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ENERGY HARVESTING FROM SHOCK ABSORBERS VIA DISPLACEMENT  
AMPLIFICATION

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## ABSTRACT

When a vehicle travels over an uneven road surface (such as a pot hole), kinetic energy is applied to the vehicle's suspension. In most vehicles, that energy is isolated from passengers by some spring-damper system that serves to limit the range and velocity of motion allowed to reach the vehicle. When this happens, some portion of the kinetic energy is dissipated out of the system by the viscosity of the damper. If one desired to capture and repurpose the shock energy imparted on the vehicle by the road, it would be necessary to eliminate the energy dissipating damper and to install some subsystem between the road surface and the vehicle.

Another obstacle to energy capture and reutilization exists as a result of the limitations of a standard electrical generator. Most generators perform best when their inputs are high speed and low torque, however the shock input supplied by the road surface is low speed and high force. Some energy transformation must take place before that input can be used to spin a generator.

This thesis solves this problem by implementing a circular assembly of buckling beams surrounding a conventional suspension spring. The nonlinear nature of buckling beam deformation is used in place of a linkage to turn a low-speed high-force input into the desired high-speed low-force output that is best suited to power a generator.

The proposed design was tested using finite element analysis to determine its optimal dimensions and parameters. It was also run through dynamic finite element analysis to determine how displacement and velocity were amplified by the buckling beam system. Input displacement was targeted at .1" for the 6" beams tested. This small displacement yielded a 4.8 gain for displacement amplification.

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## **Chapter 1**

### **Introduction**

#### **Motivation for Research**

Energy used to run devices and systems of all size and applications comes at a cost. That cost can be as simple as the monetary cost of running a generator, or as complex as the gradual damage to our environment by pollution. Either way, the electrical, chemical, and mechanical energy we use is never free. Like the law of conservation of energy, there must always be some input of resources to generate an energy output. It is for this reason that any technique capable of harnessing energy that is already available but untapped is of great interest.

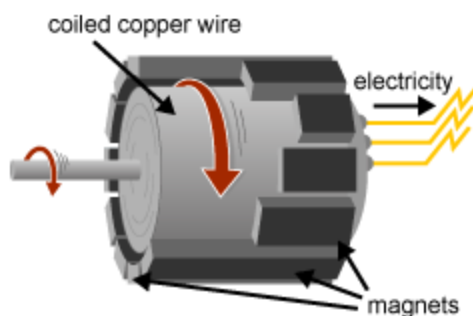
One great area of energy availability that is generally left unutilized is the energy accessible in the motion experienced by many engineered structures. While such structures are often designed to deform and move for reasons unrelated to energy harvest – such as to withstand greater forces before failure in buildings or to provide greater user comfort in vehicles – that deformation results in opportunity for energy capture and repurposing. This energy is often dissipated as heat or as viscous energy by a damper. One use of spring-damper systems is to increase user comfort by reducing the speed at which deformation occurs. However, those systems also transfer useful energy out of the system that could instead be repurposed. If those dampers could be replaced with some other system, a significant amount of the dissipated energy could instead be captured and put to use elsewhere.

Energy that is salvaged can be put to use in other devices that are integrated with the structure the energy is coming from. Energy harvested from the swaying of a bridge or high rise could be reapplied towards lights used to illuminate the outside of the structure and sensors used to measure long-term health. Energy collected from the deformation of a vehicle suspension could be harnessed to extend the

range of an electric motor, run low draw electrical systems, or to power active control systems integrated into the suspension itself. There are a multitude of opportunities present for energy harvesting technology to be integrated into existing engineering to yield an immediate positive impact. The first consideration necessary for the creation of an energy harvest technique is to choose a method of converting the available mechanical or kinetic energy into usable electrical energy.

### Conversion of Kinetic Energy to Electrical Energy Using a Generator

One of the most common ways that kinetic energy is converted to electrical energy is through an electric generator. The most common type of electric generator uses a system of magnets and wire coils to create an electric current. When the coil is rotated inside of a stationary set of magnets by some input torque, a current is induced [1]. This process allows for the conversion of kinetic energy into electrical energy; the only requirement is that the kinetic energy can be turned into a torque that will be applied to the generator's input shaft.



**Figure 1: Demonstration of torque being converted to electricity by a generator, adapted from Energy for Keeps (public domain) [1]**

Rotating electric generators are not perfect and come with limitations. This type of generator works best when its input has a low torque and high rotation rate. This follows known laws of magnetism, where the magnitude of a current induced by moving conducting wire through a magnetic field is proportional to the velocity at which the wire is moved [2].



A rotary motion generator was selected for this application over a linear generator because it is a more mature technology with regard to power generation. Linear generators serve more niche purposes and are generally not as well suited to the type of power generation necessary for the application explored in this paper. Linear electric generators function similarly to their rotating counterparts; conducting coils are passed linearly through a stationary array of magnets. Exploration of this newer technology was outside the scope of this thesis. A rotating electric generator is better than piezoelectric devices for the application explored in this paper because it works on the macroscopic scale. Piezoelectric transducers deal with electricity generation due to deformation on the microscopic scale, where input displacement is about 100 microns (.004 inches) [5].

### **Mechanical Displacement Amplification**

Since rotating electric generators yield their best results under low torque and a high rotating rate, some conversion technique is needed between the high force input to the system that will be created by shock loading and the generator's low torque input shaft. Without this step, the high force – low speed shock loading would not yield much energy generation. It was necessary for some process to take place in between the shock loading and the generator to minimize torque and maximize rotation rate. This is where mechanical displacement amplification becomes essential.

By amplifying the mechanical displacement caused by the shock load, the velocity could also be increased. Since more displacement would be occurring in the same time frame as the input displacement, the velocity was also being amplified. Due to the scientific laws governing the conservation of energy, an increase to displacement would also yield a lower output force, since the work being done by the input load to the generator must equal force times distance. Since distance was increased, force was proportionately decreased.

Any given amplification technique would only be practical if it could be performed in the very limited space available around the system whose displacement is being amplified. For the application of a vehicle suspension, this meant utilizing the limited space around the suspension spring and around the vehicle's pre-existing structures. This quickly eliminated one of the most common techniques for motion amplification: a gear train. In many applications, a train of increasingly smaller radius gears are linked together to increase rotation rate and reduce torque. However, there is not enough space available around a suspension system to achieve the gear ratios necessary to yield the desired amplification. This leaves a short list of remaining options.

### **Energy Available in Bicycle Operation**

One particular application of interest is the opportunity to capture energy available during bicycle operation as a result of shock loading due to uneven road surfaces. A bike makes effective use of the rotational kinetic energy created to spin its wheels and propel its driver. However, any energy that enters the system as a vertical translational load is either absorbed and dissipated by a shock absorber or the bike's operator. This energy is wasted; it could be saved and applied elsewhere but is instead removed from the system as quickly as possible.

A bicycle also has a few useful attributes that make it appropriate for use as a prototype system for a macroscopic energy harvesting technique. Compared to other structures on the macroscopic scale, like cars, trucks, and buildings, a bicycle is relatively small. This makes it much easier to manufacture a prototype system. It also means that a prototype will be small enough to be used with standard lab equipment like vibration shakers. Lastly, it means that the input force necessary to excite the system is small enough that it could be manually input by a human operator, removing the need for specialized equipment.

The shock absorber used for bicycles is simpler than the system used in large vehicles. Generally, a bike shock absorber is just a spring inside of some enclosure. In more complex systems, a combination of springs and dampers are used to reduce the speed and overall displacement experienced by a system. For these reasons, a bicycle is the perfect starting point for an exploration into this energy harvesting technique.

The selection of a bicycle as the vehicle of interest for this thesis was useful as it helped to inform decision making regarding the design and simulation of the proposed buckling beam amplifier. Beam length was selected as 6" in order to align with the space used by a standard rear suspension shock absorber. Forces used in the simulation of static and dynamic loading were selected to mimic the loads experienced by a bicycle during use.

## **Research Objective**

The goal of this thesis was to provide a proof of concept for the use of buckling beams as a displacement amplification technique. This research was conducted with the hope that its findings could be used as a stepping stone for future research projects related to displacement amplification or power generation. The objectives pursued to achieve this goal were the design of a suitable mechanical system using CAD modeling, the simulation of this system under realistic loads, and the manufacture of a physical prototype.

