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INVESTIGATING AN AESTHETIC UTILITY FUNCTION FOR THE COMPUTATIONAL DESIGN OF STRUCTURAL ART

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ABSTRACT

When designing and optimizing structures, engineers must make decisions based on maximizing the economy and efficiency of the structure. While these two factors are essential to a successful structural design, the aesthetic elegance of the structure is sometimes overlooked due in part to being difficult to quantify. Considering all these three criteria of economy, efficiency, and elegance can result in a design solution that can be considered “structural art”. This thesis attempts to establish a computational framework for structural art by quantifying the aesthetic beauty of truss bridges using a utility function. The utility function is generated through a review of literature about bridge aesthetics, and measurable aesthetic attributes are established for the function. The function is used in conjunction with a gradient-based topology optimization that can determine the optimal distribution of elements for a truss bridge superstructure with the goal of minimizing the structure’s compliance. The utility function and optimization are then applied to fourteen different truss bridge designs to rank designs based on compliance (or stiffness), cost, and aesthetics. Analysis of the results suggests that the proposed utility function might successfully order different bridge truss designs by their visual elegance, and that aesthetics can potentially be used as a criterion for ranking and eliminating structural design alternatives in a computational design framework.

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Chapter 1

Introduction

Structural engineers face a variety of challenges when deciding on a sufficient design for a structural system. Some of the most important decisions are made based on ensuring the structure is both safe to support the loads while also being as economical as possible. Additionally, factors such as the volume of material and aesthetics of the structure are also important when selecting a design.

The concept of "structural art" was introduced by a Professor of Civil Engineering at Princeton University named David P. Billington in the early 1980s. In his book *The Tower and the Bridge,* Billington outlines this idea of "structural art," identifying that it is something both parallel to and fully independent from architecture [1]. Structural art is achieved through success in three leading ideals: efficiency, economy, and elegance. In structural design, efficiency is achieved by using as few natural resources as possible, and thus limiting the overall volume or weight of the structure. Economy is achieved by using as few public resources as possible, meaning that the structure is designed and constructed with the lowest reasonable cost. Finally, the third ideal of elegance is established through a "conscious aesthetic motivation of the engineer" and is achieved through structural design that is visually appealing to the public [1]. Billington asserts that these three ideals all go hand in hand, and when successfully combined in a structure's overall design, then it can be considered "structural art." As described by Billington, "civil works that function reliably, cost the public as little as possible, and, when sensitively designed, become works of art [1]."

Billington's ideas concisely and accurately describe why and how infrastructure is successful in modern society. They also describe ways in which structures can be optimized and identify how trade-offs often occur in design. Structures that might be both cost effective and efficient might also turn out to be ugly, and conversely structures that look nice may cost too much or use too much material. While economy and efficiency are aspects that can and have been easily measured and optimized, the third ideal of elegance is one that is often difficult to quantify and provides challenges to current structural optimization techniques. This third ideal is different from architecture because structural elegance is established by designing aesthetic structural systems that serve a mechanical function, whereas architecture includes designing aesthetic systems to serve more of a social function. If structural optimization can somehow include this third ideal of elegance and combine it with economy and efficiency, then a framework for designing "structural art" can potentially be established.

1.1 Motivation

When structural engineers optimize structures for criteria such as cost and weight, sometimes the structural aesthetics of the building or bridge can be compromised or altogether disregarded. This can lead to designs that –while functional and economical—are not visually appealing. Billington describes that "too many ugly structures result from minimal design to support any simple formula connecting efficiency and economy to elegance [1]." Poor structural design practice can sometimes lead to an ugly structure regardless of its architectural design.

An excellent example of a structural design that led to an unpleasant visual appearance is The Brunswick Building in Chicago, Illinois, a photograph of which is shown in Figure 1. The structure was completed in 1961 and was the tallest reinforced concrete structure at the time of its completion [2]. In the design of the structure, the main lobby on the ground floor required columns to be spaced at a greater distance than columns in the upper levels of the building. As a result, a 24 foot deep transfer wall beam was necessary to transfer gravity loads from the tightly spaced columns above to more widely spaced columns below [2]. This transfer beam can be seen clearly on the building (Figure 1). Its appearance is bulky and does not integrate well with the overall appearance of the building.

Figure 1.The Brunswick Building in Chicago, IL [2]

In contrast, the Two Shell Plaza building in Houston, Texas is a structure that is designed to perform the same function as The Brunswick Building, but does so in a more aesthetic, gradual manner. This building offers a more elegant solution for transferring loads from tightly spaced columns above to widely spaced columns below using a series of haunched beams [2]. As seen in Figure 2A, the building appearance is much less bulky and more harmonious than the design of The Brunswick Building. Another example of an elegant solution, the Marine Midland Bank in Rochester, New York, (shown in Figure 2B) also uses a more aesthetic design for transferring loads to columns simply by reducing the cross-sectional dimensions of the tightly spaced columns [2].

Figure 2. (A)Two Shell Plaza (left) and (B) Marine Midland Bank (right) [2]

This example of the aesthetics of transfer beams illustrates the importance of aesthetic considerations in design that motivate this thesis and clearly shows that structural engineers can affect the aesthetics of a structure. It also shows that although people have different opinions on what looks good and what doesn't, almost everyone can agree that the structural systems used by Two Shell Plaza and Marine Midland Bank are more aesthetically pleasing than The Brunswick Building while performing the same function. This means that there are examples of times when most people can agree on what constitutes an aesthetic design, and it is not out of the realm of possibility to suggest that the aesthetic value of structures can be quantified, and that future structural designs can be optimized and evaluated based on their elegance. By assigning a quantitative value to a design for its elegance, aesthetics can potentially become a criterion that, when combined with cost and volume, allows for infrastructure to be optimized based on its adherence to "structural art."

1.2 Scope

While aesthetics in structural design is important for all types of structures, this thesis will focus on bridge design, specifically truss bridges. Truss bridges are a suitable application for investigating structural art because the structural system – that is the load transferring mechanism(s) – itself is left naked and visible to observers. This means that the placement and sizing of steel truss elements must be carefully considered in the design phase. Additionally, truss design is a common application for topology optimization problems. Combining topology optimization with an aesthetic utility function for the design of truss bridges offers a reasonable application for the illustration of the objective and methods presented in this thesis. This thesis will analyze steel truss bridge designs that have reinforced concrete piers and overall length of less than 100 meters. The aesthetic components for the utility function will also be limited to the bridge superstructure only, and not include any aesthetics resulting from the substructure.

1.3 Objective

This thesis attempts to establish a utility function for inclusion into a computational design framework for aesthetics to be a criterion by which structural systems are optimized and selected in an effort toward the design of structural art. A successful utility function will include all parts of a truss bridge superstructure that contribute to the aesthetic nature of the bridge. The utility function will establish attributes by which a bridge design can be ranked based on its aesthetic elegance and show that when these attributes are combined into one function, the overall elegance of the structure can be quantitatively determined. If incorporated correctly, an aesthetic utility function—when combined with topology optimization—is a novel idea because typically topology optimization only optimizes a structure based on attributes such as stiffness (compliance) or volume without any consideration of aesthetics. The ultimate goal of this utility function is to show that aesthetics can be an attribute incorporated into a set-based design method.

Chapter 2

Background

Over the many years of bridge construction there have been different books and manuals written about aspects that constitute an aesthetic bridge design. These have been written by both architects and structural engineers, and include criteria that explain why some bridges look nice and why others appear less visually appealing. An in-depth analysis of this past literature is essential to creating a meaningful function to quantify aesthetic bridge design. Additionally, a review of literature about utility functions is necessary to ensure that the aesthetic utility function for this thesis takes the proper form. Finally, a literature review of set-based design and the sequential decision process, as well as gradient-based topology optimization is needed to understand how an aesthetic utility function can be incorporated into structural optimization and how topology optimization can be performed to generate optimal bridge truss geometry.

2.1 Aesthetic Bridge Design

One of the most influential books written about aesthetic bridge design is *Bridges: Aesthetics and Design* written in 1984 by a German structural engineer named Fritz Leonhardt [3]. In this book, Leonhardt outlines his theories of aesthetics, including different criteria through which bridge designs can be measured for appearance. Leonhardt also includes many different examples of both aesthetic and visually displeasing bridges, and explains why these bridges succeed or fail as an aesthetic design. Leonhardt believed that, although humans all have varying opinions on what looks visually pleasing to them, all things have some form of aesthetic value and that there are ways to measure why people find some things more beautiful than others [3].

The criteria that Leonhardt identifies as the most important for establishing an aesthetic design are: proportion, order, refined form, integration into the environment, surface texture, color, character, and complexity [3]. These ideas have been very influential to the field of bridge engineering, and they have not only been adopted by many different engineers and architects, but also used as guidelines in bridge design manuals. Examples of bridge design manuals that have adopted these criteria are “Aesthetic Bridges: User’s Guide” published by the Maryland Department of Transportation in 1993 [4], and subsequently “Aesthetic Guidelines for Bridge Design” published by the Minnesota Department of Transportation in 1995 [5]. These manuals are used by state DOTs across the country to design more aesthetically pleasing bridges and include many of the concepts originally presented by Leonhardt.

The manual “Aesthetic Bridges: User’s Guide” is viewed as a pioneer for aesthetic bridge design for state DOT’s. The document iterates that structural design is vitally important to bridge aesthetics, stating that “aesthetic quality must be present in the basic structural configuration [4].” This document details aesthetic design concepts for all types of bridges and introduces new concepts that have been adapted by subsequent design manuals. The manual “Aesthetic Guidelines for Bridge Design” even further extends concepts established by Leonhardt and the Maryland Department of Transportation. The manual not only includes Leonhardt’s ideas of proportion, order, color, and refined form, but also introduces rhythm, scale, unity, and balance.

While all the criteria presented by Leonhardt, Maryland DOT, and Minnesota DOT aid in the understanding of why some bridges look nicer than others, the criteria that most interest this thesis are order, proportion, harmony, and rhythm. Criteria such as integration into the environment, color, surface texture, and character are not applicable to an aesthetic utility function for this paper because the utility function will be focused solely on the placement of steel for the truss superstructure, and not what is on the surface or how it relates to the surrounding area.

Proportion

Fritz Leonhardt suggests that proper geometric proportions of both individual parts and masses of a structure are necessary to achieving aesthetic beauty [3]. For a bridge, the important proportional relationships include the suspended superstructure and supporting columns, the depth and span of the bridge, and the height, length and width of the openings [3]. The Minnesota DOT further describes that "balanced and harmonious geometric proportions are fundamental characteristics in the development of graceful buildings and bridges alike [5]." Bridges must use proper proportions for the individual elements in the superstructure, as well as the overall appearance of the bridge. A bridge's geometry plays a key role in whether it looks visually pleasing, which is why proportioning is such an important aspect for designers to consider.

Order

Defined as an important rule to aesthetic bridge design, order in structures refers to the arrangement of elements and is established by limiting the directions of lines and edges to only a few in space [5]. Leonhardt asserts that too many directions of edges and struts "confuse the observer and arouse disagreeable emotions [3]." The principle of order is typically best demonstrated in truss bridges. Leonhardt describes that "trusses can look satisfactory if the inclinations of the truss bars are restricted to very few directions in space [3]." A truss containing few directions can look ordered and simple, whereas one that has elements going in many directions can look chaotic and confusing. This principle also relates to rhythm, which can be established through repetition of the direction of elements. Figure 3 is taken from the Minnesota DOT's "Aesthetic Guidelines for Bridge Design," and shows how order can be established from orientation of elements.

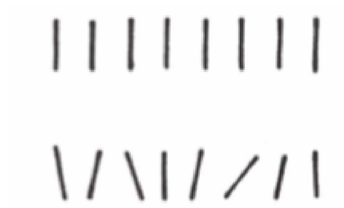


Figure 3. Order and Disorder [5]

Harmony

According to the Minnesota DOT's "Aesthetic Guidelines for Bridge Design," harmony describes the relationship of elements based on their visual characteristics [5]. The design manual also states that harmony can be created from a composition that attains order through repetition of similar elements, forms, or spaces [5]. This indicates that the same form should be maintained throughout the entire length of the bridge for it to appear harmonious. Additionally, harmony is created though keeping the form of the structure continuous with smooth curves and tapers [5]. A harmonious form for a bridge is established by having smooth lines and gradual transitions between members. Leonhardt commented on the idea of having gradual transitions between members by saying: "bodies formed by parallel straight lines appear stiff and static [3]." For a bridge, stiff, parallel chords should be avoided to prevent the bridge from looking static and rigid. A bridge with a rigid form such as a rectangle is an unharmonious form, while an arch shape is an example of a smooth, harmonious form.

2.2 Utility Functions

Since a utility function will be created to quantify an individual's aesthetic preference for a bridge structure, it is necessary to have a solid background in the formulation of utility functions and what they can be used for. The concept of "utility" is a term used often by economists, and it describes the capacity of a commodity to satisfy what a consumer wants [6]. Something that has a high utility satisfies the preferences of a certain consumer, while something with low utility does not. A utility function is a tool that can be used to quantify the utility of a certain good. Although the utility a service provides will vary from person to person, utility functions are used to find a value that approximates a utility for most people.

Every utility function has one or multiple objective(s), and each objective has attributes that can be used to indicate the degree to which something meets this objective [7]. A successful attribute is often something that is both comprehensive and measurable, meaning that the attribute affects the objective and that a commodity can be either ranked or assigned a point value based on its adherence to an attribute [7]. Therefore, an essential component of establishing a successful utility function is determining attributes that can be measured in some way.

Utility functions have been used in a variety of different engineering applications. One specific application in which a utility function has been explored in engineering to measure aesthetic preference is a paper written about quantifying aesthetic form in sports utility vehicles.

This paper is called "Quantifying Aesthetic Form Preference in a Utility Function," and provides a basis for which the utility function for this thesis can be constructed [8]. The paper indicates that utility functions offer a means to describe the relationship of attributes and can be explored to maximize a person's utility [8]." The paper establishes a utility function for measuring aesthetics in SUVs by coming up with measurable attributes that might affect a user's preference. The form of the utility function for the paper is defined as

$$U(x) = \sum_{i=0}^{n} u_i;\ u_i = f(B_i, x_i)$$

where u_i is some function representing the utility of an individual attribute x, n is the total number of attributes, and B is the attribute weights [8]. For this utility function, each attribute is weighted with a constant B which is determined from a series of experiments and customer surveys [8]. Different values are then found for each attribute based on customer preference, and summed to produce an overall utility value. While a weight will not be able to be established for this thesis, the aesthetic utility function for bridge aesthetics can be modeled off this relevant example. This example shows that it is feasible to construct a utility function that can measure customer preference based on aesthetic engineering design.

2.3 Gradient-Based Topology Optimization

This thesis will use a gradient-based topology optimization to determine what an optimized bridge truss structure might look like. Topology optimization has been successfully applied to truss structures in the past to determine the most efficient truss geometry, and these methods can be applied to bridge designs used in this paper.

The topology of a structure is defined as the arrangement of its constituting elements [9]. A topology optimization thus finds the optimal arrangement of elements based on a certain objective. To begin a topology optimization for a truss, a ground structure must be established for the structure being optimized [9]. The ground structure consists of *n* chosen nodal points and *m* possible connections, and the objective is to find the optimal substructure of the structural universe created in the ground structure [10]. The cross-sectional area and length of truss elements are variables that must be established, and to produce an objective function for the optimization, the stiffness matrix of truss elements needs to be calculated. In many cases, the objective for a truss topology optimization is to minimize the compliance (maximize stiffness) of the structure [10].

To maximize the truss' stiffness, an objective function is established for the compliance of the system with constraints on the volume of the structure. The goal of the optimization is to minimize this objective function given the restraints on other variables. A gradient-based algorithm is one type of topology optimization which seeks to modify the optimization variables that have the greatest effect on the objective function [9]. An algorithm can be developed which slightly changes the value of each variable and then measures the effect on the objective function [9]. The process of changing the variable is repeated until a termination criterion is met [9]. The termination is met when the objective function reaches a global minimum.

A gradient-based truss optimization will be the most effective method for determining the geometry of a bridge truss superstructure, which can then be analyzed for its aesthetic utility.

2.4 Set-Based Design

The main application for the aesthetic utility function is so that aesthetics can be used as a criterion for eliminating design alternatives with set-based design. Set-based design is an alternative method to point design, because set-based design is when potential design solutions are carried through the design process, as opposed to a single design that is expanded and refined [11]. With set-based design, an initial set of design alternatives is established with a low fidelity, low cost model. The goal of set-based design is to then carry a set of designs forward to more computationally expensive models before ultimately choosing an optimal solution [11]. Major decisions must be made along the way as to which design alternatives should be carried on to the next stage. This type of design can be viewed as a "sequential decision process" where the detail of modeling effort is increased while the design space of alternatives is decreased [11].

An innovative take on set-based design and the sequential decision process is presented by Dr. Mehmet Unal in his paper "A sequential decision process for the system-level design of structural frames [12]." In this paper, Unal establishes a bounding model, which is a model of lower fidelity that provides two sided limits on a decision criterion [12]. The upper bound provides a model that is always greater than the true function, and a lower bound provides a model consistently less than the true function. These bounding models can be used to guarantee that a global optimal solution is retained in the set [12]. This is a new and effective method because of how it allows designs to be eliminated and guarantees that the best designs move to the next set without having to perform expensive high-fidelity models on all the designs. Unal asserts that this method can provide the same discriminatory power as using the most detailed design model over the entire design space [12].

This concept of the sequential decision process is important to this thesis because it can be a suitable method for reducing a design space based on finding an optimal solution for structural art. The sequential decision process could potentially be used in the future in a scenario like the one analyzed in this thesis. The goal is to find an efficient way to maximize a structure's structural art by evaluating and eliminating different designs in a computationally inexpensive manner. If the aesthetic utility function presented in this paper can be successfully incorporated into set-based design, then a computational framework for structural art can potentially be established.

