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AN ANALYSIS OF LINEAR PORTFOLIO OPTIMIZATION THEORETIC ON EMPIRICAL DATA

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ABSTRACT

In the portfolio management world, numerous methodologies of optimizing on risk-adjusted returns exist with the definition of risk being the primary differentiating factor within the frameworks. The multitude of established theory that exists however has little to no robust analysis of performance on empirical data, with concrete examples of the theory presented in these papers being performed on an insubstantial amount of different investment universes, and only on single periods. Furthermore, they tend to focus on differing asset class returns, in which correlations are more concrete and return profiles of assets are tightly distributed. This paper focuses on the analysis of empirical performance of portfolio optimization techniques. Specifically, the three optimizations techniques defining risk as standard deviation, semideviation, and conditional value at risk (CVaR). Furthermore, the optimization is done on the domestic equity universe only, utilizing consensus sell-side analyst price targets as expected return estimates. Domestic equities were chosen due to the varying nature of correlations in individual equity investments, combined with the broader expected returns and risk profiles of the varying names. In order to analyze the effectiveness of the techniques, random groups of single name equities were iteratively chosen and optimized on repeatedly for two different time periods. The two periods represent a bull-market and side-ways market so as to see if different market trends lead to different results. Through generating multiple baskets of assets and optimizations for each period, a large enough sample was collected so as to derive empirical results with little to no single iteration skew as in previous papers. Ultimate results showed that in both periods, optimization frameworks with risk defined as CVaR performed best on risk adjusted and absolute return metrics. Interestingly, semivariance based optimization techniques

saw little advantage relative to standard deviation based optimizations with the two ranking fairly similar.

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Introduction

Portfolio allocation optimization represents the premier problem within financial asset management. With trillions of dollars in institutional and retail investments flowing across various asset classes, it is a constant issued faced by fund managers in how to, and to what magnitude to, allocate financial assets. Within this system, investors constantly fight the almost counterintuitive fight of searching for high return opportunities with low-risk levels. Throughout literature however, there has been constant argument over the idea of risk and what necessarily constitutes a negative-risk for an investor. This paper will try to identify the best definition of risk within a portfolio optimization framework, focusing on the risk metrics of portfolio variance, semivariance, and conditional value at risk. Numerous investment managers regularly utilize these similar yet different metrics in order to gauge both future and realized risk-levels of their investment portfolios. We will furthermore study if there are any differing characteristics of portfolios focused on optimizing on these definitions of risk, and look to identify whether any of these metrics of risk provide an absolute or relative advantage over the others within a mean-risk optimization framework.

The literature up to this point has primarily focused on the theoretic of portfolio optimization, with different optimization techniques being developed and expanded upon. Minimal comparisons of the application of these theories exist, and those that do focus on extremely long-term investment horizons, and include differing assets with fairly low correlations (bonds, equities, commodities, etc). This paper however will focus on the short-term horizon of one year, with an emphasis on equities. The time horizon of one year was chosen so as to allow a large enough period to circumvent abnormalities in the short term while simultaneously being able to incorporate divergence of asset returns from the typical normal return assumption. The period of one year thus prevents extreme convergence to the normal distribution as is typically seen in longer time horizons.

It is thus our goal to construct an analysis of short-term portfolio optimization frameworks and generate insights regarding the potential for application of these methods for institutional and other investment managers. As institutional asset managers, especially higher risk hedge funds, typically focus on quarter-to-quarter or year-to-year performance, studies that focus on extreme horizons of more than five or ten years bear minimal impact on empirical application of theory. To our knowledge, this is the first study to focus specifically on the equity space with the associated optimization techniques mentioned, and as such looks to advance the application of portfolio optimization and verify the theoretic constructed by academics within the portfolio-theory space.

It is our initial hypothesis that the highest absolute risk returns will be generated by the semivariance based optimization across all periods. Given the pure focus on negative deviation as the definition of risk, this should allow greater upside skew in returns relative to the symmetric definition of standard deviation and the limited tail-risk definition of CVaR. On a risk-adjusted basis we predict that the CVaR based optimization will perform best given its large emphasis on minimizing tail risk scenarios that typically created the largest squared deviations and thus contribute the most to respective variance and semivariance calculations used in the realized Sortino and Sharpe ratio calculations.

Literature Review

Overview

In this chapter, we will review the literature relevant to the theory of portfolio theory, and touch on the advances through time regarding progress made on both a theoretical and applied optimization perspective.

The chapter will begin with an overview of utility theory and its implications on the portfolio optimization problem. This is followed by a general discussion on the abstract concept of risk, and the nuances and different ways to define risk when it relates to financial assets. Next, the basic Mean-Variance framework will be introduced, and the benefits and reason behind optimization and diversification will be touched upon. This will then segue into the idea of the efficient frontier and the capital market line, emphasizing the two fund separation theorem that is key in separating individuals' tastes from optimal risky-portfolios. This idea is then used to show the idea of the traditional capital asset pricing model and its implications. The paper then shifts from the Mean-Variance framework to the more generalized Mean-Lower Partial Moment framework and touches upon the theory and advantages of this more nuanced method. This is followed by its extension to a more generalized Mean-Lower Partial Moment Capital Asset Pricing Model derivation, of which the Mean-Variance Capital Asset Pricing Model is just a sub-case of. The Mean-Lower Partial Moment framework is then used to derive a Mean-Semivariance optimization method and its associated Mean-Semivariance Capital Asset Pricing Model. The section then concludes with an introduction to Value at Risk and Conditional Value at Risk, and introduces an optimization technique for a Mean-Conditional Value at Risk portfolio.

Utility Theory

In order to understand the optimization criteria and arguments presented within the portfolio optimization works in existence, it is vital to first establish a base understanding to the underlying economic utility theory that is implemented in financial asset analysis and the associated risk/return optimization tradeoffs.

The initial idea of utility is an abstract concept that represents the worth that an individual receives from some form of action. In this paper, this utility refers to the value that one receives out of their investment portfolio through aspects such as return, volatility, and other modified definitions of risk. In a portfolio optimization framework, the base-line problem typically begins with the maximization of a utility function that is in and of itself a function of various statistical qualities or moments of an underlying portfolio that add or reduce an investor's overall utility.

A base framework typically utilized for utility within the optimization framework is that put forth by Von Neumann and Morgenstern (1944) in their book Theory of Games and Economic Behavior. Their work established the base preferences axioms for expected utility theory. The principles formed have become known as the Von Neumann-Morgenstern axioms and stated as the following:

1. **Completeness:** For any alternative choices A or B $\rightarrow$ $A > B$, $A < B$, or $A = B$
2. **Transivivity:** For any alternative choices A, B, C if $A \leq B$, $B \leq C \rightarrow A \leq C$
3. **Independent of Irrelavant Alternatives:** For any lotteries A, B, C with $A \geq B$, let t be the probability of a third choice beings present such that $t \in [0,1]$. If $tA+(1-t)C \geq tB + (1-t)C$ holds, then the third choice C is irrelevant and the order preferences of A and B hold independently of the presence of C

4. **Continuity:** The preference relationship (≥) is continuous if for alternative choices A, B, C →there exists some $\alpha \in [0,1]$ such that $\alpha A + (1-\alpha)C = B$
5. **Dominance:** For any alternative choices A, B, C, D if $\alpha A + (1-\alpha)B = C$, if $bA + (1-b)B = D$ and $A > B$ then $C > D$ if and only if $a > b$

Through the development of these axioms, many of the future arguments hold within the mathematical space that spans the set of operations for which these axioms hold. It is then from these ideas that the idea of risk aversion based on initial endowments of wealth are established and from this area with which much of the literature on portfolio optimization is developed.

The basic properties of these and those established are then easily extended to portfolio utility, with the idea of risk aversion that was initially established being extended by the work of Arrow (1964) and Pratt (1964). Their work transfers the idea of a lottery to represent a portfolio's return through time, incorporating the concepts of probability and thus providing a strong crossover point to incorporate utility theory into the world of mathematical finance.

Consider an initial endowment of wealth W_0 with the investor having the choice of choosing a portfolio P representing a vector of weightings placed on a portfolio of assets ($\mathbf{w_P}$). Letting R_p be the expected return of the portfolio after one period, we have $W_1 = W_0(1+R_p)$. Furthermore letting U(W) be a continuous utility function of Bernoulli functional form such that U'(W) > 0 and U''(W)<0, representing increasing absolute utility and diminishing marginal utility, we can define an investor's risk aversions as follows:

6. **Absolute risk aversion:** $\lambda_a(W) = -\frac{U''(W)}{U'(W)}$
7. **Relative risk aversion:** $\lambda_r(W) = -\frac{W\,U''(W)}{U'(W)}$

This then leads to the expected utility function being approximated as the following:

$$E[U(W)]=E[W] - 0.5\ b\ Var[W] \text{ where b is an arbitrary constant}$$

Furthermore, the framework generates the unique behavior displaying an investors utility function being globally higher or lower depending on their risk aversions. Thus a more risk averse investor demands a higher risk-premium for any change in level of risk relative to an investor that is less risk averse, and an investor has greater local risk aversion relative to another if and only if he has greater global risk aversion relative to another investor. This framework thus creates a smooth function that is applicable to many of the proceeding optimization frameworks.

The Concept of Risk

One of the most debated topics within the finance community is with regards to the definition used to model risk within quantitative models. The most commonly accepted definition for risk is the standard deviation, typically referred to as volatility, of asset returns. Investors above all wish to preserve capital and have steady growth in their investments through time in order to capitalize on the power of compounding while minimizing large swings in asset value to the downside. Through the simultaneous minimization of risk and maximization of returns, investors are able to build wealth in order to reach monetary goals such as retirement.

The definition of risk however is hotly debated, as certain characteristics utilized within financial models have been shown to not necessarily hold-up with actual data. One of the primary points that much of financial theory relies upon is the assumption of asset returns being normally distributed, with error terms that show minimal auto-correlation and homoscedasticity. Empirical studies of asset returns through time however have debunked these assumptions.

Some of these characteristics are shared across virtually all financial time-series data sets, with Cont (2000) showing that asset returns typically have numerous characteristics that are non-Gaussian, with the most notable differences being quoted as the following:

1. **Absence of autocorrelations:** asset movements exhibit minimal autocorrelation in longer-period movements
2. **Heavy tails:** asset return distributions typically have much longer tails than a normal distribution would suggest
3. **Asymmetry in returns:** large swings to the downside are typically much more common than outsized returns to the upside
4. **Aggregational Gaussianity:** as the investment horizon increases, the distribution of returns tends to approach (though not necessarily converge to) a Gaussian distribution
5. **Intermittency:** volatility of asset returns tends to show variability; that is, the variance in assets returns is not constant
6. **Volatility clustering:** volatility in asset returns tends to show characteristics of autocorrelation, with periods of high/low volatility being more likely to be followed in the immediate term by similar volatility

Given that these characteristics violate the initial assumption of normality seen in many risk estimation and portfolio optimization models, all financial models that incorporate some measure of quantitative risk are fundamentally misrepresenting the inherent risk of an investment. This stems from a need to simplify a model down to what we view as the most important areas of risk that matter to an investor in their pursuit of growth and preservation of wealth. As such, an analysis on empirical data of portfolio performance based on various risk definitions bears interest.

The Mean-Variance Framework

This establishment of the idea of risk and its relationship to utility functions thus culminated in the revolutionizing paper of Portfolio Selection published by Markowitz (1952). Let us assume that an investor has a Von Neumann-Morgenstern utility function. Consider a pool of n assets with **X** representing an nx1 vector of $x_i, \ldots, x_n$ asset weights within a portfolio such that $\sum x_i = 1$ for i = 1, …, n. Consider the expected return of asset i as being the return of an asset over a period t discounted back to the present by value $d_{i,t}$. Furthermore let $E[r_p]$ represent the total expected return of the portfolio:

$$E[r_i] = \sum_{t=1}^{\infty} d_{i,t}\, r_{i,t}$$

Furthermore, we can present the variance of an asset and covariance of the assets in the return as:

$$\sigma_i^2 = \sum_{t=1}^{\infty} d_{i,t}\,(r_{i,t} - E[r_i])^2 \quad \text{and} \quad \sigma_{i,j} = \sum_{t=1}^{\infty} d_{i,t}\,(r_{i,t} - E[r_i])(r_{j,t} - E[r_j])$$

We furthermore know that the correlation of the two assets can be presented as the following:

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i\,\sigma_j}$$

Moving to the more macro profile as a whole, we can present the characteristics of the portfolio as:

$$E[r_p] = \sum_{i=1}^{n} x_i\, E[r_i] \quad \text{and} \quad \sigma_p^2 = \mathbf{X}'\,\boldsymbol{\Omega}\,\mathbf{X}$$

Where **X'** represents the transpose of the matrix of asset weights, and **Ω** is an nxn covariance matrix such that $\Omega_{i,j} = \sigma_{i,j}$.

This basic framework provides the power and justification for the idea of optimization, as the incorporation of non-correlated assets decreases the general volatility of an underlying portfolio, and thus allows for higher expected returns for given levels of portfolio volatility.

The Efficient Frontier and Capital Market Line

The framework then developed above gives rise to the idea of the efficient frontier – that is the set of all weightings within the vector **X** such that the for a risk level, return is maximized. That is:

$$\text{Max } E[r_p] = \sum_{i=1}^{n} x_i E[r_i] \quad \text{s.t.} \quad \sigma_p = a \text{ and } \sum x_i = 1 \quad \text{with a being a feasible } \sigma_p$$

Within the mean-variance framework, this set takes the shape of the following efficient frontier for theoretical assets A through H:

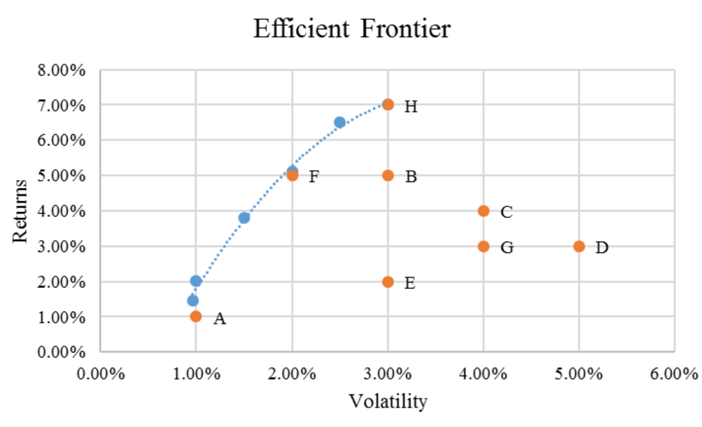

Figure 1: Efficient Frontier

Furthermore, the development of this framework provides even more insight when introducing a risk-free asset. Consider the introduction of an asset with expected return r_{rf} and σ_{rf}

= 0. Any funds can be borrowed or loaned at the rate of the risk-free asset, and it experiences no-volatility by definition as a risk free asset. Letting w now represent the weight of some risky portfolio p that is on the efficient frontier, our portfolio characteristics for the total portfolio z now become the following:

$$E[r_z] = w\,E[r_p] + (1 - w)r_{rf} \quad \text{and} \quad \sigma_z^2 = w^2\sigma_p^2 \quad \rightarrow \quad w = \frac{\sigma_z}{\sigma_p}$$

By substituting this condensed formula for w, we are left with the following equation:

$$E[r_z] = r_{rf} + \frac{E[r_p] - r_{rf}}{\sigma_p}\sigma_z$$

Thus, we have constructed a linear equation with intercept r_{rf} and the slope being the Sharpe-Ratio. This implies that there exists a choice of portfolio on the efficient frontier that maximizes the Sharpe-Ratio also creates the most efficient formulation of any assets across the board, through the ability to borrow and lend at the risk free rate we can move up or down this line depending on our volatility target. This line is known as the capital markets line. Graphically, this is presented as the following:

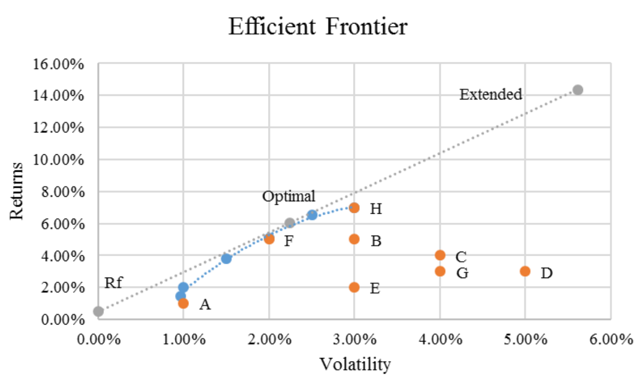

Figure 2: Efficient Frontier with Capital Market Line

Hence, in our Mean-Variance framework, the optimization techniques comes down to maximizing the Sharpe-Ratio and then lending or borrowing funds in order to move across the capital allocation line depending upon the individuals underlying utility function, thus removing a large part of the guesswork and estimation regarding the exact functional form and coefficients within one's utility function and neatly arriving at an optimal set of asset allocations within investments in the risky-portfolio investment universe.

The Mean-Variance Capital Asset Pricing Model

Further extending this idea to a more general framework were Sharpe (1964) and Litner (1965) with the derivation of the Capital Asset Pricing Model (CAPM) soon after Markowitz's portfolio selection paper. Under the assumptions of homogeneity in investor expectations and perfectly clearing markets with the ability to borrow or lend at the same rate, the only difference in investors is their general risk aversion level, thus creating a different utility function for each individual. Under this model, all investors agree, and thus the ideal portfolio for everyone must be the optimal risky portfolio discussed previously, as all investors agree that it has the highest return per unit of risk taken. As such, this portfolio must be the market portfolio. Furthermore, assuming no transaction costs and perfectly liquid markets.