## Chapter 2

### Literature Review

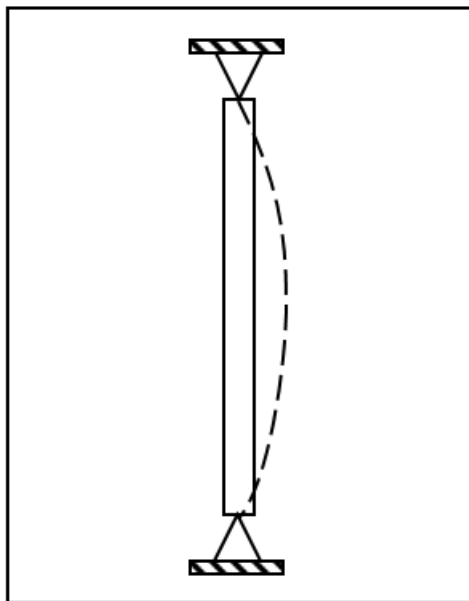
Displacement amplification used to improve power harnessing is not a novel field; many researchers have attempted to achieve displacement amplification for a variety of situations and problem statements. Before selecting a displacement amplification method for use in this paper, it was necessary to compare known methods to determine which would be best for this application. The theory defining each technique was explored, as well as how each has been applied in existing research.

### Buckling

Utilization of the displacement amplification experienced in a buckling beam could serve to improve energy capture in the suggested harnessing system. Buckling occurs in slender beams when they are exposed to compressive loading. Instead of compressing directly into itself until it becomes a flattened cross-section, a slender beam will deflect outward at some critical buckling load. This load occurs at a much smaller value than the compressive load necessary to cause plastic deformation. Once the beam is loaded beyond its critical buckling load, it quickly approaches failure since it can no longer support the load it could in its non-buckled state.

However, if material, cross section, and loading are selected carefully, failure can be avoided and deformation can be contained to be purely elastic. This allows the drawback of a buckling beam, its reduced capacity to resist loading, to be avoided. It also allows the major benefit of a buckling beam to be utilized for displacement amplification: its unique deformation profile.

A slender beam pinned at both ends and exposed to loading that pushes it downward will deflect outward when buckling occurs. The largest displacement of the beam in the outward direction will occur at half its length. This situation is demonstrated in Figure 2.



**Figure 2: Elastically deformed profile of a buckling beam**

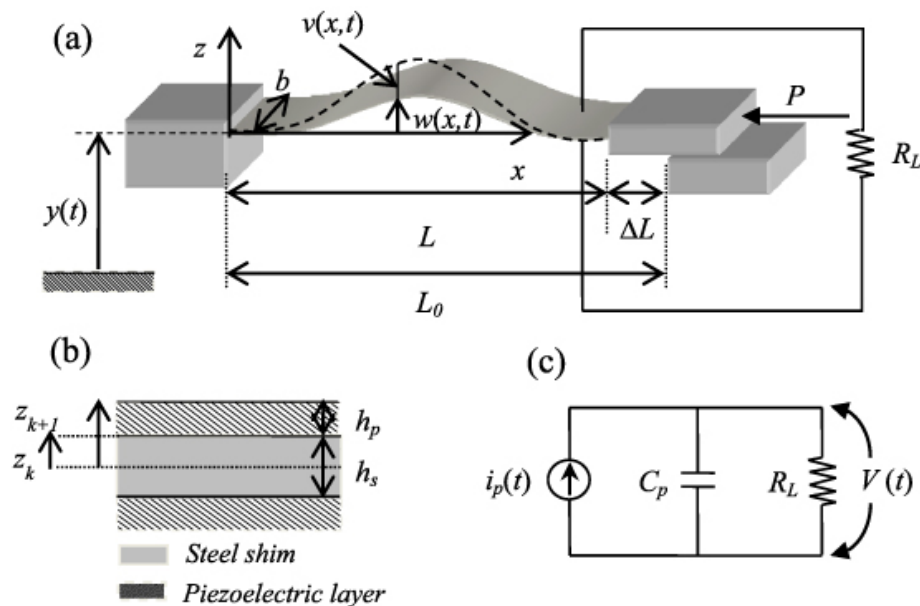
According to Euler's equation of buckling, the bending of the column can be represented by the equation [5]:

$$d^2u/dx^2 = -M/EI$$

This equation shows that the outward deflection of the beam ( $u$ ) is a function of properties of the beam, namely its Young's modulus ( $E$ ) and moment of inertia ( $I$ ), as well as the vertical displacement ( $x$ ) and moment applied to the beam ( $M$ ) [5]. The equation can be used to derive a relationship between maximum force that can be applied to a buckling beam before it will fail and the parameters describing the beam. At best, an equation that describes the shape of a buckled beam can be derived in the form of a second order ordinary differential equation. This nonlinear equation cannot realistically be solved by hand. The maximum deflection of a buckled beam is most effectively determined through the use of finite element analysis. Since the amount that a beam will deflect under load is based on the beams changing stiffness properties, a calculation must be performed by iterative solving at many time steps so that the beam's current properties can be continuously solved for and used for the next calculation.

Generally buckled beams are not used on their own in energy harvesting systems; existing research uses them alongside piezoelectric actuators [10]. A research group consisting of F. Cottone, L.

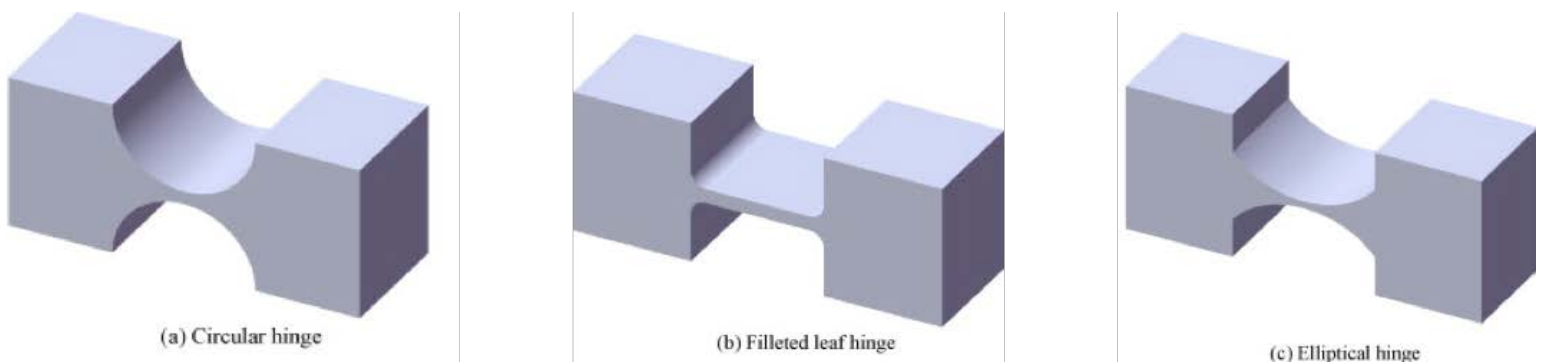
Gammaitoni, H. Vocca, M. Ferrari and V. Ferrari used buckling beams alongside piezoelectric devices to capture energy from small magnitude random vibrations. The power gathered would be used to power micro-electromechanical systems, devices that operate on the micrometer scale. The existing method for capturing this type of energy involved the use of linear resonators to capture power at a resonant frequency [10]. However, this method fails in many real-world applications where such a tight band of acceptable vibration frequencies is too narrow for the operating environment [10]. In addition, real world kinetic energy often occurs at too low of a frequency to be effectively harvested by a linear resonator [10]. The group made use of a buckled piezoelectric bridge in order to study the power output it could generate when exposed to Gaussian noise input into the system by a vibration shaker [10]. It was determined that an increased power output was obtained using the buckled beam bridge system as compared to a piezoelectric system without the bridge [10]. An image of the design concept is included in Figure 3.



**Figure 3: Design concept for piezoelectric-buckled beam energy harvester, adapted from F Cottone et al [10]**

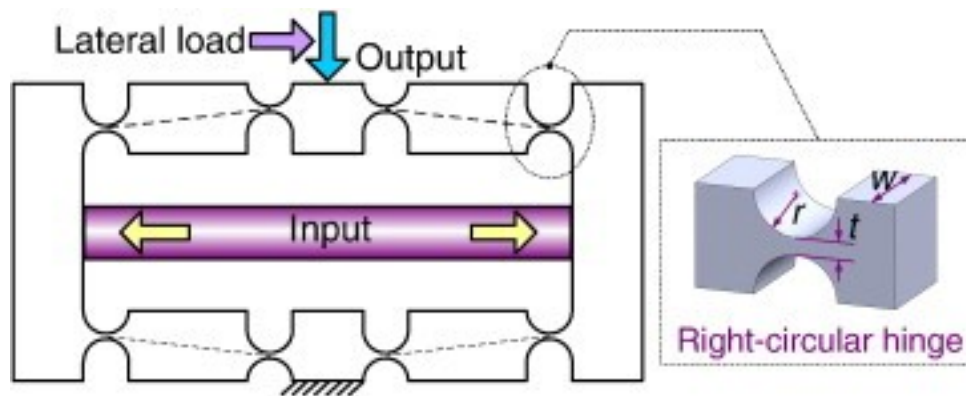
## Flexural Hinges

Another mechanism that can be used for displacement amplification is a flexural hinge. Flexural hinges are a compliant mechanism, meaning they use elastic deformation to transfer an input displacement to another output location [6]. Flexural hinges are particularly useful in many applications because they can produce a very accurate output relative to some input [6]. However, they come with the weakness of only working under a small range of motion [6]. Despite this drawback, flexural hinges can outperform their fully-rotating gear counterparts in some applications due to their lack of friction wear and maintenance requirements [6]. They can also be designed to maximize a desired system property, such as precision, output load, or output displacement [6]. Three popular flexure hinge types are circular, filleted, and elliptical, which are shown in Figure 4.