Chapter 3

Methodology

This thesis attempts to find a way to make informed design decisions with the goal of maximizing the “structural art” of a structural system. As described in Chapter 1, structures that maximizes efficiency, economy, and elegance can be best described as structural art.

While a gradient-based topology optimization is used to determine an optimal truss geometry with maximum stiffness, the main contribution of this paper is the aesthetic utility function to quantify the aesthetic value of different designs. In this thesis, the utility function is applied to the outcome of final designs and not an integral part of the optimization, however, in the future, it might be possible to include the aesthetics directly into the optimization formulation and solution algorithms. This chapter describes the methodology for determining the aesthetic utility function.

3.1 Establishing Aesthetic Utility Function

An aesthetic utility function for the design of truss bridges is developed using the concepts for aesthetic bridge design presented in Franz Leonhardt’s *Bridges: Aesthetics and Design* (1984) [3], the Minnesota DOT’s *Aesthetic Guidelines for Bridge Design* (1995) [5], and David Bennett’s *The Architecture of Bridge Design* (1997) [13]. The utility function calculates a numerical value for each bridge design based on its adherence to some of the important criteria of aesthetic bridge design presented in Chapter 2. As stated in chapter 2, the function will be limited to the criteria of proportion, order, and harmony. These three attributes can evaluate all different portions of a truss bridge superstructure. Proportion can be related to the overall dimensions of the bridge, order can be related to the struts and hangars of a truss bridge, and harmony can be related to the chord elements of a bridge.

Unlike most utility functions, this utility function is designed so that a lower value equates to a more aesthetically preferred design. Modeling the function off the utility function described in the paper "Quantifying Aesthetic Form Preference in a Utility Function" [8], the utility function for this thesis takes the form

$$U(x) = \sum_{i=0}^{n} u_i \tag{1}$$

where $U(x)$ is the overall utility function, and u_i is a function representing the utility of an individual attribute x_i with n being the total number of attributes. A total of three attributes (proportion, order, and harmony) are included in this utility function, so n = 3 and the function can be expanded to:

$$U(x) = u_{proprortion} + u_{order} + u_{harmony} \tag{2}$$

where $u_{proportion}$, u_{order}, and $u_{harmony}$ are the individual utility functions for proportion, order, and harmony, respectively.

3.2 Proportion

The first attribute considered for this utility function is proportion. For truss bridges, one of the geometric proportions that is both important to its overall appearance and easily measurable, is the depth to span ratio of the bridge superstructure. This is an essential attribute to an aesthetic bridge because it is often the most visible aspect of a truss bridge and plays a big part in determining whether the bridge has aesthetic value or not.

While there is no question that a proper depth to span ratio is important for the aesthetic quality of the bridge, the question that needs to be answered is what constitutes an aesthetic ratio. Answering this question is not easy, as there are often different proportions that will appeal to different people in different scenarios. However, a proportion that has often been used in architecture and design and has been described as aesthetically pleasing is the “Golden Ratio”. The Golden Ratio, or “Golden Rectangle” is constructed from the convergence of the Fibonacci Series [5]. The series is based on the proportion of a:b, b:(a+b), (a+b):(b+a+b), etc., and converges to a value of 1:1.618 [5]. Figure 4 shows the concept of the Golden Ratio in visual terms, as the rectangle is divided into smaller rectangles that all have a width to length ratios of 1:1.618.

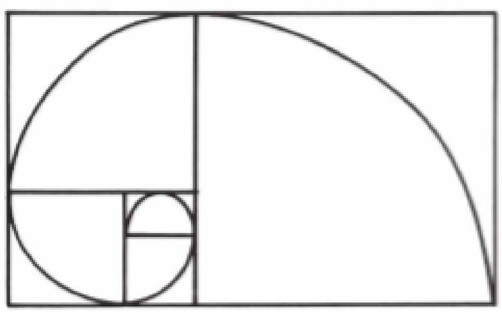


Figure 4. Golden Rectangle [5]

In his book *The Architecture of Bridge Design*, Architect David Bennett explains that the golden ratio has been used in the proportioning of windows and doors, picture frames, pages of books, etc. and also used in ancient architecture such as the Pyramids of Egypt and Gothic Cathedrals [13]. Additionally, Bennett states that the Golden ratio has been regarded for centuries as the “key to [unlocking] the mysteries of aesthetic beauty. [13]” Because of the widespread use of the golden ratio in all types of architecture and graphic design, Bennett argues that the golden rectangle can also be applied to bridges in a similar manner to create aesthetic proportions [13].

Since the Golden Ratio is a quantifiable ratio that has proven to have aesthetic value, it is reasonable to suggest that a depth to span ratio of 1:1.618 is one that can be used to quantify the aesthetic value of a bridge in terms of its proportion. Although a superstructure depth to span length ratio of 1:1.618 may sometimes be unachievable or not optimal, this ratio theoretically would provide a visually pleasing result for a bridge. Figure 5 shows how a bridge proportioned with the Golden Ratio might compare to a bridge that is not.

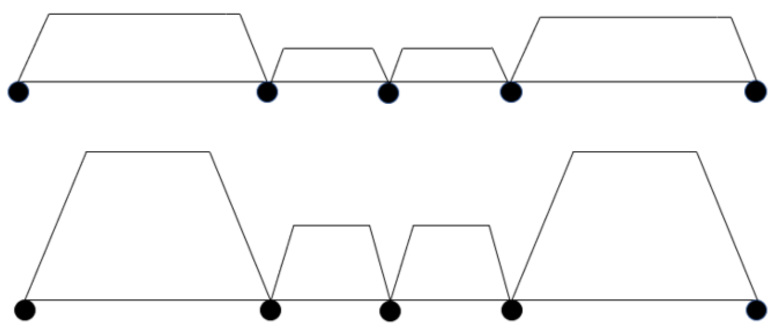


Figure 5. Golden proportioned bridge (bottom) vs non-Golden proportion (top)

With this ratio established, an aesthetic utility function for the proportion attribute can be written by calculating the percent difference of the actual span to depth ratio of the bridge to the Golden Ratio. The utility function for proportion can be written as:

$$u_{proportion} = \frac{\sum_{i=1}^{n} \frac{\frac{l_i}{h_i} - 1.618}{1.618}}{n} \tag{3}$$

where n is the total number of spans for the bridge, h_i is the depth of the ith span, and l_i is the length of the ith span. This function is the average of the percent difference from the Golden Ratio for all spans of the bridge. A span with a Golden length to depth ratio of 1.618 will produce a value of 0 for the utility function, and therefore a bridge with an average span to depth ratio closer to the Golden ratio will produce a lower value for the function, which is more desirable.

3.3 Order

As described in Chapter 2, order is another criterion for determining whether a bridge design is aesthetically pleasing, particularly for a truss bridge. Since order is achieved by limiting the lines and edges of a structure to as few directions as possible [3], measuring the order of a truss topology can be done by measuring the different directions the elements in the truss take. To measure order, the variance of the cosines of the directions taken by all struts and hangars in a truss bridge with respect to the horizontal is calculated.

Along with ensuring that the struts take as few directions as possible, the struts should also cross over each other as little as possible to prevent clutter and disorder. Figure 6 shows how elements crossing over each other can create disorder. In both images, the elements are oriented at the exact same angles, but the image on the left appears more ordered because the elements do not cross over.

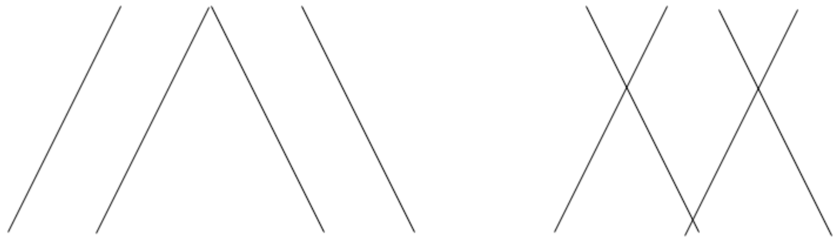


Figure 6. No crossing vs. Crossing

To incorporate this into the utility function, the function counts how many times the elements cross over and adds this as a penalty. The utility function for order can therefore be written as

$$u_{order} = \frac{\sum_{i=1}^{n}[\cos(\theta_i) - \mu]^2}{n} + \frac{X}{\alpha} \quad (4)$$

where θ_i is the angle of the ith element in the optimal topology with respect to the horizontal decking, μ is the mean angle of the population of elements, *n* is the total number of struts and hangars (excluding the upper and lower chords), *X* is the number of crosses in the topology, and α is a constant. The first term in the equation calculates the variance of the cosines taken by all struts and hangars in the topology, while the second term adds the penalty for crosses. Both terms in the equation should be weighted equally, so the constant α is determined after all designs have been evaluated. The average of the first term for a certain number of designs should be equal to the average of the second term for the same population of designs. This allows for the two terms to be as equally weighted as possible.

This equation is effective because designs with many different angles will result in a higher variance and thus a higher value for the first term of the utility function. Likewise, designs that have many struts crossing over each other will produce higher values for the second term in the function. Similar to the proportion attribute, a lower value for this function represents a more ordered, aesthetic bridge truss design.

3.4 Harmony

The concept of harmony in a structure is important because it is essential to use harmonious shapes in design, along with maintaining similar visual characteristics throughout. The shape of the bridge should be harmonious, meaning that it has smooth lines and does not change throughout the shape of the bridge.

Along with having a smooth shape, a more harmonious truss bridge design also maintains a similar shape throughout the entire bridge. A bridge design that uses an arch shape in some spans of the bridge but a trapezoidal shape in other spans is not harmonious and thus not as aesthetically pleasing as a bridge which uses the same shape throughout, as illustrated in Figure 7.

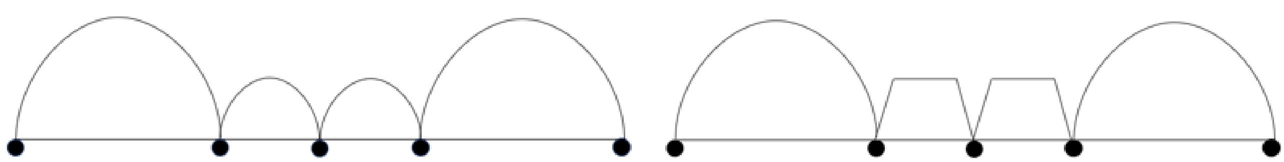


Figure 7. Two different bridge configurations

With these ideas in mind, a function for the harmony of a bridge design can be constructed based upon finding the difference in direction of adjacent chord elements in a topology. For truss bridges, the directional difference of exterior chord elements should be limited, because these elements form the overall shape of the bridge, and a smaller change in angle between adjacent elements forms a shape that is closer to a smooth arch shape. The harmony function finds the average change in angle between chord elements for each span of the bridge. The formula then uses these values to calculate a weighted average for the overall bridge. To incorporate the concept of maintaining a similar shape throughout, the variance of the averages for each span is calculated and added into the function. The harmony function takes the form

$$u_{harmony} = \left\{\sum_{i=1}^{n}\left(\frac{l_i}{L}\right) * \frac{\sum_{j=1}^{m}(\theta_j - \theta_{j+1})}{m-1}\right\} + \left\{\frac{\sum_{i=1}^{n}\{[\frac{\sum_{j=1}^{m}(\theta_j - \theta_{j+1})}{m-1}]_i - \mu\}^2}{n}\right\} \qquad (5)$$

where n is the number of spans on the bridge, l_i is the length of the ith span, L is the total length of the bridge, m is the total number of chord elements per span, θ_j is the angle that the jth element makes with respect to the horizontal in radians, and μ is the per span average of the average change in angle for the chord elements in the span. The first term in this equation is the weighted average of the average change in angle between chords, and the second term is the variance of the average change in angle between cords for each span.

This function is effective in measuring the harmony of the bridge because designs that produce lower values for the change in angle will have a shape with smoother lines. The second term of the equation also penalizes bridge designs for having differing shapes in different spans.

The three separate utility functions for proportion, order, and harmony are useful for ranking bridge designs solely based off each criterion. The three functions can then be summed to produce a single value for the aesthetic utility of a design. Combining these three functions is difficult, however, since the differing values for each function will cause some attributes to be weighted higher than others. Since there is no metric established in this thesis for determining which attribute is more important, all attributes for this utility function should be weighted equally. Therefore, a constant multiplier should be applied to each function to avoid any of the functions from being weighted higher than another. For a certain population of designs, this can be done by calculating a multiplier that allows the overall average for all three functions to be equal. The process for determining these multipliers will be described further in Chapter 4.

Chapter 4

Application to 72-meter truss bridge

4.1 Bridge Design Setup

The methods for comparing structural design alternatives based on efficiency, economy, and elegance were illustrated through application to a practical example to test the ability of the utility function to effectively rank different designs based on aesthetics and see how it might conflict with variables such as cost and compliance. The methodology was applied to a typical steel truss bridge with concrete piers and foundations.

For this application, a hypothetical situation was simulated in which a truss bridge needs to be constructed to cross a 42-meter wide river. Three different types of bridges are considered for the design. The first type of bridge is a bridge with two piers on the end for support, and three piers in the middle of the bridge over the river (Figure 8). The second type of bridge also has two supports on the end, but only one support in the center of the bridge (Figure 9). Finally, the third type of bridge is a simply supported bridge with only two supports on either end (Figure 10).

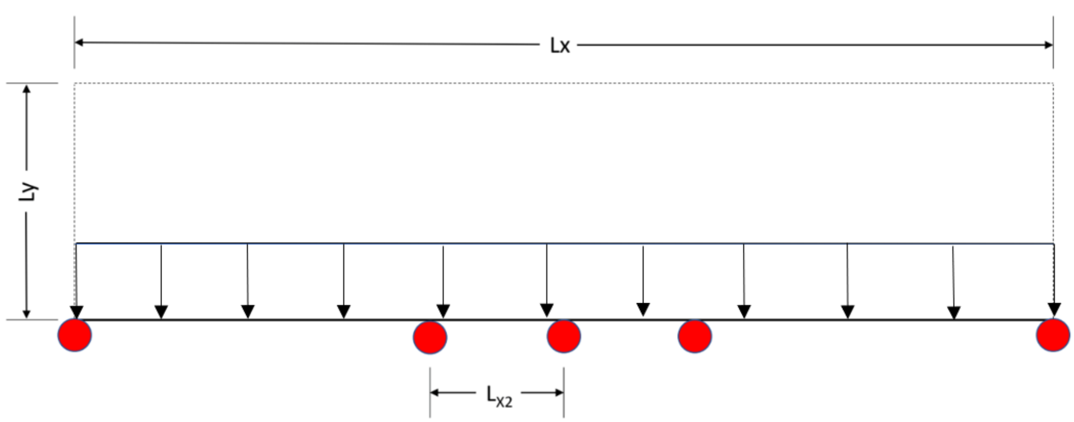


Figure 8. Setup of Three Middle Support Bridge

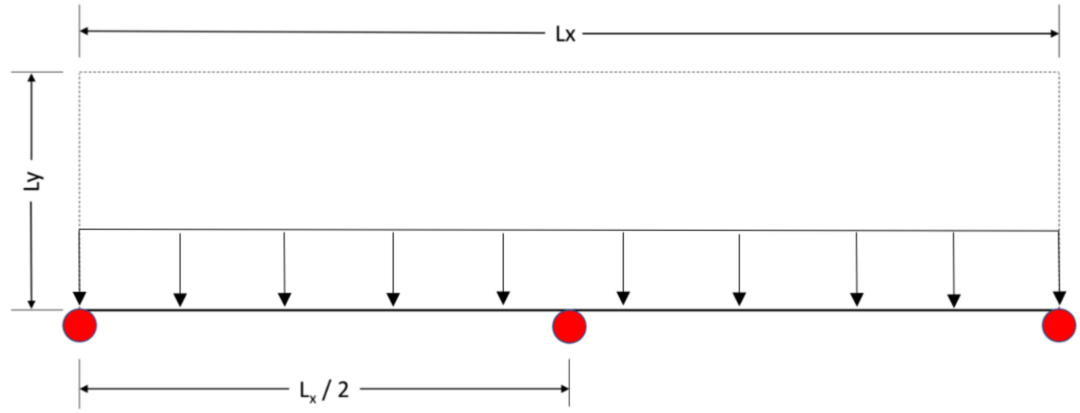


Figure 9. Setup of One Middle Support Bridge

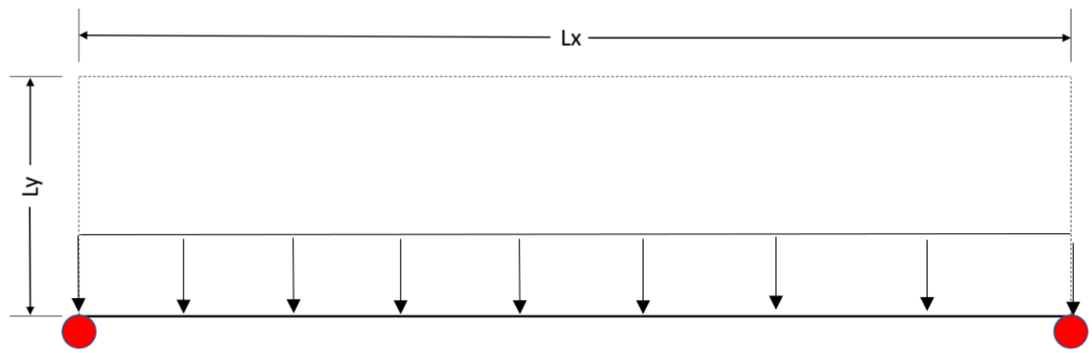


Figure 10. Setup of Simply Supported Bridge

For all bridge designs, two different conceptual design variables are determined. These variables are the total length of the bridge ground structure (L_x), and the total height of the bridge ground structure (L_y). For the bridge designs with three middle supports, a third conceptual design variable is established, which is the distance from the center support to the two other supports in the interior of the bridge span (L_{x2}). The width of the bridge deck is assumed to be equal for all designs.