Consider an investor who holds some combination (1-w) of the market portfolio and adds w of another risky asset i within the market:

$$E[r_w] = wE[r_i] + (1 - w)E[r_m] \quad \text{and} \quad \sigma_w = (w^2\sigma_i^2 + (1 - w)^2\sigma_m^2 + 2w(1 - w)\sigma_{i,m})^{.5}$$

Taking the derivative of both equations with respect to w, we form the following:

$$\frac{dE[r_w]}{dw} = E[r_i] - E[r_m] \qquad \text{and} \qquad \frac{d\sigma_w}{dw} = \frac{w\sigma_i^2 + (w-1)\sigma_m^2 + (1-2w)\sigma_{i,m}}{\sigma_w}$$

If we assume that w = 0 then:

$$\frac{d\sigma_w}{dw} = \frac{\sigma_i \ - \sigma_{i,m}^2}{\sigma_w}$$

Furthermore, dividing the two derivatives above by each other and assuming w = 0:

$$\frac{dE[w]}{d\sigma_w} = \frac{(E[r_i] - E[r_m])\ \sigma_{p,m}}{\sigma_{i,m} - \sigma_m^2}$$

This is the slope of the market portfolio. As such, it must equal the slope of the CML at this point:

$$\frac{E[r_m] - E[r_f]}{\sigma_m} = \frac{(E[r_i] - E[r_m])\ \sigma_{i,m}}{\sigma_{i,m} - \sigma_m^2} \qquad \rightarrow \qquad E[r_i] = E[r_f] + \frac{\sigma_{i,m}}{\sigma_m^2}\ (\ E[r_m] - E[r_f]\ )$$

However, we know that $\frac{\sigma_{i,m}}{\sigma_m^2}$ is equal to the beta of the market, assuming that this is in fact the market portfolio, and thus the expected return of the asset is completely based on its market beta:

$$E[r_z] = E[r_f] + \beta_i(\ E[r_m] - E[r_f]\ )$$

The Mean-Lower Partial Moment Framework

A large disadvantage of the Mean-Variance framework presented previously is that the assumptions are unrealistically strong relative to many of the modern methods. Assumptions of normal distributions in asset returns are generally untrue as based upon modern research, especially over shorter periods of time. Furthermore, excessively large constraints are placed upon the utility functional form within the mean-variance framework that appear generally untrue based on empirical data on individuals' risk-aversion to certain extreme scenarios.

Barring this in mind, Bawa and Lindenberg (1977) developed an alternative methodology for portfolio optimization and utility that emphasizes the increased negative utility of large downside movements in asset prices, alongside constructing a more robust optimization framework that allows less precise specification of one's utility function and associated coefficients in a polynomial or other form.

Let **X** be a nx1 vector of x_i, …, x_n asset weights such that $\sum x_i = 1$ for i = 1, …, n. We can further apply that $x_i \geq 0$ if we restrict the portfolio to being long-only with no short-selling. Furthermore, let F_X denote the probability distribution of r_p for portfolio X. Consider the following three classes of utility functions with r_p representing the theoretical return of a portfolio:

$$U_1 = \{u(r_p) | u'(r_p) > 0\}$$

$$U_2 = \{u(r_p) | u'(r_p) > 0 , u''(r_p) < 0 \}$$

$$U_3 = \{u(r_p) | u'(r_p) > 0 , u''(r_p) < 0 , u'''(r_p) > 0 \}$$

Through our conditions within each space, the set of feasible portfolios within each space is restricted depending upon our choice of the function form of the utility function. Thus, we can define the kth order lower partial moment of distribution F computed at point t, with a being the lower bound of the domain of function F as the following:

$$LPM_k(t; F) = \int_a^t (t - r_p)^n dF(r_p)$$

For a portfolio selection problem, it is logical to choose t as r_f, and thus we expand the LPM equation above to a more useful form, with **R** representing an nx1 vector of individual asset returns r_i for i = 1, …, n:

$$LPM_k(r_f; F) = \int_a^{r_f} (r_f - r_p)^n dF(r_p) = \int_a^{r_f} (r_f - \boldsymbol{X'R})^n dF(\boldsymbol{R})$$

And thus, portfolio selection under the Mean-LPM criteria for k = 1,2 reduces to:

$$Min\ LPM_k(r_f; \boldsymbol{X}) \quad \text{s.t.} \quad \sum x_i E[r_i] = u \ \text{and} \quad x_i \geq 0 \text{ (if no shorting)}$$

Please note, that the case of k = 3 will not be considered in this paper due to lack of evaluation tools necessarily to properly implement that case in an empirical analysis on real data.

Further of note, Bawa (1976b) has shown that the LPM set in this scenario is a convex function of **X**, and thus since we are minimizing a convex function over a convex set, we can solve the optimization problem using existing algorithms.

This further allows us to extend the Mean-LPM model to incorporate a risk-free asset into the equation. Consider X_0 is the amount of initial wealth allocated to the risk-free asset, while **Q** represents an nx1 vector of individual asset allocations as a proportion of the non-risk free capital allocated to each asset q_i. Thus, if we let **Y** be an (n+1)x1 size vector such that **Y' = (**X_0, q_1, …, q_n), the portfolio selection then reduces to the following with the inclusion of a risk-free asset:

$$Min\ LPM_k(r_f; \boldsymbol{Y}) \quad \text{s.t.} \quad X_0 r_f + \sum(1 - X_0)x_i E[r_i] = u \quad \text{and} \quad x_i \geq 0 \text{ (if no shorting)}$$

Another interesting thing to note about this space is that we can write the following:

$$LPM_k^{\frac{1}{k}}(r_f; \boldsymbol{Y}) = (1 - X_0)LPM_k^{\frac{1}{k}}(r_f; \boldsymbol{X})$$

Thus implying that the Mean-$LPM_k^{1/k}$ space is a linear combination of a portfolio of risky assets and a risk-free asset in weights (1-X_0) and X_0 respectively, thus creating a similar situation to the Mean-Variance criteria where we establish an optimal market portfolio and then can shift up and down a modified Capital Market Line, confirming that the two-funds separation theorem also holds in the Mean-LPM space.

Thus we can create an efficient frontier by minimizing the LPM for a given level of expected return. Please note, we graph relative to $LPM_k^{1/k}$ in order for this to be seen more easily:

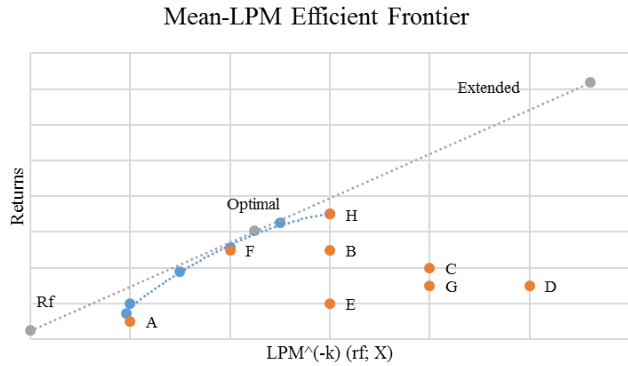

Figure 3: Mean-LPM Efficient Frontier and LPM-Capital Market Line

The Mean-Lower Partial Moment Capital Asset Pricing Model

A further point to touch upon is that similar to the derivation of the CAPM model, the LPM model extends the framework to a more general principle for k = 1, 2 , of which the CAPM is simply a special case. Let us first reconsider the initial assumptions of the CAPM model, those mainly being that all investors share homogenous beliefs regarding asset returns and statistical metrics, and the perfect liquidity of financial markets. Thus, there exists a perfect market-optimal portfolio that we can refer to as M, and this represents the tangency portfolio of the capital market line. Furthermore, the slope of this portfolio is clearly $LPM_k^{1/k}(r_f; \mathbf{M})/(E[r_m] - r_f)$. Similar to the derivation of the CAPM model, let us assume that we choose some asset j and add it to the market optimal portfolio at weight α, thus implying that the weight of portfolio M is (1-α). Thus, we can equate the tangency of the efficient frontier at portfolio M with the slope of the capital allocation line within the LPM framework, thus providing the following relationship:

$$E[r_j] - r_f = \beta_j^{MLPM_k}(E[r_m] - r_f) \quad \text{for j = 1, 2, …, M}$$

$$\beta_j^{MLPM_n} = \frac{CLPM_k(r_f; M, j)}{LPM_k(r_f; M)}$$

Where $CLPM_k(r_f; M,J)$is the colower partial moment of order k between the returns of security j and the return of the market portfolio M, defined as:

$$CLPM_k(r_f; M, j) = \int_{r_m=-\infty}^{r_f} \int_{r_j=-\infty}^{\infty} (r_f - r_m)^{k-1}(r_f - r_j) dF(r_m, r_j)$$

And $LPM_k(r_f; M)$ is defined as our initial definition of LPM. Thus, we have derived a similar formula to the initial CAPM model as the following:

$$E[r_j] = r_j + \beta_j^{MLPM_k}(E[r_m] - r_f) \quad \text{for j = 1, 2, …, M}$$

While the above is a derivation for arbitrary probability distributions F, it can be shown that the CAPM model is simply the case where F represents a symmetric, normal distribution where is simply redefined as the folowing $\beta_j^{MLPM_n} = \beta_j^{Mean-Variance} = \frac{\sigma_{j,m}}{\sigma_m^2}$, which is the typical definition of beta in the Mean-Variance framework as discussed previously.

The Mean-Semivariance Framework

Returning to the LPM framework presented previously, we shift into the model of application. Given the generality presented in the theory, we will focus on the case of k = 2, as this represents the scenario where there is a form of skew within portfolio returns and investors show a strong negative utility to large negative downswings. This sub-form is most widely applied, and the most applicable way to create an optimization framework with semivariance as the risk metric; however, as will be seen there do exist some rough approximations required in order to optimize on semivariance. Let **X** once again denote an nx1 vector of asset weights in a portfolio, and **R** represent an nx1 vector of the asset's returns.

By definition, we compute semi-variance (λ) for a portfolio as the following in a discrete space with y being a discrete number of observations for the data:

$$\lambda_{p,t}^2 = E\left[min\{\left(r_p - t\right), 0\}^2\right] = \frac{1}{y}\sum_{i=1}^{y} min\{(r_{p,i} - t), 0\}^2$$

Thus, in the Mean-Semivariance subset of the Mean-LPM set, the optimization problem's efficient frontier can be defined as:

$$min\,\lambda_{p,t}^2 = \frac{1}{y}\sum_{i=1}^{y} min\{(r_{p,i} - t), 0\}^2 \quad \text{s.t.} \quad \sum_{i=1}^{n} x_i E[r_i] = r_{target} \quad \text{and}$$

$$\sum_{i=1}^{n} x_i = 1]$$

One problem that arises from this definition however is that a construction of a semivariance matrix is inherently endogenous, as the values change depending upon the weights placed on the individual assets. For example, if the target return is 5%, and a shift in asset weights takes a periods return from 5% to 4%, in the semicovariance calculation this would result in the value being recorded as a 0% for that period and would thus inherently change the portfolio semivariance.

With this in mind, Estrada (2008) developed an alternative method through changing the definition of semicovariance to rely on the underlying assets' semicovariances rather than looking at the total portfolio semivariance at different weights, we construct a semicovariance matrix such that entry a,b is equal to:

$$\lambda_{a,b,t} = \frac{1}{y}\sum_{i=1}^{y} min\{(r_{a,i} - t), 0\}\, min\{(r_{b,i} - t), 0\}$$

Where this represents the semicovariance between asset a and b in the portfolio of n assets with target rate t. Thus, as this definition focuses on the individual asset performance rather than the total portfolio performance, the optimization problem using this definition becomes exogenous as the variables are invariant to the weights allocated to them.

Similarly, we can construct a downside correlation matrix such that entry a,b is equal to:

$$\rho_{a,b,t}^{Downside} = \frac{\lambda_{a,b,t}}{\lambda_{b,t}\lambda_{a,t}}$$

A further point to note regarding this topic, is that since this was constructed within the Mean-LPM framework, this is simply a specific case of the previously mentioned general functions, and as such the LPM-CAPM derivation as discussed previously now reverts to the following:

$$E[r_j] = r_j + \beta_j^{MeanSemivariance} \left(E[r_m] - r_f\right)$$

Where $\beta_j^{MeanSemivariance} = \frac{\lambda_{j,m,r_f}}{\lambda_{m,r_f}}$ and can be defined as the downside-beta of security j on the semivariance capital market line.

The Conditional Value at Risk Framework

Another traditional way to quantify and gauge risk in a portfolio is with the concept of Value at Risk (VaR). This is a metric that essentially quantifies the underlying value of a portfolio at risk for a given significance level δ. Mathematically, it is denoted as the following:

$$VaR = \mu + N^{-1}(\delta)\sigma$$

Where μ denotes the expected percentage return of the portfolio, σ represents the standard deviation of returns, and N^{-1} represents the normal distribution inverse function for a significant value of alpha. Typically for VaR, this number is then multiplied by some V_0, representing an initial portfolio value; however, for our assumptions and throughout this paper we will assume a V_0 of 1, thus resulting in our VaR being expressed in a percent drawdown on portfolio value rather than an arbitrary amount of notional.

One drawdown of the VaR framework however is that is explicitly does not account for the magnitude of the drawdown in portfolio value after the significance value of alpha. For instance, the portfolio VaR could have an incredibly long tail, with massive drawdowns at theVaR levels greater than α. Similarly, there could be a very dense distribution of probability right after the significance value. Clearly, the latter has significantly less risk than the former; however, the basic VaR metric does not account for this.

This leads to our discussion of Conditional Value at Risk (CVaR), which takes the magnitude of the size of the values after the VaR value α, and applies these to our risk metric so as to create a more robust and wholesome measure of tail risk. Thus given the higher quality of this metric, it is the one which shall be included in this paper in order to optimize on

Uryasev and Rockafellar (2000) give us a framework with which to apply this to portfolio theory. In order to being this framework consider a loss function f(x,y) with portfolio allocation variable x representing a portfolio of assets and y the associated random vector of movements in specific portfolio x (though the choice of x is arbitrary). Let us further assume that y has a specific probability density function p(y) representing all possible shifts in the underlying value of portfolio x.

For illustrative purposes, let x_i represent percentage of portfolio i in the portfolio at t=0, m_i represents the notional value of asset i at t=0, and let y_i represent the notional value of asset i in period t=1. The loss function for a portfolio of n assets would be the following:

$$f(\boldsymbol{X},\boldsymbol{Y}) = (x_1 m_1 + \cdots + x_n m_n) - (x_1 y_1 + \cdots + x_n y_n) = x_1(m_1 - y_1) + \cdots + x_n(m_n - y_n)$$

Furthermore, if shorts are not allowed we would additionally apply the constraint of $x_i \geq 0$ for i = 1, …, n. With this loss function in mind, let us extend this to the general framework for CVaR. Furthermore, for simplicity we assume that p(y) is such that it has a probability density function; however, the case is easily modified for the discrete case as will be done later in our methodology. We define the cumulative probability distribution function of f(**X**, **Y**) not exceeding a threshold loss of α as the following:

$$\Psi(\boldsymbol{X},\alpha) = \int_{f(x,y)\leq\alpha} p(\boldsymbol{y})d\boldsymbol{y}$$

This function is generally non-decreasing and continuous from right to left, though not necessarily from left to right due to possibility for jumps and other irregularities in the left to right behavior of the function. The literature generally assumes that the function p(y) does not have any jumps or irregularities however, and thus it is continuous everywhere with respect to α. We then define the β-VaR and β-CVaR values for the loss random variable on portfolio **X** as $\alpha_\beta(\mathbf{X})$ and $\Phi_\beta(\mathbf{X})$ respectively for any specficed probability level β in the interval (0, 1). We define these equations as the following:

$$\alpha_\beta(\boldsymbol{X}) = min\{\ \alpha \mid \Psi(\boldsymbol{X},\alpha) \geq\ \beta\ \}$$

$$\Phi_\beta(\boldsymbol{X}) = (1-\beta)^{-1}\int_{f(x,y)\geq\alpha_\beta(\boldsymbol{X})} f(x,y)p(y)dy$$

Intuitively, $\alpha_\beta(\boldsymbol{X})$ is the left endpoint of the nonempty interval consisting of values of α such that $\Phi_\beta(\mathbf{X}) = \beta$, with exceptions being ruled out based on our assumptions of the continuity and non-decreasing nature of $\Phi_\beta(\mathbf{X})$. In other words, we can restate $\Phi_\beta(\mathbf{X})$ as being the conditional expectation of the loss associated with X relative to that loss being $\alpha_\beta(\boldsymbol{X})$ or greater.

Furthermore, we can combine the previous equations into the following function F_β defined as the following:

$$F_\beta(\boldsymbol{X}, \alpha) = \alpha + (1-\beta)^{-1} \int_{-\infty}^{\infty} max[(f(x,y) - \alpha), 0] p(y) dy$$

A key characteristic of the function F_β is that it is convex and continuous as a function of α, and thus can be optimized using existing algorithms.

Another point to note is that the above continuous time formula can be reduced to a summation for discrete time and approximated for a set of q observations of the random return vector y. This is stated as the following:

$$F_\beta(\boldsymbol{X}, \alpha) \sim \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{q} max[(f(x,y) - \alpha), 0]$$

While the summation function is not necessarily continuous, it is piecewise linear with respect to α and can be minimized through linear programming methods.

Methodology

Overview

For our methodology, we have created an iterative process in order to generate a large enough sample size in order to draw realistic conclusions within the data provided. We being by choosing a random sample of 20 random securities from the S&P1500 universe as of the date that we are optimizing portfolios for. We then generate optimal portfolios for each of the optimization techniques that we are studying (Mean-Variance, Mean-Semivariance, and Mean-CVaR). We then study the performance of those portfolios over a 12-month ahead period over which the portfolios will be neither rebalanced nor have any investment moves made. We have chosen a 12-month period so as to analyze which optimization technique is most optimal to use for a medium term holding period, as distributions of returns approach approximately normal over long time horizons. Therefore a shorter time frame will likely warrant more interesting results.

Following our optimization and recording of performance for each portfolio, we will then choose another random 20 securities and repeat the process described above. We will then repeat this iteratively 13 times, so as to reduce any outlier performances of various portfolios and hopefully come to a conclusion that is statistically significant.

