

	Moderator	Zr clad	Al clad
1001	6.7118404E-02		
8016	3.3559202E-02		
5010	5.5252484E-06		
40000		3.6967821E-02	
13027			6.0198146E-02

5.2 C3 Problem Description

The C3 core is a simple 2x2 fuel assembly configuration with 2 UO_2 assemblies and 2 MOX assemblies. Reflective boundary conditions are applied around the sides in order to simulate a reflector placed around the core. Figure 5.2 below is a diagram of the layout of the C3 core.

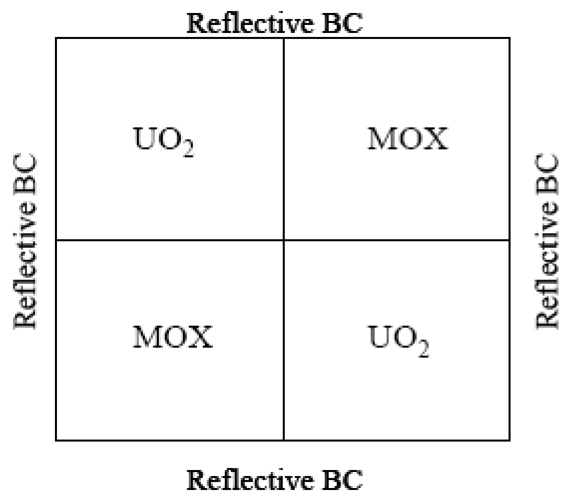

Figure 5.2: C3 Core configuration [4]

5.2.1 P-NEM Results

The results for the P-NEM using standard Assembly Discontinuity Factors (ADFs) methodology are given below in Table 5.5 for the C3 test case along with the reference results from PARAGON.

Table 5.5: k_{eff} and normalized FA power comparisons for the C3 test case [4]

k_{eff}	NEM	1.25955	
	Reference	1.25496	
	Diff, pcm	459	
2D Power Distribution	NEM	1.165	0.8347
	Reference	1.1238	0.8764
	Abs Diff, %	**4.12**	**-4.17**
	PRGN-NEM	0.8348	1.1655
	Reference	0.8764	1.1234
	Abs Diff, %	**-4.16**	**4.21**

The P-NEM results are shown not to be very accurate for UOX-MOX mini-core and this accuracy will be improved by the SA-NEM code once it is operational.

5.3 C5 Problem Description

Figure 5.3 gives a one-quarter diagram of the C5 2-D core configuration along with the boundary conditions. The overall dimensions of each fuel assembly are 21.421394×21.421394 cm, which are slightly different due to performing the calculations at Hot Zero Power (HZP) conditions instead of cold conditions as specified in the original benchmark specification [4]. A 21.421394 cm radial water reflector surrounds the core as well.

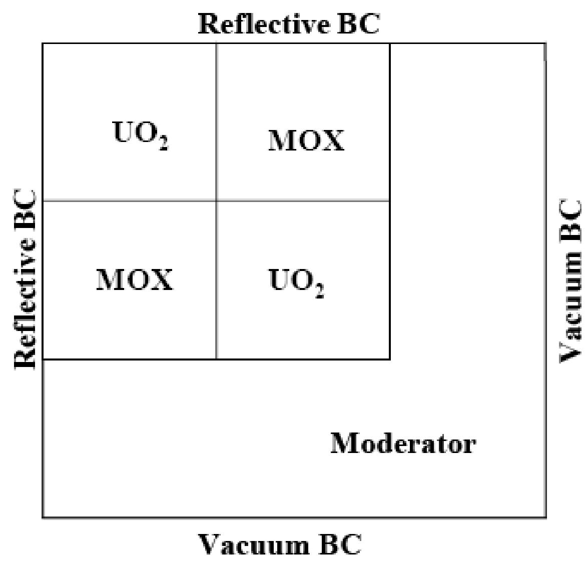

Figure 5.2: C5 core configuration [4]

5.3.1 P-NEM Results

The results for the P-NEM are given below in Table 5.6 for the C5 test case along with the reference results from PARAGON.

Table 5.6: k_{eff} and normalized FA power comparisons for the C5 test case (4)

k_{eff}	NEM	1.1932;l3	
	Reference	1.17442	
	Diff, pcm	1881	
2D Power Distribution	NEM	1.9393	0.7722
	Reference	1.7847	0.8354
	Abs Diff, %	**14.46**	**-6.32**
	NEM	0.7722	0.5363
	Reference	0.8354	0.5444
	Abs Diff, %	**-6.32**	**-1.81**

Again, the results obtained with P-NEM are very close to the PARAGON results. The SA-NEM will allow for greater accuracy on larger, more realistic problems.

6.0 Conclusions

6.1 Summary of Results

The partial current equations were used to identify the necessary elements of the response matrix equations. These elements were successfully added to the existing P-NEM code. The two new flux moments were also successfully added. The LAPACK routine that is currently used to solve the 6x6 response matrix equation is for some reason unable to solve the new response matrix equation. The code is currently being debugged in order to determine why a solution cannot currently be obtained.

6.2 Recommendations for Future Work

Recommendations for future work include using a different LAPACK routine in order to solve the response matrix equation. Many different routines exist, and the best case scenario would be to simply insert a new routine into the current code. The code would only need to be further modified to account for the passing of the necessary arrays to the LAPACK routine. Another possible solution for solving the response matrix equation could be to simplify the matrices before solving them with a matrix solving routine. The original version of NEM solved the response matrix equation without the use of LAPACK routines. It could be possible but unlikely that the SA response matrices could be simplified due to the sinh and cosh terms.

References

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3. Mathilde Christienne. 2008-2009. "Coupled TORT-TD/CTF Capability for High Fidelity LWR Core Calculations." Internship report, ENSPG, The Pennsylvania State University State University, University Park, PA.

4. Ivanov, Boyan. 2007. "Methodology For Embedded Transport Core Calculation." Ph.D. Thesis, The Pennsylvania State University, University Park, PA.

Appendix A: Response Matrix Coefficients

A matrix coefficients:

$$a_1 = 1 + \frac{1}{A_g^l \Delta x}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$a_2 = \frac{1}{A_g^l \Delta x}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$a_3 = \frac{1}{A_g^l \Delta y}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$a_4 = \frac{1}{A_g^l \Delta z}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$a_5 = \frac{1}{A_g^l \Delta x}\left(d_{gx^-}^l \Omega_\mathrm{x} - d_{gx^+}^l \Lambda_\mathrm{x}\right)$$

$$a_6 = 1 + \frac{1}{A_g^l \Delta x}\left(d_{gx^-}^l \Omega_\mathrm{x} - d_{gx^+}^l \Lambda_\mathrm{x}\right)$$

$$a_7 = \frac{1}{A_g^l \Delta y}\left(d_{gx^-}^l \Omega_\mathrm{x} - d_{gx^+}^l \Lambda_\mathrm{x}\right)$$

$$a_8 = \frac{1}{A_g^l \Delta z}\left(d_{gx^-}^l \Omega_\mathrm{x} - d_{gx^+}^l \Lambda_\mathrm{x}\right)$$