**Figure 4: Traditional flexural hinge shapes adapted from Tian, Y et al. [6]**

Qingsong Xu and Yangmin Li used a network of flexural hinges to amplify an input displacement [8]. The individual units making up the network were all right-circular flexural hinges, with parameters defined as shown in Figure 5.



**Figure 5: Flexural hinge displacement amplification system adapted from Qingon Xu and Yangmin Li [8]**

These hinges were arranged in a pattern so that they would behave similarly to a pair of beams undergoing buckling. An input displacement applied to the interior of the setup causes bending of the hinges and produces an amplified output deflection [8]. The system was tested using four different models, and it achieved a maximum amplification ratio (output/input) of about 8.25. Two of the models showed a general trend of increasing amplification as input displacement was increased. This system was only tested for input displacements up to .35 mm.

Wei Xu and Tim King focused on elliptical and corner filleted flexural hinges in their research [9]. Finite element analysis was performed in order to determine how effective each group was at amplifying the displacement of the system. A center thickness of at least 1mm was used for all scenarios [9]. The left side of the hinge was fixed so that displacement of the hinge relative to that datum could be calculated. Force applied to the right side of the hinge was varied to produce different amounts of deflection. The best setup for displacement amplification was found to be the elliptical profile hinge. A material of relatively low stiffness (Young's modulus) was used to increase hinge flexibility, which resulted in a high level of output displacement. However, these conditions also produce a system with a very large stress concentration at the center of the hinge. These large stresses have even greater consequences when applied in a fatigue loading scenario. This means that while an elliptical flexural hinge with high flexibility could provide a large displacement output, it could be dangerous to use it for

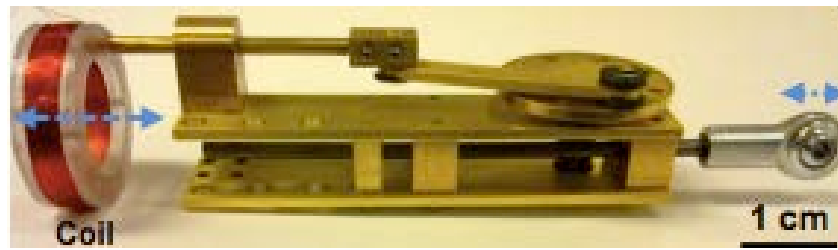


the suspension energy harnessing application since a vehicle suspension experiences frequent fatigue loading throughout its life cycle.

## **Linkages**

A linkage is an assembly made of multiple rigid bodies that is connected so that it can transmit force and motion. Since linkages can be used to modify output properties like displacement and velocity, they have been used in the design of displacement amplification systems. Linkages can be designed on both microscopic and macroscopic scales, expanding their potential applications. Common examples of linkages include pliers and scissor lifts. Pliers improve an operator's ability to apply torque to some object by using a linkage made of two connected bodies to create a larger arm so that the application of a small force can be transmitted as a large torque. A set of pliers are essentially a specialized lever, providing the user with an increased ability to apply a torque to some object instead of an increased ability to apply force. A scissor lift uses many linked bodies to allow for the large displacement needed to transport an operator to a large height while also allowing the lift to constrict into a smaller size that can fit in a much smaller volume. Linkages can be used many ways to transform a given input into a desirable output, which is why they have been used to approach the problem of displacement amplification for energy capture.

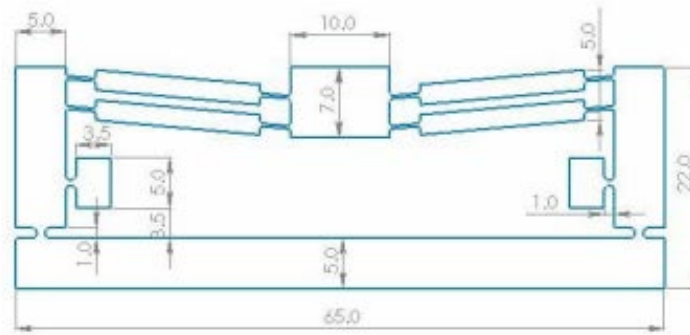
One research group made up of I. Shahosseini and K. Najafi used a system that incorporated linkages to amplify the motion of an input displacement to increase the energy available for harvest [13]. The particular application being examined included a sinusoidal input with an amplitude of 1mm and with a low frequency (less than 5 Hz) [13]. Multiple linkage designs were considered, and after comparing four potential designs the group decided to move forward with a rack and pinion input connected to a piston-like displacement amplifier. A picture of the experimental design is included in Figure 6.



**Figure 6: Prototype rack and pinion to piston displacement amplifier, adapted from I. Shahosseini and K. Najafi [13]**

Since a theoretical model of the system showed that maximum amplification would be achieved at piston rotation angles of 90 to 120 degrees, the system was designed so that a 1mm input would rotate the system to the desired angles [13]. The design was able to achieve a mechanical amplification gain of 4 [13]. The proposed design converted a small linear motion into a larger linear motion, so a linear generator was used for power generation [13]. A coil attached to the end of the displacement amplifier was passed through a ring of magnets to generate power [13]. The system was then subjected to dynamic testing to determine the effect of displacement amplification on power generated. It was found that the amplified system generated 16 times more power than an unamplified system with the same input [13].

Another group of researchers made up of P.R. Ouyang, W.J. Zhang and M.M. Gupta addressed their energy harvesting problem by combining flexural hinges, buckling beams, piezoelectric devices, and linkages [14]. The specific linkages used were a lever arm along with a four-bar linkage. The linkages themselves were made out of an assembly of buckling beams and flexural hinges with the goal of combining many amplification techniques into one even more effective amplifier [14]. Several different configurations were tested, and the proposed symmetric five bar system was found to achieve the best results in displacement amplification. A schematic of this configuration is shown in Figure 7.



**Figure 7: Schematic of a proposed five-bar displacement amplifier, adapted from P.R. Ouyang et al. [14]**

A displacement gain of about 25 was achieved for a 25 micrometer input [14]. The displacement gain was much smaller for larger force inputs; a 1 N input yielded a gain of about 4 [14]. The results showed that the proposed system worked best on the microscopic scale [14]. This was likely due to the fact that piezoelectric devices were used to capture energy, devices that function best at that scale.

## **Piezoelectric Actuators**

Piezoelectric actuators operate differently than the other discussed amplification techniques. When examined as a single, isolated unit, piezoelectric devices experience deformation as a result of some input voltage across their terminals. It is also possible for piezoelectric materials to act in the opposite direction, the input of deformation can produce a potential difference and therefore an electric current. When piezoelectric devices are used this way, it is no longer necessary to incorporate a separate generator into the energy harvesting system, as the mechanical displacement is already being converted directly into usable electric current.

Piezoelectric actuators come with their own benefits and weaknesses. Piezoelectric devices are particularly effective at yielding accurate displacements and strong force transfers [7]. However, their applications are limited due to the very small amount of elastic deformation they allow ( $\sim 0.1\%$ ). Since the

actuators are usually relatively small to start, this limit to elastic deformation sidelines piezoelectric devices in macroscopic situations. The devices are only appropriate when the problem statement they are applied to exists on the microscopic scale.

Piezoelectric actuators are not used in displacement amplification applications as a substitute for buckling beams, flexural hinges or linkages. They are generally used alongside one of those amplification structures in order to convert the achieved mechanical displacement into an electric current. Therefore, it is more accurate to say that they are used as replacements for a linear or rotational generator in some experimental designs. Since piezoelectric devices perform best on the micro-scale, linear and rotational generators are still the best solution for macroscopic situations.

### **Influence of Existing Research on this Thesis Paper**

It was determined that buckling beams would be the displacement amplification technique best suited for the application being explored in this paper. While buckling beams had previously only been explored in the study of microscopic systems, preliminary finite element simulations made it clear that they would function the same when scaled up to the macroscopic level. Buckling beams also held the advantages of being the easiest to manufacture and the least demanding of space. The most competitive alternative was displacement amplification via mechanical linkages, but that selection would have required additional development of original, specialized linkage concepts to fit the needs and constraints that exist for the bicycle application, work that would have been outside the scope of this thesis.

