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SIMULATION STUDY OF THE IMPACT OF ROAD TERRAIN VARIABILITY ON IDENTIFIABILITY OF LONGITUDINAL VEHICLE CHASSIS PARAMETERS

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ABSTRACT

This thesis applies a basic simulation study to demonstrate the relationship between mean-square road terrain variability and vehicle chassis parameter identifiability. This work is motivated by the importance of chassis parameter estimation in proper system identification, specifically parameterizing vehicle dynamics models with unknown system parameters and designing vehicle road tests which are optimized for chassis parameter identifiability. The study of vehicle chassis parameter estimation spans decades in the literature, and has been performed with the use of varying chassis dynamics models and estimation algorithms including simple linear least-squares regression. However, the quantification of the impact of terrain variability on chassis parameter identifiability remains largely unexplored. This work illustrates this relationship by performing simple linear least-squares estimation on a series of simulated driving cycles which exhibit progressively scaled sinusoidal terrain profiles. The Cramer-Rao lower bounds for the errors of the resulting parameter estimates are obtained through simple Fisher information analysis. A terrain variability metric is defined based on the mean-square of road grade across each driving cycle, and this metric is related to the estimation error values in a series of plots. The results from this simulation study demonstrate the monotonic and decreasing relationship between terrain variability and chassis parameter identifiability for a sinusoidal terrain profile. The significant degree to which terrain variability is shown to increase the quality of chassis parameter estimates can motivate the design of future experiments for estimating vehicle chassis parameters using data from on-road experiments.

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1 Statement of Objectives

The objective of this project is to illustrate the relationship between the mean-square terrain variability of a simulated sinusoidal drive cycle and the error values of longitudinal vehicle chassis parameter estimates obtained from that drive cycle. The relationship between these variables will have direct applications to the design of vehicle road tests for chassis parameter identification. This overall goal will be achieved through the procedure defined by the following objectives of this work:

1. Demonstrate an understanding of simple parameter estimation and identifiability concepts, and conduct a literature review of the use of vehicle chassis parameter estimation in automotive research.
2. Select and properly define a vehicle dynamics model which is conducive to chassis parameter estimation through simulation.
3. Design and implement a simple linear least-squares estimation algorithm with this vehicle dynamics model.
4. Incorporate simple Fisher identifiability analysis with this parameter estimation algorithm to determine the theoretical lower bounds on the errors of the parameter estimates.
5. Define a terrain variability metric, and apply parameter estimation and Fisher analysis algorithms to a series of simulated drive cycles with different values for this metric. Use this analysis to show the simulated relationship between road terrain variability and vehicle chassis parameter identifiability.

The following document will describe the motivation for this thesis project, a review of relevant literature for vehicle chassis parameter estimation, and the specific methods employed to complete these tasks.

2 Motivation

Parameter estimation refers to the use of relevant input and output data to produce estimates of model parameters for a given system. This important tool has been applied to automotive system identification for decades in the literature. Specifically, estimating vehicle chassis parameters, including the vehicle mass, drag coefficient, and rolling resistance coefficient serves several important purposes. Typically, chassis parameter estimation is conducted as a means to parameterize a vehicle dynamics model using relevant experimental data of the vehicle's longitudinal dynamics. Such data can include time-histories of the vehicle's speed and propulsion force, in addition to relevant data of the road grade. By using such data to parameterize a vehicle dynamics model, parameter estimation provides the capability to accurately simulate the dynamics of a specific vehicle, and enables accurate prediction of the vehicle's performance.

The capability of quantifying the impact of mean-square road terrain variability on the accuracy of chassis parameter estimates will serve several important purposes. The most significant impact of this capability will be the potential of this information to inform vehicle road tests which are intended to gather data to estimate vehicle chassis parameters. If an achievable degree of terrain variability is shown to increase parameter identifiability more than a perhaps costly upgrade of relevant sensors and instrumentation for data collection, significant time and financial resources can be saved.