We will then repeat the above for a separate period with different general market movements than the first studied period. This will allow us to study if the general market trends during a period cause a particular optimization to outperform. That is we will try to answer questions such as: does Mean-Semivariance outperform Mean-Variance in sideways markets, but underperform in bull markets? Graphically, our methodology is as follows:

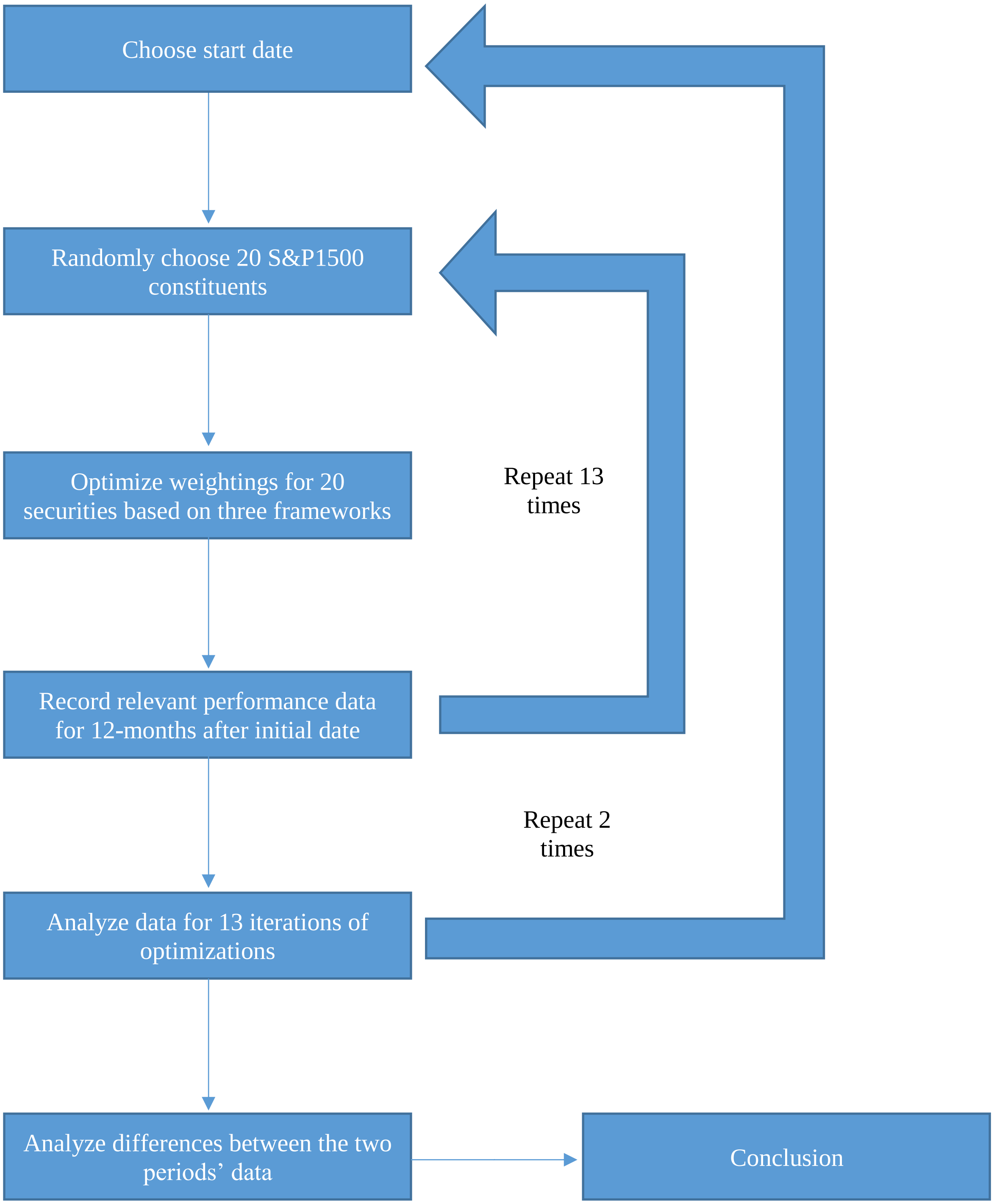

Figure 4: Graphical Representation of Methodology

Mean-Variance Optimization

For a mean-variance optimization problem, we will solve the following linear program:

$$Max\ \frac{E[r_p] - r_f}{\sigma_p}$$

$$s.t. \quad x_i \geq 0$$

$$x_i \leq 0.25$$

$$\sum_{i=1}^{20} x_i = 1$$

Where x_i represents the weighting of asset i in the portfolio, r_f represents the risk-free rate of 12-month t-bills, and $E[r_p]$ and σ_p represent the expected return and expected standard deviation of the portfolio respectively. We additionally imposing the constraints of all positive asset weights and maximum positional sizes of 25% of the portfolio so as to model out long-only and moderately diversified portfolios.

In order to illustrate this methodology, a sample optimization on a 10 asset portfolio will be performed on the following portfolio with the given expected returns and standard deviations derived from average analyst price targets at the time of the optimization and the implied volatility of 12-month at-the-money options:

Optimization security universe					
Security Number	SPX1500 Number	Ticker Symbol	Potential Upside	12M Implied Volatility	Historical Beta
1	1058	APH	20.21%	29.55%	1.058
2	800	CW	21.33%	38.09%	1.0598
3	869	BCO	14.84%	33.96%	1.0133
4	435	BOH	15.06%	32.33%	0.9362
5	453	BAC	35.49%	39.84%	1.16
6	1162	XPER	84.98%	40.86%	0.9389
7	22	POOL	13.48%	31.71%	0.965
8	1141	EXLS	16.95%	68.44%	0.968
9	1221	EBIX	47.71%	45.13%	1.0557
10	1064	CTS	49.60%	48.53%	1.0112

Figure 5: Sample Universe

The following correlation matrix was then constructed based on weekly returns for 3-years prior to the optimization date:

Correl	APH	CW	BCO	BOH	BAC	XPER	POOL	EXLS	EBIX	CTS
APH	1	0.4225719	0.3554035	0.4623024	0.4797399	0.2933223	0.4787154	0.2596479	0.2098544	0.3294307
CW	0.4225719	1	0.3126302	0.5282329	0.3897666	0.2780941	0.4753344	0.2032909	0.0807357	0.3352105
BCO	0.3554035	0.3126302	1	0.3927556	0.3465626	0.127887	0.330128	0.3197544	0.1135299	0.3345011
BOH	0.4623024	0.5282329	0.3927556	1	0.6066645	0.2886663	0.4844375	0.2417854	0.1108975	0.5208983
BAC	0.4797399	0.3897666	0.3465626	0.6066645	1	0.2034722	0.378768	0.1993708	0.2067538	0.4591414
XPER	0.2933223	0.2780941	0.127887	0.2886663	0.2034722	1	0.2750017	0.2014617	0.1500715	0.3131576
POOL	0.4787154	0.4753344	0.330128	0.4844375	0.378768	0.2750017	1	0.3922788	0.2831427	0.3050841
EXLS	0.2596479	0.2032909	0.3197544	0.2417854	0.1993708	0.2014617	0.3922788	1	0.1287722	0.1975439
EBIX	0.2098544	0.0807357	0.1135299	0.1108975	0.2067538	0.1500715	0.2831427	0.1287722	1	0.1038385
CTS	0.3294307	0.3352105	0.3345011	0.5208983	0.4591414	0.3131576	0.3050841	0.1975439	0.1038385	1

Figure 6: Sample Universe's Correlation Matrix

The implied volatilities provided by the options contract were then utilized to construct the following covariance matrix using the fact that $\sigma_{i,j} = \sigma_i \sigma_j \rho_{i,j}$:

Covariance	APH	CW	BCO	BOH	BAC	XPER	POOL	EXLS	EBIX	CTS
APH	0.087338	0.0475703	0.035668	0.0441638	0.0564884	0.0354171	0.0448659	0.052515	0.0279857	0.0472462
CW	0.0475703	0.1451	0.0404408	0.0650426	0.0591549	0.0432805	0.0574209	0.0529967	0.0138777	0.0619659
BCO	0.035668	0.0404408	0.1153214	0.0431137	0.0468909	0.0177438	0.0355529	0.0743137	0.0173973	0.0551257
BOH	0.0441638	0.0650426	0.0431137	0.1044906	0.0781338	0.0381242	0.0496608	0.0534892	0.0161762	0.0817133
BAC	0.0564884	0.0591549	0.0468909	0.0781338	0.1587465	0.0331225	0.0478589	0.0543639	0.0371726	0.0887769
XPER	0.0354171	0.0432805	0.0177438	0.0381242	0.0331225	0.1669294	0.0356319	0.0563321	0.0276683	0.0620913
POOL	0.0448659	0.0574209	0.0355529	0.0496608	0.0478589	0.0356319	0.1005714	0.0851392	0.0405191	0.0469524
EXLS	0.052515	0.0529967	0.0743137	0.0534892	0.0543639	0.0563321	0.0851392	0.468376	0.0397683	0.0656088
EBIX	0.0279857	0.0138777	0.0173973	0.0161762	0.0371726	0.0276683	0.0405191	0.0397683	0.2036266	0.0227393
CTS	0.0472462	0.0619659	0.0551257	0.0817133	0.0887769	0.0620913	0.0469524	0.0656088	0.0227393	0.2355064

Figure 7: Sample Universe's Covariance Matrix

We then utilized Excel's linear optimization solver in order to maximize the expected returns in excess of the risk-free rate (a 12-month t-bill) and generated the following portfolio:

Optimization security universe						
Security Number	SPX1500 Number	Ticker Symbol	Potential Upside	12M Implied Volatility	Historical Beta	Portfolio Weighting
1	1058	APH	20.21%	29.55%	1.058	0.02%
2	800	CW	21.33%	38.09%	1.0598	5.89%
3	869	BCO	14.84%	33.96%	1.0133	7.07%
4	435	BOH	15.06%	32.33%	0.9362	2.94%
5	453	BAC	35.49%	39.84%	1.16	11.48%
6	1162	XPER	84.98%	40.86%	0.9389	25.00%
7	22	POOL	13.48%	31.71%	0.965	3.89%
8	1141	EXLS	16.95%	68.44%	0.968	1.64%
9	1221	EBIX	47.71%	45.13%	1.0557	25.00%
10	1064	CTS	49.60%	48.53%	1.0112	17.08%

Figure 8: Sample Universe's Mean-Variance Optimized Portfolio

Furthermore, this portfolio has the following characteristics where M-CVaR, M-S, and M-V ratios are the excess return ratios respectively for the three optimization frameworks which we are studying:

M-CVaR Ratio	0.9735
M-S Ratio	3.1832
M-V Ratio	1.8922

CVaR	50.18%
Semivar	15.35%
Volatility	25.82%
Port. Beta	1.02

Exp Ret	49.27%

Figure 9: Sample Universe's Mean-Variance Optimal Portfolio Characteristics

Mean-Semivariance Optimization

For a Mean-Semivariance optimization problem, we will solve the following linear program:

$$\text{Max } \frac{E[r_p] - r_f}{\lambda_p}$$

$$\text{s.t.} \quad x_i \geq 0$$

$$x_i \leq 0.25$$

$$\sum_{i=1}^{20} x_i = 1$$

For our sample optimization in this space we will be using the same investment universe as in our Mean-Variance optimization example. We construct the following semicovariance matrix below based on three years of historical total returns of the constituents of our investment universe:

Semicov	APH	CW	BCO	BOH	BAC	XPER	POOL	EXLS	EBIX	CTS
APH	0.0002527	0.0002322	0.0001608	0.0001181	0.0001829	0.0001625	0.0001432	0.0001369	0.0002647	0.0001649
CW	0.0002322	0.0006135	0.0002484	0.000215	0.0002433	0.0002927	0.0002559	0.0002339	0.0003356	0.0002532
BCO	0.0001608	0.0002484	0.0006119	0.0001572	0.0002313	0.0001932	0.000177	0.0002583	0.0003042	0.0002129
BOH	0.0001181	0.000215	0.0001572	0.0002154	0.0002035	0.0001696	0.0001414	0.0001505	0.0002243	0.0001519
BAC	0.0001829	0.0002433	0.0002313	0.0002035	0.0004589	0.0002046	0.0001593	0.0001954	0.0003173	0.0002022
XPER	0.0001625	0.0002927	0.0001932	0.0001696	0.0002046	0.0008477	0.0001827	0.0002577	0.0004087	0.0002618
POOL	0.0001432	0.0002559	0.000177	0.0001414	0.0001593	0.0001827	0.0003078	0.0001688	0.0002999	0.0001585
EXLS	0.0001369	0.0002339	0.0002583	0.0001505	0.0001954	0.0002577	0.0001688	0.0005281	0.0004254	0.0001932
EBIX	0.0002647	0.0003356	0.0003042	0.0002243	0.0003173	0.0004087	0.0002999	0.0004254	0.0022355	0.0004016
CTS	0.0001649	0.0002532	0.0002129	0.0001519	0.0002022	0.0002618	0.0001585	0.0001932	0.0004016	0.0004371

Figure 10: Sample Universe's Semicovariance Matrix

The formula of $\lambda_p^2 = \mathbf{W}'\boldsymbol{\lambda}\mathbf{W}$ is then applied in order to calculate the portfolio semivariance, where **W** is the 10x1 vector of asset weights, **W'** is the transpose of **W**, and **λ** is the 10x10 semicovariance matrix as seen above.

Excel's solver is then once again used in order to solve the linear optimization problem presented at the beginning of this section. This leads to the following asset weights:

Optimization security universe						
Security Number	SPX1500 Number	Ticker Symbol	Potential Upside	12M Implied Volatility	Historical Beta	Portfolio Weighting
1	1058	APH	20.21%	29.55%	1.058	0.02%
2	800	CW	21.33%	38.09%	1.0598	6.89%
3	869	BCO	14.84%	33.96%	1.0133	8.32%
4	435	BOH	15.06%	32.33%	0.9362	3.46%
5	453	BAC	35.49%	39.84%	1.16	25.00%
6	1162	XPER	84.98%	40.86%	0.9389	25.00%
7	22	POOL	13.48%	31.71%	0.965	4.58%
8	1141	EXLS	16.95%	68.44%	0.968	1.72%
9	1221	EBIX	47.71%	45.13%	1.0557	0.00%
10	1064	CTS	49.60%	48.53%	1.0112	25.00%

Figure 11: Sample Universe's Mean-Semivariance Optimal Portfolio

One major point to note is the differences between the Mean-Semivariance optimized portfolio and the Mean-Variance optimized portfolio. While general directional weightings are fairly the same with regards to the most heavily weighted stocks, there are fairly significant differences in the absolute weightings. Furthermore, this portfolio has the following characteristics:

M-CVaR Ratio	1.3063
M-S Ratio	3.7612
M-V Ratio	1.6247

CVaR	35.40%
Semivar	12.29%
Volatility	28.46%
Port. Beta	1.03

Exp Ret	46.66%

Figure 12: Sample Universe's Mean-Semivariance Optimal Portfolio Characteristics

Mean-Conditional Value at Risk Optimization

For a Mean-Conditional Value at Risk optimization problem, we will solve the following linear program:

$$\text{Max } \frac{E[r_p] - r_f}{CVaR_p}$$

$$\text{s.t.} \quad x_i \geq 0$$

$$x_i \leq 0.25$$

$$\sum_{i=1}^{20} x_i = 1$$

For the Mean-CVaR framework, rather than construct a matrix, we first construct a theoretical historical portfolio with associated weightings on investments within the appropriate investment universe. VaR and CVaR is then calculated based on theoretical performance of the portfolio over a three-year period on a weekly basis. We then optimize on the Mean-CVaR ration by changing the weightings of the portfolio until the relative historical portfolio converges to the maximum ratio. CVaR is then annualized respectively and used in the calculation of objective function of the optimization. We furthermore assume a 0.05 significance level, implying a beta value of 0.95, being the 95th percentile of the loss function.

Under these assumptions, we converge to the following VaR and CVaR metrics which are then annualized:

Weekly Numbers	
Sig. Level	5.00%
Beta	95.00%
Mean Ret	0.32%
Stdev	2.31%
VaR Comp	3.49%
CVaR Comp	0.94%
CVaR	4.44%

Annual Numbers	
Sub-VaR	25.18%
Sub-CVaR	6.81%
Total CVaR	31.99%

Figure 13: Sample Universe's Total CVaR Calculation for Optimal Portfolio

Where Sub-VaR and Sub-CVaR represent α and $\frac{1}{q(1-\beta)}\sum_{k=1}^{q}\max[(f(x,y)-\alpha),0]$ respectively. This furthermore leads to the optimal portfolio weightings:

Optimization security universe						
Security Number	SPX1500 Number	Ticker Symbol	Potential Upside	12M Implied Volatility	Historical Beta	Portfolio Weighting
1	1058	APH	20.21%	29.55%	1.058	5.01%
2	800	CW	21.33%	38.09%	1.0598	22.94%
3	869	BCO	14.84%	33.96%	1.0133	0.00%
4	435	BOH	15.06%	32.33%	0.9362	0.00%
5	453	BAC	35.49%	39.84%	1.16	7.82%
6	1162	XPER	84.98%	40.86%	0.9389	25.00%
7	22	POOL	13.48%	31.71%	0.965	2.77%
8	1141	EXLS	16.95%	68.44%	0.968	8.12%
9	1221	EBIX	47.71%	45.13%	1.0557	3.34%
10	1064	CTS	49.60%	48.53%	1.0112	25.00%

Figure 14: Sample Universe's Mean-CVaR Optimal Portfolio

The portfolio furthermore possesses the following portfolio characteristics:

M-CVaR	1.4146
M-S Ratio	3.4985
M-V Sharpe	1.6001
CVaR	31.99%
Semivar	12.93%
Volatility	28.28%
Port. Beta	1.01
Exp Ret	45.67%

Figure 15: Sample Universe's Mean-CVaR Optimal Portfolio Characteristics

Data and Analysis

In our analysis, we looked at the metrics on weekly realized returns, focusing on the areas of max weekly drawdown and tradeup, 5th percentile of weekly returns, 95th percentile of weekly returns, overall returns for the entire period, standard deviation of weekly returns, semideviation of returns, Sharpe ratio, and Sortino ratio. In order to rank all the metrics, in each trial run we awarded 3, 2, and 1 points for the highest, middle, and lowest metrics for each of the 13 trials. We then summated them to form heat-map charts (with a high score in StDev and Semivar being viewed negatively).