## Chapter 3

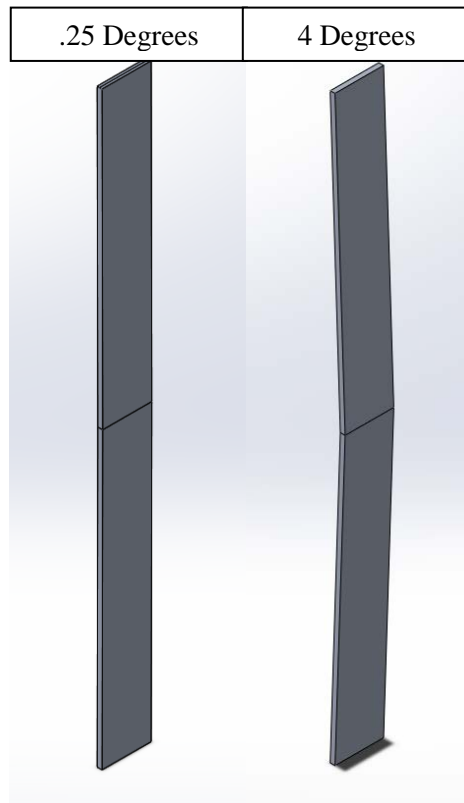
### Methods

#### Design Concept

The proposed displacement amplifier and energy capturing system were planned to consist of several slender beams placed radially around a suspension spring. The beams would be connected using a pin-pin end condition, which would allow the maximum amount of deflection during loading. The spring-beam assembly would be contained by two end plates, one that would be movable and one that would be fixed. A tensile cord would be wrapped around the beams at their vertical midpoints. When the system would be compressed, the spring would deform due to that load and the beams would deflect outward. That outward deflection would be amplified relative to the input displacement and would create a resultant tension in the radially wrapped cord. That cord would then pull on a rack and pinion attached to the rotating electric generator, supplying it with an input torque and rotational velocity. A smaller, weaker spring attached to the rack would then be used to reset the system back to its initial state once the compression load on the spring-beam assembly was removed. While the overall plan and structure of the prototype was determined, it was still necessary to determine the specific dimensions and parameters that would yield the best displacement amplification and therefore the best results.

## Beam Modeling for Parametric Study

It was first necessary to develop solid models for slender beams so that the effects that certain parameters had on displacement amplification could be investigated. Models were created using SolidWorks 2017-2018. Beams were created by sweeping a rectangular profile along a sketched path. The beams were made this way to allow for easy modification of key parameters of interest, which were stored as global variables. The parameters of interest were pre-bent angle and beam length. Two different pre-bend angles are shown in Figure 8.



**Figure 8: Comparison of different pre-bent angled beams**

A split line was used to make a “handle” on the top and bottom of each beam. In addition, the beams were purposefully oriented so that they would deflect in the positive z-direction. These considerations made it possible to set up the pin-pin end condition of the beam in SolidWorks Simulation and to easily observe the deformation results of each study. The model was then copied and modified as

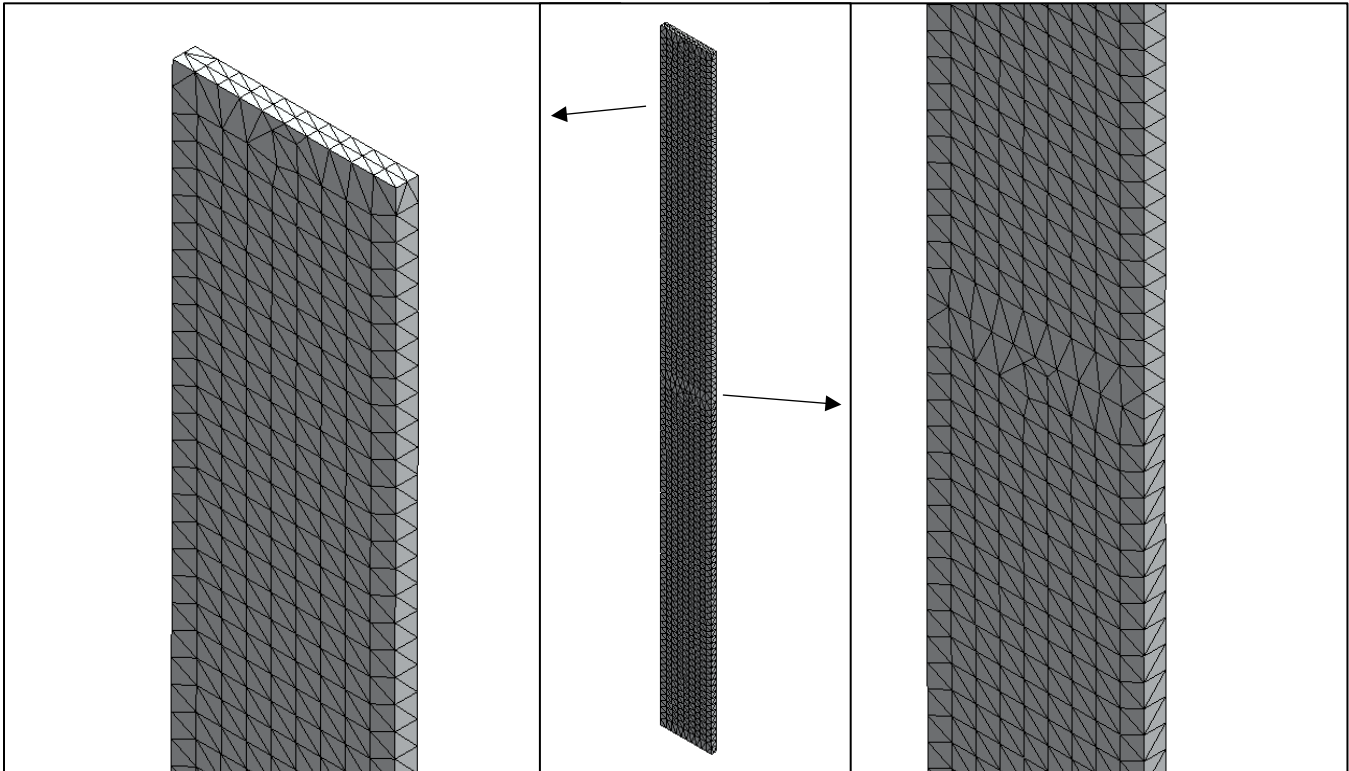
necessary to prepare a set of models that could be subjected to simulation to determine how modifying each property would affect the displacement amplification gain achieved.

### Static Nonlinear Simulation of Beams for Parametric Study

A moderately fine mesh was used in order to reduce computation time while maintaining result accuracy. Default settings were used to generate uniform tetrahedral elements along the beam. Since there was minimal curvature in the beam model, curvature-based meshing was not used. The default setting of four Jacobian points was selected; the low level of curvature present made it unnecessary for additional Jacobian points, which would have been used to correct meshing issues that occur at curved locations. Different mesh values were used for the longer length beams, since their surface areas and volumes were much larger. The meshing parameters used for 6-inch beams are shown in Figure 9. Pictures of an executed mesh are shown in Figure 10.

Mesh type	Solid Mesh
Mesher Used	Standard mesh
Automatic Transition	Off
Include Mesh Auto Loops	Off
Jacobian points	4 points
Element size	0.0531585 in
Tolerance	0.00265793 in
Mesh quality	High
Total nodes	19205
Total elements	10764
Maximum Aspect Ratio	3109.1
Percentage of elements with Aspect Ratio < 3	99.6
Percentage of elements with Aspect Ratio > 10	0.0279
% of distorted elements (Jacobian)	0

**Figure 9: Mesh settings used for 6-inch beams**



**Figure 10: Example of meshed beam**

Once the parts were meshed, a fixture was added to simulate the pinned end condition at the bottom of the beam. Since the bottom plate that the beam would be attached to is fixed, a fixed condition was applied to the split line on the bottom of the beam. The pinned connection at the top of the beam had to be implemented differently, since that pin represented the movable upper plate. To accurately simulate this connection, an advanced fixture was used. This fixture made it so that the top split line could only move up and down but was immobilized in all other directions. The desired downward displacement of the top plate could be input to that advanced fixture as the loading condition for the beam.