3 Background

Different parameter estimation techniques tend to be useful in varying scenarios, depending on the model in use and the relationships between the parameters within that model. Specifically, linear least-squares parameter estimation has several advantages and disadvantages which lend to its usefulness for vehicle chassis parameter estimation. The following sections will provide a literature review of parameter estimation in the context of vehicle chassis parameter estimation, describe the procedure for designing a linear least-squares parameter estimation algorithm, and illustrate the advantages and disadvantages of such an algorithm.

3.1 Literature Review

Several methods for estimating vehicle chassis parameters have been explored in the literature. Generally, such methods depend on the vehicle dynamics model in use, and the available data. For instance, there is a significant body of literature on estimating the chassis parameters of a vehicle using a model of its longitudinal dynamics. One example is Bae *et al*.'s application of data from on-road tests of a passenger automobile, together with a model of its longitudinal dynamics, to estimate vehicle mass, drag coefficient, and rolling resistance parameters [1]. More recent research by Altmannshofer *et al*. has further developed Bae *et al*.'s work by using a longitudinal vehicle dynamics model with a parameter estimation algorithm which accommodates non-Gaussian measurement errors and insufficient excitation from terrain and inertial properties of the experiment [2]. Vahidi *et al*. develop an estimation algorithm that estimates vehicle mass, drag coefficient, and the road grade profile itself in the absence of road grade data [3-4]. Other work also applies Bayesian parameter estimation techniques to a similar longitudinal vehicle dynamics estimation problem [5].

One can also estimate vehicle chassis parameters from vertical and lateral dynamics. For instance, Rajamani *et al*. demonstrate the estimation of vehicle states and mass through a model of suspension dynamics [6]. Pence *et al*. also estimate vehicle parameters using suspension dynamics, with a procedure which places significant emphasis on the use of low-cost sensing and instrumentation [7]. Others have applied lateral and powertrain dynamics to estimate vehicle states and chassis parameters [8].

The accuracy with which the above algorithms can estimate vehicle parameters depends on the choice of vehicle, choice of algorithm, and most importantly the design of experiment used to collect data for estimation. Estimating vehicle suspension parameters, for instance, has been shown to some extent to require data which is characterized by rich vertical road excitations. Müller *et al*. demonstrate this by showing how the addition of speed bumps in a road test increases the accuracy of the resulting parameter estimates [9]. Given this relationship between the design of experiments and the accuracy of the resulting parameter estimates, the automotive industry has developed guidelines for chassis testing for parameter identification. One common standard is SAE J2263, which describes a coast-down experiment that is designed to yield a dataset rich enough in inertial excitation to provide accurate estimates of several chassis parameters [10].

The literature indicates that the problem of vehicle chassis parameter estimation is thoroughly studied, and that the conditions under which relevant experimental data are gathered can have significant impact on the results of the parameter estimation exercise. This thesis seeks to show in simulation the specific degree to which the mean-square road grade variability of a simulated experiment directly impacts the accuracy of chassis parameter estimates obtained from the data collected from that simulation. A follow-up paper will evaluate this relationship with actual on-road experimental data gathered from an instrumented heavy-duty vehicle [11].

The following sections describe the operation and formulation of such an algorithm. Additionally, Chapter 4 further establishes the specific problem formulation given the relevant resources for this project, and describes why linear least-squares estimation is an ideal choice for this project.

3.2 Extended Example of Linear Least-Squares Parameter Estimation

Linear least-squares estimation functions on the same principles which motivate conventional linear regression. Take, for instance, the following example of a system with output y which relates to time t through the parameters θ_1 and θ_2. This example will be used to establish the equations and relationships which will be applied throughout this work, as a means to simplify the implementation. Figure 1 shows a hypothetical plot of experimental data of this system's input and output taking into account some measurement noise.