The first period studied was from 1/20/2016 through year-end. This represents a general bull-market for the S&P500 as can be seen from the below chart:

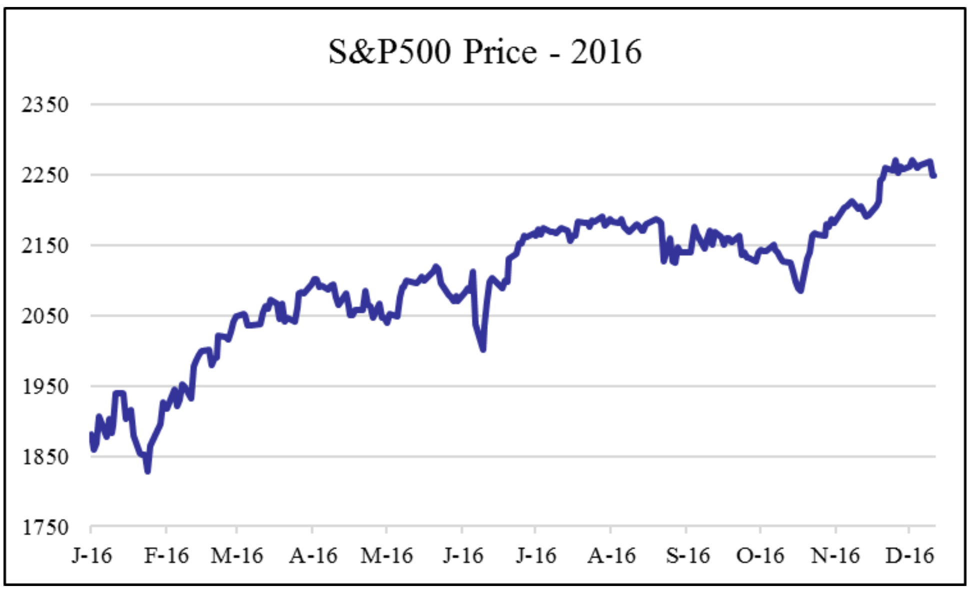

Figure 16: S&P500 Price Path for 2016

The 13 portfolios generated during this period scored the following on our metrics of study:

	Mean-Var	Mean-Semi	Mean-CVaR
Max Draw	52	40	58
Max Gain	58	54	38
5th Perc	44	50	56
95th Perc	44	60	46
Return	43	53	54
StDev	53	57	40
Semivar	51	59	40
Sharpe	48	49	53
Sortino	46	49	55

Figure 17: 1/20/2016 Optimized Portfolio Performance Metrics

Based on the above, it appears that Mean-CVaR outperforms the other two categories in virtually all categories. The only category in which it placed last was in the maximum weekly gain, which is to be expected given the focus of the metric on minimizing tail-risk of the returns. One surprising facet of the data is the generally poor performance of the mean-semivariance framework with regards to its realized levels of deviation. These portfolios scored the worst of the three optimization techniques in the deviation and semi-deviation metrics. Furthermore, these portfolios also had the highest drawdowns which seems extremely counterintuitive given that a semivariance optimization problem specifically focuses on minimizing risk defined as downside deviation.

It is my hypothesis that this apparent abnormality relative to the mean-variance framework is due to the utilization of historical semivariance relative to variance estimates being derived using forward implied volatilities as metrics of risk. As such, the more historical-based metric may create a certain bias in that it is not fully reflective of future expectations; however, as no such implied-semivariance metrics exist in the same regard as implied volatility, it appears as if mean-variance frameworks are more accurate in capturing expected risks and volatility in an applied setting.

The second period studied was from 1/20/2015 through year-end. This represents a general sideways-market from the S&P500 as can be seen from the below chart:

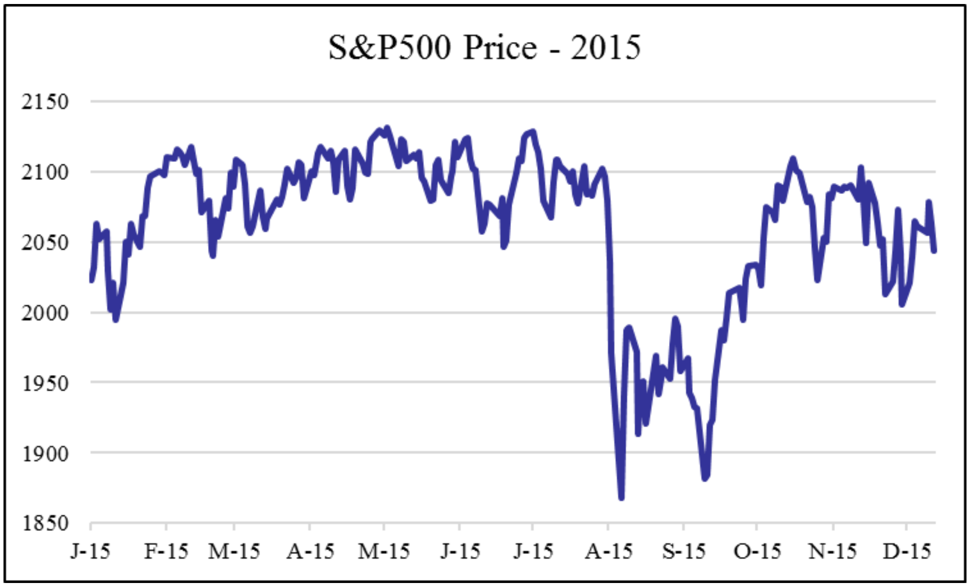

Figure 18: S&P500 Price Path for 2015

The 13 portfolios generated during this period scored the following on our metrics of study:

	Mean-Var	Mean-Semi	Mean-CVaR
Max Draw	29	18	31
Max Gain	22	29	27
5th Perc	29	19	30
95th Perc	26	29	23
Return	29	21	28
StDev	22	33	23
Semivar	25	31	22
Sharpe	28	21	29
Sortino	27	21	30

Figure 19: 1/20/2015 Optimized Portfolio Performance Metrics

For the sideways market, extremely similar results were had relative to the bull market. CVaR based optimizations continued to score best in virtually every metric. Interestingly, semivariance based optimization did worst in virtually every category in the sideways market. The confirms the thought from the optimization during the first period, suggesting that the forward implied volatility advantaged relative to historical semivariance outweighs the more investor-friendly definition of risk being semivariance rather than general variance.

Conclusion

The dilemma of asset weightings within portfolio management is a paramount issue within the asset management community. With numerous definitions of risk, it is the constant goal of the portfolio manager to minimize relevant risk metrics and exposures while simultaneously maximizing returns for given levels of risk. Bearing this in mind, numerous theoretical and applicable optimization techniques have been created in order to maximize risk-adjusted expected return for various levels of risk.

In this paper we explored and developed the theoretic behind optimization techniques, focusing on the three most commonly accepted definitions of risk by portfolio managers in the applied world and researchers in academia: standard deviation, semideviation, and conditional value at risk. We then utilized mean-variance, mean-semivariance, and mean-conditional value at risk optimization frameworks in order to analyze empirical optimization outcomes on equity portfolios over a one year period. A methodology of iterative portfolio selection was then ran for both a bull and sideways market. Our ultimate results brought us to the conclusion that conditional value at risk is the best optimization framework on both an absolute and risk-adjusted returns basis. Interestingly, there was minimal difference in performance between variance and semivariance based optimization techniques. It is our hypothesis that this is a result of having a forward looking metric of implied volatility while being forced to utilize historical semivariance due to the lack of sophisticated derivative instruments that allow us to create a forward estimate of semivariance.

Future research should build upon this paper by looking into the optimal alpha significance level to utilize within the CVaR framework, as a generic alpha of 5% was assumed in this paper. Furthermore, another area of interest would be an analysis of if optimal optimization frameworks have changed through time, as this paper primarily focused on the two one-year periods of 2015 and 2016. It is our initial thought that there would be greater divergence in performance of portfolios the farther back in time you go, when financial markets were less sophisticated; however, this is not the goal of this paper and should be researched further in the future. Finally, another area of interest would be the analysis of these optimization frameworks on different asset classes and multi-asset portfolios. Given the nuances of equity markets relative to other financial investments, there may be different rankings of methodologies for say credit and commodities versus our findings in the equity universe of CVaR optimization generally outperforming.

Appendix A

Optimized Portfolios for Period Starting 1/20/2016

The following represent the 13 investment universes and ultimate optimized portfolios for the mean-variance, mean-semivariance, and mean-CVaR frameworks (from left to right respectively):

Mean-Variance			
1	1	1	1
APH	0.00%	M-CVaR	1.18767784
CW	0.00%	M-S Ratio	3.92280599
BCO	0.00%	M-V Sharpe	2.52544918
BOH	0.00%		
BAC	1.85%	CVaR	0.44186532
XPER	25.00%	Semivar	0.13378017
POOL	0.00%	Volatility	0.2078021
EXLS	0.00%	Port. Beta	0.96792714
EBIX	9.99%	Exp Ret	0.52900365
CTS	3.59%		
MCD	0.00%	Max Draw	-2.60%
INCY	7.68%	Max Gain	6.09%
GEF	12.10%	5th Perc	-1.97%
MPWR	0.00%	95th Perc	4.23%
WLTW	6.08%	Return	53.61%
CTL	8.47%	Stdev.s	15.17%
NEM	10.65%	Semivar	13.48%
MHO	9.04%	Sharpe	3.51
CCI	5.56%	Sortino	3.94
AON	0.00%		
2	2	2	2
CORT	16.50%	M-CVaR	1.53207584
TXN	0.00%	M-S Ratio	3.52769346
MDC	0.00%	M-V Sharpe	2.92448273
EME	0.00%		
ACET	20.69%	CVaR	0.40939391
KEY	0.00%	Semivar	0.17779961
IR	0.00%	Volatility	0.21447298
MATX	14.20%	Port. Beta	1.18527023
PDCE	5.83%	Exp Ret	0.63143252
TCBI	0.00%		
CCC	0.00%	Max Draw	-4.07%
DDD	5.75%	Max Gain	8.43%
VMI	0.00%	5th Perc	-3.50%
NJR	0.00%	95th Perc	5.03%
DRE	19.04%	Return	46.86%
TTI	0.00%	Stdev.s	19.77%
SANM	18.00%	Semivar	16.36%
ALE	0.00%	Sharpe	2.35
INT	0.00%	Sortino	2.84
BA	0.00%		

Mean-Semivariance			
1	1	1	1
APH	0.00%	M-CVaR	1.32898337
CW	0.00%	M-S Ratio	4.24570549
BCO	0.00%	M-V Sharpe	2.33118413
BOH	0.00%		
BAC	1.83%	CVaR	0.38402665
XPER	25.00%	Semivar	0.12020735
POOL	0.00%	Volatility	0.21892952
EXLS	0.00%	Port. Beta	0.96459872
EBIX	3.64%	Exp Ret	0.51457503
CTS	3.68%		
MCD	0.00%	Max Draw	-2.93%
INCY	4.87%	Max Gain	5.88%
GEF	25.00%	5th Perc	-1.69%
MPWR	0.00%	95th Perc	4.79%
WLTW	6.54%	Return	57.00%
CTL	8.19%	Stdev.s	15.60%
NEM	8.20%	Semivar	14.04%
MHO	7.96%	Sharpe	3.63
CCI	5.08%	Sortino	4.03
AON	0.00%		
2	2	2	2
CORT	16.80%	M-CVaR	1.47037603
TXN	0.00%	M-S Ratio	3.77587912
MDC	0.00%	M-V Sharpe	2.476863
EME	0.00%		
ACET	21.37%	CVaR	0.45698766
KEY	0.00%	Semivar	0.17795689
IR	0.00%	Volatility	0.2712882
MATX	5.22%	Port. Beta	1.18995109
PDCE	0.00%	Exp Ret	0.67615371
TCBI	9.80%		
CCC	0.00%	Max Draw	-6.03%
DDD	9.15%	Max Gain	13.72%
VMI	0.00%	5th Perc	-3.94%
NJR	0.00%	95th Perc	5.69%
DRE	0.00%	Return	49.01%
TTI	3.71%	Stdev.s	25.09%
SANM	8.30%	Semivar	21.09%
ALE	0.00%	Sharpe	1.94
INT	0.66%	Sortino	2.30
BA	25.00%		

Mean-CVaR			
1	1	1	1
APH	0.00%	M-CVaR	1.71991034
CW	9.85%	M-S Ratio	4.22925503
BCO	0.00%	M-V Sharpe	2.20637154
BOH	0.00%		
BAC	0.94%	CVaR	0.28199993
XPER	25.00%	Semivar	0.11468086
POOL	0.00%	Volatility	0.21982454
EXLS	0.23%	Port. Beta	0.94654002
EBIX	0.00%	Exp Ret	0.4892246
CTS	9.16%		
MCD	0.00%	Max Draw	-3.49%
INCY	0.00%	Max Gain	5.72%
GEF	19.45%	5th Perc	-2.14%
MPWR	0.00%	95th Perc	4.91%
WLTW	10.32%	Return	57.51%
CTL	0.00%	Stdev.s	16.29%
NEM	8.92%	Semivar	14.30%
MHO	11.70%	Sharpe	3.50
CCI	4.16%	Sortino	3.99
AON	0.29%		
2	2	2	2
CORT	4.64%	M-CVaR	1.73447354
TXN	0.00%	M-S Ratio	3.32487985
MDC	0.00%	M-V Sharpe	2.48201936
EME	0.00%		
ACET	25.00%	CVaR	0.29735404
KEY	0.00%	Semivar	0.15511921
IR	0.00%	Volatility	0.20779561
MATX	9.74%	Port. Beta	1.13062562
PDCE	9.55%	Exp Ret	0.51996272
TCBI	4.34%		
CCC	0.00%	Max Draw	-4.79%
DDD	5.76%	Max Gain	6.03%
VMI	0.00%	5th Perc	-3.24%
NJR	0.00%	95th Perc	4.84%
DRE	25.00%	Return	38.99%
TTI	3.22%	Stdev.s	18.87%
SANM	12.11%	Semivar	14.79%
ALE	0.00%	Sharpe	2.04
INT	0.63%	Sortino	2.61
BA	0.00%		

3	3	3	3	3	3	3	3	3	3	3	3	3	3
NWBI	0.00%	M-CVaR	1.38471166		NWBI	0.00%	M-CVaR	1.34908614		NWBI	0.00%	M-CVaR	1.59950961
NYCB	5.26%	M-S Ratio	3.82813002		NYCB	5.22%	M-S Ratio	4.7776368		NYCB	0.00%	M-S Ratio	4.20070358
OSIS	4.69%	M-V Sharpe	1.88004878		OSIS	0.00%	M-V Sharpe	1.61495624		OSIS	0.00%	M-V Sharpe	1.72514845
UMPQ	0.00%				UMPQ	1.18%				UMPQ	0.00%		
TEX	0.62%	CVaR	0.37196451		TEX	0.59%	CVaR	0.43202642		TEX	0.11%	CVaR	0.33639274
MED	8.67%	Semivar	0.13454705		MED	0.00%	Semivar	0.12199355		MED	1.65%	Semivar	0.12808888
POST	1.49%	Volatility	0.27396289		POST	1.44%	Volatility	0.36090195		POST	1.44%	Volatility	0.31189398
TISI	25.00%	Port. Beta	0.97328785		TISI	25.00%	Port. Beta	0.97594857		TISI	16.68%	Port. Beta	0.94826741
PII	0.00%	Exp Ret	0.5192736		PII	0.03%	Exp Ret	0.58705086		PII	0.00%	Exp Ret	0.54227341
CRY	8.79%				CRY	1.24%				CRY	14.52%		
LPT	0.00%	Max Draw	-6.11%		LPT	0.00%	Max Draw	-6.89%		LPT	0.00%	Max Draw	-5.84%
KMI	4.09%	Max Gain	10.06%		KMI	25.00%	Max Gain	8.58%		KMI	16.78%	Max Gain	8.77%
TER	0.00%	5th Perc	-3.95%		TER	0.00%	5th Perc	-3.47%		TER	0.01%	5th Perc	-3.40%
AFL	0.00%	95th Perc	7.24%		AFL	11.90%	95th Perc	6.45%		AFL	0.00%	95th Perc	6.94%
BOH	0.00%	Return	52.98%		BOH	0.00%	Return	45.21%		BOH	0.00%	Return	53.64%
MPW	3.72%	Stdev.s	23.64%		MPW	5.55%	Stdev.s	23.60%		MPW	14.33%	Stdev.s	22.46%
ASTE	0.00%	Semivar	19.66%		ASTE	0.00%	Semivar	18.93%		ASTE	0.00%	Semivar	18.77%
NPO	25.00%	Sharpe	2.22		NPO	22.73%	Sharpe	1.90		NPO	20.78%	Sharpe	2.37
EBS	0.50%	Sortino	2.67		EBS	0.11%	Sortino	2.37		EBS	0.11%	Sortino	2.84
CHTR	12.17%				CHTR	0.00%				CHTR	13.58%		
4	4	4	4	4	4	4	4	4	4	4	4	4	4
TREX	0.93%	M-CVaR	0.93434022		TREX	0.74%	M-CVaR	0.98886098		TREX	1.00%	M-CVaR	1.32099691
SANM	0.31%	M-S Ratio	3.53325827		SANM	0.00%	M-S Ratio	3.86373623		SANM	0.00%	M-S Ratio	3.07872208
CXW	3.83%	M-V Sharpe	1.9654932		CXW	0.00%	M-V Sharpe	1.8376184		CXW	1.12%	M-V Sharpe	1.68256664
IFF	0.00%				IFF	0.00%				IFF	12.21%		
BEL	7.10%	CVaR	0.44640416		BEL	25.00%	CVaR	0.45485675		BEL	9.73%	CVaR	0.25821593
UTX	2.59%	Semivar	0.1180478		UTX	16.65%	Semivar	0.11641325		UTX	5.61%	Semivar	0.11079352
FSP	25.00%	Volatility	0.21220799		FSP	19.05%	Volatility	0.24476795		FSP	6.29%	Volatility	0.20272745
WST	25.00%	Port. Beta	0.93064343		WST	25.00%	Port. Beta	0.96345838		WST	25.00%	Port. Beta	0.95117815
FAST	0.00%	Exp Ret	0.42130336		FAST	0.00%	Exp Ret	0.45400009		FAST	0.00%	Exp Ret	0.34531245
PKI	0.00%				PKI	0.00%				PKI	0.00%		
MDCO	0.75%	Max Draw	-5.31%		MDCO	0.00%	Max Draw	-4.16%		MDCO	7.90%	Max Draw	-3.76%
BSX	2.54%	Max Gain	7.77%		BSX	0.00%	Max Gain	8.09%		BSX	1.42%	Max Gain	5.69%
GDOT	0.00%	5th Perc	-3.02%		GDOT	0.00%	5th Perc	-2.11%		GDOT	0.00%	5th Perc	-2.52%
BOFI	15.04%	95th Perc	5.64%		BOFI	13.55%	95th Perc	5.91%		BOFI	3.69%	95th Perc	3.98%
WTFC	0.00%	Return	36.43%		WTFC	0.00%	Return	46.00%		WTFC	0.00%	Return	25.84%
ETH	0.00%	Stdev.s	19.13%		ETH	0.00%	Stdev.s	18.63%		ETH	1.31%	Stdev.s	14.46%
INT	0.63%	Semivar	15.52%		INT	0.00%	Semivar	16.00%		INT	0.00%	Semivar	11.02%
AEP	5.47%	Sharpe	1.88		AEP	0.00%	Sharpe	2.45		AEP	13.89%	Sharpe	1.76
LEN	10.81%	Sortino	2.32		LEN	0.00%	Sortino	2.85		LEN	10.84%	Sortino	2.31
AIZ	0.00%				AIZ	0.00%				AIZ	0.00%		
5	5	5	5	5	5	5	5	5	5	5	5	5	5
MMC	1.00%	M-CVaR	4.44441259		MMC	1.00%	M-CVaR	4.40977004		MMC	1.00%	M-CVaR	4.88843737
UNH	3.12%	M-S Ratio	11.3659621		UNH	2.94%	M-S Ratio	11.7902738		UNH	15.80%	M-S Ratio	11.100871
SKX	0.00%	M-V Sharpe	5.80274352		SKX	0.00%	M-V Sharpe	5.7553574		SKX	0.00%	M-V Sharpe	5.5745903
HSIC	2.53%				HSIC	5.61%				HSIC	4.29%		
CHK	0.00%	CVaR	0.32931297		CHK	0.00%	CVaR	0.32474497		CHK	0.00%	CVaR	0.29162116
TNC	25.00%	Semivar	0.12877068		TNC	25.00%	Semivar	0.12146034		TNC	19.49%	Semivar	0.12841981
CUB	2.20%	Volatility	0.25222598		CUB	0.00%	Volatility	0.24882046		CUB	0.00%	Volatility	0.25572674
SKX	0.00%	Port. Beta	0.97148144		SKX	0.00%	Port. Beta	0.9591107		SKX	0.06%	Port. Beta	0.97745344
PSB	8.96%	Exp Ret	1.4678127		PSB	6.93%	Exp Ret	1.43626066		PSB	1.37%	Exp Ret	1.42978179
NCI	0.00%				NCI	0.00%				NCI	0.85%		
TECH	5.01%	Max Draw	-4.57%		TECH	13.58%	Max Draw	-4.45%		TECH	11.76%	Max Draw	-3.99%
AIZ	0.00%	Max Gain	10.29%		AIZ	0.00%	Max Gain	9.62%		AIZ	0.00%	Max Gain	9.50%
FRED	3.22%	5th Perc	-3.25%		FRED	0.00%	5th Perc	-2.60%		FRED	0.00%	5th Perc	-2.95%
CY	2.34%	95th Perc	4.19%		CY	0.00%	95th Perc	4.25%		CY	0.00%	95th Perc	3.61%
HOLX	8.10%	Return	27.36%		HOLX	3.79%	Return	26.72%		HOLX	14.06%	Return	25.80%
IBM	8.86%	Stdev.s	18.70%		IBM	16.15%	Stdev.s	17.51%		IBM	0.00%	Stdev.s	17.67%
FLR	0.00%	Semivar	14.99%		FLR	0.00%	Semivar	14.09%		FLR	0.00%	Semivar	13.87%
FF	25.00%	Sharpe	1.44		FF	25.00%	Sharpe	1.50		FF	25.00%	Sharpe	1.44
ATGE	4.65%	Sortino	1.80		ATGE	0.00%	Sortino	1.87		ATGE	6.32%	Sortino	1.83
NUS	0.00%				NUS	0.00%				NUS	0.00%		