The nonlinear simulation type was used for all simulations that calculated displacement amplification gain. The linear buckling simulation module could not be used because it is incapable of calculating displacement of a beam in real world units. The linear buckling simulation is only useful for finding a rough buckling safety factor and a normalized buckling mode shape. The standard static

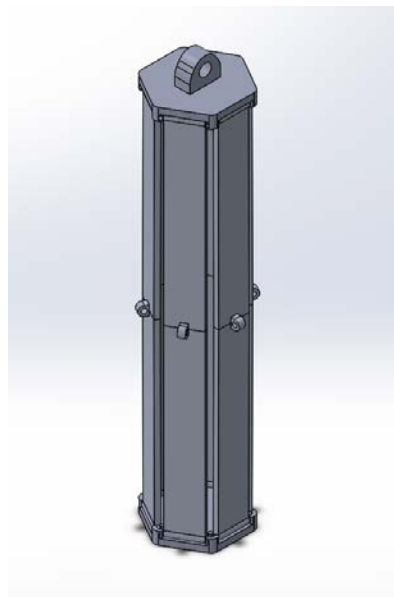


simulation module was also insufficient since it did not take into account the changing properties of the beam as it underwent buckling.

Results of early nonlinear simulations were validated using a real-world model. The real model was subjected to a vertical displacement and its outward deflection was recorded. The real model was seen to exhibit a displacement amplification gain of about 4.5. This value lined up well with similar nonlinear studies that were performed.

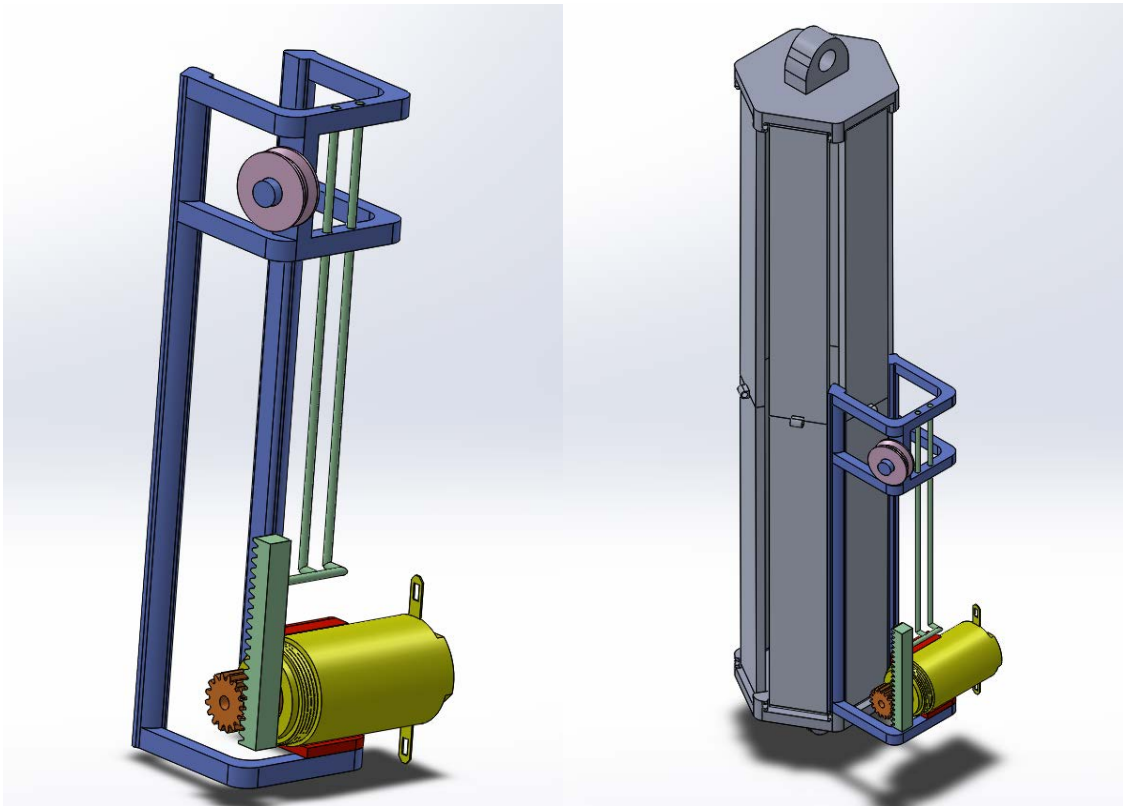
### **Full System Design**

Design constraints were chosen so that a prototype system would be about approximately the same size as a standard rear bicycle suspension. To meet this requirement, total beam length was chosen to be 6". End plate radius was set at 1.25". Based on the data gathered during the parametric study, a beam pre-bent angle of .25 degrees was selected. Each beam was fitted with thin attachment loops to hold the tensile cord in place while not limiting the elastic deformation of the beams. A picture of the beam-endplate subassembly is shown in Figure 11.

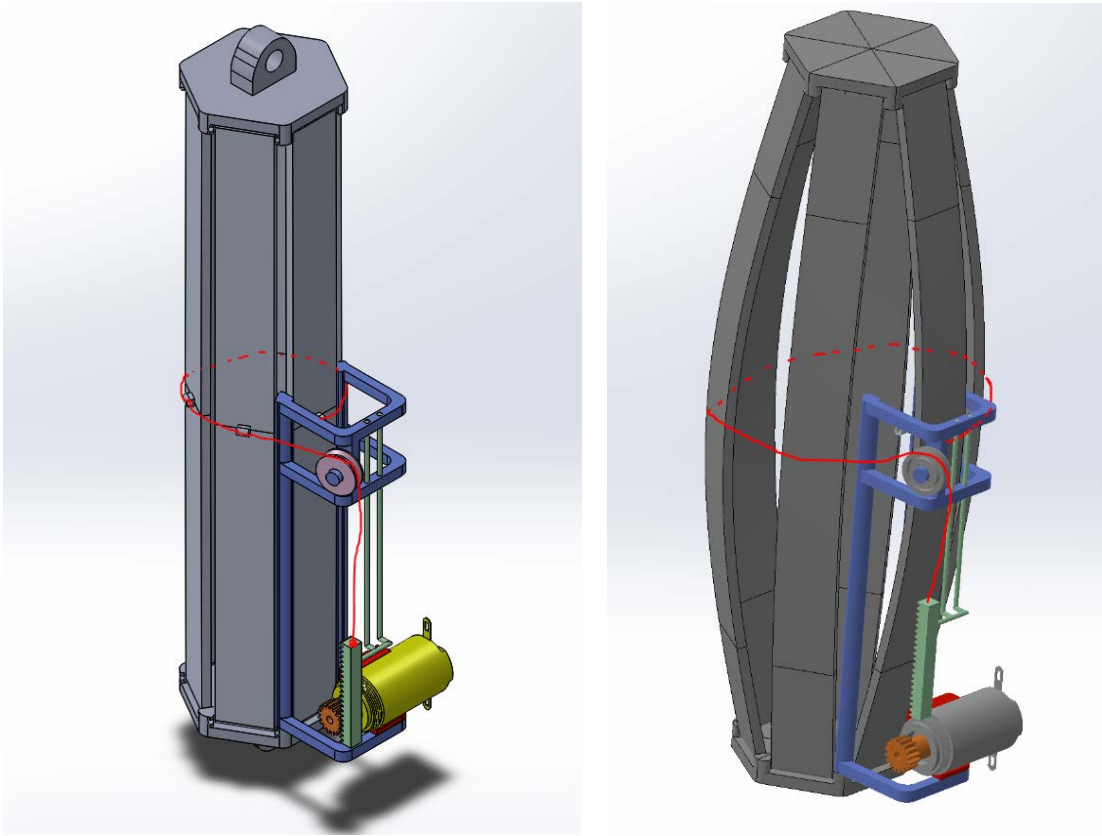


**Figure 11: Beam-Endplate subassembly**

In order to minimize the space requirement of the assembly, a pulley was used to redirect the tensile cord so that it was parallel to the vertical axis of the beams. This enabled the rack, pinion, and generator to all be placed alongside the beam-endplate subassembly. Without redirection of the cord, the generator would have been placed radially outward from the beam-endplate subassembly, causing a sizeable increase in the space required for the energy harvesting system. All platforms and attachment points were build off of the geometry of the bottom endplate. The bottom plate was used since it was fixed relative to the assembly. A picture of the isolated generator subassembly as well as an image of the fully assembled system are included in Figure 12. A mockup of how the tensile cord would connect and react to deformation of the system is roughly sketched in Figure 13.