Due to time constraints, only fourteen total designs are analyzed for this application. The length of the bridge (L_x) is fixed at 72 meters for all designs to simplify the analysis and gain a better understanding of the trade-offs between cost, compliance, and aesthetics. 72 meters was determined to be an appropriate length for a bridge crossing a 42-meter river and is a realistic length for a truss bridge. The height of the ground structure (L_y) is varied between three discrete values. These values are 24 m, 36 m, and 48 m. This generates three different one-middle support designs and three different simply supported designs. The distance between middle supports (L_{x2}) is varied between three discrete values of 12 m, 15 m, and 18 m. This generates nine different designs for the bridges with three middle supports, however, due to time constraints and insufficient computational power, the design with an L_y = 48 m and L_{x2} = 18 m has to be eliminated, so that only eight different designs with three middle supports are evaluated.

4.2 Compliance-based Topology Optimization

To determine the optimal topology of each design, a compliance-based topology optimization is necessary. The goal of the optimization is to determine the proper allocation of volume in the ground structure with the goal of minimizing the compliance of the design. A MATLAB optimization code for finding an optimal topology with minimum compliance was written by Navid Changizi and adapted by the author for this thesis. The optimization problem is defined as:

$$min_a \; c(d) = f^T d(a) \tag{6}$$

$$s.t. K(a)d(a) = f$$

$$a^T 1 \leq v$$

$$a_{min} < a \leq a_{max}$$

where *K(a)* is the global stiffness matrix, and *d(a)* is the displacement vector under the load *f* that is derived from finite element analysis [14]. The global stiffness matrix *K(a)* is an assembly of element stiffness matrices and is further described in [14]. The design variable is the cross-sectional area of each individual member, *a*, which is stored in the vector *a*. The minimum area that the member can take is a_{min} and the maximum area is a_{max}. [14]. These values are determined from the minimum and maximum dimensions of W-shaped steel sections in the AISC Steel Design Manual [15]. The variable *v* denotes the total volume of material [14]. Additionally, the 1 is the vector of element lengths.

The objective function for this problem is *c(d),* which is the compliance of the domain, and the goal of the optimization process is to minimize this objective function by changing the variables composing the vector *a*. The ground structure used for this problem is defined as a partially full connectivity, which means that all elements are connected to each node within a specified radius. The nodes for the ground structure are placed every three meters in the x and y directions, and the radius for the partially full connectivity is 6.71 meters so that elements are connected to all nodes within 6.71 m. The ground structures for each of the fourteen designs are displayed in Appendix A. The load applied on each bridge design is assumed to be a distributed load of 1 lb/in on the bridge deck, and the modulus of elasticity, E, for each member is assumed to be 1 ksi to simplify the optimization.

The partially connected ground structure used for this problem is a lower fidelity model when compared to a full connectivity where elements are connected to every node in the ground structure. Figure 11 shows the partially full connectivity as compared to a full connectivity.

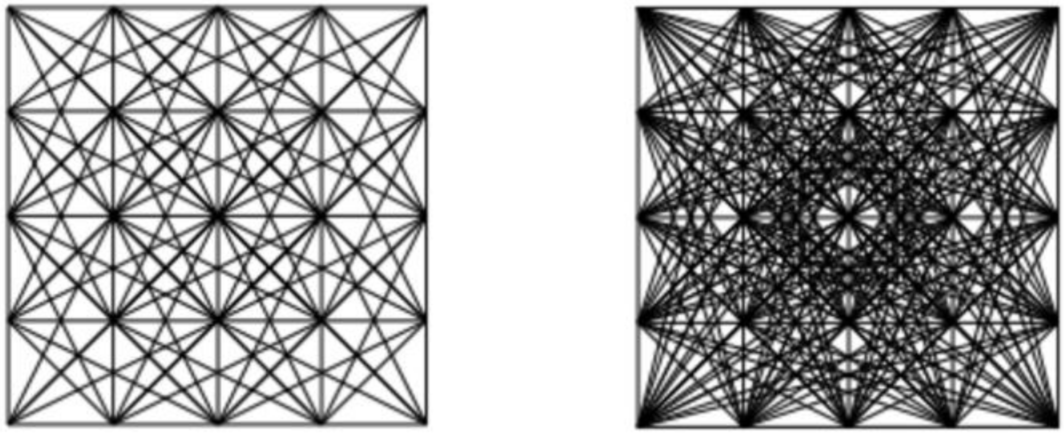


Figure 11. Partially full connectivity (left) vs. full connectivity (right) [14]

The reason for using this low fidelity model is so that money and time can be saved in the optimization process, and designs can be eliminated based on the results obtained from the less costly method. Also, as described in [14], a partially connected mesh serves as an upper bound to the full connected mesh, and when combined with the lower bound method described in [14], the results from the full connected mesh are guaranteed to be between the two bounds. The non-dominated designs can then be evaluated in the future at the more computationally expensive, higher fidelity models such as the full connected mesh.

The constraint for the optimization function, v, is the total amount of material and is determined for each design based on the width and height of the ground structure. For this problem, the volume constraint is determined by the equation

$$v = \alpha * L_x * L_y \tag{7}$$

where α is a constant multiplier, and L_x and L_y are the width and height of the ground structure, respectively. The value of α is set as 0.5 for this problem.

Table 1 shows all fourteen designs considered for this problem, as well as the volume constraint for each design.

Table 1. Bridge Designs

Design Number	Design	Volume Constraint (in^3)
1	Three Middle Supports: L_y = 24 m, L_{x2} = 12 m	1.34 x 10^6
2	Three Middle Supports: L_y = 36 m, L_{x2} = 12 m	2.01 x 10^6
3	Three Middle Supports: L_y = 48 m, L_{x2} = 12 m	2.68 x 10^6
4	Three Middle Supports: L_y = 24 m, L_{x2} = 15 m	1.34 x 10^6
5	Three Middle Supports: L_y = 36 m, L_{x2} = 15 m	2.01 x 10^6
6	Three Middle Supports: L_y = 48 m, L_{x2} = 15 m	2.68 x 10^6
7	Three Middle Supports: L_y = 24 m, L_{x2} = 18 m	1.34 x 10^6
8	Three Middle Supports: L_y = 36 m, L_{x2} = 18 m	2.01 x 10^6
9	Simply Supported: L_y = 24 m	1.34 x 10^6
10	Simply Supported: L_y = 36 m	2.01 x 10^6
11	Simply Supported: L_y = 48 m	2.68 x 10^6
12	One Middle Support: L_y =24 m	1.34 x 10^6
13	One Middle Support: L_y = 36 m	2.01 x 10^6
14	One Middle Support: L_y = 48 m	2.68 x 10^6

The MATLAB program then performs an iterative gradient-based optimization by changing the values of *a* for each iteration and measuring the resulting effect on the objective compliance function. When the objective function becomes non-decreasing, the iteration stops, and the compliance is measured. Elements in the ground structure who have very small *a*-values are eliminated from the topology so only elements that have sufficient area remain. This produces an optimal topology, which represents the optimal geometry of a truss superstructure if its compliance is to be minimized (maximum stiffness).

The optimal topologies established by the compliance-based optimization are shown in Appendix B. It is important to note that the L_y values for each design are not the overall height of the bridge but instead the overall height of the bridge ground structure. For most of the designs, the actual height of the optimal geometry is a lot smaller than the L_y value.

4.3 Applying Aesthetic Utility Function

With the optimal topologies established from the optimization, the utilities for proportion, order, and harmony are calculated. A sample calculation for the utility function will be shown for the first design (Three Middle Supports: L_y = 24 m, L_{x2} = 12 m), and then the values for each design will be ranked and tabulated.

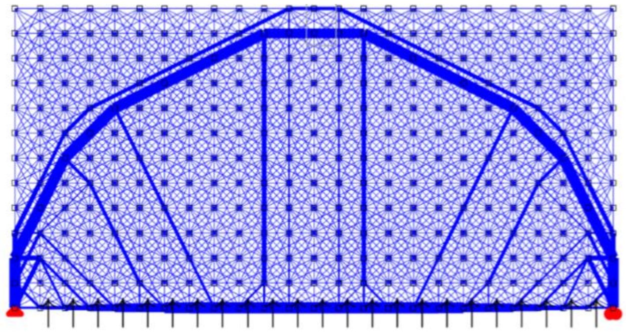
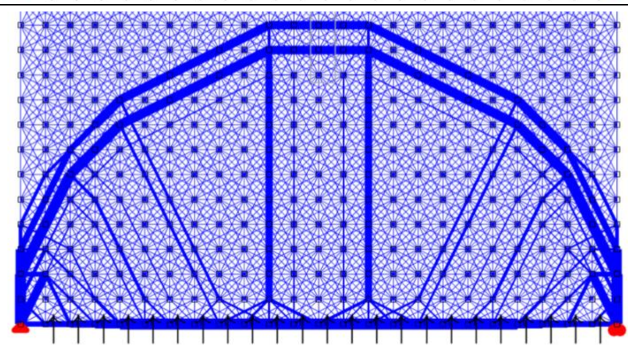
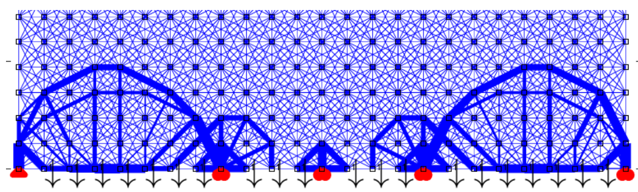
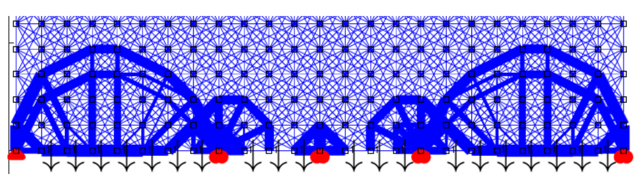
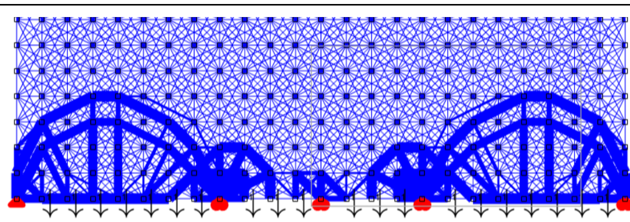
Proportion

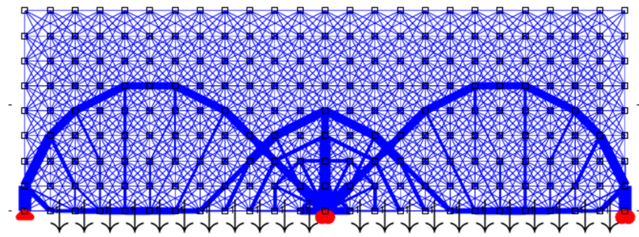
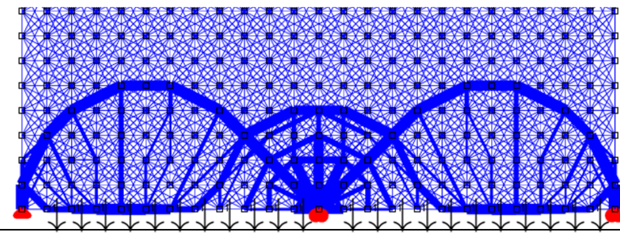
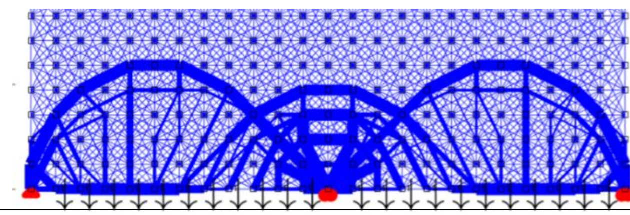
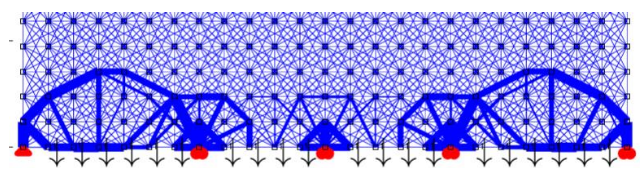
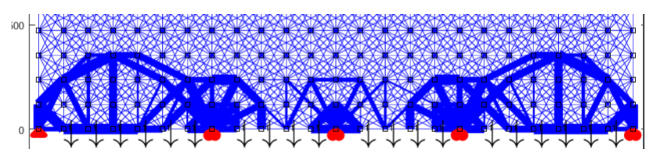
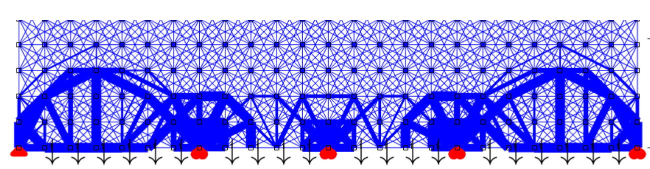
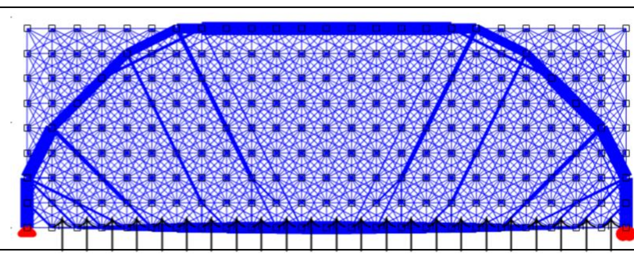
As described in Chapter 3, the utility for proportion is determined by comparing the average span to depth ratio of the bridge to the Golden ratio of 1.618. For the first design, the height of the two outer spans is 12 meters, and the average height of the two inner spans is (6 m + 3 m) / 2 = 4.5 meters. The length of both inner spans is L_{x2} = 12 meters, and the outer spans are both (72 m – 12m – 12m) / 2 = 24 meters. The number of spans, *n*, is equal to 4. With this data, the utility function is then calculated with Equation (3):

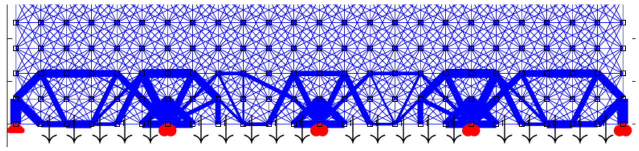
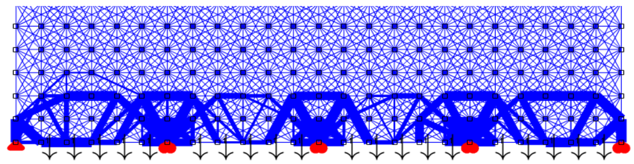
$$u_{proportion,1} = \frac{2 * \left[\frac{\left(\frac{24\ m}{12\ m}\right) - 1.618}{1.618}\right] + 2 * \left[\frac{\left(\frac{12\ m}{4.5\ m}\right) - 1.618}{1.618}\right]}{4} = \mathbf{0.442}$$

This function is applied to the remaining topologies, and the results are tabulated in Table 2 and designs are ranked from best to worst in terms of their proportion utility, $u_{proportion}$.

Table 2. Aesthetic Design Ranking by Proportion

Rank	Design	Utility	Topology
1	Simply supported: Ly = 36m	0.236	
1	Simply supported: Ly = 48m	0.236	
3	Three Middle Supports: Ly = 24 m, Lx2 = 12 m	0.442	
3	Three Middle Supports: Ly = 36 m, Lx2 = 12 m	0.442	
3	Three Middle Supports: Ly = 48 m, Lx2 = 12 m	0.442	

6	One Middle Support: Ly = 24 m	0.483	
6	One Middle Support: Ly = 36 m	0.483	
6	One Middle Support: Ly = 48 m	0.483	
9	Three Middle Supports: Ly = 24 m, Lx2 = 15 m	0.494	
9	Three Middle Supports: Ly = 36 m, Lx2 = 15 m	0.494	
9	Three Middle Supports: Ly = 48 m, Lx2 = 15 m	0.494	
12	Simply Supported: Ly = 24 m	0.854	

12	Three Middle Supports: Ly = 36 m, Lx2 = 18 m	0.854	
12	Three Middle Supports: Ly = 36 m, Lx2 = 18 m	0.854	

According to the function, the best designs for proportion are the simply supported designs with an L_y = 36 m and L_y = 48 m. The optimal topologies of these two designs have a height of 36 meters, producing a span to depth ratio of 2, which is the closest any of the designs come to the Golden ratio of 1.618. The worst designs are the three middle support designs with L_{x2} = 18 m, as these optimal topologies have an average span to depth ratio of 3. This function appears to be successful in quantifying the proportional design of the bridges, as the higher ranked bridges appear more proportional and greater in height than the lower ranked bridges, which are low in height and thin.

One challenge that arises from the utility function for proportion is that it is difficult to compare the proportion of the simply supported bridges to the bridges with three middle supports. The supports in the middle prevent the optimal topology for the three middle support bridges from having a higher height, while the simply supported bridges need a higher height since their span is so much longer. The utility function also only measures the proportion on a local scale as opposed to a global height to length ratio for the bridge. This is why the simply supported bridges appear so different from the other bridges (in terms of proportion) and may be why they are ranked much higher than the other designs. Nonetheless, the simply supported bridges with a very large depth appear aesthetic and seem to be proportioned well.