6	6	6	6	6	6	6	6	6	6	6	6	6	6
ARI	0.00%	M-CVaR	1.12757111		ARI	1.04%	M-CVaR	1.07776918		ARI	1.00%	M-CVaR	1.1998215
EW	0.00%	M-S Ratio	2.81978896		EW	0.00%	M-S Ratio	3.32497667		EW	0.00%	M-S Ratio	3.10467752
OMI	0.00%	M-V Sharpe	1.54237894		OMI	0.00%	M-V Sharpe	1.33970925		OMI	0.00%	M-V Sharpe	1.45191412
MED	13.17%				MED	0.00%				MED	0.00%		
NTRS	0.00%	CVaR	0.26247752		NTRS	0.00%	CVaR	0.30653239		NTRS	0.00%	CVaR	0.26690652
IPG	0.00%	Semivar	0.10495894		IPG	0.00%	Semivar	0.09936045		IPG	0.00%	Semivar	0.10314765
JEC	5.00%	Volatility	0.19188674		JEC	0.79%	Volatility	0.24659915		JEC	0.58%	Volatility	0.22056413
TAP	17.88%	Port. Beta	0.93787377		TAP	6.61%	Port. Beta	0.99277689		TAP	18.35%	Port. Beta	0.94736511
AMT	25.00%	Exp Ret	0.30017206		AMT	25.00%	Exp Ret	0.33458116		AMT	21.85%	Exp Ret	0.32445018
CHTR	8.98%				CHTR	0.00%				CHTR	7.84%		
CNMD	5.62%	Max Draw	-4.37%		CNMD	1.51%	Max Draw	-4.89%		CNMD	9.48%	Max Draw	-4.69%
HOMB	2.51%	Max Gain	4.58%		HOMB	0.00%	Max Gain	5.96%		HOMB	0.00%	Max Gain	5.12%
HAYN	6.10%	5th Perc	-2.70%		HAYN	15.24%	5th Perc	-3.17%		HAYN	11.63%	5th Perc	-3.02%
V	0.00%	95th Perc	3.46%		V	0.00%	95th Perc	4.47%		V	0.00%	95th Perc	3.49%
HOMB	2.51%	Return	24.78%		HOMB	0.00%	Return	28.24%		HOMB	0.00%	Return	18.82%
URI	0.00%	Stdev.s	14.29%		URI	0.00%	Stdev.s	17.98%		URI	0.00%	Stdev.s	16.37%
VLO	0.00%	Semivar	10.16%		VLO	0.00%	Semivar	13.64%		VLO	0.00%	Semivar	11.70%
FOXA	13.25%	Sharpe	1.70		FOXA	25.00%	Sharpe	1.55		FOXA	22.14%	Sharpe	1.12
MAR	0.00%	Sortino	2.40		MAR	12.67%	Sortino	2.04		MAR	7.13%	Sortino	1.57
HBHC	0.00%				HBHC	12.14%				HBHC	0.00%		
7	7	7	7	7	7	7	7	7	7	7	7	7	7
MPWR	0.00%	M-CVaR	0.63609444		MPWR	0.00%	M-CVaR	1.06561707		MPWR	0.00%	M-CVaR	1.21359072
STI	0.00%	M-S Ratio	2.30342284		STI	14.80%	M-S Ratio	3.4345594		STI	8.08%	M-S Ratio	3.23975995
FTI	0.12%	M-V Sharpe	1.71600271		FTI	16.12%	M-V Sharpe	1.15207025		FTI	11.84%	M-V Sharpe	1.11745164
EHC	8.60%				EHC	0.00%				EHC	11.08%		
RRC	4.82%	CVaR	0.61825231		RRC	25.00%	CVaR	0.41246405		RRC	25.00%	CVaR	0.32561648
PJC	0.00%	Semivar	0.17073151		PJC	0.00%	Semivar	0.12797238		PJC	0.00%	Semivar	0.12197359
MCD	0.42%	Volatility	0.22917613		MCD	0.00%	Volatility	0.38151209		MCD	0.00%	Volatility	0.35363064
BEAT	17.76%	Port. Beta	0.98154093		BEAT	1.50%	Port. Beta	1.05571181		BEAT	2.52%	Port. Beta	0.9909063
EL	0.00%	Exp Ret	0.39747686		EL	0.00%	Exp Ret	0.44373873		EL	4.80%	Exp Ret	0.39937514
SEE	0.00%				SEE	0.89%				SEE	0.85%		
ACM	14.12%	Max Draw	-17.11%		ACM	10.39%	Max Draw	-4.68%		ACM	4.74%	Max Draw	-5.35%
GPN	0.00%	Max Gain	9.75%		GPN	0.00%	Max Gain	11.85%		GPN	11.21%	Max Gain	10.54%
AMT	25.00%	5th Perc	-3.55%		AMT	3.32%	5th Perc	-4.24%		AMT	3.94%	5th Perc	-3.66%
SPGI	25.00%	95th Perc	6.18%		SPGI	17.33%	95th Perc	5.24%		SPGI	3.37%	95th Perc	3.90%
HII	0.00%	Return	21.37%		HII	0.00%	Return	33.86%		HII	0.00%	Return	28.38%
PII	0.00%	Stdev.s	29.55%		PII	10.66%	Stdev.s	21.98%		PII	6.37%	Stdev.s	20.19%
ALG	0.00%	Semivar	19.45%		ALG	0.00%	Semivar	17.62%		ALG	0.00%	Semivar	15.66%
NWBI	0.00%	Sharpe	0.71		NWBI	0.00%	Sharpe	1.52		NWBI	0.00%	Sharpe	1.39
T	4.15%	Sortino	1.08		T	0.00%	Sortino	1.90		T	6.13%	Sortino	1.79
CATO	0.00%				CATO	0.00%				CATO	0.07%		
8	8	8	8	8	8	8	8	8	8	8	8	8	8
F	0.00%	M-CVaR	1.653227		F	13.85%	M-CVaR	1.76363454		F	1.00%	M-CVaR	1.93735153
DECK	25.00%	M-S Ratio	4.8160507		DECK	0.00%	M-S Ratio	5.71145894		DECK	8.80%	M-S Ratio	5.05932008
RLI	0.00%	M-V Sharpe	1.86608196		RLI	0.00%	M-V Sharpe	1.85833215		RLI	0.00%	M-V Sharpe	1.75472813
CVGW	0.00%				CVGW	0.00%				CVGW	8.26%		
OGE	0.00%	CVaR	0.45753861		OGE	0.00%	CVaR	0.51217474		OGE	0.00%	CVaR	0.41497289
HAYN	0.00%	Semivar	0.1570613		HAYN	0.00%	Semivar	0.15815382		HAYN	0.00%	Semivar	0.15890443
HNI	25.00%	Volatility	0.40534939		HNI	25.00%	Volatility	0.48607514		HNI	15.68%	Volatility	0.45816121
VRSN	0.00%	Port. Beta	1.11580461		VRSN	0.00%	Port. Beta	1.16925615		VRSN	0.00%	Port. Beta	1.11350459
GPS	0.00%	Exp Ret	0.76062518		GPS	0.00%	Exp Ret	0.90749906		GPS	0.00%	Exp Ret	0.80815837
EGN	11.97%				EGN	25.00%				EGN	23.22%		
MCRI	0.00%	Max Draw	-6.52%		MCRI	0.00%	Max Draw	-8.42%		MCRI	0.00%	Max Draw	-8.13%
AAPL	21.43%	Max Gain	11.84%		AAPL	9.91%	Max Gain	11.50%		AAPL	4.97%	Max Gain	11.31%
CREE	0.00%	5th Perc	-5.57%		CREE	0.00%	5th Perc	-5.60%		CREE	0.00%	5th Perc	-4.56%
KOPN	14.64%	95th Perc	5.70%		KOPN	25.00%	95th Perc	6.67%		KOPN	25.00%	95th Perc	6.45%
FNGN	1.96%	Return	44.92%		FNGN	1.24%	Return	54.89%		FNGN	0.00%	Return	51.51%
IVC	0.00%	Stdev.s	26.45%		IVC	0.00%	Stdev.s	28.38%		IVC	0.00%	Stdev.s	26.11%
ALGT	0.00%	Semivar	20.50%		ALGT	0.00%	Semivar	22.06%		ALGT	0.00%	Semivar	20.68%
EGHT	0.00%	Sharpe	1.68		EGHT	0.00%	Sharpe	1.92		EGHT	0.00%	Sharpe	1.96
APOG	0.00%	Sortino	2.17		APOG	0.00%	Sortino	2.47		APOG	3.28%	Sortino	2.47
WDFC	0.00%				WDFC	0.00%				WDFC	9.78%		

9	9	9	9	9	9	9	9	9	9	9	9	9	9
MED	1.00%	M-CVaR	1.25570199		MED	0.00%	M-CVaR	1.36867332		MED	1.00%	M-CVaR	2.07580196
MMC	0.00%	M-S Ratio	4.53400869		MMC	0.00%	M-S Ratio	5.72400413		MMC	0.00%	M-S Ratio	4.72254247
ABM	0.00%	M-V Sharpe	2.16621636		ABM	0.00%	M-V Sharpe	1.88203941		ABM	0.00%	M-V Sharpe	1.81798697
GHL	0.00%				GHL	7.39%				GHL	0.00%		
TGI	5.72%	CVaR	0.57349538		TGI	15.04%	CVaR	0.59974963		TGI	0.04%	CVaR	0.2972434
GILD	8.77%	Semivar	0.15883059		GILD	0.00%	Semivar	0.14340683		GILD	0.00%	Semivar	0.13065386
VVI	24.79%	Volatility	0.33244107		VVI	8.31%	Volatility	0.43615522		VVI	25.00%	Volatility	0.33939651
CCL	0.00%	Port. Beta	1.02261286		CCL	0.00%	Port. Beta	1.07050297		CCL	0.24%	Port. Beta	0.96353409
HUBG	0.00%	Exp Ret	0.72434929		HUBG	0.00%	Exp Ret	0.82507131		HUBG	0.00%	Exp Ret	0.62122842
EL	0.00%				EL	0.00%				EL	0.00%		
IRDM	25.00%	Max Draw	-4.98%		IRDM	15.07%	Max Draw	-5.98%		IRDM	16.08%	Max Draw	-3.68%
EXR	0.00%	Max Gain	10.17%		EXR	0.00%	Max Gain	10.40%		EXR	20.89%	Max Gain	6.27%
NWE	0.00%	5th Perc	-3.06%		NWE	0.00%	5th Perc	-3.82%		NWE	5.90%	5th Perc	-1.92%
CF	11.87%	95th Perc	5.05%		CF	25.00%	95th Perc	5.98%		CF	5.85%	95th Perc	5.87%
TREE	13.67%	Return	37.75%		TREE	4.20%	Return	38.01%		TREE	0.00%	Return	43.95%
INTU	0.00%	Stdev.s	21.30%		INTU	0.00%	Stdev.s	23.04%		INTU	0.00%	Stdev.s	16.11%
NWN	0.00%	Semivar	17.54%		NWN	0.00%	Semivar	18.34%		NWN	0.00%	Semivar	13.88%
APC	8.65%	Sharpe	1.75		APC	25.00%	Sharpe	1.63		APC	25.00%	Sharpe	2.70
UNFI	0.00%	Sortino	2.13		UNFI	0.00%	Sortino	2.05		UNFI	0.00%	Sortino	3.14
SANM	0.53%				SANM	0.00%				SANM	0.00%		
10	10	10	10	10	10	10	10	10	10	10	10	10	10
CECO	10.05%	M-CVaR	0.80664776		CECO	1.00%	M-CVaR	0.94213465		CECO	1.01%	M-CVaR	1.20081425
CBOE	0.00%	M-S Ratio	3.0369116		CBOE	0.00%	M-S Ratio	3.95649966		CBOE	0.00%	M-S Ratio	3.17233086
IDCC	12.16%	M-V Sharpe	2.04326295		IDCC	7.34%	M-V Sharpe	1.70391524		IDCC	25.00%	M-V Sharpe	1.56070728
ORLY	1.07%				ORLY	2.84%				ORLY	8.11%		
TSCO	0.00%	CVaR	0.64200745		TSCO	0.00%	CVaR	0.40866671		TSCO	2.65%	CVaR	0.29906685
BANC	0.00%	Semivar	0.17052649		BANC	0.00%	Semivar	0.09731306		BANC	0.00%	Semivar	0.11320501
UVE	0.00%	Volatility	0.25345434		UVE	0.00%	Volatility	0.2259614		UVE	1.05%	Volatility	0.2301032
ROCK	0.00%	Port. Beta	0.96138383		ROCK	3.93%	Port. Beta	0.89491603		ROCK	7.73%	Port. Beta	0.88829426
WABC	0.00%	Exp Ret	0.52208387		WABC	0.00%	Exp Ret	0.38922907		WABC	0.00%	Exp Ret	0.36333374
OZRK	0.00%				OZRK	0.00%				OZRK	0.00%		
ACIW	0.00%	Max Draw	-5.60%		ACIW	10.60%	Max Draw	-3.51%		ACIW	3.63%	Max Draw	-4.59%
PACW	5.10%	Max Gain	10.66%		PACW	13.04%	Max Gain	7.02%		PACW	0.89%	Max Gain	7.44%
AJG	14.44%	5th Perc	-3.40%		AJG	25.00%	5th Perc	-2.97%		AJG	17.88%	5th Perc	-3.39%
IVR	25.00%	95th Perc	5.96%		IVR	25.00%	95th Perc	5.37%		IVR	24.70%	95th Perc	5.86%
ALB	0.00%	Return	72.64%		ALB	1.58%	Return	43.75%		ALB	0.00%	Return	57.72%
TREE	14.79%	Stdev.s	22.90%		TREE	6.59%	Stdev.s	17.81%		TREE	0.03%	Stdev.s	19.26%
SSTK	17.40%	Semivar	20.29%		SSTK	0.00%	Semivar	14.59%		SSTK	4.28%	Semivar	16.30%
EFX	0.00%	Sharpe	3.15		EFX	1.02%	Sharpe	2.43		EFX	0.00%	Sharpe	2.97
WD	0.00%	Sortino	3.56		WD	0.00%	Sortino	2.97		WD	0.00%	Sortino	3.51
AIV	0.00%				AIV	2.06%				AIV	3.03%		
11	11	11	11	11	11	11	11	11	11	11	11	11	11
LNN	0.00%	M-CVaR	1.35771675		LNN	0.00%	M-CVaR	1.43604171		LNN	0.99%	M-CVaR	1.53325287
MYL	4.83%	M-S Ratio	3.69165239		MYL	4.18%	M-S Ratio	4.18000092		MYL	4.96%	M-S Ratio	3.87868349
CLB	1.29%	M-V Sharpe	1.95742614		CLB	1.25%	M-V Sharpe	1.52226453		CLB	9.34%	M-V Sharpe	1.66367191
TILE	13.32%				TILE	12.27%				TILE	10.86%		
MANT	0.00%	CVaR	0.41044381		MANT	0.00%	CVaR	0.39539119		MANT	0.00%	CVaR	0.35359378
MMSI	25.00%	Semivar	0.15095312		MMSI	3.95%	Semivar	0.13583687		MMSI	9.22%	Semivar	0.13977647
ALEX	0.00%	Volatility	0.28469347		ALEX	25.00%	Volatility	0.37299577		ALEX	12.88%	Volatility	0.32587475
LION	0.00%	Port. Beta	0.98108346		LION	0.00%	Port. Beta	1.02171337		LION	0.05%	Port. Beta	1.00012521
MRTN	0.00%	Exp Ret	0.56147643		MRTN	0.00%	Exp Ret	0.57200823		MRTN	0.00%	Exp Ret	0.54635867
SLG	16.63%				SLG	15.25%				SLG	7.87%		
BBT	0.00%	Max Draw	-6.54%		BBT	0.00%	Max Draw	-6.60%		BBT	0.00%	Max Draw	-6.91%
BRO	0.00%	Max Gain	12.51%		BRO	0.00%	Max Gain	11.74%		BRO	0.00%	Max Gain	11.25%
POOL	0.00%	5th Perc	-4.18%		POOL	0.00%	5th Perc	-3.70%		POOL	0.00%	5th Perc	-3.55%
CHFC	0.00%	95th Perc	5.76%		CHFC	0.00%	95th Perc	5.60%		CHFC	0.00%	95th Perc	6.00%
ATO	0.00%	Return	43.54%		ATO	0.00%	Return	41.50%		ATO	0.00%	Return	46.67%
CATM	9.42%	Stdev.s	23.90%		CATM	7.44%	Stdev.s	23.07%		CATM	17.61%	Stdev.s	22.83%
HNI	20.77%	Semivar	19.45%		HNI	25.00%	Semivar	18.56%		HNI	24.95%	Semivar	18.45%
CSL	0.00%	Sharpe	1.80		CSL	0.00%	Sharpe	1.78		CSL	0.00%	Sharpe	2.03
CVX	0.00%	Sortino	2.22		CVX	0.00%	Sortino	2.21		CVX	0.00%	Sortino	2.51
CBM	8.73%				CBM	5.66%				CBM	1.26%		