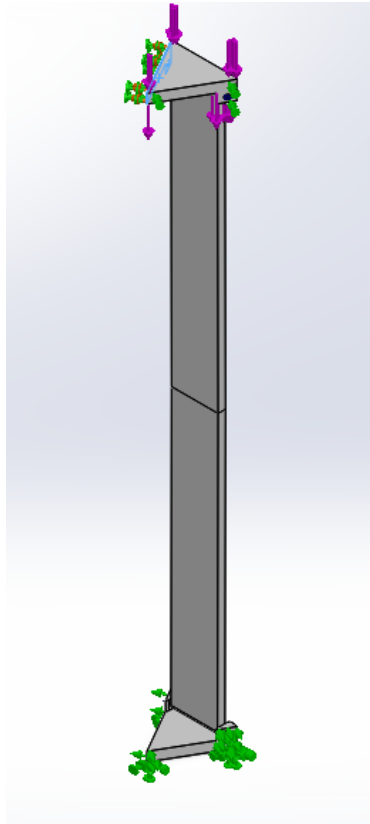
**Figure 12: Generator subassembly isolated and with full assembly**



**Figure 13: Mockup of tensile cord path**

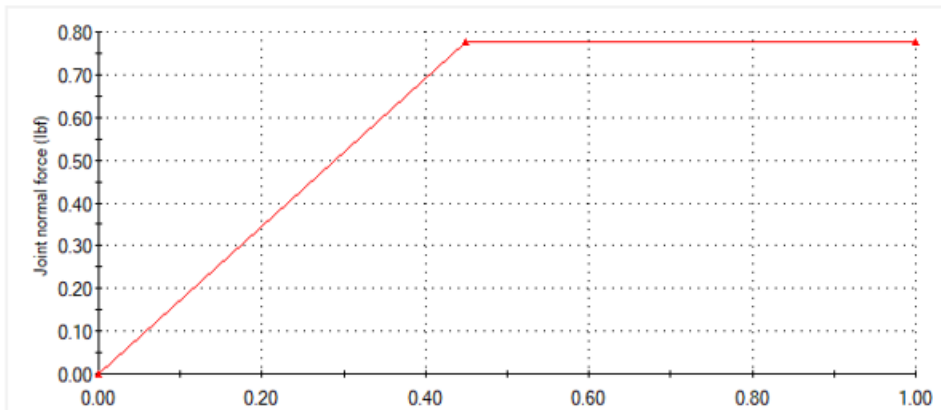
## **Dynamic Simulation**

A nonlinear dynamic simulation was used to determine how the assembled buckling beam system would respond to shock loading. A dynamic simulation was the appropriate simulation method since the desired loading was time dependent. The assembly tested consisted of six beams attached to two end plates. The goal of the simulation was to obtain displacements and velocities experienced by the assembly when it was exposed to loading created to mimic the real-world conditions it would be exposed to when integrated with a bike in order to generate energy. The un-deformed assembly used in the simulation is shown in Figure 14.

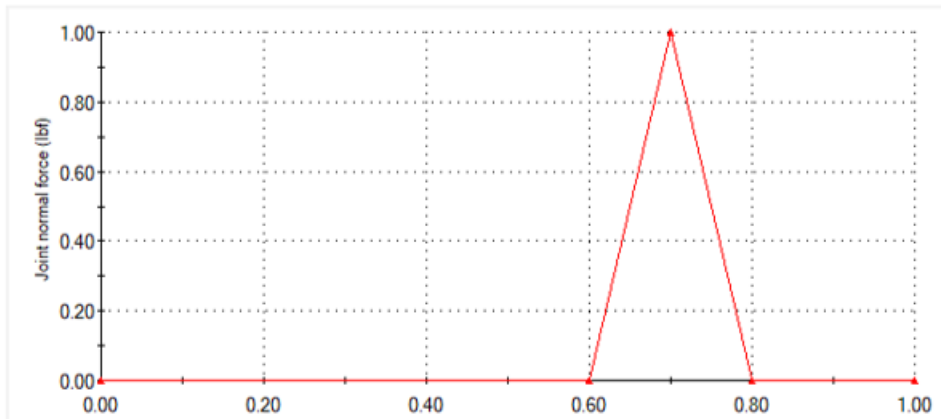


**Figure 14: Un-deformed beam endplate assembly for dynamic simulation**

Realistic loading was simulated using two separate loading conditions, both of which were applied to the movable top endplate. The first load was a static load meant to approximate the weight of a cyclist sitting on the bicycle. The load was applied as a linearly increasing force that grew until reaching a maximum value, at which point it continued to be applied as a constant force. The second load implemented was the time-dependent shock load. This load began after the static load had already reached its maximum value and quickly increased to its maximum value. After reaching its maximum value, the load quickly decreased back to zero. The magnitudes of both loads were set to the pounds force equivalent of the weight of an average cyclist, which was assumed to be 150 lbs. The load plots from SolidWorks are shown in Figure 15 and 16.



**Figure 15: Static load for dynamic simulation**



**Figure 16: Shock load for dynamic simulation**

In order to reduce simulation time, the cyclic symmetry advanced fixture was used. This fixture allowed for a one sixth assembly – containing one beam, its end pins, and its accompanying section of the endplate – to be circularly patterned around the assembly’s central axis. This simplification of the simulated model allowed for several variations of the simulation to be run. These variations featured different load magnitudes, materials, and springs.

## Chapter 4

### Results

#### Parametric Study

During the parametric study, three distinct groups of data were collected. Each group was defined by the alteration of one important parameter so that the effects of that parameter on displacement amplification could be determined. The three variables that were modified were input displacement, beam length, and pre-bent angle. The results have been collected and are represented in Table 1, 2 and 3.

**Table 1: Varied input displacements for 6-inch beams**

Input Displacement (in)	Output Displacement (in)	Amplification Gain
0.15	5.93E-01	3.952
0.1	4.84E-01	4.841
0.05	3.41E-01	6.822
0.025	2.39E-01	9.568
0.01	1.48E-01	14.840
0.001	3.63E-02	36.250
0.0001	1.89E-03	18.860
0.0005	1.89E-02	37.840
0.00075	2.85E-02	38.000
0.00085	3.18E-02	37.376

**Table 2: Varied beam lengths**

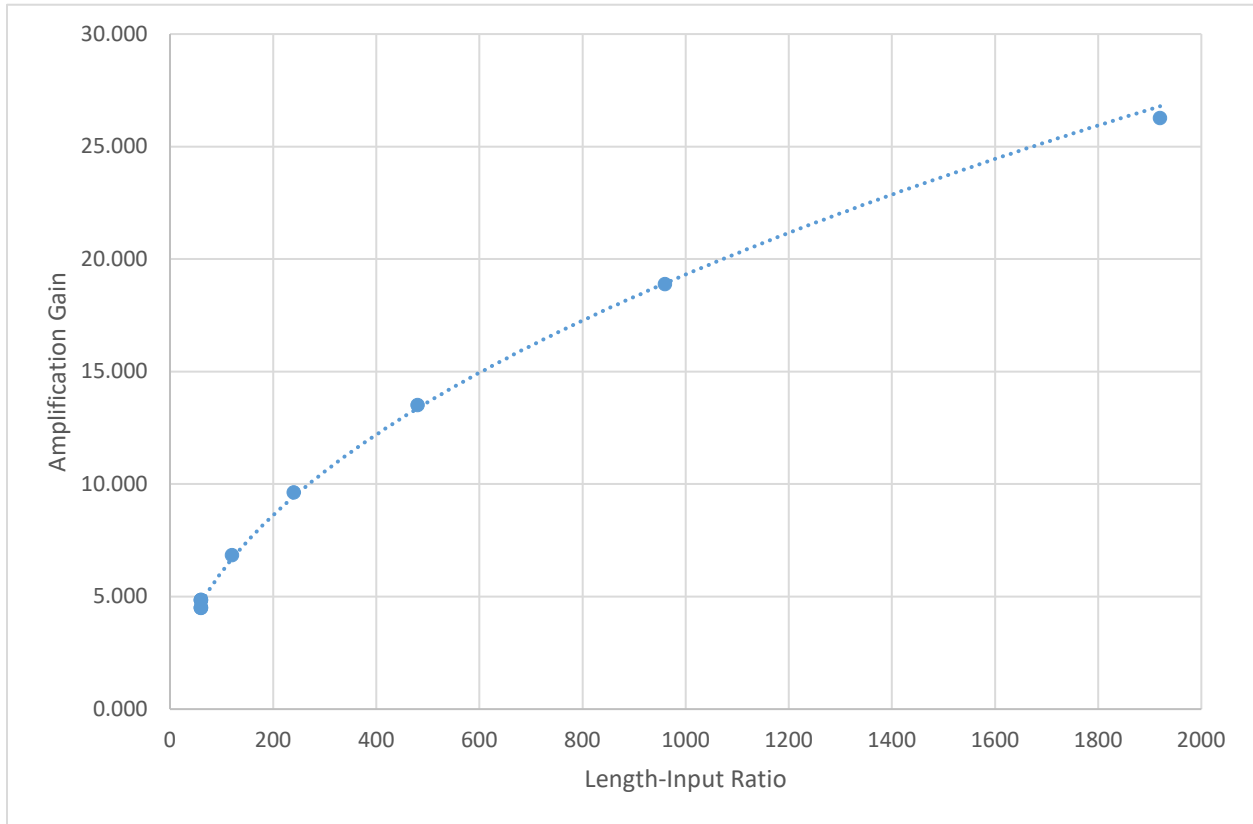
Length (in)	Input Displacement (in)	Output Displacement (in)	Amplification Gain	Length-Input Ratio
6	0.1	4.86E-01	4.855	60
12	0.1	6.85E-01	6.845	120
12	0.2	9.70E-01	4.851	60
24	0.1	9.64E-01	9.638	240
24	0.4	1.94E+00	4.853	60
48	0.1	1.35E+00	13.520	480
48	0.8	3.61E+00	4.513	60
96	0.1	1.89E+00	18.900	960
96	1.6	7.22E+00	4.511	60
192	0.1	2.63E+00	26.270	1920
192	3.2	1.44E+01	4.506	60