Order

The order for each design is determined by finding the deviation of directions of all strut elements in the topology and determining the number of times elements cross over each other. Since the ground structure is a partially connected mesh, the elements are limited in the amount of directions they can take. The elements are angled at either 0°, 26.565°, 45°, 63.43°, or 90° from the horizontal decking. For the first design, there is a total of 46 strut elements, with 16 taking a 45° angle (cosine = 0.707), 12 at a 63.43° angle (cosine = 0.447), six at a 26.565° angle (cosine = 0.894), 10 hangars at a 90° angle (cosine = 1.0), and two horizontal struts with angles of 0° (cosine = 0). The variance of the cosines of these angles is calculated to be **0.101.** Additionally, from an inspection of the optimal topology, the elements cross over each other sixteen times, so X = 16 for the equation.

After finding the variance of the cosines and number of crosses for each of the fourteen designs, the average variance for all designs was found to be 0.094, and the average number of crosses was 27.71. To make both terms in the utility function equally weighted, the value of α was calculated to be 300, so that the average for the second term of the equation is equal to 27.71 / 300 = 0.092, which is close to 0.094. Using Equation (4), the order utility function for this set of designs to be

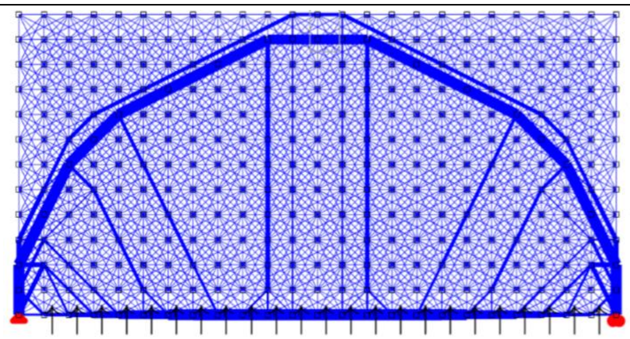
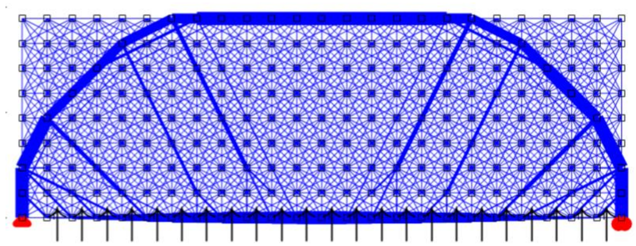
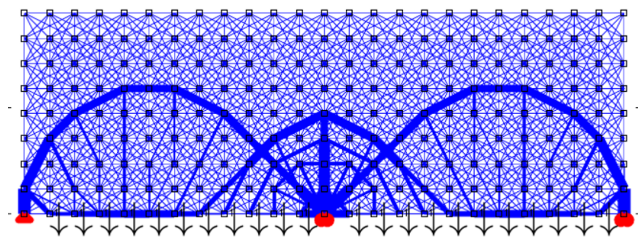
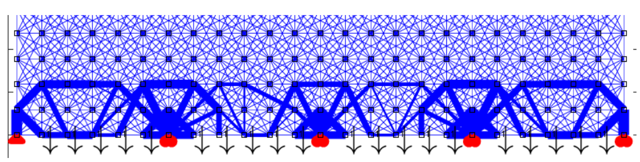
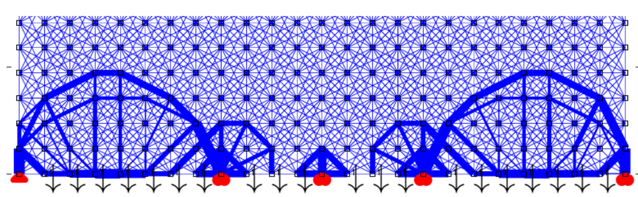
$$u_{order} = \frac{\sum_{i=1}^{n}[\cos(\theta_i) - \mu]^2}{n} + \frac{X}{300}$$

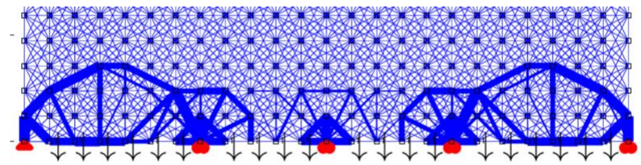
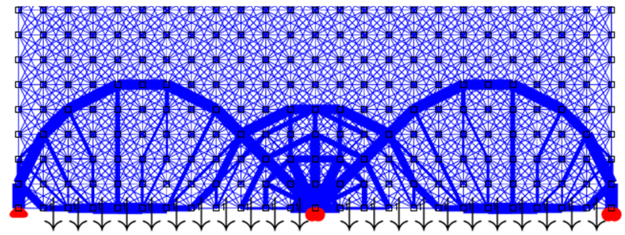
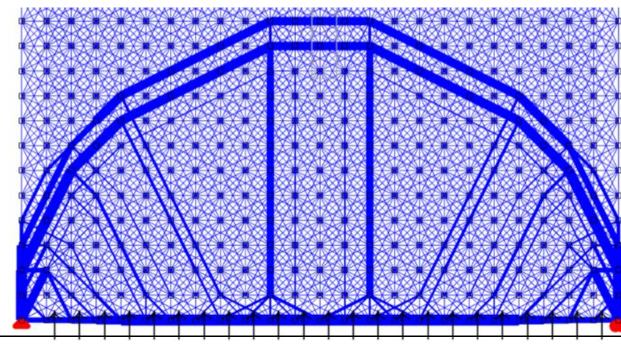
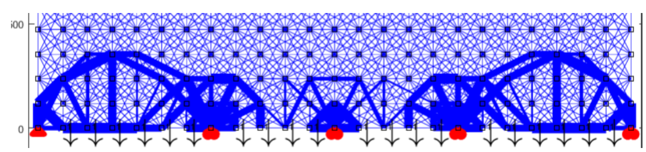
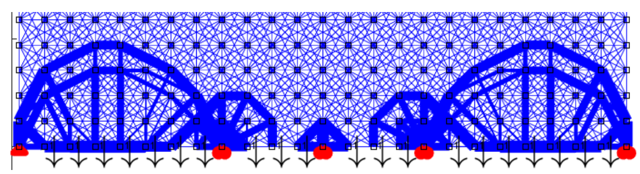
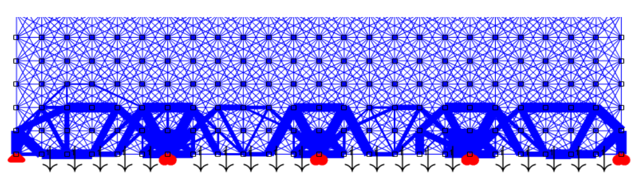
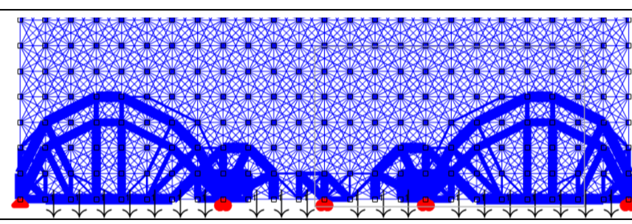
so that the order for the first design can be quantified as:

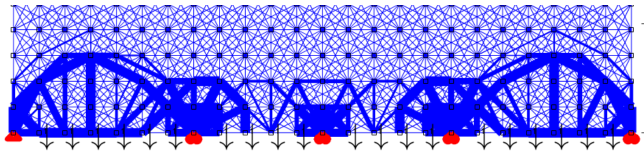
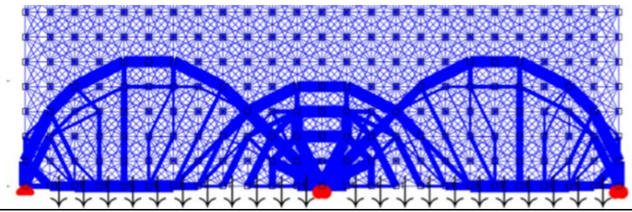
$$u_{order,1} = 0.101 + \frac{16}{300} = \mathbf{0.154}$$

The results for all fourteen designs are tabulated in Table 3 and designs are ranked from best to worst in terms of order.

Table 3. Aesthetic Design Ranking by Order

Rank	Design	Utility	Topology
1	Simply supported: Ly = 36m	0.108	
2	Simply Supported: Ly = 24 m	0.121	
3	One Middle Support: Ly = 24 m	0.127	
4	Three Middle Supports: Ly = 24 m, Lx2 = 18 m	0.144	
5	Three Middle Supports: Ly = 24 m, Lx2 = 12 m	0.154	

6	Three Middle Supports: Ly = 24 m Lx2 =15 m	0.165	
7	One Middle Support: Ly = 36 m	0.166	
8	Simply supported: Ly = 48m	0.187	
9	Three Middle Supports: Ly = 36 m, Lx2 = 15 m	0.214	
10	Three Middle Supports: Ly = 36 m, Lx2 = 12 m	0.223	
11	Three Middle Supports: Ly = 36 m, Lx2 = 18 m	0.227	
12	Three Middle Supports: Ly = 48 m, Lx2 = 12 m	0.257	

13	Three Middle Supports: Ly = 48 m, Lx2 = 15 m	0.260	
14	One Middle Support: Ly = 48 m	0.262	

The highest ranked designs by the order function are the simply supported designs with L_y = 36 m and L_y = 24 m (Designs 9 and 10). These designs appear to be the most orderly because they have fewer strut elements and fewer crossings, as well as elements which are oriented in similar directions. The least ordered designs are Design 14 and 6, which are the one middle support with L_y = 48 m and the three middle supports with L_y = 48 m and L_{x2} = 15 m. These designs have many struts that overlap and seem to be oriented in many different directions. The lowest ranked designs also appear very cluttered and confusing, which is a sign of disorder. Overall, the aesthetic utility function for order seems to have done a good job with separating the less confusing designs from the more confusing. The designs with lower values for order tend to be the designs with L_y = 24 m, which means they have less volume and thus less elements to cause disorder, while the less ordered designs have L_y = 48 m, indicating more potential for elements to cause disorder since there is more volume.

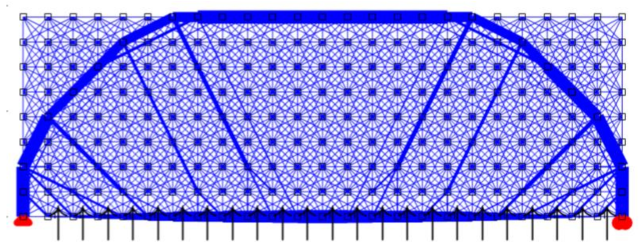
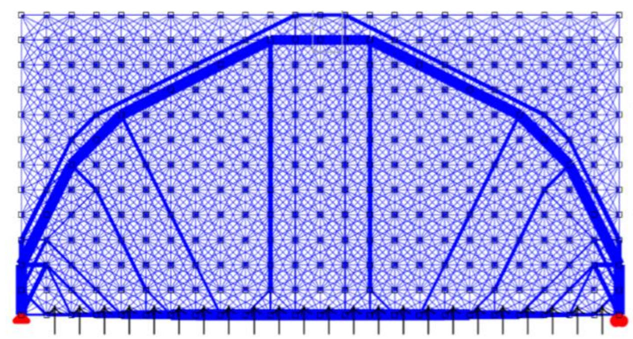
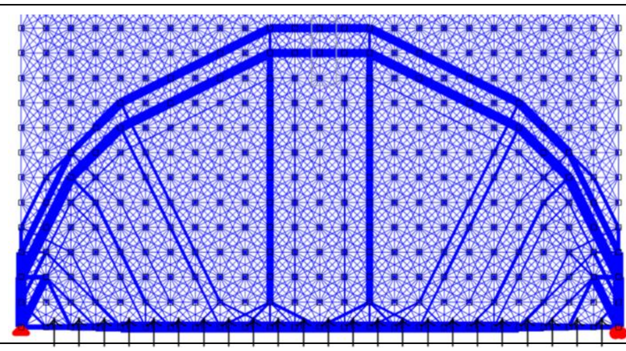
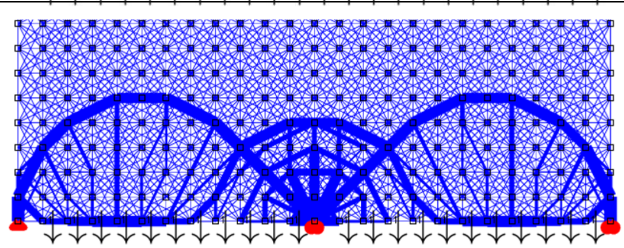
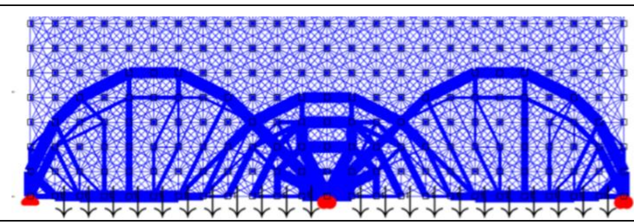
Harmony

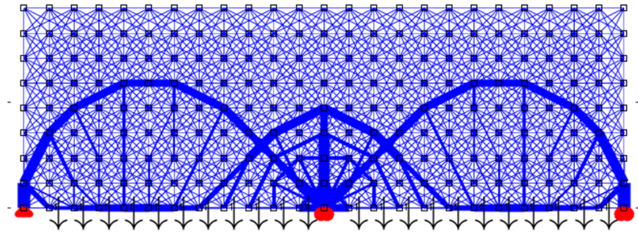
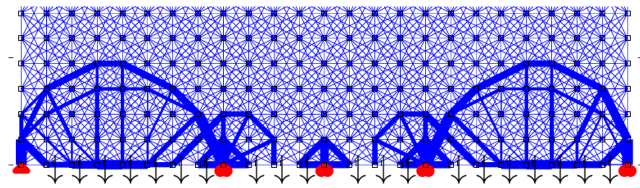
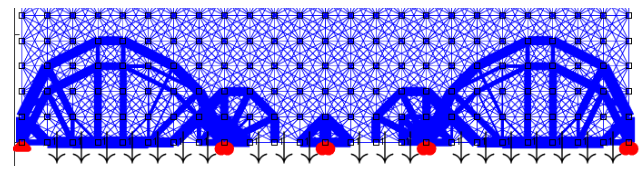
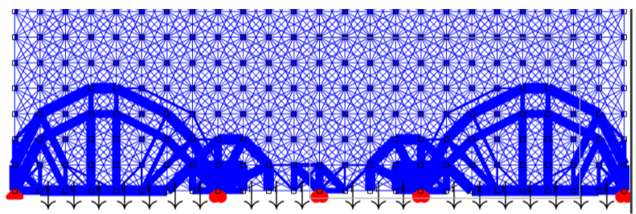
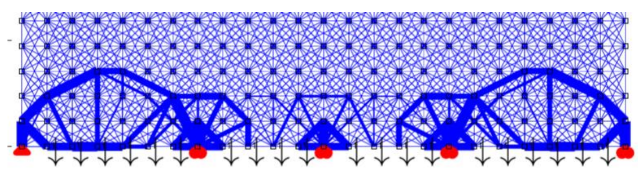
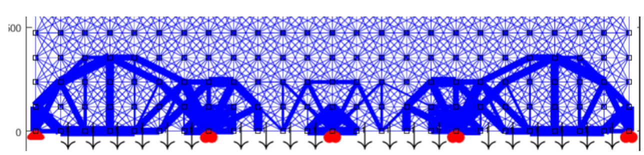
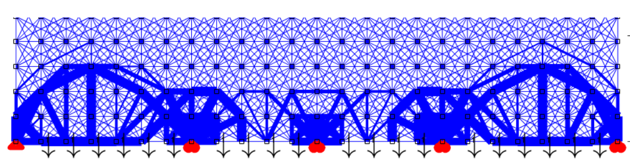
The utility function for harmony measures how smooth the shape of the bridge is, as well as whether it maintains the same shape throughout the entire length of the bridge. The function takes the difference in angle between adjacent cords and finds the average change in angle for each span. In the case of Design 1, there are two outer spans of equal length (24 m) and shape and two inner spans of equal length (12 m) and shape. Going from left to right from the far left support, the exterior chord elements in the outer span have angles of 90°, 63.43°, 26.565°, 0°, 26.565°, and 45° with respect to the horizontal. This gives angular differences between adjacent elements of: 26.565°, 36.865°, 26.565°, 26.565°, and 18.435°. These angles are then converted to radians by multiplying each by π/180, and then the average of the five angles is determined to be 0.470 radians. The same process is then performed for the inner span, which has exterior chord elements of 45°, 0°, 45°, and 90°. This produces an average change in angle of 0.785 radians for the inner span. With these averages calculated, the first term in equation (5), (a weighted average for the entire bridge) is computed as:

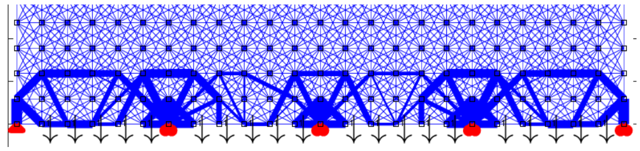
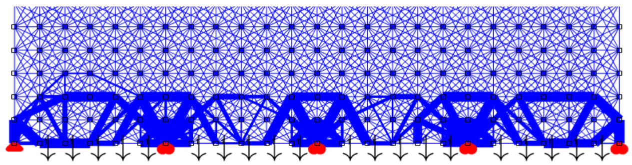
$$\left[2 * \left(\frac{12\ m}{72\ m}\right) * (0.785)\right] + \left[2 * \left(\frac{24\ m}{72\ m}\right) * (0.470)\right] = 0.575.$$

The second term for the harmony utility function is the variance of the averages calculated for each span, so the variance of 0.785 and 0.470 is calculated, and is equal to 0.025. Adding these two values together produces a utility value for harmony of 0.575 + 0.025 = 0.600 for the first design. This process is performed on the remaining 13 designs, and the results are tabulated in Table 4. Table 4 also ranks all designs in terms of harmony.