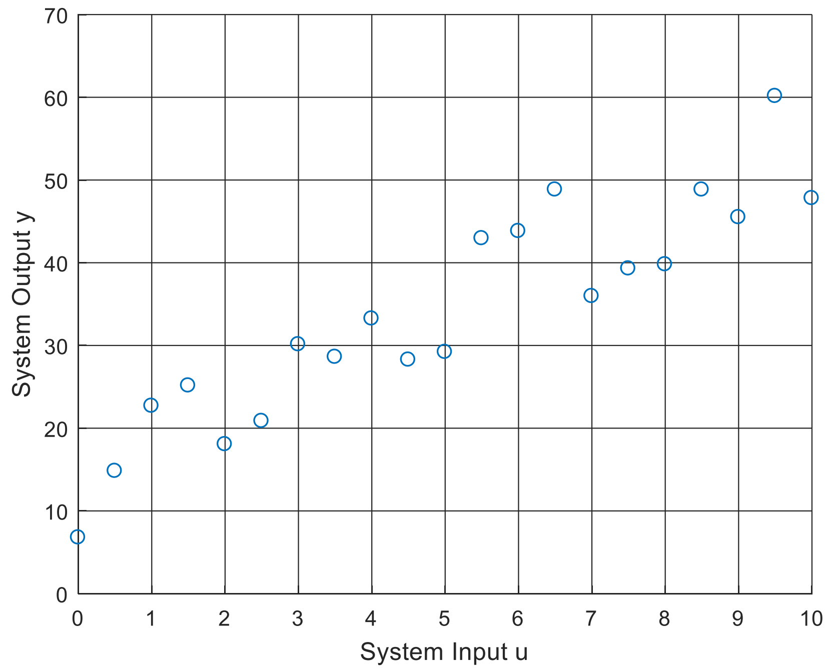


Figure 1. Example System Measurements

For this example, we define a model:

$$y = \theta_1 + \theta_2 t \tag{1}$$

For future reference, in this case study $\theta_{1;true} = 15$ and $\theta_{2;true} = 4$.

In order to estimate this system's parameters without bias, a few assumptions need to be made. First, the measurement noise must be *independent and identically distributed* with known statistics and zero mean. This means that the noise has the same probability distribution at every instance of measurement, and that the realization of noise at each measurement is independent of that at other measurements. Second, the noise is assumed to only affect the output signal y. In order to find the likelihood that the differences between the output measurements y_i and predicted output y are only a result of the measurement noise, the following variable is defined:

$$v_i = y_i - (\theta_1 + \theta_2 t_i) \tag{2}$$

From linear regression, the likelihood that this difference is only a result of the measurement noise is:

$$Likelihood\ of\ v_i = \frac{1}{\sqrt{2\sigma^2\pi}}\exp\left(\frac{-v_i^2}{2\sigma^2}\right) \tag{3}$$

where σ is the known standard deviation of the measurement error. The total likelihood of v_i, denoted $L(\vec{\theta})$, is then:

$$L(\vec{\theta}) = \prod_{i=1}^{N}\frac{1}{\sqrt{2\sigma^2\pi}}\exp\left(\frac{-v_i^2}{2\sigma^2}\right) \tag{4}$$

$$L(\vec{\theta}) = \left(\frac{1}{\sqrt{2\sigma^2\pi}}\right)^N\exp\left(\frac{-1}{2\sigma^2}\sum_{i=1}^{N} v_i^2\right) \tag{5}$$

where N is the final measurement index, and:

$$\vec{\theta} = \begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix} \tag{6}$$

For linear least-squares estimation (or maximum likelihood estimation), the objective is to *maximize* this total likelihood $L(\vec{\theta})$. We can simplify (5) by taking the natural logarithm of $L(\vec{\theta})$ and eliminating constant coefficients to give the following expression:

$$\max_{\vec{\theta}} \quad -\sum_{i=1}^{N} v_i^2 \tag{7}$$

which further simplifies into the following objective:

$$\min_{\vec{\theta}} \quad f(\vec{\theta}) = \sum_{i=1}^{N} v_i^2 = \sum_{i=1}^{N}(y_i - (\theta_1 + \theta_2 t_i))^2 \tag{8}$$