**Table 3: Varied pre-bent angles**

Pre-bent Angle (degrees)	Input Displacement	Output Displacement (in)	Amplification Gain
4	.1 in	4.14E-01	4.140
2	.1 in	4.51E-01	4.506
1	.1 in	4.70E-01	4.702
0.5	.1 in	4.80E-01	4.803
0.25	.1 in	4.86E-01	4.855
0.15	.1 in	4.53E-01	4.534
0.1	.1 in	1.61E-01	1.610
0	.1 in	1.63E-01	1.633

The parametric study proved that maximum displacement amplification was achieved by a beam that was pre-bent .25 degrees. It also showed that maximum displacement amplification is achieved at the onset of buckling, since very small input displacements achieved very large amplification ratios.

Additionally, it became clear that the input displacement value was less important than how that value compared to the overall length of the beam. A plot was constructed to compare the beam length to input displacement ratio with the achieved amplification gain. This plot is depicted in Figure 17.



**Figure 17: Amplification gain as a function of beam length to input ratio**

There are several data points clustered at the leftmost area of the plot. These data points represent every situation where input displacement was purposely set as a proportion to beam length. For example, some of the points at this location are the result of a .1" displacement for a 6" beam, a .2" displacement for a 12" Beam, and a .4" displacement for a 24" beam. All of these scenarios create an approximately equal displacement gain.



### Dynamic Simulation

Displacement and velocity information was acquired using nonlinear dynamic performed in SolidWorks Simulation. Plots for the displacement experienced by beams from the assembly simulated are shown in Figure 16 and 17.

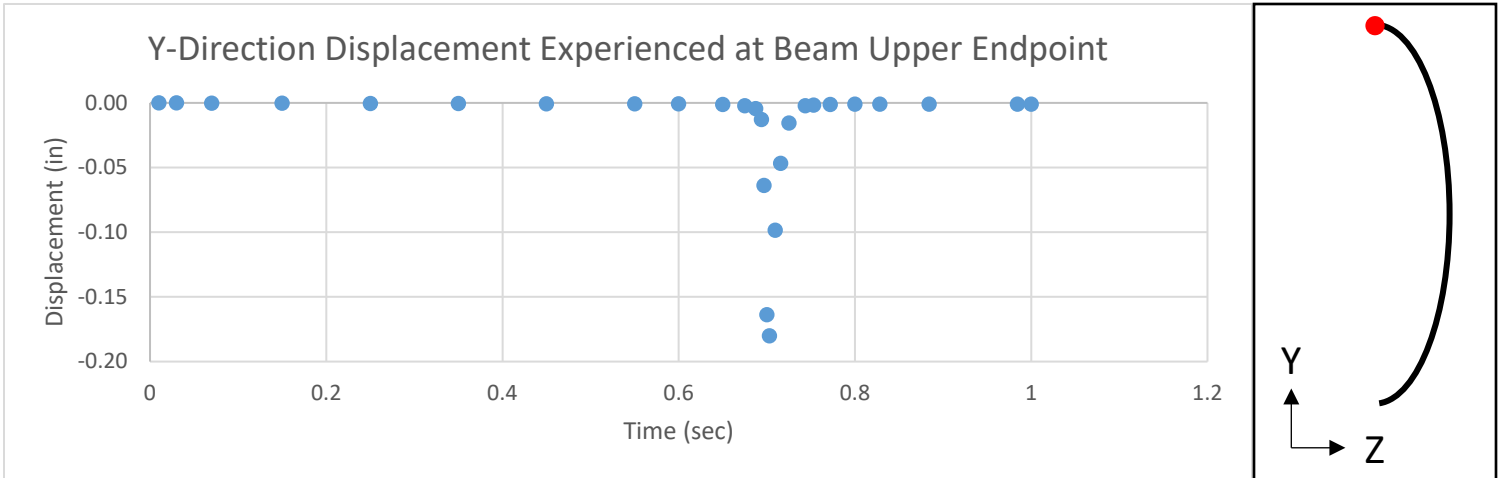


Figure 18: Y Displacement of beam upper endpoint

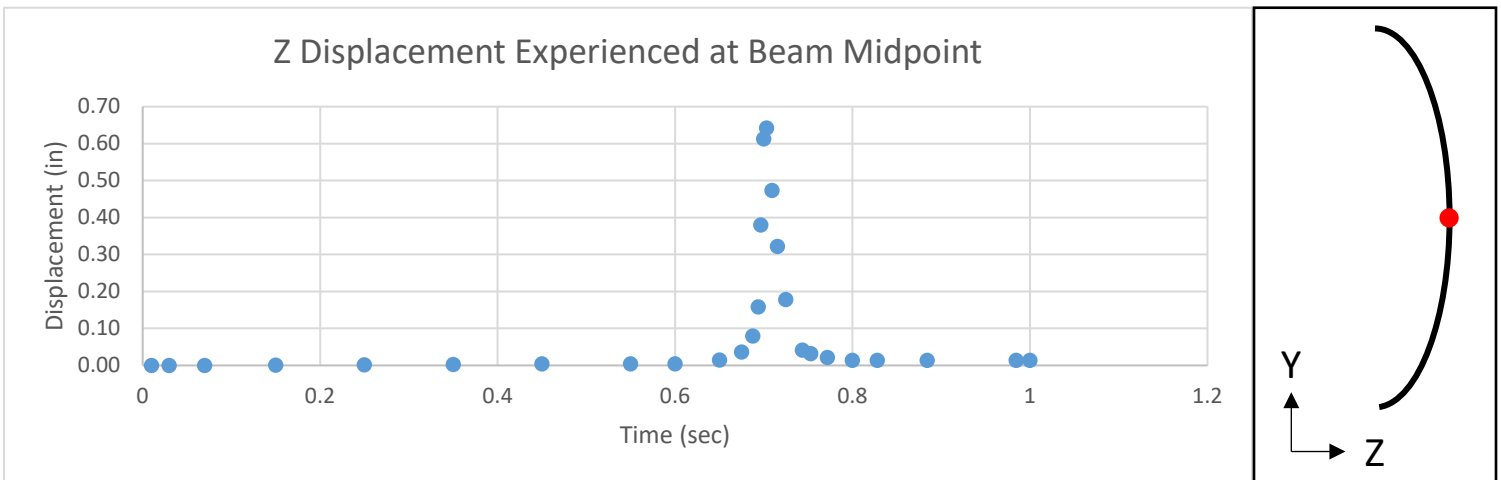
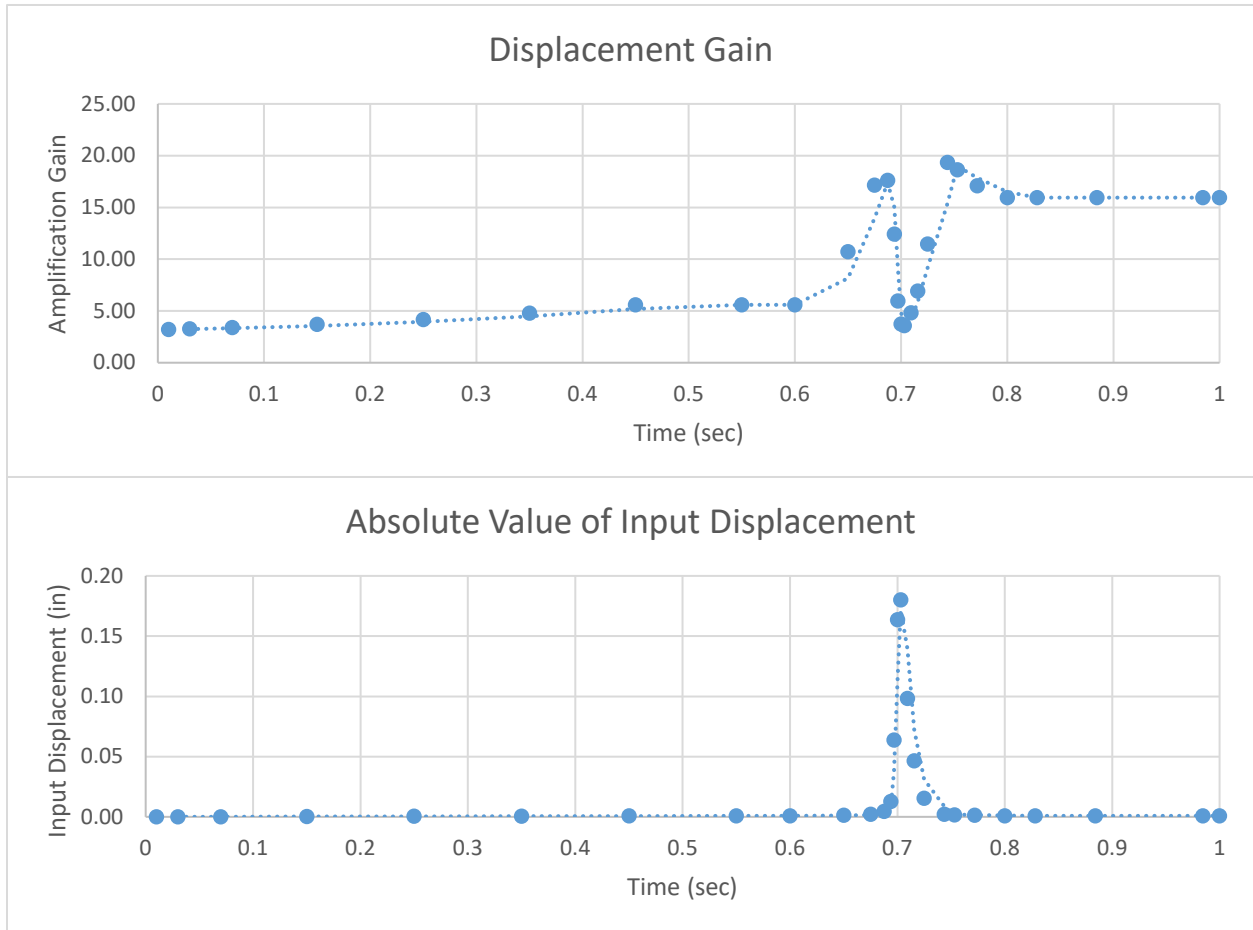