12	12	12	12	12	12	12	12	12	12	12	12	12	12
PDFS	5.70%	M-CVaR	1.55597616		PDFS	1.01%	M-CVaR	1.46827401		PDFS	1.01%	M-CVaR	1.63628593
TG	0.00%	M-S Ratio	4.9407424		TG	0.00%	M-S Ratio	5.03922791		TG	0.00%	M-S Ratio	4.4272637
TGNA	0.00%	M-V Sharpe	2.52203265		TGNA	0.00%	M-V Sharpe	2.26360777		TGNA	0.00%	M-V Sharpe	2.09704154
CUB	3.49%				CUB	0.00%				CUB	25.00%		
MNTA	15.34%	CVaR	0.37226598		MNTA	8.18%	CVaR	0.37990635		MNTA	3.86%	CVaR	0.30567036
AMZN	0.00%	Semivar	0.11723683		AMZN	0.00%	Semivar	0.11069287		AMZN	1.38%	Semivar	0.11297365
IVC	0.00%	Volatility	0.2296707		IVC	0.00%	Volatility	0.24642371		IVC	3.25%	Volatility	0.2385094
ESIO	25.00%	Port. Beta	0.96280234		ESIO	25.00%	Port. Beta	0.98389345		ESIO	24.60%	Port. Beta	0.93706157
MDRX	0.00%	Exp Ret	0.583447		MDRX	0.00%	Exp Ret	0.56201662		MDRX	0.00%	Exp Ret	0.50437412
WD	0.00%				WD	0.00%				WD	0.00%		
CLI	9.39%	Max Draw	-6.49%		CLI	1.14%	Max Draw	-7.72%		CLI	0.00%	Max Draw	-6.10%
ATO	6.85%	Max Gain	10.27%		ATO	0.00%	Max Gain	10.13%		ATO	6.75%	Max Gain	10.80%
PWR	14.07%	5th Perc	-3.51%		PWR	20.08%	5th Perc	-3.17%		PWR	15.63%	5th Perc	-3.15%
TDG	0.00%	95th Perc	4.33%		TDG	0.00%	95th Perc	5.13%		TDG	2.90%	95th Perc	3.67%
GTLS	0.00%	Return	29.18%		GTLS	0.00%	Return	25.08%		GTLS	0.00%	Return	24.17%
FOXA	6.08%	Stdev.s	20.05%		FOXA	15.37%	Stdev.s	20.64%		FOXA	7.54%	Stdev.s	20.40%
INGR	0.00%	Semivar	15.65%		INGR	0.00%	Semivar	15.82%		INGR	0.00%	Semivar	15.77%
JPM	0.00%	Sharpe	1.43		JPM	4.21%	Sharpe	1.19		JPM	0.00%	Sharpe	1.16
ATGE	0.00%	Sortino	1.84		ATGE	0.00%	Sortino	1.56		ATGE	0.00%	Sortino	1.51
VFC	14.08%				VFC	25.00%				VFC	8.07%		

13	13	13	13	13	13	13	13	13	13	13	13	13	13
PGNX	1.07%	M-CVaR	1.2633854		PGNX	1.01%	M-CVaR	1.45081049		PGNX	1.01%	M-CVaR	1.55945624
CHRW	0.00%	M-S Ratio	3.62406408		CHRW	0.00%	M-S Ratio	4.45403935		CHRW	0.00%	M-S Ratio	4.00192964
ROCK	0.00%	M-V Sharpe	2.35335703		ROCK	0.00%	M-V Sharpe	1.91313843		ROCK	0.00%	M-V Sharpe	2.04063917
DSPG	6.25%				DSPG	0.00%				DSPG	10.37%		
SIX	0.00%	CVaR	0.3839074		SIX	0.00%	CVaR	0.34750751		SIX	11.98%	CVaR	0.31091262
PFS	0.00%	Semivar	0.13383401		PFS	0.00%	Semivar	0.11319333		PFS	0.00%	Semivar	0.12115521
MTH	0.00%	Volatility	0.20609835		MTH	0.00%	Volatility	0.26352904		MTH	0.00%	Volatility	0.23759939
FULT	0.00%	Port. Beta	0.98338992		FULT	0.00%	Port. Beta	0.9701958		FULT	0.00%	Port. Beta	0.95611394
WIRE	0.00%	Exp Ret	0.48923301		WIRE	0.00%	Exp Ret	0.50837754		WIRE	0.00%	Exp Ret	0.48906463
PFE	25.00%				PFE	25.00%				PFE	15.36%		
RBC	0.91%	Max Draw	-4.81%		RBC	18.30%	Max Draw	-6.44%		RBC	5.80%	Max Draw	-4.39%
NTRS	0.00%	Max Gain	11.27%		NTRS	0.87%	Max Gain	13.51%		NTRS	0.00%	Max Gain	10.98%
HT	0.00%	5th Perc	-3.63%		HT	20.99%	5th Perc	-3.62%		HT	11.92%	5th Perc	-2.65%
MKSI	0.00%	95th Perc	5.12%		MKSI	0.00%	95th Perc	5.44%		MKSI	0.00%	95th Perc	5.11%
CLH	6.90%	Return	29.11%		CLH	0.96%	Return	27.84%		CLH	8.06%	Return	30.61%
ANIK	10.43%	Stdev.s	20.93%		ANIK	0.00%	Stdev.s	22.96%		ANIK	0.99%	Stdev.s	20.44%
XOXO	20.30%	Semivar	17.09%		XOXO	0.00%	Semivar	18.66%		XOXO	1.61%	Semivar	16.98%
PERY	25.00%	Sharpe	1.37		PERY	25.00%	Sharpe	1.19		PERY	25.00%	Sharpe	1.48
WHR	0.00%	Sortino	1.68		WHR	3.13%	Sortino	1.47		WHR	2.10%	Sortino	1.78
MTDR	4.14%				MTDR	4.74%				MTDR	5.81%		

Appendix B

Optimized Portfolios for Period Starting 1/20/2015

The following represent the 13 investment universes and ultimate optimized portfolios for the mean-variance, mean-semivariance, and mean-CVaR frameworks (from left to right respectively):

Mean-Variance					Mean-Semivariance					Mean-CVaR			
1	1	1	1	1	1	1	1	1	1	1	1	1	1
OA	1.00%	M-CVaR	0.7886963		OA	1.00%	M-CVaR	0.8112777		OA	1.00%	M-CVaR	0.8425537
TDG	0.00%	M-S Ratio	2.0510081		TDG	0.00%	M-S Ratio	2.1841497		TDG	0.00%	M-S Ratio	1.9473783
OXM	25.00%	M-V Sharpe	1.5448566		OXM	25.00%	M-V Sharpe	1.5436098		OXM	25.00%	M-V Sharpe	1.4340103
AMAT	0.00%				AMAT	0.00%				AMAT	0.00%		
NI	0.00%	CVaR	0.3516437		NI	0.00%	CVaR	0.3696612		NI	0.00%	CVaR	0.3282615
MDC	0.00%	Semivar	0.1352213		MDC	0.00%	Semivar	0.1373065		MDC	0.00%	Semivar	0.1420258
VIAB	13.01%	Volatility	0.1795248		VIAB	21.88%	Volatility	0.1942835		VIAB	25.00%	Volatility	0.1928702
CCC	7.69%	Port. Beta	0.9848402		CCC	13.84%	Port. Beta	1.0070423		CCC	5.85%	Port. Beta	1.0103392
SMTC	3.27%	Exp Ret	0.2789401		SMTC	1.88%	Exp Ret	0.3014979		SMTC	1.29%	Exp Ret	0.2781779
STT	2.19%				STT	0.00%				STT	1.63%		
SWK	3.18%	Max Draw	-7.84%		SWK	2.39%	Max Draw	-8.06%		SWK	0.46%	Max Draw	-8.59%
GIII	2.14%	Max Gain	6.66%		GIII	0.00%	Max Gain	6.73%		GIII	7.04%	Max Gain	7.03%
DUK	6.84%	5th Perc	-4.31%		DUK	1.01%	5th Perc	-4.90%		DUK	0.74%	5th Perc	-5.01%
ITG	0.74%	95th Perc	4.34%		ITG	0.00%	95th Perc	4.51%		ITG	0.00%	95th Perc	4.54%
ELY	4.77%	Return	-0.69%		ELY	3.46%	Return	-3.41%		ELY	11.99%	Return	-3.14%
ESS	0.00%	Stdev.s	20.15%		ESS	0.00%	Stdev.s	20.90%		ESS	0.00%	Stdev.s	21.57%
BLL	0.00%	Semivar	13.23%		BLL	0.00%	Semivar	13.47%		BLL	0.00%	Semivar	13.76%
HON	5.19%	Sharpe	-0.04		HON	4.55%	Sharpe	-0.17		HON	2.50%	Sharpe	-0.15
NTGR	0.00%	Sortino	-0.06		NTGR	0.00%	Sortino	-0.26		NTGR	0.00%	Sortino	-0.24
TDS	25.00%				TDS	25.00%				TDS	17.49%		
2	2	2	2	2	2	2	2	2	2	2	2	2	2
MUR	0.00%	M-CVaR	0.9101278		MUR	0.00%	M-CVaR	0.9793458		MUR	1.00%	M-CVaR	1.0308479
MCS	25.00%	M-S Ratio	1.9671641		MCS	0.00%	M-S Ratio	2.3061657		MCS	25.00%	M-S Ratio	1.9204862
CBL	0.00%	M-V Sharpe	1.3471197		CBL	0.00%	M-V Sharpe	1.2247231		CBL	0.00%	M-V Sharpe	1.22728
POL	25.00%				POL	0.85%				POL	6.04%		
K	0.00%	CVaR	0.3655857		K	0.00%	CVaR	0.3441005		K	0.00%	CVaR	0.3042639
VSH	0.00%	Semivar	0.1691418		VSH	0.00%	Semivar	0.1461271		VSH	0.00%	Semivar	0.1633179
RRTS	8.39%	Volatility	0.2469934		RRTS	11.20%	Volatility	0.2751589		RRTS	6.79%	Volatility	0.255565
TKR	1.32%	Port. Beta	1.2149726		TKR	1.76%	Port. Beta	1.100593		TKR	0.00%	Port. Beta	1.1474065
SCI	9.30%	Exp Ret	0.3343297		SCI	21.70%	Exp Ret	0.3385934		SCI	6.30%	Exp Ret	0.3152498
CCOI	0.20%				CCOI	0.00%				CCOI	0.80%		
LXP	0.00%	Max Draw	-5.45%		LXP	0.00%	Max Draw	-6.48%		LXP	0.00%	Max Draw	-5.77%
RIG	0.00%	Max Gain	8.54%		RIG	0.00%	Max Gain	11.53%		RIG	0.00%	Max Gain	10.05%
LHCG	0.00%	5th Perc	-4.35%		LHCG	0.00%	5th Perc	-4.99%		LHCG	5.84%	5th Perc	-4.30%
KALU	2.21%	95th Perc	5.74%		KALU	14.34%	95th Perc	5.33%		KALU	3.40%	95th Perc	5.89%
ICUI	0.00%	Return	2.33%		ICUI	0.00%	Return	3.72%		ICUI	6.52%	Return	11.75%
MDU	0.27%	Stdev.s	21.37%		MDU	22.34%	Stdev.s	25.82%		MDU	1.59%	Stdev.s	22.55%
TTI	19.30%	Semivar	16.33%		TTI	25.00%	Semivar	19.69%		TTI	25.00%	Semivar	17.88%
MTRN	0.00%	Sharpe	0.10		MTRN	0.00%	Sharpe	0.14		MTRN	0.00%	Sharpe	0.51
DG	9.00%	Sortino	0.13		DG	2.80%	Sortino	0.18		DG	11.73%	Sortino	0.65
BEN	0.00%				BEN	0.00%				BEN	0.00%		

3	3	3	3	3	3	3	3	3	3	3	3	3	3
CNMD	0.96%	M-CVaR	0.6374948		CNMD	0.64%	M-CVaR	0.5949761		CNMD	0.64%	M-CVaR	0.6449802
WNC	0.00%	M-S Ratio	1.8648495		WNC	0.00%	M-S Ratio	1.9301728		WNC	0.00%	M-S Ratio	1.8181805
EQIX	1.57%	M-V Sharpe	1.2813896		EQIX	0.00%	M-V Sharpe	1.2504298		EQIX	0.00%	M-V Sharpe	1.2411065
RYN	0.00%				RYN	0.00%				RYN	0.00%		
MAA	0.00%	CVaR	0.418616		MAA	0.00%	CVaR	0.4881332		MAA	0.00%	CVaR	0.4162737
PVH	25.00%	Semivar	0.143103		PVH	25.00%	Semivar	0.1504672		PVH	25.00%	Semivar	0.1476687
MTRN	0.00%	Volatility	0.2082626		MTRN	0.00%	Volatility	0.2322622		MTRN	0.00%	Volatility	0.2163298
NTCT	4.26%	Port. Beta	1.0154245		NTCT	0.58%	Port. Beta	0.9993681		NTCT	0.58%	Port. Beta	0.9624235
WWE	15.58%	Exp Ret	0.2684655		WWE	20.38%	Exp Ret	0.2920276		WWE	16.57%	Exp Ret	0.2700883
SBSI	3.31%				SBSI	5.42%				SBSI	5.44%		
OA	0.00%	Max Draw	-6.37%		OA	0.00%	Max Draw	-6.73%		OA	0.00%	Max Draw	-5.97%
JBT	0.00%	Max Gain	5.59%		JBT	0.00%	Max Gain	6.36%		JBT	0.00%	Max Gain	6.08%
ZBRA	0.00%	5th Perc	-3.69%		ZBRA	0.00%	5th Perc	-4.85%		ZBRA	0.00%	5th Perc	-2.91%
BANC	13.89%	95th Perc	4.63%		BANC	20.86%	95th Perc	5.24%		BANC	20.62%	95th Perc	4.60%
KEY	15.21%	Return	10.68%		KEY	25.00%	Return	15.47%		KEY	4.03%	Return	15.65%
WAFD	0.00%	Stdev.s	18.57%		WAFD	0.00%	Stdev.s	21.30%		WAFD	0.00%	Stdev.s	17.99%
CME	0.00%	Semivar	13.30%		CME	0.00%	Semivar	15.57%		CME	0.00%	Semivar	13.47%
TTC	9.91%	Sharpe	0.57		TTC	0.00%	Sharpe	0.72		TTC	25.00%	Sharpe	0.86
KFY	0.00%	Sortino	0.79		KFY	0.00%	Sortino	0.98		KFY	0.00%	Sortino	1.15
UNM	10.30%				UNM	2.12%				UNM	2.12%		