This is the so-called least squares formulation, in that the optimal set of parameters minimizes the sum of squares of the residuals between the experimental measurements and the model prediction. The first-order condition for optimality can be used to find the optimal parameter values $\vec{\theta}^*$ for the model. This is achieved by setting $\frac{df}{d\vec{\theta}} = \vec{0}$:

$$\frac{df}{d\vec{\theta}} = \begin{bmatrix} 2\sum_{i=1}^{N}(y_i - (\theta_1 + \theta_2 t_i))(-1) \\ 2\sum_{i=1}^{N}(y_i - (\theta_1 + \theta_2 t_i))(-t_i) \end{bmatrix} = \vec{0} \tag{9}$$

$$\begin{bmatrix} \sum_{i=1}^{N} 1 & \sum_{i=1}^{N} t_i \\ \sum_{i=1}^{N} t_i & \sum_{i=1}^{N} t_i^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} y_i t_i \end{bmatrix} \tag{10}$$

$$\vec{\theta}^* = \begin{bmatrix} \sum_{i=1}^{N} 1 & \sum_{i=1}^{N} t_i \\ \sum_{i=1}^{N} t_i & \sum_{i=1}^{N} t_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} y_i t_i \end{bmatrix} \tag{11}$$

Using (11) in Matlab yields the following parameter estimates for this example:

$$\vec{\theta}^* = \begin{bmatrix} 14.24 \\ 3.92 \end{bmatrix} \tag{12}$$

This derivation demonstrates how this form of parameter estimation works on the same principles which motivate regression. The functionality of (10), however, can be replicated in a simpler procedure. This procedure simply reformats (1) into a matrix equation, and solves for $\vec{\theta}$ with matrix algebra. To do this, we define a system output vector and a regressor matrix:

$$\vec{y} = \begin{bmatrix} y_i \\ y_{i+1} \\ \vdots \\ y_N \end{bmatrix} \tag{13}$$

$$[Reg] = \begin{bmatrix} 1 & t_i \\ 1 & t_{i+1} \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix} \tag{14}$$

This regressor matrix represents the terms which are multiplied by each parameter at every measurement instance. Now, after rewriting (1) as a matrix equation:

$$[Reg]\vec{\theta} = \vec{y} \tag{15}$$

which in expanded form is:

$$\begin{bmatrix} 1 & t_i \\ 1 & t_{i+1} \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} y_i \\ y_{i+1} \\ \vdots \\ y_N \end{bmatrix} \tag{16}$$

we can solve for the vector $\vec{\theta}$. The following calculations demonstrate this procedure. First, we multiply both sides by the transpose of the regressor matrix:

$$[Reg]^T[Reg]\vec{\theta} = [Reg]^T\vec{y} \tag{17}$$

This allows us to calculate a square, invertible matrix $[Reg]^T[Reg]$, ensuring $\vec{\theta}$ can be isolated. Next, we go about solving for $\vec{\theta}$:

$$[[Reg]^T[Reg]]^{-1}[[Reg]^T[Reg]]\vec{\theta} = [[Reg]^T[Reg]]^{-1}[Reg]^T\vec{y} \tag{19}$$

$$\vec{\theta} = [[Reg]^T[Reg]]^{-1}[Reg]^T\vec{y} \tag{20}$$

This solution is functionally identical to (11), and yields the same parameter estimates. This is important because (20) can be generalized to different systems with minimal effort for deriving a unique solution like that shown in (11) for (1).

If the assumptions defined on page 6 are true, then on average $\vec{\theta}^*$ will equal $\vec{\theta}_{true}$. This procedure will work for any static or dynamic system given (i) such assumptions hold true, (ii) the relationships between each parameter and the terms in the model are linear, (iii) each parameter relates to a unique contribution to the system's output, (iv) data is available for all relevant inputs and outputs, and (v) the system model is correct in its description of the system dynamics.