$$a_9 = \frac{1}{A_g^l \Delta x}\left(d_{gy^+}^l \Psi_\mathrm{y} - d_{gy^-}^l \Theta_\mathrm{y}\right)$$

$$a_{10} = 1 + \frac{1}{A_g^l \Delta y}\left(d_{gy^+}^l \Psi_\mathrm{y} - d_{gy^-}^l \Theta_\mathrm{y}\right)$$

$$a_{11} = \frac{1}{A_g^l \Delta y}\left(d_{gy^+}^l \Psi_\mathrm{y} - d_{gy^-}^l \Theta_\mathrm{y}\right)$$

$$a_{12} = \frac{1}{A_g^l \Delta z}\left(d_{gy^+}^l \Psi_\mathrm{y} - d_{gy^-}^l \Theta_\mathrm{y}\right)$$

$$a_{13} = \frac{1}{A_g^l \Delta x}\left(d_{gy^-}^l \Omega_\mathrm{y} - d_{gy^+}^l \Lambda_\mathrm{y}\right)$$

$$a_{14} = \frac{1}{A_g^l \Delta y}\left(d_{gy^-}^l \Omega_\mathrm{y} - d_{gy^+}^l \Lambda_\mathrm{y}\right)$$

$$a_{15} = 1 + \frac{1}{A_g^l \Delta y}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$a_{16} = \frac{1}{A_g^l \Delta z}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$a_{17} = \frac{1}{A_g^l \Delta x}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$a_{18} = \frac{1}{A_g^l \Delta y}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$a_{19} = 1 + \frac{1}{A_g^l \Delta z}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$a_{20} = \frac{1}{A_g^l \Delta z}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$a_{21} = \frac{1}{A_g^l \Delta x}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$a_{22} = \frac{1}{A_g^l \Delta y}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$a_{23} = \frac{1}{A_g^l \Delta z}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$a_{24} = 1 + \frac{1}{A_g^l \Delta z}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

Temporary b coefficient variables:

$$bs_{x1} = \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Psi_{\mathrm{x}} + \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Theta_{\mathrm{x}} - \frac{D_g^l}{\Delta x}$$

$$bs_{x2} = \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Psi_{\mathrm{x}} - \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Theta_{\mathrm{x}} - \frac{3D_g^l}{\Delta x}$$

$$bs_{x3} = \frac{D_g^l}{A_g^l \Delta x}(\Psi_{\mathrm{x}} + \Theta_{\mathrm{x}}) - \frac{D_g^l}{2A_g^l \Delta x}$$

$$bs_{x4} = \frac{2D_g^l}{5A_g^l \Delta x}(\Psi_{\mathrm{x}} - \Theta_{\mathrm{x}}) - \frac{D_g^l}{5A_g^l \Delta x}$$

$$bs_{x5} = \frac{D_g^l}{\Delta x} - \left(\frac{d_{gx^+}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Lambda_\mathrm{x} - \left(\frac{d_{gx^-}^l}{2} + \frac{2D_g^l}{\Delta x}\right)\Omega_\mathrm{x}$$

$$bs_{x6} = \left(\frac{d_{gx-}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Omega_\mathrm{x} - \left(\frac{d_{gx^+}^l}{2} + \frac{6D_g^l}{\Delta x}\right)\Lambda_\mathrm{x} - \frac{3D_g^l}{\Delta x}$$

$$bs_{x7} = \frac{D_g^l}{A_g^l \Delta x}\left(\frac{1}{2} - \Lambda_\mathrm{x} - \Omega_\mathrm{x}\right)$$

$$bs_{x8} = \frac{D_g^l}{5A_g^l \Delta x}(2\Omega_\mathrm{x} - 2\Lambda_\mathrm{x} - 1)$$

B matrix coefficients:

$$b_1 = \frac{1}{A_g^l}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$b_2 = \frac{1}{A_g^l} bs_{1x}$$

$$b_3 = \frac{1}{A_g^l} bs_{2x}$$

$$b_4 = \frac{6}{A_g^l \Delta x^2 k_{gl}^2} bs_{1x} + bs_{3x}$$

$$b_5 = \frac{4}{A_g^l \Delta x^2 k_{gl}^2} bs_{2x} + bs_{4x}$$

$$b_6 = \frac{1}{A_g^l}\left(d_{gx^-}^l \Omega_\mathrm{x} - d_{gx^+}^l \Lambda_\mathrm{x}\right)$$

$$b_7 = \frac{1}{A_g^l} bs_{5x}$$

$$b_8 = \frac{1}{A_g^l} bs_{6x}$$

$$b_9 = \frac{6}{A_g^l \Delta x^2 k_{gl}^2} bs_{5x} + bs_{7x}$$

$$b_{10} = \frac{4}{A_g^l \Delta x^2 k_{gl}^2} bs_{6x} + bs_{8x}$$

$$b_{11} = \frac{1}{A_g^l}\left(d_{gy^+}^l \Psi_\mathrm{y} - d_{gy^-}^l \Theta_\mathrm{y}\right)$$

$$b_{12} = \frac{1}{A_g^l} bs_{1y}$$

$$b_{13} = \frac{1}{A_g^l} bs_{2y}$$

$$b_{14} = \frac{6}{A_g^l \Delta y^2 k_{gl}^2} bs_{1y} + bs_{3y}$$

$$b_{15} = \frac{4}{A_g^l \Delta y^2 k_{gl}^2} bs_{2y} + bs_{4y}$$

$$b_{16} = \frac{1}{A_g^l}\left(d_{gy^-}^l \Omega_\mathrm{y} - d_{gy^+}^l \Lambda_\mathrm{y}\right)$$