4	4	4	4	4	4	4	4	4	4	4	4	4	4
LH	1.00%	M-CVaR	0.8814039		LH	1.00%	M-CVaR	0.8431278		LH	1.00%	M-CVaR	0.9057172
ZION	25.00%	M-S Ratio	2.2663531		ZION	25.00%	M-S Ratio	2.3051694		ZION	23.58%	M-S Ratio	2.2488982
CACI	0.00%	M-V Sharpe	1.5549833		CACI	0.00%	M-V Sharpe	1.4952028		CACI	0.00%	M-V Sharpe	1.5387951
VNO	0.00%				VNO	0.00%				VNO	0.00%		
MAS	0.00%	CVaR	0.4101055		MAS	0.00%	CVaR	0.4363881		MAS	0.00%	CVaR	0.3893289
SBH	21.42%	Semivar	0.1594935		SBH	13.60%	Semivar	0.1596112		SBH	25.00%	Semivar	0.1567976
APC	5.13%	Volatility	0.2324582		APC	9.20%	Volatility	0.2460743		APC	3.28%	Volatility	0.2291545
X	25.00%	Port. Beta	1.1111541		X	25.00%	Port. Beta	1.1379438		X	25.00%	Port. Beta	1.0980054
RHI	0.00%	Exp Ret	0.3630686		RHI	0.00%	Exp Ret	0.369531		RHI	0.00%	Exp Ret	0.3542218
SCHW	0.00%				SCHW	0.00%				SCHW	0.00%		
KMPR	2.58%	Max Draw	-8.02%		KMPR	4.93%	Max Draw	-8.47%		KMPR	4.63%	Max Draw	-7.84%
SF	0.00%	Max Gain	5.12%		SF	0.00%	Max Gain	5.70%		SF	0.00%	Max Gain	5.01%
LTC	0.00%	5th Perc	-4.34%		LTC	0.00%	5th Perc	-4.50%		LTC	0.53%	5th Perc	-4.24%
WAB	3.93%	95th Perc	4.12%		WAB	3.89%	95th Perc	4.41%		WAB	6.36%	95th Perc	4.25%
SNV	0.34%	Return	-18.48%		SNV	0.00%	Return	-20.27%		SNV	0.00%	Return	-18.57%
PKE	13.24%	Stdev.s	19.79%		PKE	11.86%	Stdev.s	20.98%		PKE	10.62%	Stdev.s	19.86%
SMG	0.00%	Semivar	11.93%		SMG	0.00%	Semivar	12.83%		SMG	0.00%	Semivar	12.08%
DISCK	0.00%	Sharpe	-0.94		DISCK	5.51%	Sharpe	-0.97		DISCK	0.00%	Sharpe	-0.94
WM	0.00%	Sortino	-1.56		WM	0.00%	Sortino	-1.59		WM	0.00%	Sortino	-1.55
EQIX	2.35%				EQIX	0.00%				EQIX	0.00%		

5	5	5	5	5	5	5	5	5	5	5	5	5	5
EAT	1.00%	M-CVaR	0.8938234		EAT	0.99%	M-CVaR	0.8516572		EAT	0.10%	M-CVaR	0.9752935
SMTC	0.00%	M-S Ratio	1.8065155		SMTC	0.00%	M-S Ratio	2.0256496		SMTC	0.00%	M-S Ratio	1.8549416
SAFM	0.00%	M-V Sharpe	1.3110326		SAFM	0.00%	M-V Sharpe	1.2428071		SAFM	0.05%	M-V Sharpe	1.2930668
DVA	0.00%				DVA	0.00%				DVA	0.00%		
SXI	25.00%	CVaR	0.2726738		SXI	9.07%	CVaR	0.2872429		SXI	21.41%	CVaR	0.2446153
KRC	0.00%	Semivar	0.1349129		KRC	0.00%	Semivar	0.1207674		KRC	0.00%	Semivar	0.1286141
CDR	0.00%	Volatility	0.1859009		CDR	0.00%	Volatility	0.1968386		CDR	0.00%	Volatility	0.1845007
DIOD	1.13%	Port. Beta	1.0857497		DIOD	0.00%	Port. Beta	1.0833896		DIOD	0.00%	Port. Beta	1.0611981
MYL	0.00%	Exp Ret	0.2453222		MYL	0.00%	Exp Ret	0.2462324		MYL	0.00%	Exp Ret	0.2401717
EQT	25.00%				EQT	25.00%				EQT	25.00%		
HCI	3.34%	Max Draw	-6.35%		HCI	0.00%	Max Draw	-6.69%		HCI	2.81%	Max Draw	-5.97%
RJF	6.39%	Max Gain	5.39%		RJF	1.94%	Max Gain	6.35%		RJF	3.81%	Max Gain	5.43%
CHD	0.00%	5th Perc	-3.86%		CHD	0.00%	5th Perc	-4.92%		CHD	0.00%	5th Perc	-4.72%
UNM	7.93%	95th Perc	3.74%		UNM	25.00%	95th Perc	3.21%		UNM	0.00%	95th Perc	3.56%
ODFL	4.18%	Return	-3.44%		ODFL	0.00%	Return	-0.56%		ODFL	3.98%	Return	-2.14%
DST	9.44%	Stdev.s	17.35%		DST	25.00%	Stdev.s	18.30%		DST	17.81%	Stdev.s	17.73%
ABT	5.77%	Semivar	11.14%		ABT	0.00%	Semivar	11.61%		ABT	8.07%	Semivar	10.91%
BAX	6.00%	Sharpe	-0.21		BAX	0.00%	Sharpe	-0.04		BAX	0.35%	Sharpe	-0.13
MDT	4.82%	Sortino	-0.32		MDT	13.00%	Sortino	-0.06		MDT	16.60%	Sortino	-0.21
CREE	0.00%				CREE	0.00%				CREE	0.00%		

6	6	6	6	6	6	6	6	6	6	6	6	6	6
BCO	0.01%	M-CVaR	0.5744984		BCO	0.01%	M-CVaR	0.6845253		BCO	0.01%	M-CVaR	0.8409729
NWE	0.00%	M-S Ratio	1.6345699		NWE	0.00%	M-S Ratio	2.071179		NWE	0.00%	M-S Ratio	1.8222852
ISRG	0.00%	M-V Sharpe	1.2283196		ISRG	0.00%	M-V Sharpe	1.1584962		ISRG	0.00%	M-V Sharpe	1.1136195
TRV	0.00%				TRV	0.00%				TRV	0.00%		
KMPR	3.05%	CVaR	0.5487589		KMPR	25.00%	CVaR	0.5287316		KMPR	0.00%	CVaR	0.4072723
WYNN	25.00%	Semivar	0.192871		WYNN	24.99%	Semivar	0.174746		WYNN	25.00%	Semivar	0.1879536
ZUMZ	0.00%	Volatility	0.2566605		ZUMZ	0.00%	Volatility	0.3124138		ZUMZ	11.99%	Volatility	0.3075601
DSW	5.44%	Port. Beta	1.303412		DSW	1.07%	Port. Beta	1.295091		DSW	4.78%	Port. Beta	1.2370616
JLL	0.00%	Exp Ret	0.3168611		JLL	0.00%	Exp Ret	0.3635301		JLL	2.83%	Exp Ret	0.344105
AKS	11.81%				AKS	21.08%				AKS	25.00%		
DVN	0.00%	Max Draw	-7.01%		DVN	5.49%	Max Draw	-8.16%		DVN	5.59%	Max Draw	-8.33%
WWW	0.00%	Max Gain	8.14%		WWW	0.04%	Max Gain	10.79%		WWW	1.46%	Max Gain	11.50%
WEN	0.00%	5th Perc	-5.31%		WEN	0.00%	5th Perc	-5.76%		WEN	0.00%	5th Perc	-6.68%
PGTI	14.90%	95th Perc	4.69%		PGTI	0.00%	95th Perc	6.07%		PGTI	0.00%	95th Perc	5.20%
URI	12.03%	Return	-13.34%		URI	8.86%	Return	-25.18%		URI	0.00%	Return	-36.00%
PLT	0.00%	Stdev.s	22.13%		PLT	1.30%	Stdev.s	26.31%		PLT	0.00%	Stdev.s	26.89%
NDAQ	10.85%	Semivar	14.57%		NDAQ	4.09%	Semivar	17.92%		NDAQ	4.20%	Semivar	17.35%
STI	16.62%	Sharpe	-0.61		STI	3.32%	Sharpe	-0.96		STI	8.44%	Sharpe	-1.34
KRG	0.00%	Sortino	-0.93		KRG	0.47%	Sortino	-1.41		KRG	6.99%	Sortino	-2.08
ILG	0.29%				ILG	4.29%				ILG	3.71%		
7	7	7	7	7	7	7	7	7	7	7	7	7	7
ISRG	0.01%	M-CVaR	0.9381802		ISRG	0.01%	M-CVaR	1.0122467		ISRG	0.01%	M-CVaR	1.0626528
MDCO	25.00%	M-S Ratio	1.7869281		MDCO	25.00%	M-S Ratio	2.1376051		MDCO	25.00%	M-S Ratio	1.9342019
DCI	0.00%	M-V Sharpe	1.348132		DCI	0.00%	M-V Sharpe	1.3141806		DCI	0.00%	M-V Sharpe	1.2890468
LYB	0.00%				LYB	2.14%				LYB	2.71%		
CASY	0.00%	CVaR	0.3240773		CASY	0.00%	CVaR	0.333616		CASY	0.00%	CVaR	0.2878487
FHN	0.00%	Semivar	0.1701484		FHN	0.00%	Semivar	0.1579813		FHN	0.00%	Semivar	0.1581444
ROG	25.00%	Volatility	0.225529		ROG	5.80%	Volatility	0.2569675		ROG	11.38%	Volatility	0.2372941
MOH	2.09%	Port. Beta	1.1414313		MOH	0.00%	Port. Beta	1.1364756		MOH	0.00%	Port. Beta	1.1036552
CY	0.00%	Exp Ret	0.3056429		CY	0.00%	Exp Ret	0.3393017		CY	0.00%	Exp Ret	0.3074833
TXRH	0.00%				TXRH	0.00%				TXRH	0.00%		
TRIP	6.95%	Max Draw	-5.80%		TRIP	2.19%	Max Draw	-6.43%		TRIP	0.89%	Max Draw	-5.59%
SRE	2.83%	Max Gain	7.07%		SRE	2.19%	Max Gain	5.70%		SRE	6.99%	Max Gain	6.80%
RS	12.24%	5th Perc	-4.04%		RS	23.41%	5th Perc	-4.67%		RS	12.44%	5th Perc	-4.75%
WFC	2.28%	95th Perc	4.64%		WFC	2.09%	95th Perc	4.37%		WFC	0.00%	95th Perc	3.72%
TSCO	0.00%	Return	-11.04%		TSCO	0.00%	Return	-15.41%		TSCO	0.00%	Return	-13.95%
CHSP	0.00%	Stdev.s	18.82%		CHSP	0.00%	Stdev.s	19.56%		CHSP	0.00%	Stdev.s	19.13%
APC	6.63%	Semivar	13.19%		APC	12.17%	Semivar	12.30%		APC	6.99%	Semivar	12.32%
SXC	15.36%	Sharpe	-0.60		SXC	25.00%	Sharpe	-0.80		SXC	25.00%	Sharpe	-0.74
LION	1.61%	Sortino	-0.85		LION	0.00%	Sortino	-1.27		LION	8.58%	Sortino	-1.15
BMI	0.00%				BMI	0.00%				BMI	0.00%		
8	8	8	8	8	8	8	8	8	8	8	8	8	8
NVDA	0.00%	M-CVaR	0.8193845		NVDA	0.00%	M-CVaR	1.0331278		NVDA	0.00%	M-CVaR	1.2119601
GOOGL	25.00%	M-S Ratio	2.2704519		GOOGL	25.00%	M-S Ratio	2.4886052		GOOGL	25.00%	M-S Ratio	2.31454
CBS	0.00%	M-V Sharpe	1.7839786		CBS	2.62%	M-V Sharpe	1.5213535		CBS	4.91%	M-V Sharpe	1.420147
PCAR	0.00%				PCAR	0.00%				PCAR	0.00%		
CVLT	0.00%	CVaR	0.4769506		CVLT	0.00%	CVaR	0.3387635		CVLT	0.00%	CVaR	0.2836394
LXP	0.00%	Semivar	0.1721269		LXP	0.00%	Semivar	0.1406354		LXP	0.00%	Semivar	0.1485218
AES	0.00%	Volatility	0.2190643		AES	9.02%	Volatility	0.2300491		AES	8.68%	Volatility	0.2420592
ATGE	0.00%	Port. Beta	1.2160476		ATGE	0.00%	Port. Beta	1.155161		ATGE	0.00%	Port. Beta	1.1283367
ROG	25.00%	Exp Ret	0.3924059		ROG	6.92%	Exp Ret	0.351586		ROG	14.98%	Exp Ret	0.3453596
HSY	0.00%				HSY	1.32%				HSY	0.00%		
ONB	0.00%	Max Draw	-9.64%		ONB	4.42%	Max Draw	-8.44%		ONB	0.00%	Max Draw	-7.89%
NWN	0.00%	Max Gain	8.45%		NWN	0.00%	Max Gain	8.53%		NWN	0.00%	Max Gain	7.08%
IPXL	0.00%	5th Perc	-6.00%		IPXL	0.00%	5th Perc	-6.42%		IPXL	0.00%	5th Perc	-5.49%
FCX	23.09%	95th Perc	5.85%		FCX	25.00%	95th Perc	4.89%		FCX	21.85%	95th Perc	4.25%
HZO	0.00%	Return	-21.99%		HZO	0.00%	Return	-14.53%		HZO	0.00%	Return	-6.90%
MAC	0.00%	Stdev.s	26.50%		MAC	0.00%	Stdev.s	24.51%		MAC	0.00%	Stdev.s	22.23%
EGOV	4.89%	Semivar	16.81%		EGOV	10.23%	Semivar	15.16%		EGOV	23.58%	Semivar	13.73%
MU	22.02%	Sharpe	-0.84		MU	4.70%	Sharpe	-0.60		MU	1.00%	Sharpe	-0.32
APC	0.00%	Sortino	-1.32		APC	10.76%	Sortino	-0.97		APC	0.00%	Sortino	-0.51
EZPW	0.00%				EZPW	0.00%				EZPW	0.00%		

9	9	9	9	9	9	9	9	9	9	9	9	9	9
CMG	0.00%	M-CVaR	0.8173994		CMG	0.00%	M-CVaR	0.9841745		CMG	9.08%	M-CVaR	1.1430702
NWN	0.00%	M-S Ratio	1.8627482		NWN	0.00%	M-S Ratio	2.1149534		NWN	0.00%	M-S Ratio	1.858681
VLY	1.89%	M-V Sharpe	1.23074		VLY	0.00%	M-V Sharpe	1.0961115		VLY	0.00%	M-V Sharpe	1.1007033
XEC	3.71%				XEC	0.00%				XEC	0.00%		
BRO	0.00%	CVaR	0.3798611		BRO	25.00%	CVaR	0.3103838		BRO	5.46%	CVaR	0.2296674
CALM	15.50%	Semivar	0.1666882		CALM	21.10%	Semivar	0.1444343		CALM	10.57%	Semivar	0.1412431
WST	0.00%	Volatility	0.2522858		WST	0.00%	Volatility	0.2786868		WST	7.00%	Volatility	0.2385074
DIN	0.00%	Port. Beta	1.1840157		DIN	0.00%	Port. Beta	1.1052884		DIN	2.99%	Port. Beta	1.0784993
T	6.20%	Exp Ret	0.3120982		T	1.29%	Exp Ret	0.3070718		T	9.49%	Exp Ret	0.2641259
HT	0.00%				HT	0.00%				HT	0.00%		
SBUX	25.00%	Max Draw	-6.79%		SBUX	16.95%	Max Draw	-6.28%		SBUX	13.92%	Max Draw	-5.71%
SPTN	0.00%	Max Gain	8.05%		SPTN	0.00%	Max Gain	7.85%		SPTN	0.00%	Max Gain	8.20%
HIW	0.00%	5th Perc	-4.37%		HIW	0.00%	5th Perc	-4.36%		HIW	0.00%	5th Perc	-3.77%
TTI	25.00%	95th Perc	5.32%		TTI	25.00%	95th Perc	5.81%		TTI	25.00%	95th Perc	4.99%
T	6.20%	Return	21.87%		T	1.29%	Return	26.42%		T	9.49%	Return	20.19%
POOL	0.00%	Stdev.s	23.18%		POOL	0.00%	Stdev.s	22.40%		POOL	0.00%	Stdev.s	20.20%
LRCX	0.00%	Semivar	17.58%		LRCX	2.05%	Semivar	17.23%		LRCX	0.00%	Semivar	15.70%
PH	0.00%	Sharpe	0.94		PH	0.00%	Sharpe	1.17		PH	0.00%	Sharpe	0.99
EXLS	0.00%	Sortino	1.23		EXLS	7.32%	Sortino	1.52		EXLS	7.00%	Sortino	1.28
PAY	16.51%				PAY	0.00%				PAY	0.00%		
10	10	10	10	10	10	10	10	10	10	10	10	10	10
OCLR	0.00%	M-CVaR	0.9354913		OCLR	0.00%	M-CVaR	0.8883198		OCLR	0.00%	M-CVaR	0.9563868
ESV	0.00%	M-S Ratio	2.1041048		ESV	0.00%	M-S Ratio	2.324704		ESV	0.00%	M-S Ratio	2.1762741
JPM	25.00%	M-V Sharpe	1.8141693		JPM	25.00%	M-V Sharpe	1.6275402		JPM	25.00%	M-V Sharpe	1.7171814
LLY	0.00%				LLY	0.00%				LLY	1.48%		
HIG	0.00%	CVaR	0.3857722		HIG	0.00%	CVaR	0.3586927		HIG	0.00%	CVaR	0.3557942
SONC	0.00%	Semivar	0.1715155		SONC	0.00%	Semivar	0.1370643		SONC	0.57%	Semivar	0.1563575
MTD	0.00%	Volatility	0.1989266		MTD	0.00%	Volatility	0.1957763		MTD	0.00%	Volatility	0.1981601
ITG	0.00%	Port. Beta	1.1589204		ITG	0.00%	Port. Beta	1.1133378		ITG	0.68%	Port. Beta	1.1470388
CCMP	0.00%	Exp Ret	0.3624865		CCMP	0.00%	Exp Ret	0.3202338		CCMP	2.41%	Exp Ret	0.3418768
LNN	0.00%				LNN	0.00%				LNN	0.00%		
HUBB	0.00%	Max Draw	-5.85%		HUBB	8.08%	Max Draw	-6.54%		HUBB	1.70%	Max Draw	-6.10%
CYTK	16.26%	Max Gain	8.14%		CYTK	7.15%	Max Gain	6.09%		CYTK	11.67%	Max Gain	7.33%
APOG	8.52%	5th Perc	-3.10%		APOG	0.83%	5th Perc	-2.66%		APOG	6.66%	5th Perc	-2.81%
PM	8.02%	95th Perc	4.61%		PM	5.91%	95th Perc	4.20%		PM	2.59%	95th Perc	4.01%
MTB	6.56%	Return	12.90%		MTB	11.06%	Return	7.23%		MTB	8.39%	Return	10.94%
LHCG	0.00%	Stdev.s	18.76%		LHCG	0.00%	Stdev.s	17.54%		LHCG	0.00%	Stdev.s	18.00%
KEY	4.53%	Semivar	14.33%		KEY	10.73%	Semivar	12.44%		KEY	6.43%	Semivar	13.36%
CI	4.53%	Sharpe	0.68		CI	0.97%	Sharpe	0.40		CI	5.65%	Sharpe	0.60
MMC	1.58%	Sortino	0.89		MMC	5.27%	Sortino	0.57		MMC	1.77%	Sortino	0.81
HAYN	25.00%				HAYN	25.00%				HAYN	25.00%		
11	11	11	11	11	11	11	11	11	11	11	11	11	11
ADBE	25.00%	M-CVaR	0.9630812		ADBE	12.76%	M-CVaR	1.1512006		ADBE	25.00%	M-CVaR	1.1872401
CTSH	0.00%	M-S Ratio	2.1246893		CTSH	0.00%	M-S Ratio	2.4797456		CTSH	0.00%	M-S Ratio	2.3222939
A	0.00%	M-V Sharpe	1.4088538		A	0.00%	M-V Sharpe	1.3445359		A	3.31%	M-V Sharpe	1.3257688
DIN	0.00%				DIN	0.00%				DIN	0.00%		
IT	0.00%	CVaR	0.3436788		IT	0.00%	CVaR	0.3171737		IT	5.45%	CVaR	0.3030349
LPSN	25.00%	Semivar	0.155783		LPSN	24.99%	Semivar	0.1472451		LPSN	19.98%	Semivar	0.1457881
OSK	5.73%	Volatility	0.2349361		OSK	0.00%	Volatility	0.2715662		OSK	1.27%	Volatility	0.2553709
VMC	1.50%	Port. Beta	1.0785956		VMC	0.00%	Port. Beta	1.0179427		VMC	0.00%	Port. Beta	1.0741606
DISH	3.20%	Exp Ret	0.3325906		DISH	0.00%	Exp Ret	0.3667305		DISH	1.30%	Exp Ret	0.3401627
AMAG	0.00%				AMAG	0.00%				AMAG	1.29%		
HRS	0.00%	Max Draw	-6.07%		HRS	0.00%	Max Draw	-7.39%		HRS	0.00%	Max Draw	-5.74%
RRC	16.43%	Max Gain	4.82%		RRC	25.00%	Max Gain	5.62%		RRC	25.00%	Max Gain	4.36%
PCAR	0.00%	5th Perc	-4.75%		PCAR	0.00%	5th Perc	-5.46%		PCAR	0.00%	5th Perc	-4.28%
ALB	9.67%	95th Perc	3.62%		ALB	22.61%	95th Perc	4.18%		ALB	4.19%	95th Perc	3.64%
BRO	0.00%	Return	-13.86%		BRO	14.64%	Return	-18.19%		BRO	7.39%	Return	-14.66%
DIOD	5.64%	Stdev.s	17.86%		DIOD	0.00%	Stdev.s	20.60%		DIOD	1.79%	Stdev.s	17.25%
GIFI	0.00%	Semivar	10.60%		GIFI	0.00%	Semivar	11.69%		GIFI	4.03%	Semivar	10.06%
IBM	1.70%	Sharpe	-0.78		IBM	0.00%	Sharpe	-0.89		IBM	0.00%	Sharpe	-0.86
HPQ	6.13%	Sortino	-1.32		HPQ	0.00%	Sortino	-1.57		HPQ	0.00%	Sortino	-1.47
JWN	0.00%				JWN	0.00%				JWN	0.00%		