3.3 Example of Parameter Identifiability using Fisher Information

Continuing the example in Section 3.2, we can now explore how to evaluate the accuracy of the parameter estimates shown in (12). This type of question references a concept called parameter identifiability, which is the overall ability for a system's parameters to be estimated.

The theoretical variances of the estimates obtained from either (11) or (20), given all of the requisite assumptions hold true, are defined by their respective Cramer-Rao lower bounds, which can be found from the system's covariance matrix. In order to calculate the covariance matrix, the concept of Fisher information must be defined. Generally speaking, Fisher information is a representation of the excitation demonstrated in the available data. Fisher information increases with higher degrees of sample excitation, leading to an increase in parameter identifiability. For the system example given by (1), the Fisher information matrix is given by the following relationship:

$$F = \frac{[Reg]^T[Reg]}{\sigma^2} \tag{21}$$

where σ is the standard deviation of the measurement error, which for this example we assume to be equal to 7.5. An important note is that the numerator of (21) is equivalent to the first term in (10), namely:

$$F = \frac{\begin{bmatrix} \sum_{i=1}^{N} 1 & \sum_{i=1}^{N} t_i \\ \sum_{i=1}^{N} t_i & \sum_{i=1}^{N} t_i^2 \end{bmatrix}}{\sigma^2} \tag{22}$$

The inverse of this Fisher information matrix is the system's covariance matrix, and the diagonal terms of the covariance matrix are the Cramer-Rao lower bounds for the variances of each parameter θ_1 and θ_2. In reality, the true variance of the parameter estimates will tend to be larger than the value given by these lower bounds. Continuing the above example with (22), the 3σ errors of the parameter estimates shown in (11) are:

$$\vec{\theta}^* = \begin{bmatrix} 14.24 \ {}^{+}/{-}\ 3.994 \\ 3.92 \ {}^{+}/{-}\ 0.117 \end{bmatrix} \tag{23}$$

The Matlab code for this extended example is included in Appendix A.

Given that the inverse of the Fisher matrix is proportional to the errors of the parameter estimates, it follows that a 'larger' Fisher matrix will yield better, more accurate estimates. From (22), it also follows that decreasing the measurement error σ can also lead to better estimates.

3.4 Advantages and Disadvantages of Linear Least-Squares Estimation

The most significant advantage of linear least-squares estimation is the ease at which it can be implemented. While the example in 3.1 is true only for the specific system defined by (1), the procedure can be generalized easily and meaningful results can be obtained in a relatively short amount of time.

The largest disadvantage of linear least-squares estimation is its inherent limitations regarding parameter identifiability. This issue was discussed briefly in Section 3.2. A detailed example where linear least-squares estimation will struggle to correctly estimate a system's parameters is shown in the model below:

$$y(\vec{\theta}, t) = \theta_1 + \theta_2 + \theta_3 t \tag{24}$$

In this example, there are multiple parameters which relate to similar contributions to the output, namely θ_1 and θ_2. This type of relationship impacts the parameters' structural identifiability. In this case, the calculations of Section 3.2 will yield a Fisher information matrix which is singular, meaning none of the parameters can be effectively estimated. Both θ_1 and θ_2 would have to be combined into a single parameter for parameter estimation to be possible. This would reduce the capability of parameter estimation to yield meaningful insights into the properties and behavior of the system.

Additionally, linear least-squares estimation requires that all of the parameters share linear relationships throughout the system model. This limitation reduces the overall applicability of this estimation method, and often requires several parrameters to be combined into a single parameter, or for the estimation of some algebraic manipulation of a parameter. Both cases introduce estimation bias when attempting to isolate the parameter estimate. Fortunately, for the purposes of this thesis, the vehicle dynamics model described in Chapter 4 fulfills most of these significant requirements.

4 System Model and Drive Cycle Formulation

In order to demonstrate successful estimation of vehicle chassis parameters, a vehicle chassis model will be implemented in Simulink. This model will need time histories for several inputs corresponding to a simulated drive cycle, or route from which data will be generated. The following sections discuss the chassis model formulation, and the generation of sample drive cycles for the parameter estimation exercise.