Table 4. Aesthetic Design Ranking by Harmony

Rank	**Design**	**Utility**	**Topology**
1	Simply supported: Ly = 24 m	0.393	
1	Simply supported: Ly = 36 m	0.393	
1	Simply supported: Ly = 48 m	0.393	
4	One Middle Support: Ly = 36 m	0.409	
4	One Middle Support: Ly = 48 m	0.409	

6	One Middle Support: Ly = 24 m	0.418	
7	Three Middle Supports: Ly = 24 m, Lx2 = 12 m	0.600	
7	Three Middle Supports: Ly = 36 m, Lx2 = 12 m	0.600	
7	Three Middle Supports: Ly = 48 m, Lx2 = 12 m	0.600	
10	Three Middle Supports: Ly = 24 m, Lx2 = 15 m	0.766	
10	Three Middle Supports: Ly = 36 m, Lx2 = 15 m	0.766	
10	Three Middle Supports: Ly = 48 m, Lx2 = 15 m	0.766	

13	Three Middle Supports: Ly = 24 m, Lx2 = 18 m	0.969	
13	Three Middle Supports: Ly = 36 m, Lx2 = 18 m	0.969	

From analysis of Table 4, the most harmonic designs are the simply supported designs, as all have smooth cords that produce an arch-like shape. These designs also have no variance between spans because there is only one span. The one-middle-support bridge designs follow closely behind in terms of their harmonic utility value. This makes sense because these designs also have arch-like shapes and appear very smooth and harmonious. On the other end of the spectrum, the designs with L_{x2}=18m are the least harmonic according to the utility function. This makes sense too because these designs are flat and have sharp edges. The harmony utility function appears to do a good job overall of ranking designs with smooth, arch-like shapes ahead of designs with sharp edges and rectangular shapes. One shortcoming of the harmony function, however, is it does not take into consideration the relative distance of adjacent chord elements. In the case of the simple L_y = 24 m bridge, the harmony function calculates a very low value due to minimal change in angle between elements, but the fact that the top chord element is so much longer than the others causes the shape to not be nearly as smooth. This shortcoming could be potentially fixed for a future application of the utility function.

Overall Aesthetic Utility

With the values for all portions of the utility function established, the values can now be combined to determine an overall aesthetic for each design and rank them based on their elegance. Here it is assumed that each attribute – proportion, order, and harmony—have equal weight, although a survey of individuals could provide a better idea of the general ordering of these attributes. To ensure that all three criteria are weighted equally, the average utility amongst all fourteen designs is found for each criterion. The average for proportion is 0.521, order is 0.187, and harmony is 0.604. To make all averages equal, the order values are multiplied by (0.521 / 0.187) = 2.8, and harmony values multiplied by (0.521 / 0.604) = 0.863 so that all averages are equal to 0.521. This ensures that no one criterion dominates over another. The results are then tabulated in Table 5 and summed to produce the overall aesthetic utility for each design.

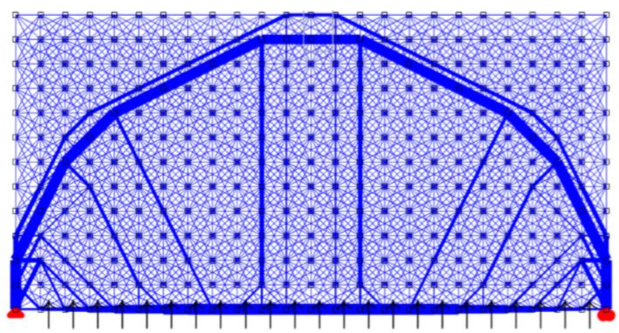
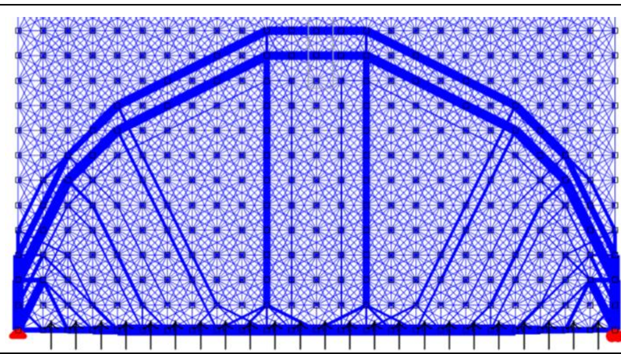
Table 5. Results of Aesthetic Utility Function

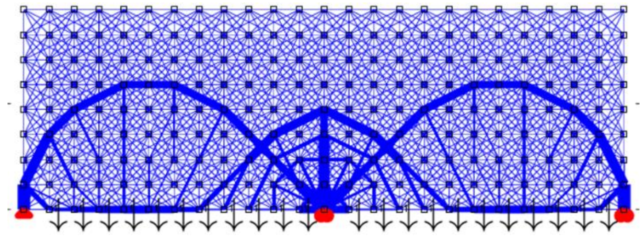
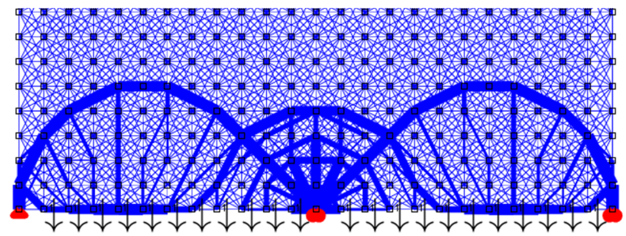
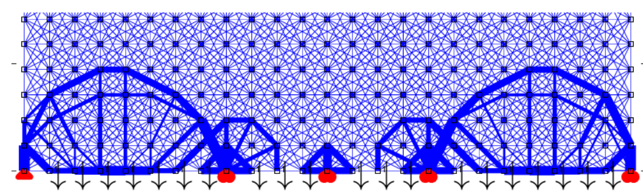
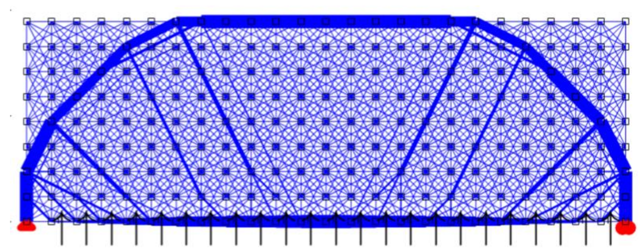
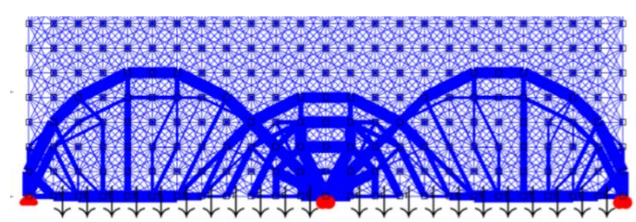
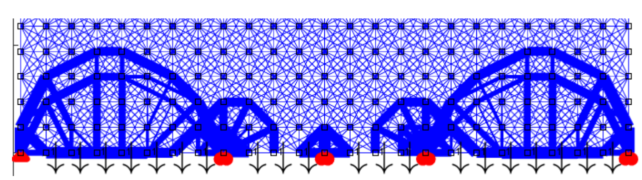
Design #	Design	u_{order}	$u_{proportion}$	$u_{harmony}$	U(x)
1	Ly = 24, Lx2 = 12	0.431	0.442	0.518	**1.391**
2	Ly = 36, Lx2 = 12	0.624	0.442	0.518	**1.584**
3	Ly = 48, Lx2 = 12	0.720	0.442	0.518	**1.680**
4	Ly = 24, Lx2 = 15	0.462	0.494	0.661	**1.617**
5	Ly =36, Lx2 = 15	0.599	0.494	0.661	**1.754**
6	Ly = 48, Lx2 = 15	0.728	0.494	0.661	**1.883**
7	Ly = 24, Lx2 = 18	0.403	0.854	0.836	**2.094**
8	Ly = 36, Lx2 = 18	0.636	0.854	0.836	**2.326**

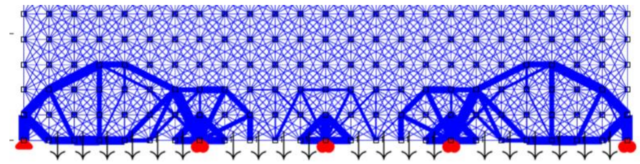
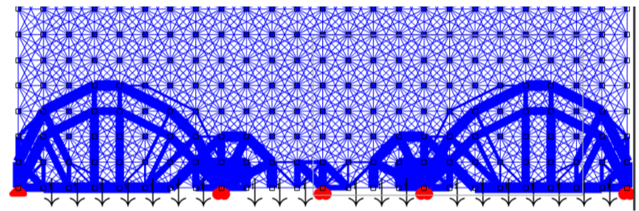
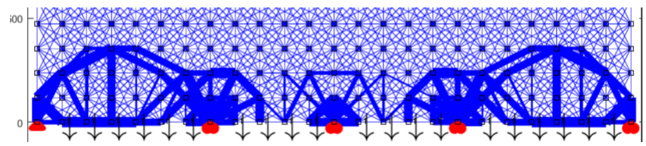
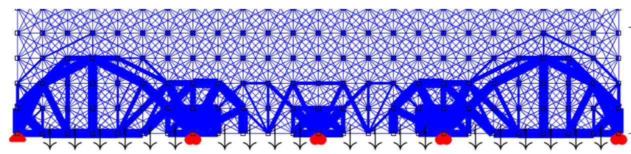
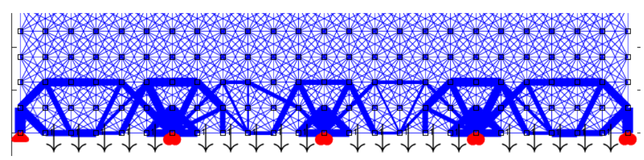
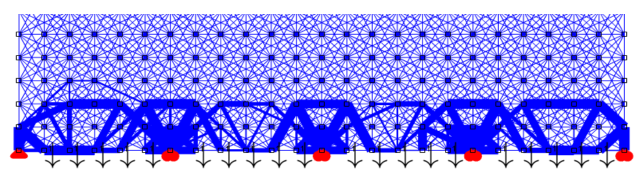
9	Simple: Ly = 24	0.339	0.854	0.339	**1.532**
10	Simple: Ly = 36	0.302	0.236	0.339	**0.877**
11	Simple: Ly = 48	0.524	0.236	0.339	**1.098**
12	One-Mid: Ly = 24	0.356	0.483	0.361	**1.200**
13	One-Mid: Ly = 36	0.465	0.483	0.353	**1.301**
14	One-Mid: Ly = 48	0.734	0.483	0.353	**1.570**

With the aesthetic utility values established, the fourteen designs are ranked from most aesthetic to least aesthetic in Table 6.

Table 6. Overall Aesthetic Utility Ranking

Rank	**Design**	**Utility**	**Topology**
1	Simply supported: Ly = 36m	0.877	
2	Simply supported: Ly = 48m	1.098	

3	One Middle Support: Ly = 24 m	1.200	
4	One Middle Support: Ly = 36 m	1.301	
5	Three Middle Supports: Ly = 24 m, Lx2 = 12 m	1.391	
6	Simply Supported: Ly = 24 m	1.532	
7	One Middle Support: Ly = 48 m	1.570	
8	Three Middle Supports: Ly = 36 m, Lx2 = 12 m	1.584	

9	Three Middle Supports: Ly = 24 m, Lx2 = 15 m	1.617	
10	Three Middle Supports: Ly = 48 m, Lx2 = 12 m	1.680	
11	Three Middle Supports: Ly = 36 m, Lx2 = 15 m	1.754	
12	Three Middle Supports: Ly = 48 m Lx = 15 m	1.883	
12	Three Middle Supports: Ly = 24 m, Lx2 = 18 m	2.094	
14	Three Middle Supports: Ly = 36 m, Lx2 = 18 m	2.326	

As expected, the designs with the lowest values are the simply supported designs with L_y = 36 m and L_y = 48 m (Designs 10 and 11). Design 10 has the lowest values in all three categories, and both 10 and 11 have significantly better proportioning than any of the other designs, according to the function. Designs 12 and 13 are the third and fourth best designs in terms of aesthetics and this is understandable because these designs are both harmonious and ordered. The worst overall designs by far based on the function are Designs 7 and 8, which are the L_{x2}=18 m designs. This is not surprising either because although these designs are somewhat ordered, they have very bad proportioning and sharp, unharmonious shapes.

Looking at the rankings shows that the aesthetic utility function overall does a reasonably good job of ranking and separating designs based upon their aesthetic elegance. While people will obviously have differing preferences on what they think looks the best and what looks the worst, most people should generally agree with which designs that the function ranks as more aesthetic and which designs the function grades as less aesthetic. It is also interesting to see how differing values that the function assigns to each design actually represent visual differences between designs. Designs that have more arch-like shapes have lower values for the harmony function, and designs which are more cluttered actually have lower values for the order function. The utility function in total successfully applies the three criteria described by Leonhardt, and appears to be a good start in ranking designs for their elegance.

Although the utility function provides an initial attempt and general idea about how structural aesthetics can be quantified, there are areas that require improvement. A significant challenge is with the overall portioning aspect because of the difficulty in comparing designs with different span lengths, or single span versus multi-span and yet, early in the conceptual phase of the design, engineers might be considering each of these options. For example, it is difficult to compare a three-middle support span with a smaller overall height to a simply supported bridge with a large overall height. The Golden ratio also might not be the most ideal proportion for span length to depth because of how it sometimes produces bulky bridge designs. The harmony function also requires some improvement because of not taking relative length of adjacent elements in consideration. Despite these limitations, we proceed by showing how the values generated by the utility function for the fourteen different designs can be used in conjunction with compliance and cost to broadly compare structural design alternatives and reveal trade-offs that might occur between the three facets of structural art.

4.4 Cost Function

To compare the cost of each bridge generated by the optimization process, an overall cost function for each bridge is determined. The cost function is not meant to be the actual cost of each bridge, but is used to generate a realistic cost that can allow for designs to be compared. For these bridge designs, many assumptions are made about the type of material as well as the dimensions of the bridge deck, bridge piers and foundation. To simplify the cost of the bridge, the bridge is broken into four main components: bridge foundation/piers, superstructure/steel, bridge decking, and earth excavation. The cost for each of these four components is added together to determine an overall bridge cost. The overall cost function used for determining the cost of each bridge is

$$Total\ Cost\ (\$) = \sum_{i=1}^{n} C_i = C_{foundation} + C_{superstructure} + C_{deck} + C_{excavation} \qquad (8)$$

Foundations and Bridge Piers

All the bridge designs analyzed for this problem have a certain number of bridge piers and foundations depending on the design. The piers and foundations are assumed to be constructed with normal weight concrete, so to find the total cost for these components, the volume of concrete needed and material cost for the concrete is determined.

The total volume of concrete can be determined by summing the volume of piers and the volume of the foundations. The foundations for each pier are assumed square spread footings with dimensions of 4 m x 4 m x 2 m. These dimensions are determined based on typical footing sizes for a bridge of this type and are a size that these footings could be in real life. The volume for each of these foundations are calculated simply with the equation

$$V_{ftg} = (4\ m) * (4m) * (2m) = 32\ m^3 \tag{9}$$

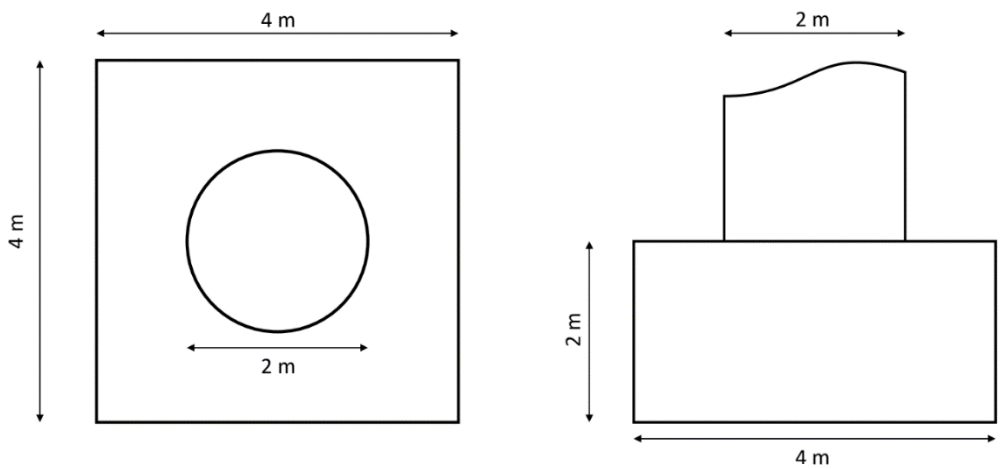


Figure 12. Bridge Foundations

Each square spread footing supports a cylindrical bridge pier that has an assumed radius of 1 meter. Like the size of the foundations, the 1-meter radius was determined based off typical dimensions for a bridge pier and is a realistic size of a bridge pier for this type of bridge. The bridge piers supported by the footings are assumed cylindrical piers with a radius of 1 meter. Figure 12 shows the plan (left) and section (right) views for the foundations.