$$b_{17} = \frac{1}{A_g^l} bs_{5y}$$

$$b_{18} = \frac{1}{A_g^l} bs_{6y}$$

$$b_{19} = \frac{6}{A_g^l \Delta y^2 k_{gl}^2} bs_{5y} + bs_{7y}$$

$$b_{20} = \frac{4}{A_g^l \Delta y^2 k_{gl}^2} bs_{6y} + bs_{8y}$$

$$b_{21} = \frac{1}{A_g^l}(\Psi_\mathrm{z} - \Theta_\mathrm{z})$$

$$b_{22} = \frac{1}{A_g^l} bs_{1z}$$

$$b_{23} = \frac{1}{A_g^l} bs_{2z}$$

$$b_{24} = \frac{6}{A_g^l \Delta z^2 k_{gl}^2} bs_{1z} + bs_{3z}$$

$$b_{25} = \frac{4}{A_g^l \Delta z^2 k_{gl}^2} bs_{2z} + bs_{4z}$$

$$b_{26} = \frac{1}{A_g^l}(\Omega_z - \Lambda_z)$$

$$b_{27} = \frac{1}{A_g^l} bs_{5z}$$

$$b_{28} = \frac{1}{A_g^l} bs_{6z}$$

$$b_{29} = \frac{6}{A_g^l \Delta z^2 k_{gl}^2} bs_{5z} + bs_{7z}$$

$$b_{30} = \frac{4}{A_g^l \Delta z^2 k_{gl}^2} bs_{6z} + bs_{8z}$$

$$b_{31} = -\frac{12}{\Delta y} b_2$$

$$b_{32} = -\frac{60}{\Delta y} b_3$$

$$b_{33} = -\frac{12}{\Delta z} b_2$$

$$b_{34} = -\frac{60}{\Delta z} b_3$$

$$b_{35} = -\frac{12}{\Delta y} b_7$$

$$b_{36} = -\frac{60}{\Delta y} b_8$$

$$b_{37} = -\frac{12}{\Delta z} b_7$$

$$b_{38} = -\frac{60}{\Delta z} b_8$$

$$b_{39} = -\frac{12}{\Delta x} b_{12}$$

$$b_{40} = -\frac{60}{\Delta x} b_{13}$$

$$b_{41} = -\frac{12}{\Delta z} b_{12}$$

$$b_{42} = -\frac{60}{\Delta z} b_{13}$$

$$b_{43} = -\frac{12}{\Delta x} b_{17}$$

$$b_{44} = -\frac{60}{\Delta x} b_{18}$$

$$b_{45} = -\frac{12}{\Delta z} b_{17}$$

$$b_{46} = -\frac{60}{\Delta z} b_{18}$$

$$b_{47} = -\frac{12}{\Delta x} b_{22}$$

$$b_{48} = -\frac{60}{\Delta x} b_{23}$$

$$b_{49} = -\frac{12}{\Delta y} b_{22}$$

$$b_{50} = -\frac{60}{\Delta y} b_{23}$$

$$b_{51} = -\frac{12}{\Delta x} b_{27}$$

$$b_{52} = -\frac{60}{\Delta x} b_{28}$$

$$b_{53} = -\frac{12}{\Delta y} b_{27}$$

$$b_{54} = -\frac{60}{\Delta y} b_{28}$$

C matrix coefficients:

$$c_1 = 1 - 4\Psi_\mathrm{x} + \frac{1}{A_g^l \Delta x}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$c_2 = 4\Theta_\mathrm{x} + \frac{1}{A_g^l \Delta x}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$c_3 = \frac{1}{A_g^l \Delta y}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$c_4 = \frac{1}{A_g^l \Delta z}\left(d_{gx^+}^l \Psi_\mathrm{x} - d_{gx^-}^l \Theta_\mathrm{x}\right)$$

$$c_5 = 4\Lambda_{\mathrm{x}} + \frac{1}{A_g^l \Delta x}\left(d_{gx^-}^l \Omega_{\mathrm{x}} - d_{gx^+}^l \Lambda_{\mathrm{x}}\right)$$

$$c_6 = 1 - 4\Omega_{\mathrm{x}} + \frac{1}{A_g^l \Delta x}\left(d_{gx^-}^l \Omega_{\mathrm{x}} - d_{gx^+}^l \Lambda_{\mathrm{x}}\right)$$

$$c_7 = \frac{1}{A_g^l \Delta y}\left(d_{gx^-}^l \Omega_{\mathrm{x}} - d_{gx^+}^l \Lambda_{\mathrm{x}}\right)$$

$$c_8 = \frac{1}{A_g^l \Delta z}\left(d_{gx^-}^l \Omega_{\mathrm{x}} - d_{gx^+}^l \Lambda_{\mathrm{x}}\right)$$

$$c_9 = \frac{1}{A_g^l \Delta x}\left(d_{gy^+}^l \Psi_{\mathrm{y}} - d_{gy^-}^l \Theta_{\mathrm{y}}\right)$$

$$c_{10} = 1 - 4\Psi_{\mathrm{y}} + \frac{1}{A_g^l \Delta y}\left(d_{gy^+}^l \Psi_{\mathrm{y}} - d_{gy^-}^l \Theta_{\mathrm{y}}\right)$$

$$c_{11} = 4\Theta_{\mathrm{y}} + \frac{1}{A_g^l \Delta y}\left(d_{gy^+}^l \Psi_{\mathrm{y}} - d_{gy^-}^l \Theta_{\mathrm{y}}\right)$$

$$c_{12} = \frac{1}{A_g^l \Delta z}\left(d_{gy^+}^l \Psi_{\mathrm{y}} - d_{gy^-}^l \Theta_{\mathrm{y}}\right)$$

$$c_{13} = \frac{1}{A_g^l \Delta x}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$c_{14} = 4\Lambda_{\mathrm{y}} + \frac{1}{A_g^l \Delta y}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$c_{15} = 1 - 4\Omega_{\mathrm{y}} + \frac{1}{A_g^l \Delta y}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$c_{16} = \frac{1}{A_g^l \Delta z}\left(d_{gy^-}^l \Omega_{\mathrm{y}} - d_{gy^+}^l \Lambda_{\mathrm{y}}\right)$$

$$c_{17} = \frac{1}{A_g^l \Delta x}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$c_{18} = \frac{1}{A_g^l \Delta y}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$c_{19} = 1 - 4\Psi_{\mathrm{z}} + \frac{1}{A_g^l \Delta z}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$c_{20} = 4\Theta_{\mathrm{y}} + \frac{1}{A_g^l \Delta z}(\Psi_{\mathrm{z}} - \Theta_{\mathrm{z}})$$

$$c_{21} = \frac{1}{A_g^l \Delta x}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$c_{22} = \frac{1}{A_g^l \Delta y}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$c_{23} = 4\Lambda_{\mathrm{z}} + \frac{1}{A_g^l \Delta z}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

$$c_{24} = 1 - 4\Omega_{\mathrm{z}} + \frac{1}{A_g^l \Delta z}(\Omega_{\mathrm{z}} - \Lambda_{\mathrm{z}})$$