4.1 Vehicle Chassis Model Formulation

The parameter estimation algorithms described in this thesis are designed and validated based on the format of a conventional longitudinal chassis dynamics model. The relevant parameters for this study are listed in Table 1.

Table 1. Relevant Vehicle Parameters

Parameter	*Description*
M	vehicle mass [kg]
M_{eff}	effective mass given reflected driveline inertia [kg]
g	gravitational acceleration constant [m/s2]
τ_{FD}	torque at final drive [(kg.m2)/s2]
R_{FD}	final drive gear ratio [-]
r_{wheel}	wheel radius [m]
ρ_{air}	ambient air density [kg/m3]
C_d	drag coefficient [-]
A_{ref}	frontal area of the truck [m2]
C_r	rolling resistance coefficient [-]
θ_{road}	road grade [rad]
V_{veh}	vehicle velocity [m/s]
X	vehicle position [m]

A longitudinal chassis model accounts for the vehicle's propulsion force, braking force, force from aerodynamic drag, forces from rolling resistance at the tires, and a force from the road grade. This model formulation is described by the following equations:

State variables: $x_1 = X,\ x_2 = V_{veh}$

Input variables: $u = \tau_{FD}$

Disturbances: $w_1 = \theta_{road}$

Model: $\dot{x}_1 = x_2,$

$$\dot{x}_2 = \frac{1}{M}[F_{prop} - F_{brake} - F_{drag} - F_{roll} - F_{grade}] \quad (25)$$

where each term is represented by:

$$F_{prop} = \frac{\tau_{FD} R_{FD}}{r_{wheel}} \quad (26)$$

$$F_{drag} = \frac{1}{2}\rho_{air} C_d A_{ref} {V_{veh}}^2 \quad (27)$$

$$F_{roll} = C_r M g cos(w_1) \quad (28)$$

$$F_{grade} = M g sin(w_1). \quad (29)$$

A free-body diagram demonstrating these forces is shown in Figure 2. The braking force throughout this analysis will be assumed to be zero.

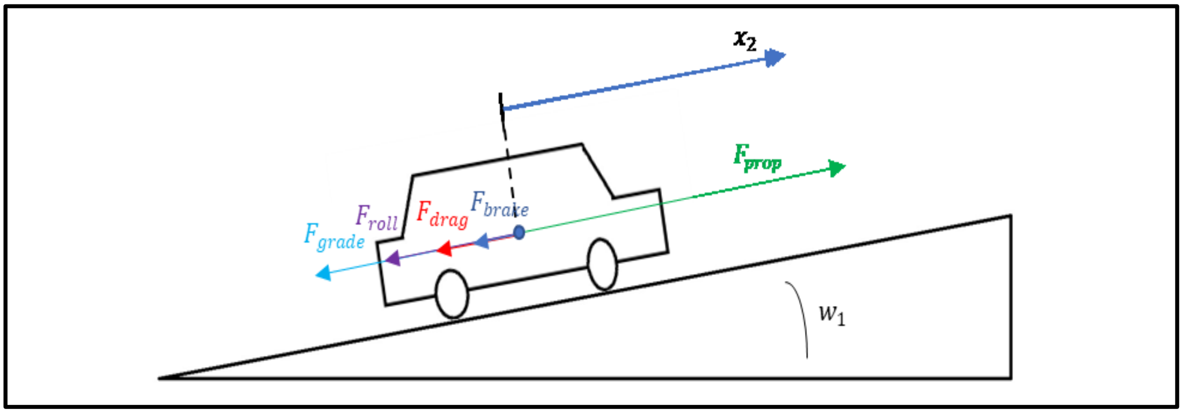


Figure 2. Free-Body Diagram of Longitudinal Dynamics Model

The longitudinal model will be implemented in Simulink and provided time histories for τ_{FD} and w_1. In the discussion of results included in Chapter 6, a series of similar simulated drive cycles with progressively scaled terrain profiles will be used to estimate vehicle chassis parameters, and a comparison of the accuracy of these estimates will be based on a defined metric for the terrain variability of each sample.