12	12	12	12	12	12	12	12	12	12	12	12	12	12
ZBRA	0.00%	M-CVaR	0.6331418		ZBRA	0.00%	M-CVaR	0.6954415		ZBRA	0.00%	M-CVaR	0.719255
TDG	0.00%	M-S Ratio	1.7305719		TDG	0.00%	M-S Ratio	1.8002186		TDG	0.32%	M-S Ratio	1.7064572
LABL	0.00%	M-V Sharpe	1.3179042		LABL	0.00%	M-V Sharpe	1.2507009		LABL	0.00%	M-V Sharpe	1.2121036
SNV	0.00%				SNV	0.00%				SNV	0.00%		
C	25.00%	CVaR	0.3560386		C	25.00%	CVaR	0.362275		C	25.00%	CVaR	0.3330277
ADI	0.00%	Semivar	0.1302592		ADI	0.00%	Semivar	0.1399503		ADI	0.00%	Semivar	0.1403679
GM	7.07%	Volatility	0.1710465		GM	0.00%	Volatility	0.2014399		GM	0.00%	Volatility	0.1976166
JKHY	0.00%	Port. Beta	1.0618064		JKHY	0.00%	Port. Beta	1.0997633		JKHY	0.00%	Port. Beta	1.0915642
ANSS	0.00%	Exp Ret	0.2270229		ANSS	0.00%	Exp Ret	0.2535411		ANSS	0.00%	Exp Ret	0.2411318
TTMI	9.14%				TTMI	23.67%				TTMI	25.00%		
WYNN	23.76%	Max Draw	-5.83%		WYNN	25.00%	Max Draw	-7.27%		WYNN	17.15%	Max Draw	-7.45%
NYCB	0.00%	Max Gain	5.69%		NYCB	0.00%	Max Gain	6.78%		NYCB	0.00%	Max Gain	6.46%
AVY	0.00%	5th Perc	-4.20%		AVY	0.00%	5th Perc	-4.64%		AVY	0.00%	5th Perc	-4.61%
KLIC	0.00%	95th Perc	3.80%		KLIC	0.00%	95th Perc	4.66%		KLIC	0.00%	95th Perc	5.11%
PGR	0.00%	Return	-8.40%		PGR	0.00%	Return	-10.59%		PGR	0.00%	Return	-6.82%
T	10.03%	Stdev.s	17.12%		T	4.69%	Stdev.s	20.61%		T	3.78%	Stdev.s	19.80%
MTRN	0.00%	Semivar	10.79%		MTRN	0.00%	Semivar	13.47%		MTRN	3.96%	Semivar	13.08%
NWL	0.00%	Sharpe	-0.50		NWL	0.00%	Sharpe	-0.52		NWL	0.00%	Sharpe	-0.35
PSA	0.00%	Sortino	-0.79		PSA	0.00%	Sortino	-0.80		PSA	0.00%	Sortino	-0.53
PRA	25.00%				PRA	21.64%				PRA	24.78%		
13	13	13	13	13	13	13	13	13	13	13	13	13	13
AGN	5.42%	M-CVaR	1.1168895		AGN	0.00%	M-CVaR	0.9462587		AGN	14.71%	M-CVaR	1.2409512
NVDA	0.00%	M-S Ratio	2.3731091		NVDA	0.00%	M-S Ratio	2.6330281		NVDA	0.00%	M-S Ratio	2.2745079
YUM	0.00%	M-V Sharpe	1.5569845		YUM	0.00%	M-V Sharpe	1.3102061		YUM	0.00%	M-V Sharpe	1.5188729
PWR	25.00%				PWR	25.00%				PWR	25.00%		
KND	23.38%	CVaR	0.3678558		KND	9.41%	CVaR	0.4321092		KND	25.00%	CVaR	0.3312019
UMBF	1.22%	Semivar	0.1731291		UMBF	0.00%	Semivar	0.1552916		UMBF	17.29%	Semivar	0.1807008
HSC	0.00%	Volatility	0.2638782		HSC	25.00%	Volatility	0.3120784		HSC	3.25%	Volatility	0.270599
KEX	3.25%	Port. Beta	1.1812527		KEX	17.10%	Port. Beta	1.2583106		KEX	0.00%	Port. Beta	1.1771285
CPE	15.04%	Exp Ret	0.4124543		CPE	5.37%	Exp Ret	0.4104871		CPE	13.38%	Exp Ret	0.4126055
MTB	8.18%				MTB	0.00%				MTB	0.00%		
DBD	4.59%	Max Draw	-7.55%		DBD	0.00%	Max Draw	-7.94%		DBD	0.00%	Max Draw	-7.46%
EFII	5.67%	Max Gain	10.88%		EFII	15.00%	Max Gain	8.82%		EFII	1.37%	Max Gain	11.34%
GVA	0.00%	5th Perc	-5.46%		GVA	0.00%	5th Perc	-6.23%		GVA	0.00%	5th Perc	-5.42%
ATO	0.00%	95th Perc	4.32%		ATO	0.00%	95th Perc	4.16%		ATO	0.00%	95th Perc	3.79%
HT	0.00%	Return	-7.97%		HT	0.00%	Return	-20.11%		HT	0.00%	Return	-9.64%
PTEN	0.00%	Stdev.s	23.61%		PTEN	3.13%	Stdev.s	24.78%		PTEN	0.00%	Stdev.s	23.14%
IDCC	0.00%	Semivar	15.79%		IDCC	0.00%	Semivar	15.70%		IDCC	0.00%	Semivar	15.51%
AEP	0.00%	Sharpe	-0.34		AEP	0.00%	Sharpe	-0.82		AEP	0.00%	Sharpe	-0.42
PXD	0.00%	Sortino	-0.51		PXD	0.00%	Sortino	-1.29		PXD	0.00%	Sortino	-0.63
ESL	8.24%				ESL	0.00%				ESL	0.00%		

Bibliography

Markowitz, Harry. “Portfolio Selection.” The Journal of Finance, vol. 7, no. 1, 1952, p. 77., doi:10.2307/2975974.

Von Neumann, John, and Oskar Morgenstern. Theory of Games and Economic Behavior. Princeton University Press, 1944.

Cont, R. “Empirical properties of asset returns: stylized facts and statistical issues.” Quantitative Finance, vol. 1, no. 2, Jan. 2001, pp. 223–236., doi:10.1088/1469-7688/1/2/304.

Pratt, John W. “Risk Aversion in the Small and in the Large.” Econometrica, vol. 32, no. 1/2, 1964, p. 122., doi:10.2307/1913738.

Ozga, S. A., and Kenneth J. Arrow. “Aspects of the Theory of Risk-Bearing.” Economica, vol. 33, no. 130, 1966, p. 251., doi:10.2307/2552628.

Lai, Kin Keung, et al. “Mean-Variance-Skewness-Kurtosis-Based Portfolio Optimization.” First International Multi-Symposiums on Computer and Computational Sciences (IMSCCS06), 2006, doi:10.1109/imsccs.2006.239.

Mamoghli, Chokri, and Sami Daboussi. “Portfolio Optimization in a Downside Risk Framework (Optimisation De Portefeuille Dans Le Cadre Du Downside Risk).” SSRN Electronic Journal, 2008, doi:10.2139/ssrn.1237322.

Doganoglu, Toker, et al. “Portfolio optimization when risk factors are conditionally varying and heavy tailed.” Computational Economics, vol. 29, no. 3-4, June 2007, pp. 333–354., doi:10.1007/s10614-006-9071-1.

Bawa, Vijay S., and Eric B. Lindenberg. “Abstract: Capital Market Equilibrium in a Mean-Lower Partial Moment Framework.” Journal of Financial and Quantitative Analysis, vol. 12, no. 04, 1977, p. 635., doi:10.1017/s0022109000023346.

Unser, Matthias. "Lower partial moments as measures of perceived risk: An experimental study." Journal of Economic Psychology, vol. 21, no. 3, 2000, pp. 253–280., doi:10.1016/s0167-4870(00)00004-0.

Bawa, Vijay S. "Optimal rules for ordering uncertain prospects." Journal of Financial Economics, vol. 2, no. 1, 1975, pp. 95–121., doi:10.1016/0304-405x(75)90025-2.

Estrada, Javier. "Mean-Semivariance Optimization: A Heuristic Approach." SSRN Electronic Journal, 2007, doi:10.2139/ssrn.1028206.

Sharpe, William F. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." The Journal of Finance, vol. 19, no. 3, 1964, p. 425., doi:10.2307/2977928.

Lintner, John. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets: A Reply." The Review of Economics and Statistics, vol. 51, no. 2, 1969, p. 222., doi:10.2307/1926735.

Rockafellar, R. Tyrrell, and Stanislav Uryasev. "Optimization of conditional value-at-Risk." The Journal of Risk, vol. 2, no. 3, 2000, pp. 21–41., doi:10.21314/jor.2000.038.

Krokhmal, Pavlo, et al. "Portfolio optimization with conditional value-at-Risk objective and constraints." The Journal of Risk, vol. 4, no. 2, 2001, pp. 43–68., doi:10.21314/jor.2002.057.

Uryasev, S. "Conditional value-at-Risk: optimization algorithms and applications." Proceedings of the IEEE/IAFE/INFORMS 2000 Conference on Computational Intelligence for Financial Engineering (CIFEr) (Cat. No.00TH8520), doi:10.1109/cifer.2000.844598.

ACADEMIC VITA

BENJAMIN A. ROWLES

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EDUCATION

The Pennsylvania State University | Schreyer Honors College — **University Park, PA**
Smeal College of Business | Bachelor of Science in Finance — *Class of 2018*
Eberly College of Science | Bachelor of Science in Mathematics
College of the Liberal Arts | Bachelor of Science in Economics

RELEVANT EXPERIENCE

Citi — **New York City, NY**
Summer Analyst, Markets Division — *Jun 2017 – Aug 2017*
- Interned on three desks for 3 weeks each within the company's markets division including the CLO Structuring, CMBS Structuring, and Rates Sales desks
- Created excel models for the cash flows of CMBS and CLO structured products and developed sensitivity analyses to defaults in the underlying securities
- Pitched multiple trade ideas to various full-time Citi markets employees focusing on assets in the high yield, structured credit, CMBS, and rates spaces

Nittany Lion Fund, LLC — **University Park, PA**
Director of Risk Analytics — *Jan 2017 – Jan 2018*
- Created and implemented quantitative risk models to analyze exposures and generate data for factor weightings and portfolio risk-adjusted return metrics
- Implemented portfolio theory to generate forward sharpe-ratios and created optimization models so as to best select subsector single-name equity weights
- Worked alongside the Chief Investment Officer in analyzing portfolio risks on a qualitative and quantitative basis and communicated concerns to leads

Lead Fund Manager, Materials Sector — *Aug 2016 – Jan 2017*
- Managed $225 thousand in the Materials sector of the $8.5 million Nittany Lion Fund equity portfolio which is composed similarly to the S&P 500 index
- Generated 8.03% return relative to the S&P Material Index's 5.21% over a 6-month period with a portfolio beta of 1.13 relative to the index's 1.35 beta
- Created a basic net asset value model (NAV) to value natural resource-based companies in the mining and energy universes on an asset valuation basis
- Constructed and presented stock pitches that utilized comparables analysis, discount cash flow valuation, and drivers of forward looking price growth

Associate Fund Manager, Energy Sector & Financials Sector — *Aug 2015 – Aug 2016*
- Employed energy specific valuation methods such as net asset value models for E&P companies and sum-of-the-parts analyses for major integrated stocks
- Utilized financials specific valuation techniques such as excess equity models in order to value insurance companies, banks, and similar institutions

Leveraged Lion Capital — **University Park, PA**
Co-Founder — *Oct 2016*
- Assisted in the founding and set-up of a collegiate student investment club recognized by both Smeal and Penn State focused on the leveraged loan universe
- Partnered with members of Bank of America Merrill Lynch, the LSTA, and S&P Global in order to obtain leveraged loan pricing data and issuance info
- Interviewed approximately 100 qualified candidates of which 22 were chosen to be the organization's first leveraged loan analysts and portfolio managers

Chief Investment Officer, Executive Board — *Oct 2016 – Jan 2018*
- Allocated capital in a paper portfolio of leveraged loans across seven sub-sectors with the goal of outperforming the LSTA 100 index's total return
- Collaborated with portfolio managers in each subsector in order to decide on planned investment moves within subsectors' individual loan investments
- Monitored macro trends and portfolio risk levels to position the portfolio to take advantage of developing trends in the leveraged loan and high yield spaces
- Educated new members on fundamental topics such as cash flow analysis, three-statement flow modeling, covenant analysis, and general credit analysis

Seeking Alpha — **New York City, NY**
Premium and PRO+ Contributor — *Dec 2017 – Present*
- Write fundamental research on various equities and investment ideas on a public website and receive feedback from editors and other public investors
- Author works for Seeking Alpha's retail (Premium) free service, consistently receiving designation as "Editor's Pick," resulting in increased visibility
- Contribute to Seeking Alpha's institutional (PRO+) subscription-only service, comprised of particularly compelling articles with asymmetric risk-profiles
- Cover investments within the semiconductor, materials, energy, and utilities spaces with an emphasis on value and contrarian ideas in distressed sectors

Convergent Wealth Advisors — **Washington, DC**
Intern, Private Wealth Management — *Jun 2016 – Aug 2016*
- Scrutinized investment portfolios and determined optimal asset allocation in such a way that would allow the client to reach retirement spending goals
- Created individual equity and ETF evaluations so as to identify opportunities in individual stocks and minimize expenses related to various ETF fees

OTHER ACTIVITIES

Penn State Powerlifting Team — **University Park, PA**
Competition Team Member — *Jan 2015 – Present*
- Compete against other schools, both individually and as a team, in the squat, bench press, and deadlift events in order to determine individual/team strength
- Practice daily lifting regimes which incorporate the concepts of progressive overload and periodization with the ultimate goal of increasing power
- Teach new members of the club correct form on the three main lifts and proper utilization of the lifting belt, knee wraps, squat suit, and benching shirt

Theta Delta Chi Fraternity, Sigma Triton Charge — **University Park, PA**
Member — *Sep 2015 – Present*
- Attend weekly meetings at which fraternity topics and ideas are brought forth in front of all active members so as to plan and discuss future events
- Participate in philanthropic events over the course of the year in order to raise money for the Penn State Panhellenic Dance MaraTHON and Autism Speaks

ADDITIONAL INFORMATION

Honors and Awards: 5th Place 2017 USA National Collegiate Powerlifting Championship 105 kg, 5th Place 2016 USA National Collegiate Powerlifting Championship 105 kg, 1st place 2015 USA State Powerlifting New Jersey Championship 93 kg, Pennsylvania Collegiate Powerlifting bench press, squat, and three-lift combined total state records 105 kg
Interests: Game of Thrones, Piano, Entourage, EDM Music, Seattle Seahawks, Mathematics, Powerlifting, Personal Portfolio Management