Appendix B: Additional Code/Code Changes

The icc and ibb arrays set up the C and B matrices in bbmain with the proper elements that are defined in bbpara

```
c  set up response matrix position arrays
c
c  cart or cyl geom
c
      if(igeom.ne.2) then
c
c  icc array for cart geom
c
        if(igeom.eq.0) then
          icc(1,1)=1
          icc(2,1)=2
          icc(3,1)=3
          icc(4,1)=3
          icc(5,1)=4
          icc(6,1)=4
          icc(1,2)=5
          icc(2,2)=6
          icc(3,2)=7
          icc(4,2)=7
          icc(5,2)=8
          icc(6,2)=8
          icc(1,3)=9
          icc(2,3)=9
          icc(3,3)=10
          icc(4,3)=11
          icc(5,3)=12
          icc(6,3)=12
          icc(1,4)=13
          icc(2,4)=13
          icc(3,4)=14
          icc(4,4)=15
          icc(5,4)=16
          icc(6,4)=16
          icc(1,5)=17
          icc(2,5)=17
          icc(3,5)=18
          icc(4,5)=18
          icc(5,5)=19
          icc(6,5)=20
          icc(1,6)=21
          icc(2,6)=21
          icc(3,6)=22
          icc(4,6)=22
          icc(5,6)=23
          icc(6,6)=24

endif
        ibb(1,1)=1
        ibb(2,1)=2
        ibb(3,1)=3
```

```
ibb(4,1)=4
ibb(5,1)=5
ibb(6,1)=0
ibb(7,1)=0
ibb(8,1)=0
ibb(9,1)=0
ibb(10,1)=0
ibb(11,1)=0
ibb(12,1)=0
ibb(13,1)=0
ibb(14,1)=31
ibb(15,1)=32
ibb(16,1)=33
ibb(17,1)=34
ibb(18,1)=0
ibb(19,1)=0
ibb(20,1)=0
ibb(21,1)=0
ibb(22,1)=0
ibb(23,1)=0
ibb(24,1)=0
ibb(25,1)=0
ibb(1,2)=6
ibb(2,2)=7
ibb(3,2)=8
ibb(4,2)=9
ibb(5,2)=10
ibb(6,2)=0
ibb(7,2)=0
ibb(8,2)=0
ibb(9,2)=0
ibb(10,2)=0
ibb(11,2)=0
ibb(12,2)=0
ibb(13,2)=0
ibb(14,2)=35
ibb(15,2)=36
ibb(16,2)=37
ibb(17,2)=38
ibb(18,2)=0
ibb(19,2)=0
ibb(20,2)=0
ibb(21,2)=0
ibb(22,2)=0
ibb(23,2)=0
ibb(24,2)=0
ibb(25,2)=0
ibb(1,3)=11
ibb(2,3)=0
ibb(3,3)=0
ibb(4,3)=0
ibb(5,3)=0
ibb(6,3)=12
ibb(7,3)=13
ibb(8,3)=14
ibb(9,3)=15
ibb(10,3)=0
```

```
ibb(11,3)=0
ibb(12,3)=0
ibb(13,3)=0
ibb(14,3)=0
ibb(15,3)=0
ibb(16,3)=0
ibb(17,3)=0
ibb(18,3)=39
ibb(19,3)=40
ibb(20,3)=41
ibb(21,3)=42
ibb(22,3)=0
ibb(23,3)=0
ibb(24,3)=0
ibb(25,3)=0
ibb(1,4)=16
ibb(2,4)=0
ibb(3,4)=0
ibb(4,4)=0
ibb(5,4)=0
ibb(6,4)=17
ibb(7,4)=18
ibb(8,4)=19
ibb(9,4)=20
ibb(10,4)=0
ibb(11,4)=0
ibb(12,4)=0
ibb(13,4)=0
ibb(14,4)=0
ibb(15,4)=0
ibb(16,4)=0
ibb(17,4)=0
ibb(18,4)=43
ibb(19,4)=44
ibb(20,4)=45
ibb(21,4)=46
ibb(22,4)=0
ibb(23,4)=0
ibb(24,4)=0
ibb(25,4)=0
ibb(1,5)=21
ibb(2,5)=0
ibb(3,5)=0
ibb(4,5)=0
ibb(5,5)=0
ibb(6,5)=0
ibb(7,5)=0
ibb(8,5)=0
ibb(9,5)=0
ibb(10,5)=22
ibb(11,5)=23
ibb(12,5)=24
ibb(13,5)=25
ibb(14,5)=0
ibb(15,5)=0
ibb(16,5)=0
ibb(17,5)=0
```

```
      ibb(18,5)=0
      ibb(19,5)=0
      ibb(20,5)=0
      ibb(21,5)=0
      ibb(22,5)=47
      ibb(23,5)=48
      ibb(24,5)=49
      ibb(25,5)=50
      ibb(1,6)=26
      ibb(2,6)=0
      ibb(3,6)=0
      ibb(4,6)=0
      ibb(5,6)=0
      ibb(6,6)=0
      ibb(7,6)=0
      ibb(8,6)=0
      ibb(9,6)=0
      ibb(10,6)=27
      ibb(11,6)=28
      ibb(12,6)=29
      ibb(13,6)=30
      ibb(14,6)=0
      ibb(15,6)=0
      ibb(16,6)=0
      ibb(17,6)=0
      ibb(18,6)=0
      ibb(19,6)=0
      ibb(20,6)=0
      ibb(21,6)=0
      ibb(22,6)=51
      ibb(23,6)=52
      ibb(24,6)=53
      ibb(25,6)=54
```