4.2 Generating Simulated Data

For the purposes of evaluating the effects of terrain variability on the parameter estimation process, the drive cycles developed in this section will exhibit a similar terrain profile with a broad spectrum of terrain. These terrain profiles need to be developed with consideration given to the propulsion force being exerted by the simulated vehicle.

Throughout this exercise, a propulsion force which exhibits minor sinusoidal variation will be applied to every drive cycle. The following expression describes this force:

$$F_{prop} = \frac{R_{FD}}{r_{wheel}}(225 * (1 + 0.1\sin(0.1 * t))) \quad (30)$$

4.2.1 Generalized Drive Cycle and Terrain Profile

Each drive cycle will be based off of a parabolic trajectory representing a single hill for the vehicle to traverse. This profile is defined by the following relationship:

$$f(x) = h * \cos^2(\frac{X}{300}) \tag{31}$$

where h is the overall height from peak to trough of the sinusoidal road profile, and x is the horizontal distance the vehicle can travel. The road grade of this terrain profile is approximated with a forward-Euler relation. Figure 3 show examples of elevation and road grade from the spectrum of drive cycles.

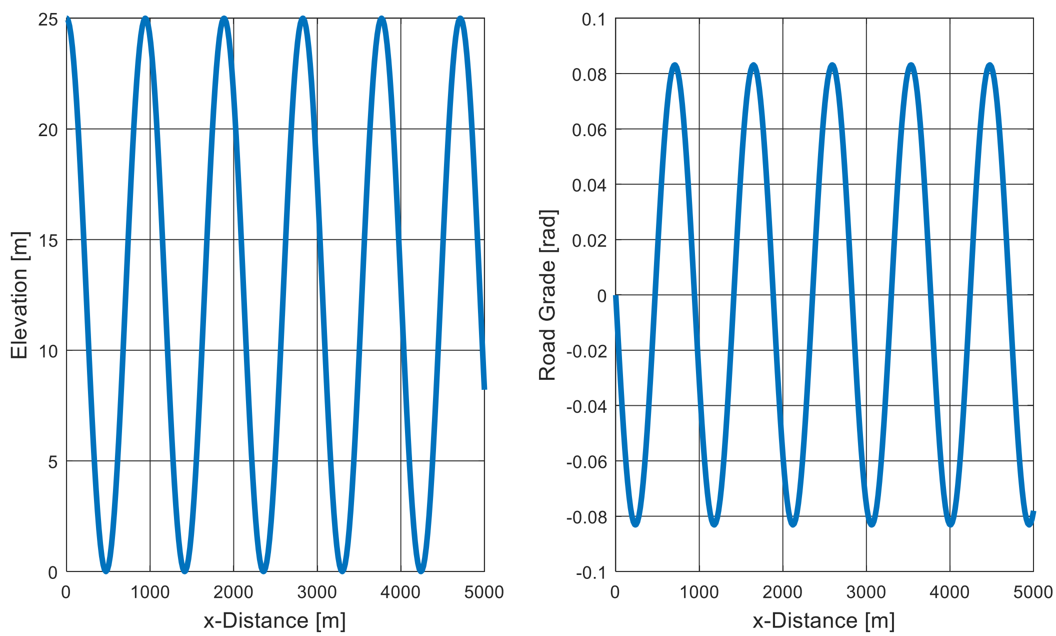


Figure 3. (left) Sample Terrain Profile and (right) Corresponding Road Grade Profile

This profile will be progressively scaled, and each instance will be used to evaluate the parameter estimation algorithm developed in Chapter 5. This work assumes no measurement error is associated with the values for the road grade of each sample.