As stated earlier, the river width for all bridge designs is 42 meters. The river profile is assumed to have a steep slope, with the land beneath the river sloping at 45 degrees from the center of the river to the edges. The two outer supports at each end of the bridge are located at a height of 5 meters above the ground, and 15 meters from the start of the 45-degree slope of the river. The volume of concrete for the bridge piers is calculated by taking the volume of a cylinder, which is $\boldsymbol{\pi}*R^2*h$ where R is the radius of the pier in meters and h is the height of the pier in meters. Therefore, the volume of the substructure for all bridge designs analyzed is calculated with the equation

$$V_{substructure} = \left(n * V_{ftg}\right) + 2 * (\pi * R^2 * h_o) + (\pi * R^2 * h_c) + 2 * (\pi * R^2 * h_i) \qquad (10)$$

where n is the total number of footings for the bridge, h_o is the height of the outer piers (5 m), h_c is the height of the pier in the middle of the bridge, and h_i is the height of the interior bridge piers. For simply supported designs, h_c and h_i are equal to 0 because there are no interior piers. For the designs with one middle support, h_i is equal to 0 because there are no interior bridge piers.

Simply Supported Designs

Figure 13 shows the layout and dimensions of the simply supported bridge designs. With only two footings (n=2) supporting two five-meter high piers on the outer edges of the bridge, the volume of concrete is calculated easily with Equation (10):

$$V_{substructure,simple} = (2 * 32m^3) + [2 * \pi * (1m)^2 * 5m] = \mathbf{95.4\ m^3}\ .$$

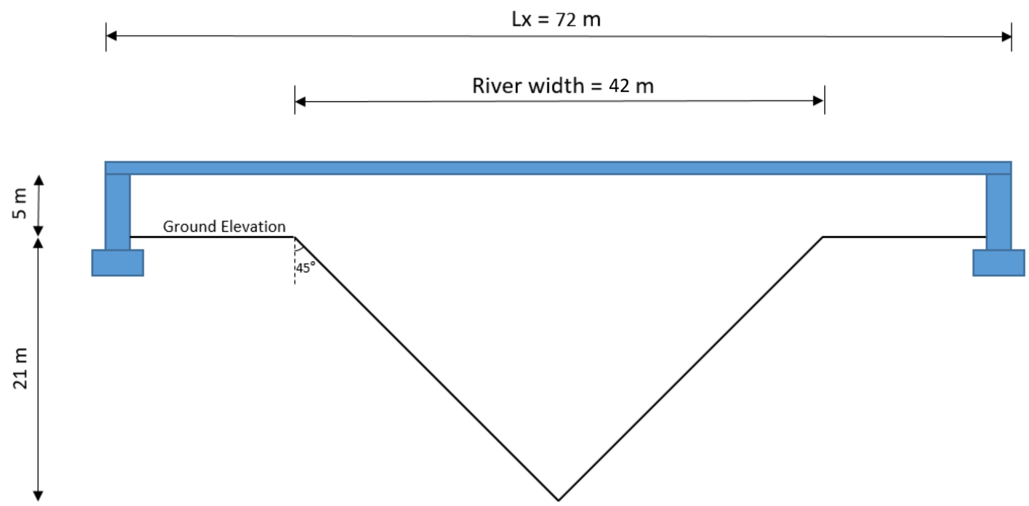


Figure 13: Simply Supported Bridge Elevation

One Middle Support Designs

Figure 14 shows the layout and dimensions of the one-middle support bridge designs. The designs have three footings (n=3) that support three piers. The two outer piers have heights of five meters, while the interior pier has a height of [(64 m) / 2]*tan(45°) = 32 meters (h_c = 32 m). Therefore, the volume of concrete for the substructure is calculated with equation (10):

$$V_{substructure,onemid} = (3 * 32m^3) + [2 * \pi * (1m)^2 * 5m] + \pi * (1m)^2 * (21m)$$

$$= \mathbf{193.4\ m^3}\ .$$

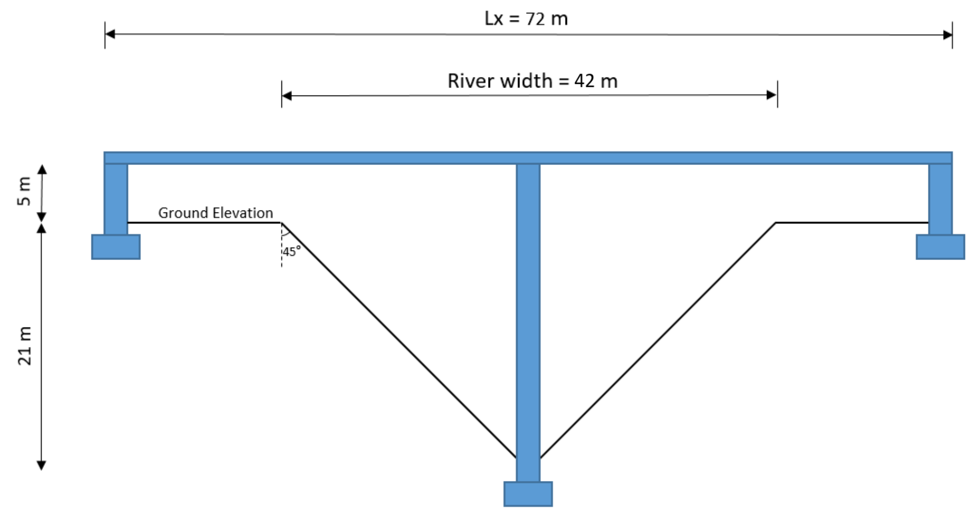


Figure 14: One Middle Support Bridge Elevation

Three Middle Support Designs

Unlike the simply supported and one middle support designs, the three middle support designs have a more complicated concrete substructure cost function. The function is directly related to the horizontal distance of the supports from the center of the bridge (L_{x2}).

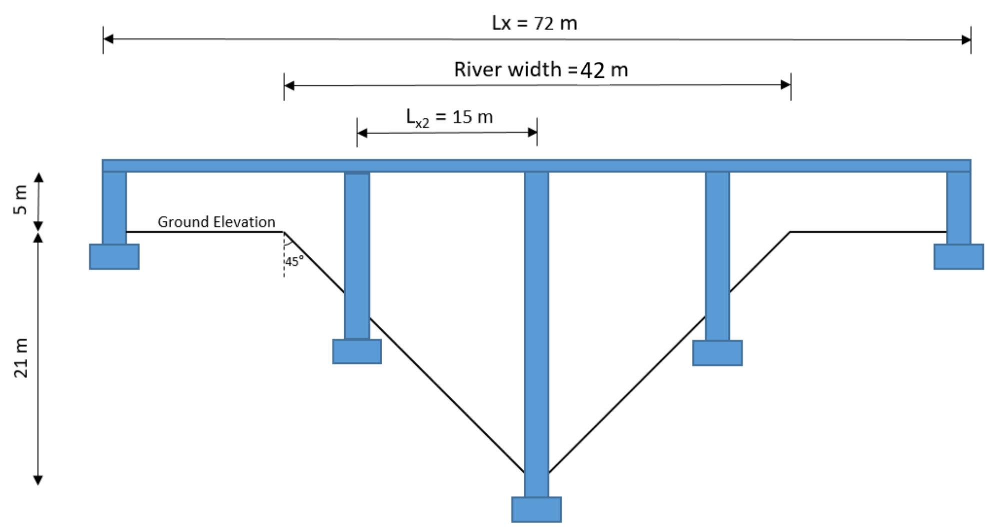


Figure 15: Elevation of Three Middle Support Bridge

Figure 15 shows a typical layout of a three-middle-support design and shows how the height of the two interior piers varies based on L_{x2}. All designs have five footings (n=5) supporting five piers. The exterior piers have heights of h_o = 5 m, and the middle bridge pier has a height of h_c = 32 m.

The interior bridge piers have a height that is equal to (64 m /2) – L_{x2} due to the 45 degree slope of the river profile as shown in the figure. Therefore, the volume of the concrete substructure for these designs can be calculated with Equation (10) which comes out to:

$$V_{substructure,3mid} = (5 * 32\ m^3) + 2 * (\pi * (1m)^2 * 5m) + (\pi * (1m)^2 * 21\ m) + 2 * [\pi * (1m)^2 * (21 - L_{x2})] = \mathbf{257.4} + \boldsymbol{2\pi} * (\mathbf{21} - \boldsymbol{L_{x2}})$$

Cost of Substructure

The cost of the concrete for the bridge foundation and piers is determined through research of material costs for bridge construction. Although material cost is constantly changing and differs for different locations, for this problem only an average, reasonable price for concrete is needed. Using the Washington State Department of Transportation (WSDOT) *Bridge Design Manual M23-50.17*, June 2017 edition, a price for concrete is determined [16]. On Page 12-36 of the manual, the price of concrete class 4000 for footings is given as a low of $400 / cubic yard and a high price of $600 / cubic yard [16]. An average cost of $500 / cubic yard is taken, and then converted to $ / cubic meter by multiplying by 1.31 cubic yards / 1m^3. Finally, this unit cost is multiplied by the volume of concrete to determine the overall substructure cost.

Steel/Superstructure

The superstructure of each bridge consists of a steel truss with an optimal topology determined by the optimization algorithm. The volume of steel for the truss portion of the bridge is already determined based on L_x and Ly. Therefore, the only information that needs to be found to estimate cost is the unit price of steel. For this example, the steel is assumed to be structural carbon steel which is a material typically used for bridge girders. Using the WSDOT *Bridge Design Manual Edition M23-50.17*, a typical low unit cost for structural carbon steel is $1.80/lb. and the high unit cost is $2.80/lb. [16]. The average of the low and high unit cost is taken, so the unit cost used for this problem is determined to be $2.30 / lb. of steel.

With the unit cost of steel in dollars per pound, the weight needs to be converted into volume by multiplying by the density of steel. The density of structural carbon steel is assumed to be about 484 lb./ft^3. Therefore, the cost of steel can then be calculated by the following function:

$$C_{superstructure} = \frac{\$2.30}{lb} * \frac{484\ lb}{ft^3} * V_{steel}\,(in^3) * \frac{1\ ft^3}{1728\ in^3} \tag{11}$$

where $C_{superstructure}$ is the total superstructure cost for the bridge, and V_{steel} is the volume of the bridge steel in cubic inches.

Bridge Deck

With the cost of the bridge foundation/piers and steel superstructure determined, the cost of the bridge decking needs to be determined. For this example, since all designs have the same total length, this cost will not vary from bridge to bridge. However, it is still useful to have this number in order to obtain a more reasonable cost for the bridge designs. Using the WSDOT *Bridge Design Manual M23-50.17*, it was determined that the type of concrete that can be used for a bridge deck is Class 4000D, which has a low unit cost of \$1000 / yd^3 and a high unit cost of \$1800 / yd^3 [16]. The average of these values is taken, so the unit cost for the bridge decking for this problem is \$1400 / yd^3. This is then converted to \$1974 / m^3 by multiplying by the conversion factor 1.31 yd^3 / m^3.

To find the volume of concrete needed for the bridge deck, a standard slab thickness and bridge deck width needs to be determined and set for all bridge designs. Taking data from the Montana Department of Transportation's *Bridge Design Standards*, a standard width for a bridge supporting a roadway is 11.4 m [17], so this is used as the width for this problem. Taking additional data from the Caltrans *Bridge Design Practice Manual*, the thickness of a typical bridge deck is between 7 inches and 10 inches [18]. Converting to meters, this is a thickness of 0.18 m to 0.25 m. The upper bound of this range is taken, so the thickness for the bridge deck for this problem is 0.25 m. With this data, the cost of the concrete needed for the bridge deck slab can be calculated with the following equation.

$$C_{deck} = \frac{\$1400}{yd^3} * \frac{1.31\ yd^3}{m^3} * 11.4\ m * 0.25\ m * 72\ m = \$376{,}337 \qquad (12)$$

Excavation

An additional cost for the excavation of earth and soil to lay the foundations for the bridge piers needs to be estimated to obtain a more accurate representation of the overall cost of constructing the bridge. The cost of excavation is given as a unit cost per volume of excavation needed. Thus, the volume of excavation for each case needs to be calculated.

According to the WSDOT *Bridge Design Manual*, excavation for a bridge is typically 1 foot extra on either side of the foundation [16]. To simplify the calculations for this problem, the excavation was assumed to be 0.5 m on either side of the foundation, which is slightly higher than 1ft. Each footing was also assumed to be 1m below the finished ground line.

With the dimensions of the foundation already set as 4m x 4m x 2m, the volume of the excavation can then be calculated for each type of footing (the two edge footings, two interior footings, and one middle footing).

Edge Footings

The calculation for the edge footings is simple because they are assumed to be beneath flat ground. Figure 16 shows the volume of excavation that is needed around these footings. The equation for the volume of edge footings is:

$$V_{edge} = (4\,m + 0.5\,m + 0.5\,m) * (4m + 0.5\,m + 0.5\,m\,) * 3\,m = 75\,m^3 \qquad (13)$$

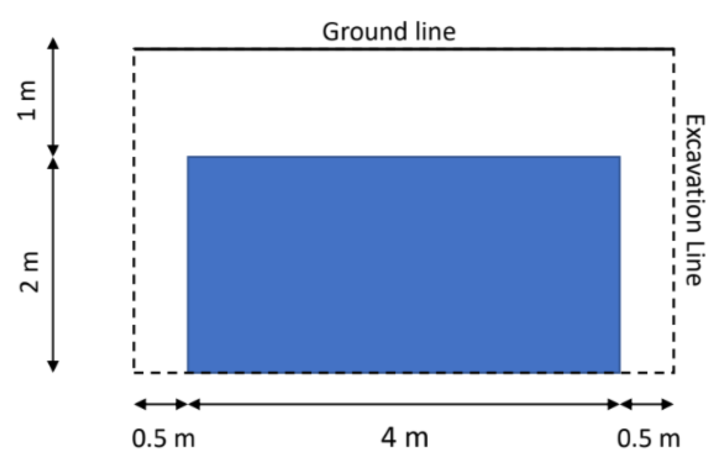


Figure 16: Exterior Footing Excavation

Interior Footings

For the interior footings, the calculation becomes more complicated because the footing is situated underneath a 45° sloped surface. Since the footing is assumed to be at least 1m below the ground elevation, one edge of the footing is 1m below the surface, while the other edge is 5m below the surface (due to the 45° slope of the land). Figure 17 shows the excavation needed for these footings. This causes the volume of the excavation to be:

$$V_{interior} = \frac{1}{2}(5\,m * 5\,m * 5\,m) + (2\,m * 5\,m * 5\,m) = 112.5\,m^3 \qquad (14)$$

Middle Footing

For the footing in the middle of the bridge span, the center of the footing is 1 m below the ground surface, while the right edges are 3m below the ground due to the 45° slope of the land. Figure 18 shows the volume of excavation needed, and the calculation for the excavation volume is:

$$V_{middle} = 2 * \frac{1}{2} * (2.5\,m * 2.5\,m * 5\,m) + (5\,m * 3\,m * 5\,m) = 106\,m^3 \qquad (15)$$

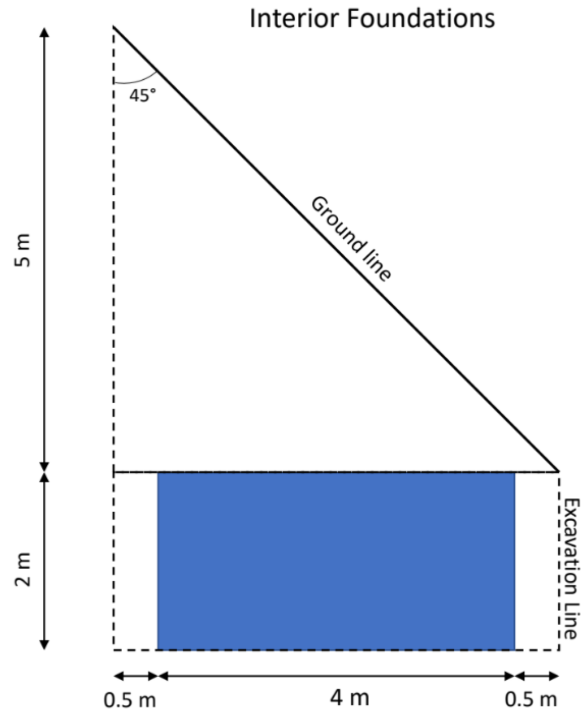


Figure 17: Interior Footing Excavation

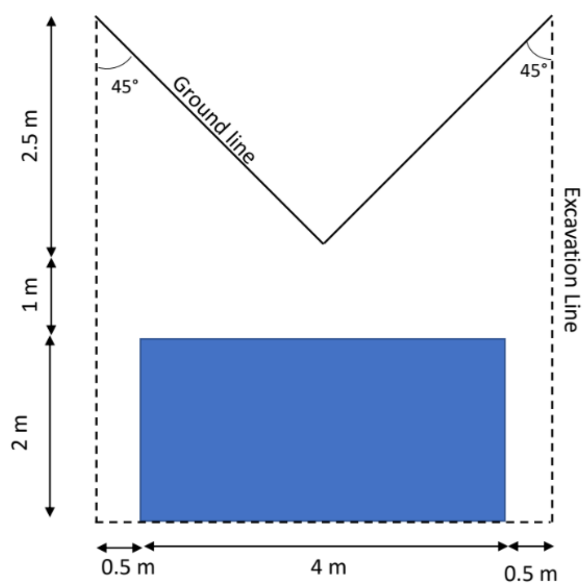


Figure 18: Middle Footing Excavation

Total Excavation Volume

The total excavation volume for each design is dependent on how many pier footings the bridge design has. For the simply supported bridge designs with only two outer bridge piers, the total excavation volume is:

$$V_{tot,simple} = 2V_{edge} = 2 * 75m^3 = \mathbf{150\ m^3} \quad (16)$$

For the designs with one bridge pier in the center, the total excavation volume is:

$$V_{tot,1mid} = 2V_{edge} + V_{middle} = 2 * (75\ m^3) + 106\ m^3 = \mathbf{256\ m^3} \quad (17)$$

For the designs with three interior piers, the total excavation volume is:

$$V_{tot,3mid} = 2V_{edge} + 2V_{interior} + V_{middle} = 2 * (75\ m^3) + 2 * (112.5\ m^3) + 106\ m^3 = \mathbf{481\ m^3} \quad (18)$$

Excavation Cost Function

With the volume of excavation calculated for the bridge, the unit cost for earth excavation can be applied to find the overall cost of excavation for each design. According to the WSDOT Bridge Design Manual Appendix 12.3-A2, the typical cost for rock and earth excavation is between \$100 / cubic yard and \$220 / cubic yard [16]. Therefore, the cost per cubic yard of excavation was assumed to be \$150 / cubic yard for this problem. To convert to meters, the \$150 is multiplied by 1.31 yd^3 / m^3 to get a unit cost of \$196.5 / m^3. With this unit price, the cost for the excavation is calculated simply by multiplying the volume of the excavation by the unit cost.