The following code is the temporary variables that are calculated in bbpara for the matrix coefficients

```
c jar temporary parameters for SA
      do 468 l=1,ncr
         rx11(l)=adfe(l,k,lg)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))
     1          *delx(l)/2.e0)+2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/
     2          difn(l,k,lg))*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
     3          *delx(l)/2.e0)
         rx12(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
     1          sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
     2          -adfe(l,k,lg)/(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l))
     3          *sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
     4          +adfe(l,k,lg)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
     5          *delx(l)/2.e0)
         rx21(l)=-adfw(l,k,lg)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg)))
     1          *delx(l)/2.e0-2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/
     2          difn(l,k,lg))*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
     3          *delx(l)/2.e0)
         rx22(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
     1          sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
     2          -((2.e0*adfw(l,k,lg))/(sqrt(aa(l,k,lg)/difn(l,k,lg))
     3          *delx(l)))*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))
```

```
4           *delx(l)/2.e0)+adfw(l,k,lg)*cosh(sqrt(aa(l,k,lg)
5           /difn(l,k,lg))*delx(l)/2.e0)
        ry11(l)=adfn(l,k,lg)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))
1           *dely(l)/2.e0)+2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)
2           /difn(l,k,lg))*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
3           *dely(l)/2.e0)
       ry12(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
1           sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
2           -adfn(l,k,lg)/(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l))
3           *sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
4           +adfn(l,k,lg)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
5           *dely(l)/2.e0)
        ry21(l)=-adfs(l,k,lg)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))
1           *dely(l)/2.e0)-2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)
2           /difn(l,k,lg))*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))
3           *dely(l)/2.e0)
         ry22(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
1           sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
2           -((2.e0*adfs(l,k,lg))/(sqrt(aa(l,k,lg)/difn(l,k,lg))
3           *dely(l)))*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*
4           dely(l)/2.e0)+adfs(l,k,lg)*cosh(sqrt(aa(l,k,lg)
5           /difn(l,k,lg))*dely(l)/2.e0)
         rz11(l)=sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
1           +2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))
2           *cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
         rz12(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
1           sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
2           -2.e0/(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l))
3           *sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
4           +cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
         rz21(l)=-sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
1           -2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))
2           *cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
         rz22(l)=2.e0*difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*
1           sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
2           -((2.e0)/(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)))
3           *sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
4           +cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
         gamx(l) = 1/(rx11(l)*rx22(l)-rx12(l)*rx21(l))
         gamy(l) = 1/(ry11(l)*ry22(l)-ry12(l)*ry21(l))
         gamz(l) = 1/(rz11(l)*rz22(l)-rz12(l)*rz21(l))
        psix(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamx(l))
 1          *(rx22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
 2          -rx21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0))
        thetax(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamx(l))
 1          *(rx12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
 2          -rx11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0))
        psiy(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamy(l))
 1          *(ry22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
 2          -ry21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0))
        thetay(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamy(l))
 1          *(ry12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
 2          -ry11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0))
        psiz(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamz(l))*
 1           (rz22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
 2          -rz21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0))
        thetaz(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamz(l))*
```

```
     1         (rz12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
     2         -rz11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0))
      omegax(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamx(l))
     1         *(rx12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
     2         +rx11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0))
      lambdax(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamx(l))
     1         *(rx22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0)
     2         +rx21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delx(l)/2.e0))
      omegay(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamy(l))
     1         *(ry12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
     2         +ry11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0))
      lambday(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamy(l))
     1         *(ry22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0)
     2         +ry21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*dely(l)/2.e0))
      omegaz(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamz(l))
     1         *(rz12(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
     2         +rz11(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0))
      lambdaz(l) = (difn(l,k,lg)*sqrt(aa(l,k,lg)/difn(l,k,lg))*gamz(l))
     1         *(rz22(l)*cosh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0)
     2         +rz21(l)*sinh(sqrt(aa(l,k,lg)/difn(l,k,lg))*delz(l)/2.e0))
      bs1x(l) = (adfe(l,k,lg)/2.0 + 2.0*difn(l,k,lg)/delx(l))*psix(l) +
     1         (adfw(l,k,lg)/2.0 +2.0*difn(l,k,lg)/delx(l))*thetax(l) -
     2         difn(l,k,lg)/delx(l)
      bs2x(l) = (adfe(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/delx(l))*psix(l) +
     1         (adfw(l,k,lg)/2.0 +6.0*difn(l,k,lg)/delx(l))*thetax(l) -
     2         3.0*difn(l,k,lg)/delx(l)
      bs3x(l) = difn(l,k,lg)/(aa(l,k,lg)*delx(l))*(psix(l) + thetax(l))
     1          - difn(l,k,lg)/(2.0*delx(l)*aa(l,k,lg))
      bs4x(l) = 2.0*difn(l,k,lg)/(5.0*aa(l,k,lg)*delx(l))*(psix(l) -
     1         thetax(l)) - difn(l,k,lg)/(5.0*delx(l)*aa(l,k,lg))
      bs5x(l) = difn(l,k,lg)/delx(l) - lambdax(l)*(adfe(l,k,lg)/2.0 +
     1         2.0*difn(l,k,lg)/delx(l)) - omegax(l)*(adfw(l,k,lg)/2.0 +
     2         2.0*difn(l,k,lg)/delx(l))
      bs6x(l) = omegax(l)*(adfw(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/delx(l))
     1         -lambdax(l)*(adfe(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/delx(l))
     2         -3.0*difn(l,k,lg)/delx(l)
      bs7x(l) = difn(l,k,lg)/(aa(l,k,lg)*delx(l))*(.5 - lambdax(l)
     1          - omegax(l))
      bs8x(l) = difn(l,k,lg)/(5.0*aa(l,k,lg)*delx(l))*(2.0*omegax(l)
     1          - 2.0*lambdax(l) -1.0)
      bs1y(l) = (adfn(l,k,lg)/2.0 + 2.0*difn(l,k,lg)/dely(l))*psiy(l) +
     1         (adfs(l,k,lg)/2.0 +2.0*difn(l,k,lg)/dely(l))*thetay(l) -
     2         difn(l,k,lg)/dely(l)
      bs2y(l) = (adfn(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/dely(l))*psiy(l) +
     1         (adfs(l,k,lg)/2.0 +6.0*difn(l,k,lg)/dely(l))*thetay(l) -
     2         3.0*difn(l,k,lg)/dely(l)
      bs3y(l) = difn(l,k,lg)/(aa(l,k,lg)*dely(l))*(psiy(l) + thetay(l))
     1          - difn(l,k,lg)/(2.0*dely(l)*aa(l,k,lg))
      bs4y(l) = 2.0*difn(l,k,lg)/(5.0*aa(l,k,lg)*dely(l))*(psiy(l) -
     1         thetay(l)) - difn(l,k,lg)/(5.0*dely(l)*aa(l,k,lg))
      bs5y(l) = difn(l,k,lg)/dely(l) - lambday(l)*(adfn(l,k,lg)/2.0 +
     1         2.0*difn(l,k,lg)/dely(l)) - omegay(l)*(adfs(l,k,lg)/2.0 +
     2         2.0*difn(l,k,lg)/dely(l))
      bs6y(l) = omegay(l)*(adfs(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/dely(l))
     1         -lambday(l)*(adfn(l,k,lg)/2.0 + 6.0*difn(l,k,lg)/dely(l))
     2         -3.0*difn(l,k,lg)/dely(l)
      bs7y(l) = difn(l,k,lg)/(aa(l,k,lg)*dely(l))*(.5 - lambday(l)
```

```
     1            - omegay(l))
            bs8y(l) = difn(l,k,lg)/(5.0*aa(l,k,lg)*dely(l))*(2.0*omegay(l)
     1            - 2.0*lambday(l) -1.0)
            bs1z(l) = (1.0/2.0 + 2.0*difn(l,k,lg)/delz(l))*psiz(l) +
     1            (1.0/2.0 +2.0*difn(l,k,lg)/delz(l))*thetaz(l) -
     2            difn(l,k,lg)/delz(l)
            bs2z(l) = (1.0/2.0 + 6.0*difn(l,k,lg)/delz(l))*psiz(l) + (1.0/2.0
     1            +6.0*difn(l,k,lg)/delz(l))*thetaz(l) - 3.0*difn(l,k,lg)
     2            /delz(l)
            bs3z(l) = difn(l,k,lg)/(aa(l,k,lg)*delz(l))*(psiz(l) + thetaz(l))
     1            - difn(l,k,lg)/(2.0*delz(l)*aa(l,k,lg))
            bs4z(l) = 2.0*difn(l,k,lg)/(5.0*aa(l,k,lg)*delz(l))*(psiz(l) -
     1            thetaz(l)) - difn(l,k,lg)/(5.0*delz(l)*aa(l,k,lg))
            bs5z(l) = difn(l,k,lg)/delz(l) - lambdaz(l)*(1.0/2.0 + 2.0*
     1            difn(l,k,lg)/delz(l)) - omegaz(l)*(adfs(l,k,lg)/2.0 + 2.0
     2            *difn(l,k,lg)/delz(l))
            bs6z(l) = omegaz(l)*(1.0/2.0 + 6.0*difn(l,k,lg)/delz(l)) -
     1            lambdaz(l)*(1.0/2.0 + 6.0*difn(l,k,lg)/delz(l)) -
     2            3.0*difn(l,k,lg)/delz(l)
            bs7z(l) = difn(l,k,lg)/(aa(l,k,lg)*delz(l))*(.5 - lambdaz(l)
     1            - omegaz(l))
            bs8z(l) = difn(l,k,lg)/(5.0*aa(l,k,lg)*delz(l))*(2.0*omegaz(l)
     1            - 2.0*lambdaz(l) -1.0)

 468        continue
```