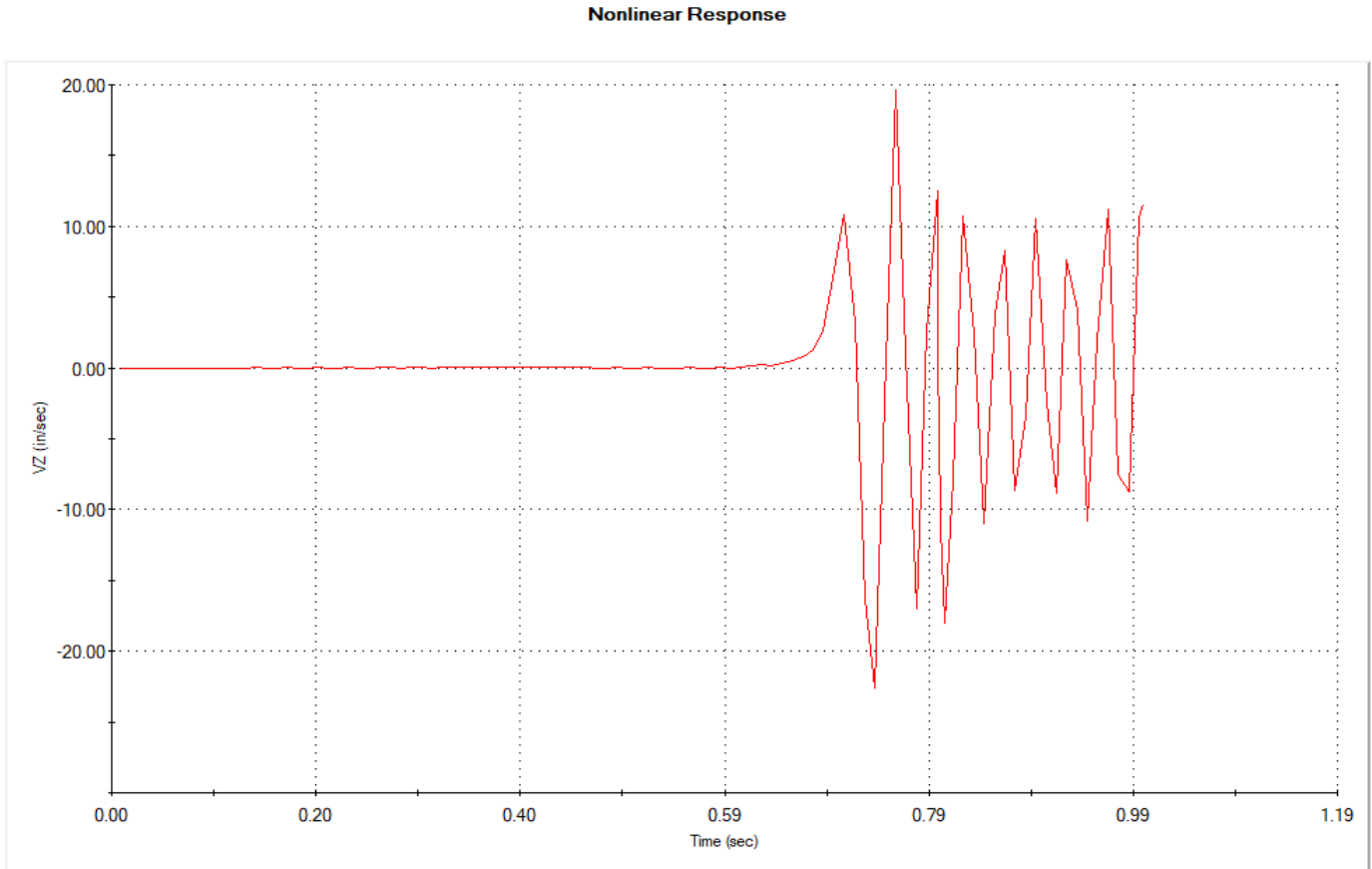
Figure 19: Z Displacement at beam midpoint

Displacement data was then used to calculate the amplification gain yielded by the assembly as a function of time. This plot, along with the relative input displacement being applied as a function of time, is shown in Figure 20.



**Figure 18: Displacement gain experienced by the beam midpoint relative to input displacement**

Velocity information was also extracted for the beam midpoint location, since that would be the part of the assembly applying tension to its cord and imparting a velocity onto the rack used to activate the generator. A plot of the velocity experienced by this beam location is shown in Figure 21.



**Figure 19: Velocity experienced at beam midpoint**

Recall that the maximum input force created by the combination of shock loading and static loading occurred at 0.7 seconds. At peak shock loading a maximum instantaneous velocity of 10.12 inches per second was achieved. Due to the geometry employed by the assembly, this beam deflection velocity would result in a magnified cord pull velocity, which is what would be applied to the generator. To determine how deflection of one beam would affect the deflection of the surrounding cord, a physical model was constructed and measured.

## **Chapter 5**

### **Discussion**

The overall goal in the creation of the buckling beam assembly was to design a system that could accept a high-force low-velocity input and transform it into a low-force high-velocity output. This change was desirable as the output load would be much better suited toward creating a current through a generator. Optimization of the design using a parametric study and static finite element analysis was employed so that the ideal parameters for displacement and velocity magnification could be determined. While this analysis produced a quantitative solution to the question of preferred pre-bent angle, it also corroborated some known truths about buckling beams. When only input displacement was altered, it became clear that input loads closest to the onset of buckling yielded the greatest displacement amplifications at the beam arch. When different beam lengths were tested with both a standard .1” input displacement as well as a proportional input based on their length, it became clear that the magnitude of the input was less important than the ratio of that magnitude to the overall beam length. The standard .1” input produced a much larger amplification gain on a longer beam, which lines up with the assertion that amplification is greatest at buckling onset. These findings imply that the buckling beam displacement amplification technique could be scaled up for use with a larger structure and larger loads.

Testing using dynamic finite element analysis was necessary to provide a proof of concept that the proposed assembly could accept the expected loading and return a desired amplified output. Dynamic analysis was especially useful because it allowed for the creation of a time dependent velocity plot for the assembly. This plot is particularly important because the potential for power generation in a generator is defined by the velocity input applied to it. Future investigation could be performed to determine the power generated by the proposed loading by constructing a physical prototype and running it through a compressive loading device.

In the dynamic simulation there was a period of snap-back that occurred after shock loading was removed. In order to prevent this behavior from occurring, it is recommended that a low stiffness spring be added to the middle of the assembly to reduce oscillation and to ensure that the assembly returns to its initial uncompressed state.

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