Total Costs

Tables 7 through 9 show the tabulated results of the costs for each bridge design evaluated. The total cost is found by summing the substructure cost, superstructure cost, bridge deck cost, and excavation cost. The total cost values will be used later to judge each design on its agreement with the structural art concept established in Chapter 1.

Table 7. Costs for Three Middle Support Bridge Designs

Lx (m)	Ly (m)	Lx2 (m)	Volume (in^3)	Substructure Cost	Superstructure Cost	Bridge Deck Cost	Excavation Cost	Total Cost (USD)
72	24	12	$1.34x10^6$	$242,669	$862,730	$376,337	$94,517	**$1,576,252**
72	36	12	$2.10x10^6$	$242,669	$1,294,092	$376,337	$94,517	**$2,007,614**
72	48	12	$2.68x10^6$	$242,669	$1,725,202	$376,337	$94,517	**$2,438,724**
72	24	15	$1.34x10^6$	$230,322	$862,730	$376,337	$94,517	**$1,563,906**
72	36	15	$2.10x10^6$	$230,322	$1,294,092	$376,337	$94,517	**$1,995,267**
72	48	15	$2.68x10^6$	$230,322	$1,725,202	$376,337	$94,517	**$2,426,378**
72	24	18	$1.34x10^6$	$217,976	$862,730	$376,337	$94,517	**$1,551,559**
72	36	18	$2.10x10^6$	$217,976	$1,294,092	$376,337	$94,517	**$1,982,921**
72	48	18	$2.68x10^6$	$217,976	$1,725,202	$376,337	$94,517	**$2,414,031**

Table 8. Simply Supported Bridge Costs

Lx (m)	Ly (m)	Lx2 (m)	Volume (in^3)	Substructure Cost	Superstructure Cost	Bridge Deck Cost	Excavation Cost	Total Cost (USD)
72	24	0	$1.34x10^6$	$62,247	$862,730	$376,337	$29,475	**$1,331,039**
72	36	0	$2.10x10^6$	$62,247	$1,294,868	$376,337	$29,475	**$1,762,401**
72	48	0	$2.68x10^6$	$62,247	$1,725,202	$376,337	$29,475	**$2,193,512**

Table 9. One Middle Support Bridge Costs

Lx (m)	Ly (m)	Lx2 (m)	Volume (in^3)	Substructure Cost	Superstructure Cost	Bridge Deck Cost	Excavation Cost	Total Cost (USD)
72	24	0	1.34×10^6	$126,670	$862,730	$376,337	$50,304	**$1,416,556**
72	36	0	2.10×10^6	$126,670	$1,294,868	$376,337	$50,304	**$1,848,179**
72	48	0	2.68×10^6	$126,670	$1,725,202	$376,337	$50,304	**$2,278,513**

4.5 Structural Art Application

With the aesthetic utility (elegance) and cost (economy) established for the bridge designs, the last variable that must be determined is the compliance, which represents the efficiency. The compliance is determined from the topology optimization. Once the compliance is calculated, the structural art values for each design can be tabulated, as in Table 10.

Table 10. Structural Art Values

Design #	Design	Compliance (Efficiency) (in^3/lb)	Cost (Economy) (USD)	Aesthetics (Elegance)
1	Ly = 24, Lx2 = 12	2.23×10^6	$1,576,252	1.332
2	Ly = 36, Lx2 = 12	1.52×10^6	$2,007,614	1.526
3	Ly = 48, Lx2 = 12	1.18×10^6	$2,438,724	1.621
4	Ly = 24, Lx2 = 15	1.75×10^6	$1,563,906	1.641
5	Ly =36, Lx2 = 15	1.19×10^6	$1,995,267	1.778
6	Ly = 48, Lx2 = 15	9.23×10^5	$2,426,378	1.907
7	Ly = 24, Lx2 = 18	1.51×10^6	$1,551,559	2.125
8	Ly = 36, Lx2 = 18	1.03×10^6	$1,982,921	2.357

9	Simple: Ly = 24	2.85×10^7	$1,331,039	1.544
10	Simple: Ly = 36	1.82×10^7	$1,762,401	0.890
11	Simple: Ly = 48	1.40×10^7	$2,193,512	1.111
12	One-Mid: Ly = 24	6.55×10^6	$1,416,556	1.314
13	One-Mid: Ly = 36	4.52×10^6	$1,848,179	1.314
14	One-Mid: Ly = 48	2.91×10^6	$2,278,513	1.583

A design which maximizes its structural art is one that minimizes all three values in Table 10. To determine the extent to which each design does this, the three structural art variables can be combined to produce three separate plots, which can also be used to determine "dominated" designs. Dominated designs are ones that are clearly not the optimal solution because there are other designs that have lower X and Y values. In order for a design to be dominated, it has to be dominated in all three plots. If a design is dominated, it can be eliminated from consideration for the final design of the bridge. The three plots that are applicable for this problem are compliance vs. cost, compliance vs. aesthetics, and finally aesthetics vs. cost.

The first graph plots compliance on the Y axis and cost on the X axis (Figure 19). The data points are all labelled based on their design numbers established in Table 1. In this plot, there is a clear trade-off between compliance and cost, which is expected. More costly designs have lower compliances and vice versa. There are five non-dominated designs in this plot: 9, 12, 7, 8, and 6. These designs form an optimal Pareto front, and if a design were to be optimized for compliance and cost, any of these designs could be chosen. Design 7 offers the most optimal design, as it is the closest to the origin.

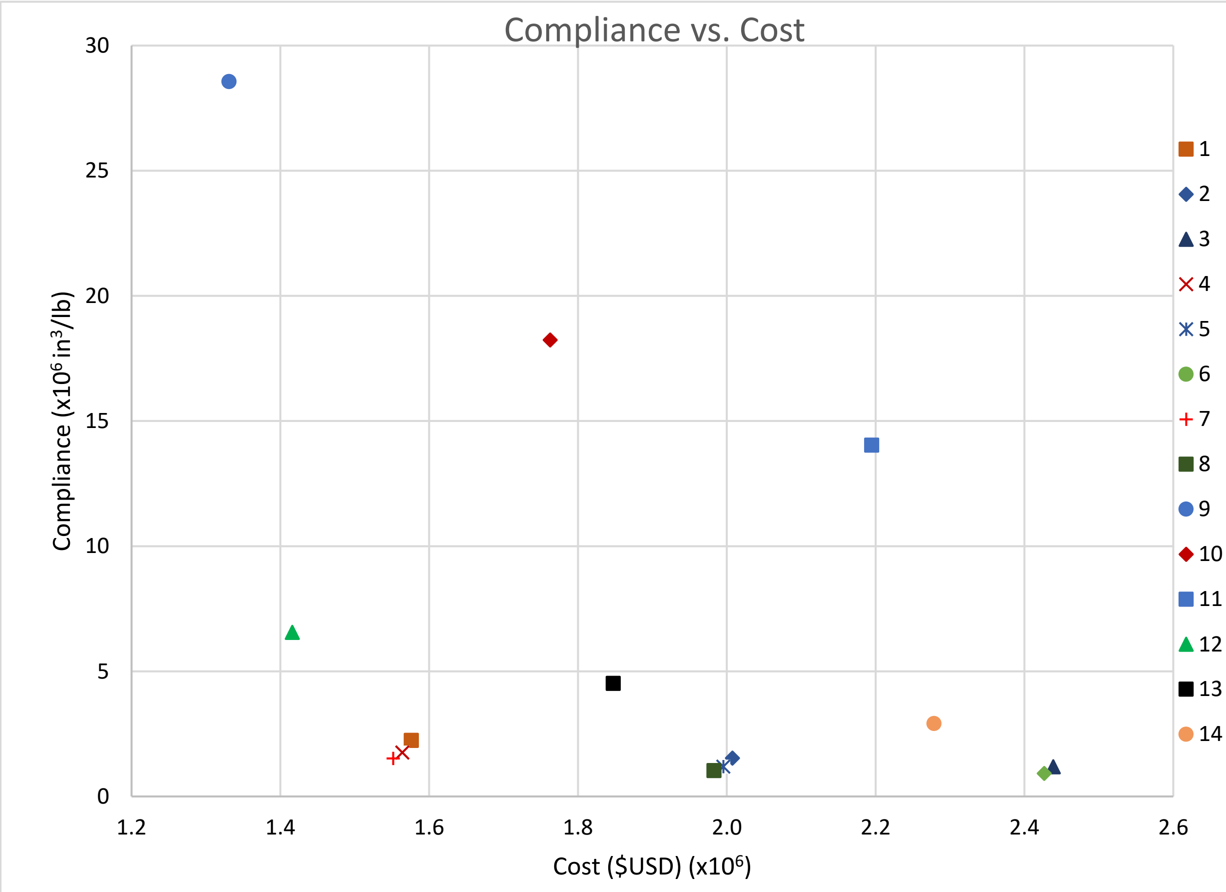


Figure 19. Compliance vs. Cost

The second graph plots compliance on the Y axis and aesthetics on the X axis (Figure 20). This is a very interesting plot because there is a clear tradeoff between compliance and aesthetics. More aesthetic designs tend to have higher compliances, and vice versa. The data points have high correlation apart from one outlier, which means that there are very few dominated designs. The only dominated designs in this plot are 9, 14, 4, 5 and 7. The rest of the designs form the optimal

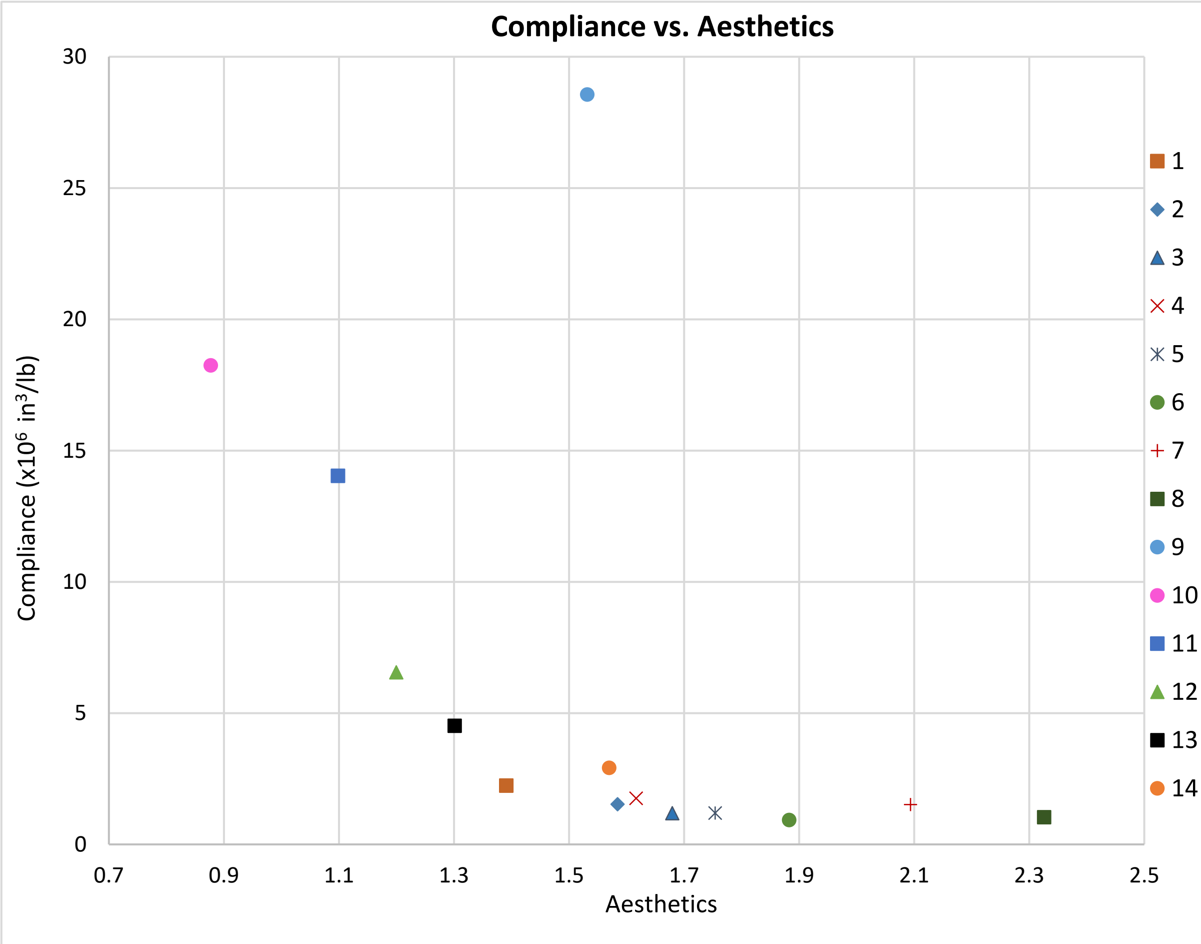


Figure 20. Compliance vs. Aesthetics

Pareto front, and any could be chosen as an optimal design if the design were to be optimized for only aesthetics and compliance. Designs 1, 12, or 13 offer the most optimal solution as they are closest to the origin.

The third graph plots aesthetics on the Y axis and cost on the X axis. This plot is different from the other two plots in the fact that there is no clear trade-off between cost and aesthetics. The data points appear scattered throughout the graph with no real correlation. This likely occurs because the designs with less volume (thus lower material cost) tend to have a better order due to less elements crossing over each other and causing clutter. There are also only three non-dominated designs on this plot: 9, 12 and 10. These three designs appear to form the optimal Pareto front.

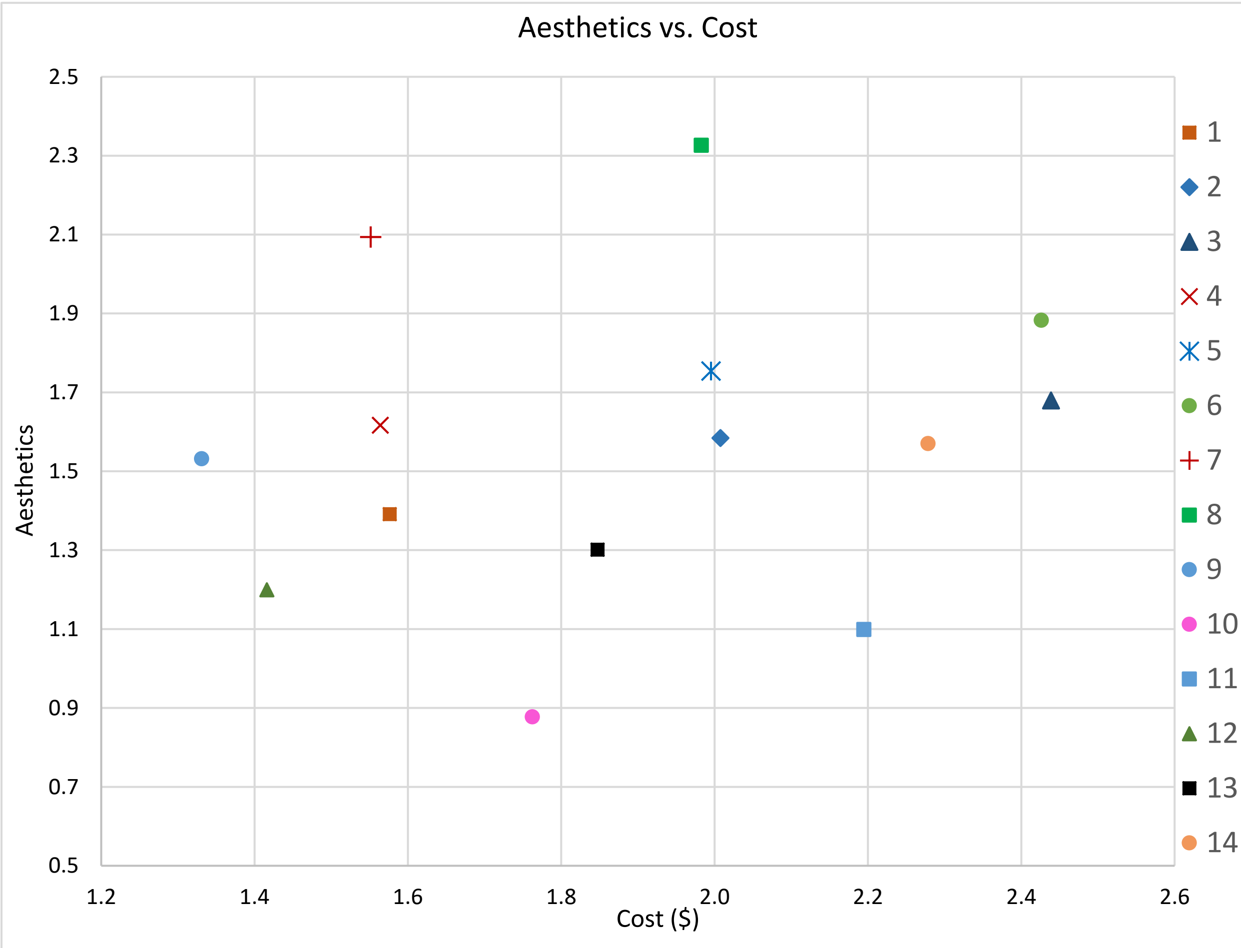


Figure 21. Aesthetics vs. Cost

These three plots can then be used to rank designs based on how closely they adhere to the notion of structural art presented by Billington. This can be done by finding the radial distance of the data points from the origin on all three plots, and then summing the three distances for each design. Designs with the smallest overall distance represent designs that most closely align to the ideals of structural art. In order to properly calculate the distances, the distances are normalized by using the plot units. For the compliance axis, the lowest value is 0 on both plots, and each graphical unit represents a change of 5.0 in^3/lb ($x10^6$) for compliance. For the aesthetics axis, the lowest value is 0.7 on both plots, and each graphical unit represents a 0.2 change in the utility function. For the cost axis, the lowest value is ($x10^6$ USD) on both plots, and each graphical unit represents a change of 0.2 ($x10^6$ USD). These distances are used so that no one distance dominates over another, and allows for the distances to be summed from the three plots.