The following code contains the matrix elements

b elements

```
                  b(l,k,1,lg) = 1/aa(l,k,lg) * (adfe(l,k,lg)*psix(l) -
     1                              adfw(l,k,lg)*thetax(l))
                  b(l,k,2,lg) = 1/aa(l,k,lg) * bs1x(l)
                  b(l,k,3,lg) = 1/aa(l,k,lg) * bs2x(l)
                  b(l,k,4,lg) = 6.0/(aa(l,k,lg)*delx(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs1x(l) + bs3x(l)
                  b(l,k,5,lg) = 4.0/(aa(l,k,lg)*delx(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs2x(l) + bs4x(l)
                  b(l,k,6,lg) = 1/aa(l,k,lg) * (adfw(l,k,lg)*omegax(l) -
     1                              adfe(l,k,lg)*lambdax(l))
                  b(l,k,7,lg) = 1/aa(l,k,lg) * bs5x(l)
                  b(l,k,8,lg) = 1/aa(l,k,lg) * bs6x(l)
                  b(l,k,9,lg) = 6.0/(aa(l,k,lg)*delx(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs5x(l) + bs7x(l)
                  b(l,k,10,lg) = 4.0/(aa(l,k,lg)*delx(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs6x(l) + bs8x(l)
                  b(l,k,11,lg) = 1/aa(l,k,lg) * (adfn(l,k,lg)*psiy(l) -
     1                              adfs(l,k,lg)*thetay(l))
                  b(l,k,12,lg) = 1/aa(l,k,lg) * bs1y(l)
                  b(l,k,13,lg) = 1/aa(l,k,lg) * bs2y(l)
                  b(l,k,14,lg) = 6.0/(aa(l,k,lg)*dely(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs1y(l) + bs3y(l)
                  b(l,k,15,lg) = 4.0/(aa(l,k,lg)*dely(l)**2*sqrt(aa(l,k,lg)/
     1                              difn(l,k,lg))**2)*bs2y(l) + bs4y(l)
                  b(l,k,16,lg) = 1/aa(l,k,lg) * (adfs(l,k,lg)*omegay(l) -
     1                              adfn(l,k,lg)*lambday(l))
```

```
                 b(l,k,17,lg) = 1/aa(l,k,lg) * bs5y(l)
                 b(l,k,18,lg) = 1/aa(l,k,lg) * bs6y(l)
                 b(l,k,19,lg) = 6.0/(aa(l,k,lg)*dely(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs5y(l) + bs7y(l)
                 b(l,k,20,lg) = 4.0/(aa(l,k,lg)*dely(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs6y(l) + bs8y(l)
                 b(l,k,21,lg) = 1/aa(l,k,lg) * (psiz(l) - thetaz(l))
                 b(l,k,22,lg) = 1/aa(l,k,lg) * bs1z(l)
                 b(l,k,23,lg) = 1/aa(l,k,lg) * bs2z(l)
                 b(l,k,24,lg) = 6.0/(aa(l,k,lg)*delz(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs1z(l) + bs3z(l)
                 b(l,k,25,lg) = 4.0/(aa(l,k,lg)*delz(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs2z(l) + bs4z(l)
                 b(l,k,26,lg) = 1/aa(l,k,lg) * (omegaz(l) - lambdaz(l))
                 b(l,k,27,lg) = 1/aa(l,k,lg) * bs5z(l)
                 b(l,k,28,lg) = 1/aa(l,k,lg) * bs6z(l)
                 b(l,k,29,lg) = 6.0/(aa(l,k,lg)*delz(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs5z(l) + bs7z(l)
                 b(l,k,30,lg) = 4.0/(aa(l,k,lg)*delz(l)**2*sqrt(aa(l,k,lg)/
     1                          difn(l,k,lg))**2)*bs6z(l) + bs8z(l)
                 b(l,k,31,lg) = -12/dely(l)*b(l,k,1,lg)
                 b(l,k,32,lg) = -60/dely(l)*b(l,k,2,lg)
                 b(l,k,33,lg) = -12/delz(l)*b(l,k,1,lg)
                 b(l,k,34,lg) = -60/delz(l)*b(l,k,2,lg)
                 b(l,k,35,lg) = -12/dely(l)*b(l,k,7,lg)
                 b(l,k,36,lg) = -60/dely(l)*b(l,k,8,lg)
                 b(l,k,37,lg) = -12/delz(l)*b(l,k,7,lg)
                 b(l,k,38,lg) = -60/delz(l)*b(l,k,8,lg)
                 b(l,k,39,lg) = -12/delx(l)*b(l,k,11,lg)
                 b(l,k,40,lg) = -60/delx(l)*b(l,k,12,lg)
                 b(l,k,41,lg) = -12/delz(l)*b(l,k,11,lg)
                 b(l,k,42,lg) = -60/delz(l)*b(l,k,12,lg)
                 b(l,k,43,lg) = -12/delx(l)*b(l,k,17,lg)
                 b(l,k,44,lg) = -60/delx(l)*b(l,k,18,lg)
                 b(l,k,45,lg) = -12/delz(l)*b(l,k,17,lg)
                 b(l,k,46,lg) = -60/delz(l)*b(l,k,18,lg)
                 b(l,k,47,lg) = -12/delx(l)*b(l,k,21,lg)
                 b(l,k,48,lg) = -60/delx(l)*b(l,k,22,lg)
                 b(l,k,49,lg) = -12/dely(l)*b(l,k,21,lg)
                 b(l,k,50,lg) = -60/dely(l)*b(l,k,22,lg)
                 b(l,k,51,lg) = -12/delx(l)*b(l,k,27,lg)
                 b(l,k,52,lg) = -60/delx(l)*b(l,k,28,lg)
                 b(l,k,53,lg) = -12/dely(l)*b(l,k,27,lg)
                 b(l,k,54,lg) = -60/dely(l)*b(l,k,28,lg)
```

a elements

```
              ba1 (l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*delx(l))
     1                   *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
              ba2 (l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1                   *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
     2                   *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
              ba3 (l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1                   *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
              ba4 (l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1                   *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
```

```
            ba5 (l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1               *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
            ba6 (l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*delx(l))

            ba7 (l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1               *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
            ba8 (l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1               *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
            ba9 (l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1               *(adfn(l,k,lg)*psiy(l)-adfs(l,k,lg)*thetay(l))
            ba10(l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*dely(l))
     1               *(adfn(l,k,lg)*psiy(l)-adfs(l,k,lg)*thetay(l))
            ba11(l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1               *(adfn(l,k,lg)*psiy(l)-adfs(l,k,lg)*thetay(l))
            ba12(l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1               *(adfn(l,k,lg)*psiy(l)-adfs(l,k,lg)*thetay(l))
            ba13(l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1               *(adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))
            ba14(l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1               *(adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))
            ba15(l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*dely(l))
     1               *(adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))
            ba16(l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1               *(adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))

            ba17(l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1               *(psiz(l)-thetaz(l))
            ba18(l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1               *(psiz(l)-thetaz(l))
            ba19(l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*delz(l))
     1               *(psiz(l)-thetaz(l))
            ba20(l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1               *(psiz(l)-thetaz(l))
            ba21(l,k,lg)   = 1.0/(aa(l,k,lg)*delx(l))
     1               *(omegaz(l)-lambdaz(l))
            ba22(l,k,lg)   = 1.0/(aa(l,k,lg)*dely(l))
     1               *(psiz(l)-thetaz(l))
            ba23(l,k,lg)   = 1.0/(aa(l,k,lg)*delz(l))
     1               *(psiz(l)-thetaz(l))
            ba24(l,k,lg)   = 1.0 + 1.0/(aa(l,k,lg)*delz(l))
     1               *(psiz(l)-thetaz(l))
```

c elements

```
             bc(l,k,1, lg)  = 1.0 -4.0*psix(l)+1/(aa(l,k,lg)*delx(l))
     1               *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
             bc(l,k,2, lg)  = 4.0*thetax(l) + 1/(aa(l,k,lg)*delx(l))
     1               *(adfe(l,k,lg)*psix(l) - adfw(l,k,lg)
     2               *thetax(l)) + 1.0 - 4.0*omegax(l)
             bc(l,k,3, lg)  = 1.0/(aa(l,k,lg)*dely(l))
     1           *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
             bc(l,k,4, lg)  = 1.0/(aa(l,k,lg)*delz(l))
     1           *(adfe(l,k,lg)*psix(l)-adfw(l,k,lg)*thetax(l))
             bc(l,k,5, lg)  = 4.0*lambdax(l) + 1/(aa(l,k,lg)*delx(l))
     1               *(adfw(l,k,lg)*omegax(l) - adfe(l,k,lg)*lambdax(l))
             bc(l,k,6, lg)  =1.0-4.0*omegax(l)+1/(aa(l,k,lg)*delx(l))
```

```
      1                 *(adfw(l,k,lg)*omegax(l) - adfe(l,k,lg)*lambdax(l))
              bc(l,k,7, lg)  = 1.0/(aa(l,k,lg)*dely(l))
      1             *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
              bc(l,k,8, lg)  = 1.0/(aa(l,k,lg)*delz(l))
      1             *(adfw(l,k,lg)*omegax(l)-adfe(l,k,lg)*lambdax(l))
              bc(l,k,9, lg)  = 1.0/(aa(l,k,lg)*delx(l))*(adfn(l,k,lg)
      1             *psiy(l)-adfs(l,k,lg)*thetay(l))
              bc(l,k,10,lg)  = 1.0 - 4.0*psiy(l) + 1/(aa(l,k,lg)
      1             *dely(l))*(adfn(l,k,lg)*psiy(l) -adfs(l,k,lg)*thetay(l))
              bc(l,k,11,lg)  = 4.0*thetay(l) + 1/(aa(l,k,lg)*
      1              dely(l))*(adfn(l,k,lg)*psiy(l) - adfs(l,k,lg)*thetay(l))
              bc(l,k,12,lg)  = 1.0/(aa(l,k,lg)
      1              *delz(l))*(adfn(l,k,lg)*psiy(l)-adfs(l,k,lg)*thetay(l))
              bc(l,k,13,lg)  = 1.0/(aa(l,k,lg)*delx(l))
      1              *(adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))
              bc(l,k,14,lg)  = 4.0*lambday(l) + 1/(aa(l,k,lg)*dely(l))
      1              *(adfs(l,k,lg)*omegay(l)- adfn(l,k,lg)*lambday(l))
              bc(l,k,15,lg)  =1.0-4.0*omegay(l)+1/(aa(l,k,lg)*dely(l))
      1              *(adfs(l,k,lg)*omegay(l) - adfn(l,k,lg)*lambday(l))
              bc(l,k,16,lg)  = 1.0/(aa(l,k,lg)*delz(l))*
      1              (adfs(l,k,lg)*omegay(l)-adfn(l,k,lg)*lambday(l))
              bc(l,k,17,lg)  = 1.0/(aa(l,k,lg)
      1              *delx(l))*(psiz(l)-thetaz(l))
              bc(l,k,18,lg)  = 1.0/(aa(l,k,lg)*dely(l))*(psiz(l)
      1               -thetaz(l))
              bc(l,k,19,lg)  = 1.0 - 4.0*psiz(l) + 1/(aa(l,k,lg)
      1              *delz(l))*(psiz(l) - thetaz(l))
              bc(l,k,20,lg)  = 4.0*thetaz(l) + 1/(aa(l,k,lg)*delz(l))
      1              *(psiz(l) - thetaz(l))
              bc(l,k,21,lg)  = 1.0/(aa(l,k,lg)*delx(l))
      1              *(omegaz(l)-lambdaz(l))
              bc(l,k,22,lg)  = 1.0/(aa(l,k,lg)*dely(l))*(psiz(l)-
      1              thetaz(l))
              bc(l,k,23,lg)  = 4.0*lambdaz(l) +
      1              1/(aa(l,k,lg)*delz(l))*(omegaz(l) - lambdaz(l))
              bc(l,k,24,lg)  = 1.0 - 4.0*omegaz(l)
      1               + 1/(aa(l,k,lg)*delz(l))*(omegaz(l) - lambdaz(l))
```

The following code setups the A matrix with the appropriate elements in bbguts

```
c bdi New procedure for solving the responce matrix which in this
c bdi case is 6x6 (tested only for cartesian)
          if(igeom.ne.1) then
            do 302 lc=1,numco(icol,k)
              l=iocol(lc,icol,k)

c             RHS
             brhs(1) = bvmg(lc,1)
             brhs(2) = bvmg(lc,2)
             brhs(3) = bvmg(lc,3)
             brhs(4) = bvmg(lc,4)
             brhs(5) = bvmg(lc,5)
             brhs(6) = bvmg(lc,6)

c             Matrix
              ann(1,1) = ba1 (l,k,lg)
```

```
               ann(1,2) = ba2 (l,k,lg)
               ann(1,3) = ba3 (l,k,lg)
               ann(1,4) = ba3 (l,k,lg)
               ann(1,5) = ba4 (l,k,lg)
               ann(1,6) = ba4 (l,k,lg)
c
               ann(2,1) = ba5 (l,k,lg)
               ann(2,2) = ba6 (l,k,lg)
               ann(2,3) = ba7 (l,k,lg)
               ann(2,4) = ba7 (l,k,lg)
               ann(2,5) = ba8 (l,k,lg)
               ann(2,6) = ba8 (l,k,lg)
c
               ann(3,1) = ba9 (l,k,lg)
               ann(3,2) = ba9 (l,k,lg)
               ann(3,3) = ba10(l,k,lg)
               ann(3,4) = ba11(l,k,lg)
               ann(3,5) = ba12(l,k,lg)
               ann(3,6) = ba12(l,k,lg)
c
               ann(4,1) = ba13(l,k,lg)
               ann(4,2) = ba13(l,k,lg)
               ann(4,3) = ba14(l,k,lg)
               ann(4,4) = ba15(l,k,lg)
               ann(4,5) = ba16(l,k,lg)
               ann(4,6) = ba16(l,k,lg)
c
               ann(5,1) = ba17(l,k,lg)
               ann(5,2) = ba17(l,k,lg)
               ann(5,3) = ba18(l,k,lg)
               ann(5,4) = ba18(l,k,lg)
               ann(5,5) = ba19(l,k,lg)
               ann(5,6) = ba20(l,k,lg)
c
               ann(6,1) = ba21(l,k,lg)
               ann(6,2) = ba21(l,k,lg)
               ann(6,3) = ba22(l,k,lg)
               ann(6,4) = ba22(l,k,lg)
               ann(6,5) = ba23(l,k,lg)
               ann(6,6) = ba24(l,k,lg)
```