Table 11 shows the sum of the radial distances for each design, and ranks designs from lowest distance to highest distance (best to worst in terms of structural art).

Table 11. Ranking Designs by Structural Art

Rank	Design Number	Design	Total Radial Distance (Units)
1	#12	One Middle Support: Ly = 24 m	7.24
2	#1	Three Middle Supports: Ly = 24m, Lx2 = 12 m	9.35
3	#13	One Middle Support: Ly = 36 m	10.92
4	#10	Simply Supported: Ly = 36 m	11.31
5	#4	Three Middle Supports: Ly = 24 m, Lx2 = 15 m	11.38
6	#2	Three Middle Supports: Ly = 36 m, Lx2 = 12 m	14.47
7	#11	Simply Supported: Ly = 48 m	14.51
8	#5	Three Middle Supports: Ly = 36 m, Lx2 = 15 m	15.86
9	#7	Three Middle Supports: Ly = 24 m, Lx2 = 18 m	15.95
10	#14	One Middle Support: Ly = 48 m	16.74
11	#9	Simply Supported: Ly = 24 m	17.03

12	#3	Three Middle Supports: Ly = 48 m, Lx2 = 12 m	19.00
13	#6	Three Middle Supports: Ly = 48 m, Lx2 = 15 m	20.57
14	#8	Three Middle Supports: Ly = 48 m, Lx2 = 18 m	21.08

The data in this table suggest that the design that most closely adheres to Billington's definition of "structural art" is the design with one middle support and L_y = 24 m (Design 12). This design has the smallest total radial distance, and thus offers the best balance between all three criteria. The three-middle support design with L_y = 24 m and L_{x2} = 12 m is the second-best design, as it also offers a good balance between the three structural art ideals. The worst designs are the three middle support designs with L_y = 48 m, which have the greatest overall distance from the origin. These designs do not balance the three criteria well and are not optimal designs, according to this analysis.

Furthermore, an analysis of the plots shows that designs 14, 4, and 5 are dominated in all three plots. These designs can thus be fully eliminated from consideration because there is guaranteed to be a better design which more closely adheres to the ideals of structural art. This is an interesting observation because designs 4 and 5 actually rank fairly high in terms of total radial distance.

In a set-based design approach, the remaining 11 designs can subsequently be analyzed with a more computationally expensive method. The use of the inexpensive partially connected mesh allows for three designs to be eliminated and thus can save time and money that might have been spent on evaluating these dominated designs with costlier algorithms. Although the graphs generated from the partially connected meshes have small discriminatory power, it is still interesting to see that aesthetics can be incorporated with cost and compliance to eliminate some designs based on their structural art. It is also interesting to see that many bridge designs which would have been eliminated if only compliance and cost were considered, are still kept with the addition of aesthetics. A design like number 10 might be eliminated if only considered for cost and compliance, but its high aesthetic value allows it to remain in consideration. Furthermore, design 7, which is the optimal design in the compliance vs cost plot, is actually not a great design when aesthetics is incorporated because of its very high value in the utility function. If aesthetics were not incorporated, this design might be chosen, and a visually displeasing bridge might be constructed. These results show how aesthetics can certainly play a key role in design optimization, and may have a trade-off between typical measureable characteristics like compliance.

Chapter 5

Conclusion

If structural aesthetics can successfully be incorporated into existing design and optimization techniques, then it is possible for structural systems to be derived through computation, yet also align with Billington's idea of "structural art." This thesis investigated aesthetic attributes of truss bridges and attempted to develop an aesthetic utility function for truss bridges to see if there is any value in using aesthetic elegance as a criterion for the structural design of bridges. While the function was rudimentary and included basic calculations with only three attributes, the utility function was successful in ranking bridge designs based on the aesthetic attributes of proportion, order, and harmony. The designs that produced lower values of the respective and overall utility function also generally appeared to be more gradual and easy on the eyes, meaning that the function can potentially be applied to additional truss structures to determine their aesthetic value.

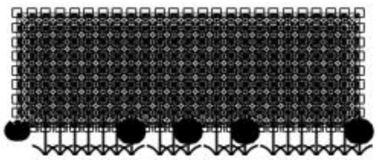
The utility function can be refined and improved in multiple different ways to provide greater levels of discerning the aesthetic quality of a given design alternative and even including weights to reflect general consensus. Instead of just limiting the function to three attributes, the function could be expanded to five or six attributes by including criteria such as rhythm or scale, along with considering the bridge substructure as part of the function. Additionally, the existing functions for the three attributes could be expanded so that more aspects of the bridge superstructure are measured. The proportion attribute could be extended to include proportions between individual elements, and the order attribute could include relative sizing of members. A weight for each attribute could also be included to determine which attributes might be more important than others. This can be done by surveying the public to figure out what aspects of a truss bridge they find to be important for aesthetics.

Although this thesis shows that an aesthetic utility function can possibly be used to quantify the aesthetic elegance of truss bridges, it has not been derived or demonstrated in the general sense, such as for the design of buildings, and other types of bridges like plate girder or suspension bridges. The results obtained in this thesis are promising though and could inspire the continued development and refinement of similar functions that can be specific to these other types of bridges.

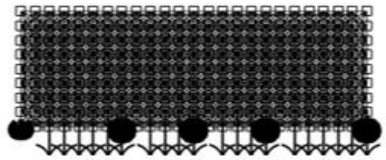
Quantifying aesthetic utility will always be a very difficult challenge due to individuals having differing opinions. This problem will never be fully solved, and no perfect solution can ever be produced. However, this thesis shows that aesthetics can potentially be quantified in a reasonable manner, and if this utility function is further refined, it could be a groundbreaking measure in allowing the aesthetic design of bridges to be considered during the conceptual design phase along with other important criteria, such as cost and efficiency, in an effort to identify design alternatives that closely adhere to Billington's notion of "structural art."

Appendix A

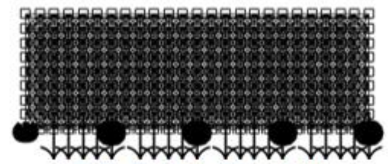
Ground Structures



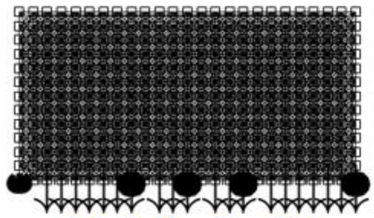
1. Lx2=12 m, Ly = 24m



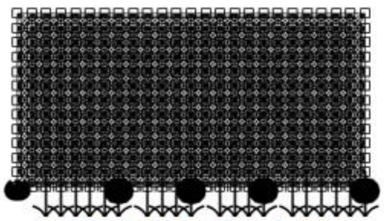
4. Lx2=15 m, Ly = 24m



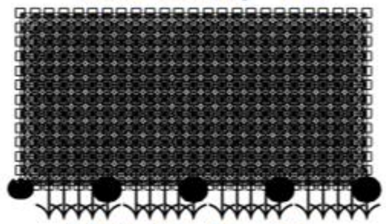
7. Lx2=18 m, Ly = 24m



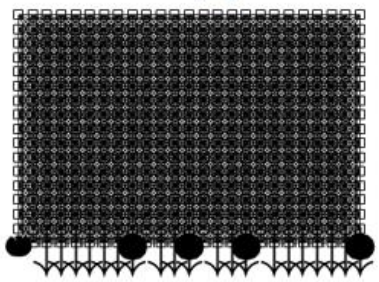
2. Lx2=12 m, Ly = 36m



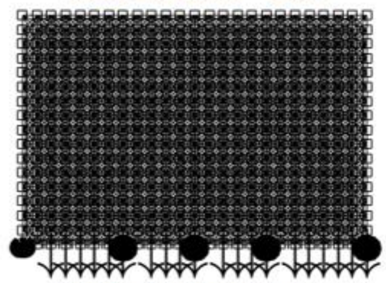
5. Lx2=12 m, Ly = 24m



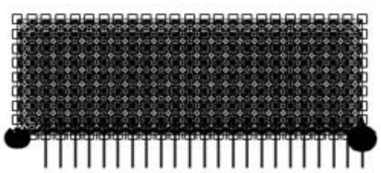
8. Lx2=12 m, Ly = 24m



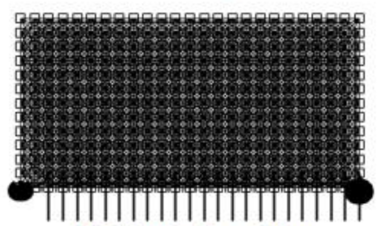
3. Lx2=12 m, Ly = 48m



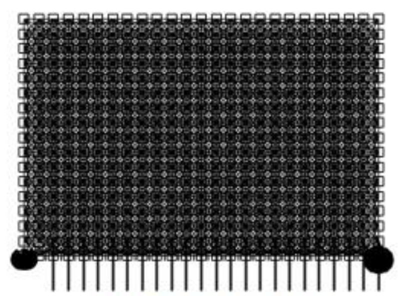
6. Lx2=15 m, Ly = 48m



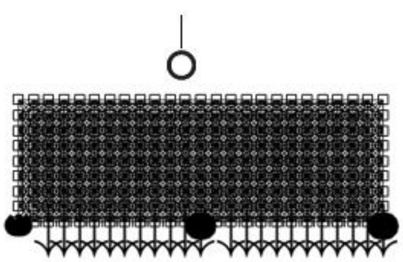
9. Simple: Ly = 24m



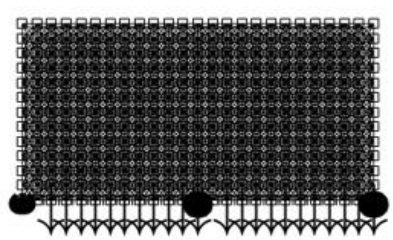
10. Simple: Ly = 36 m



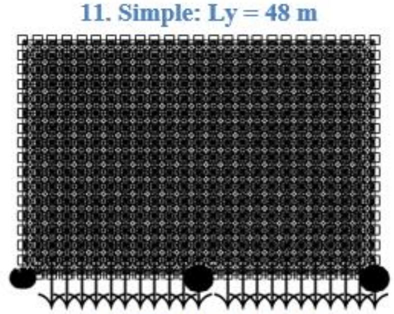
11. Simple: Ly = 48 m



12. One-Mid Ly = 24m



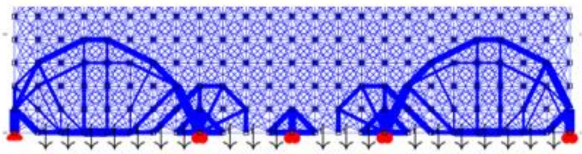
13. One-Mid Ly = 36m



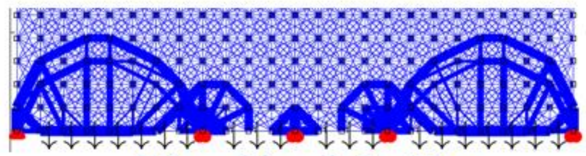
14. One-Mid: Ly = 48 m

Appendix B

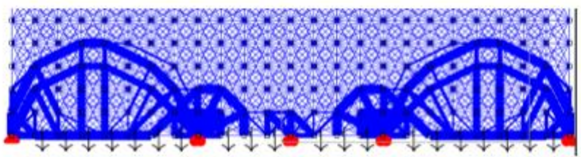
Optimal Topologies



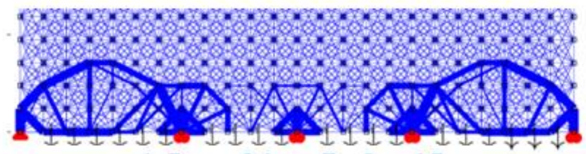
1. Ly = 24 m, Lx2 = 12 m



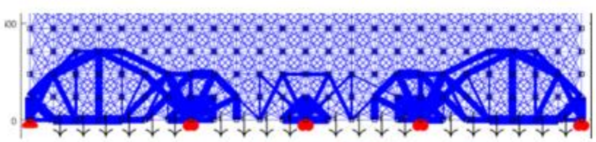
2. Ly = 36 m, Lx2 = 12 m



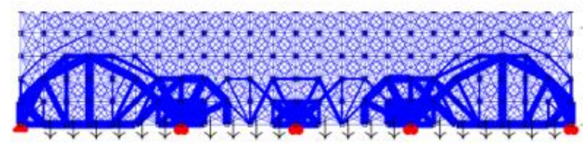
3. Ly = 48 m, Lx2 = 12 m



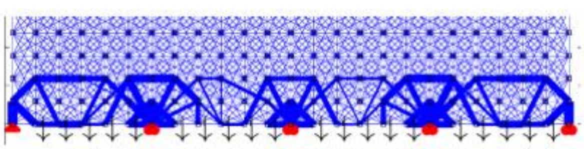
4. Ly = 24 m, Lx2 = 15 m



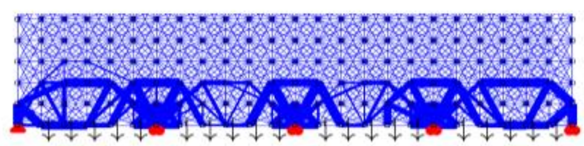
5. Ly = 36 m, Lx2 = 15 m



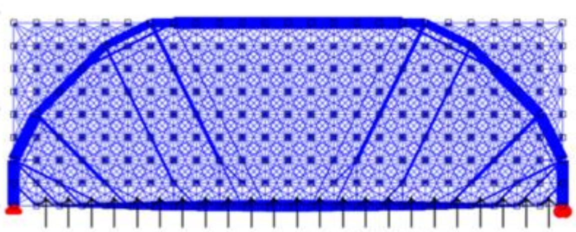
6. Ly = 48 m, Lx2 = 15 m



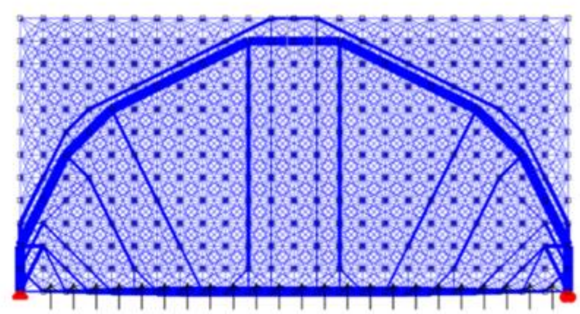
7. Ly = 24 m, Lx2 = 18 m



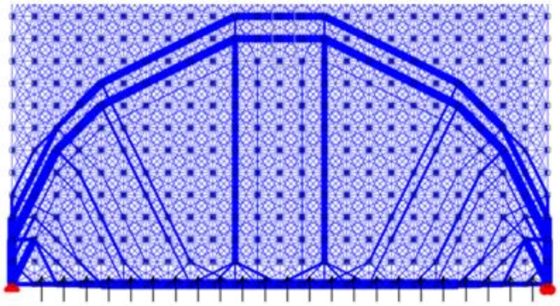
8. Ly = 36 m, Lx2 = 18 m



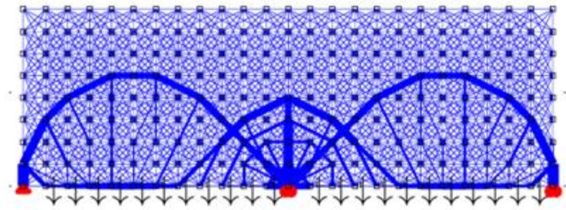
9. Simply Supported: Ly=24 m



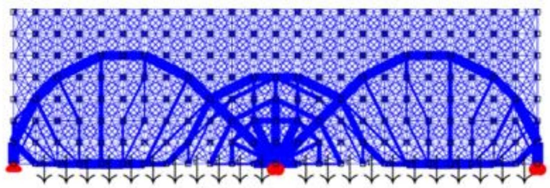
10. Simply Supported: Ly=36 m



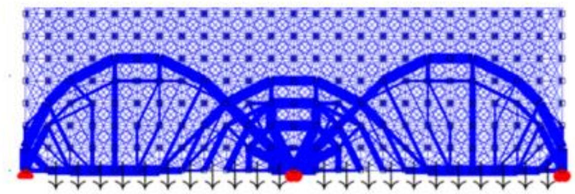
11. Simply Supported: Ly=48 m



12. One Middle Support: Ly=24 m



13. One Middle Support: Ly=36 m



14. One Middle Support: Ly=48 m

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