The following code contains the new flux moments

```
           phi(l,k,2,lg)=-1.0/aa(l,k,lg)
     1       *(cpx11*jout(l,k,1,lg) - cmx11*jout(l,k,2,lg)
     2       + cmx12*jin(l,k,2,lg) - cpx12*jin(l,k,1,lg)
     3       + qou(l,k,14,lg)/dely(l) + qou(l,k,16,lg)/delz(k)
     4       -qou(l,k,2,lg))
           phi(l,k,3,lg)=-1.0/aa(l,k,lg)
     1       *(cpx21*jout(l,k,1,lg) + cmx21*jout(l,k,2,lg)
     2       - cpx22*jin(l,k,1,lg) - cmx22*jin(l,k,2,lg)
     3       - 6.0*difn(l,k,lg)*phi(l,k,1,lg)/delx(l)**2
     4       - qou(l,k,3,lg)
     5       + qou(l,k,15,lg)/dely(l) +qou(l,k,17,lg)/delz(k))
          phi(l,k,4,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(2.0*delx(k)**2)*
     1            (2.0/adfe(l,k,g)*(jout(l,k,1,lg)+jin(l,k,1,lg))-2/
```

```
     2           adfw(l,k,g)*(jout(l,k,2,lg)+jin(l,k,2,lg)))-qou(l,k,4,lg))
      phi(l,k,5,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(5.0*delx(k)**2)*
     1           (2.0/adfe(l,k,g)*(jout(l,k,1,lg)+jin(l,k,1,lg))+2/
     2           adfw(l,k,g)*(jout(l,k,2,lg)+jin(l,k,2,lg)))-qou(l,k,5,lg))
      phi(l,k,6,lg)=-1.0/aa(l,k,lg)
     1     *(cpy11*jout(l,k,3,lg) - cmy11*jout(l,k,4,lg)
     2     + cmy12*jin(l,k,4,lg) - cpy12*jin(l,k,3,lg)
     3     + qou(l,k,18,lg)/delx(l) + qou(l,k,20,lg)/delz(k)
     4     -qou(l,k,6,lg))
      phi(l,k,7,lg)=-1.0/aa(l,k,lg)
     1     *(cpy21*jout(l,k,3,lg) + cmy21*jout(l,k,4,lg)
     2     - cpy22*jin(l,k,3,lg) - cmy22*jin(l,k,4,lg)
     3     - 6.0*difn(l,k,lg)*phi(l,k,1,lg)/dely(l)**2
     4     - qou(l,k,7,lg)
     5     + qou(l,k,19,lg)/delx(l) + qou(l,k,21,lg)/delz(k))
      phi(l,k,8,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(2.0*dely(k)**2)*
     1           (2.0/adfn(l,k,g)*(jout(l,k,3,lg)+jin(l,k,3,lg))-2/
     2           adfs(l,k,g)*(jout(l,k,4,lg)+jin(l,k,4,lg)))-qou(l,k,8,lg))
      phi(l,k,9,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(5.0*dely(k)**2)*
     1           (2.0/adfn(l,k,g)*(jout(l,k,3,lg)+jin(l,k,3,lg))+2/
     2           adfs(l,k,g)*(jout(l,k,4,lg)+jin(l,k,4,lg)))-qou(l,k,9,lg))
      phi(l,k,10,lg)=-1.0/aa(l,k,lg)*(cpz1*(jout(l,k,5,lg) -
     1     jout(l,k,6,lg)) + cmz1*(jin(l,k,6,lg) - jin(l,k,5,lg))
     2     + qou(l,k,22,lg)/delx(l) + qou(l,k,24,lg)/dely(l)
     3     -qou(l,k,10,lg))
      phi(l,k,11,lg)=-1.0/aa(l,k,lg)*(cpz2*(jout(l,k,5,lg) +
     1     jout(l,k,6,lg)) - cmz2*(jin(l,k,5,lg) + jin(l,k,6,lg))
     2     - 6.0*difn(l,k,lg)*phi(l,k,1,lg)/delz(k)**2
     3     - qou(l,k,11,lg)
     4     + qou(l,k,23,lg)/delx(l) + qou(l,k,25,lg)/dely(l))
      phi(l,k,12,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(2.0*delz(k)**2)*
     1           (2.0*(jout(l,k,5,lg)+jin(l,k,5,lg))-2*(jout(l,k,6,lg)+
     2           jin(l,k,6,lg)))-qou(l,k,12,lg))
      phi(l,k,13,lg)=-1.0/aa(l,k,lg)*(difn(l,k,lg)/(5.0*delz(k)**2)*
     1           (2.0*(jout(l,k,5,lg)+jin(l,k,5,lg))+2*(jout(l,k,6,lg)+
     2           jin(l,k,6,lg)))-qou(l,k,13,lg))
```

Academic Vita

Education

Concurrent Majors: Nuclear Engineering and Mechanical Engineering with honors in Nuclear Engineering - Expected Graduation Date – Spring 2010

Relevant Experience

- **Research Assistant in the Reactor Dynamics and Fuel Management Group** — *5/2009 – Present*, *State College, PA*
    - Working on the development of a computer code for the Semi-Analytical Nodal Expansion Method (SA-NEM) 3D Kinetics Package for neutron diffusion
    - In the process of completing a Senior Thesis with Dr. Kostadin Ivanov for the Schreyer Honors College based on the SA-NEM
- **Teaching Intern for the Mechanical and Nuclear Engineering Department** — *8/2009 – Present*, *State College, PA*
    - Assisting a professor in teaching Introduction to Reactor Design
    - Holding weekly office hours and grading/creating homework assignments
- **Engineering Intern for Exelon Nuclear** — 5/2008 – 8/2008, *Kennett Square,PA*
    - Worked in the Equipment Reliability Department
    - Developed a standardized performance monitoring plan template for the feedwater system
    - Gained unescorted access to Exelon's fleet of nuclear power plants and toured each of their eastern plants
    - Worked with the Reactor Engineering Department at the Peach Bottom Atomic Power Station during a shift in the Unit 2 control room for a power maneuver
- **Co-op Project Engineer for the Valspar Corporation** — *6/2007 – 8/2007*, *Pittsburgh, PA*
    - Led and Managed capital projects by overseeing the design, analysis, funding, contractor relations, safety/environmental evaluation, and completion phases
    - Analyzed data from erroneous batches of various material coatings in order to identify problems in the preparation and quality of raw materials

Skills, Awards, and Extracurricular Activities

- Experience using SolidWorks, Matlab, C++, Java, Visual Basic, Fortran, and HTML
- Scholar in the Schreyer Honors College — *9/2006 – Present*
    - Recipient of the Academic Excellence Scholarship
- Member of the American Nuclear Society — *9/2007 – Present*
    - Active student and national member
- Tutor for the Morgan Academic Support Center — *10/2007 - 8/2009*
    - Tutored student athletes in math, physics, and chemistry
- Scholarship Chairman of the Kappa Alpha Order — *3/2007 – Present*
    - Organized events and monitored the chapter's academic performance
- Volunteer Work at Church Soup Kitchen — *9/2005 – Present*
    - Assist other church members in running the weekly soup kitchen
- 3 year Member and 2 time Captain of the Rube Goldberg Engineering Team — *10/2002 - 12/2005*
    - Created complicated machines that performed tasks such as
- Greek Sing Stage Crew Captain — *10/2007 - 11/2007*
    - Managed the backstage area for a show consisting of short musicals