4.2.2 Creating Data from Simulated Drive Cycles

In order to obtain time histories for x_2 and $\dot{x}_2$, the road profiles established in Section 4.2.1 and a propulsion force trajectory need to be run through a simulation of the longitudinal model. This simulation is implemented in a Simulink block diagram which interfaces with this project's Matlab code. Appendix B shows the format of this block diagram. The nominal longitudinal chassis model parameters used in this simulation are shown in Table 2.

Table 2. Nominal Chassis Parameter Values

Parameter	*Value*
$\mathbf{M}$	1800 [kg]
$\mathbf{C_d}$	0.35 [-]
$\mathbf{C_r}$	0.0125 [-]
$\boldsymbol{A_{ref}}$	7 [m^2]
$\boldsymbol{\rho_{air}}$	1.2 [kg/m^3]
$\boldsymbol{R_{FD}}$	2.7 [-]
$\boldsymbol{r_{wheel}}$	0.3 [m]

Artificial random gaussian noise with $\sigma = 1\ \frac{m}{s}$ is added to the velocity data which is output from this longitudinal model simulation.

5 Linear Least-Squares Problem Formulation

This work follows the procedure established in Section 3.2 in order to design a linear least-squares parameter estimation algorithm. But first, we must verify that the conditions for linear least-squares parameter estimation are satisfied with the model described in Chapter 4.

5.1 Conditions and Assumptions for Parameter Estimation Algorithm

This work seeks to estimate vehicle mass M, drag coefficient C_d, and rolling resistance coefficient C_r from the Volvo model. Given the longitudinal model formulation in (17), we can verify that the broad conditions given in Section 3.2 on page 9 are satisfied before designing an estimation algorithm:

(i) Is the measurement noise independent and identically distributed, with zero mean?

The measurement noise will be generated exclusively in Matlab, with known properties. As a result, this assumption is justified.

(ii) Are the relationships between the parameters and terms in the model linear?

In (17-21), the desired parameters M, C_d, and C_r each share linear relationships with different terms.

(iii) Does each desired parameter relate to a unique contribution to the system's output?

Analyzing the relationships which are shared between the model parameters, relevant constants and data, and the model output reveals that each parameter does indeed relate to a unique contribution to the model output.

(iv) Is data available for all relevant inputs and outputs?

All relevant data will be synthesized.

(v) Is the model correct in its description of the system dynamics?

The same model formulation included in the simulation will be utilized by the parameter estimation algorithm in this thesis. By sharing the same basis for describing and predicting chassis dynamics, the model in use by the estimation algorithm will in theory identically describe the simulated vehicle chassis dynamics.

While these assumptions have been validated largely by the design of this analysis, many would not necessarily hold nearly as true when estimating vehicle chassis parameters from actual experimental data. For instance, the longitudinal vehicle dynamics model does not represent disturbances including wind velocity, which could create bias in estimating the drag coefficient of a vehicle. Similarly, instrumentation and sensors which could be used to gather experimental data on metrics including final drive torque, road grade, and vehicle velocity may not necessarily exhibit independent and identically distributed noise with zero mean. Broadly speaking, analyzing real-world experiments poses its own set of unique challenges for obtaining accurate chassis parameter estimates. Many of these challenges are discussed in my follow-up paper, which evaluates the relationship between mean-square road grade variability and vehicle chassis parameter identifiability with actual experimental data [11]. Given that, however, these general requirements hold true in this specific application, the linear least-squares parameter estimation algorithm is an ideal choice for parameter estimation.

5.2 Linear Least-Squares Parameter Estimation Problem Formulation

The calculations shown in this section will be implemented in a Matlab program in conjunction with the Volvo simulation. First, all of the terms which depend on the desired parameters are grouped on the right-hand side, and the desired parameters are factored out of their respective terms:

$$\dot{x}_2 + gsin(w_1) =$$

$$\frac{1}{M}\left(F_{prop} - F_{brake}\right) - \frac{C_d}{M}\left(\frac{1}{2}\rho_{air}A_{ref}{V_{veh}}^2\right) - C_r(gcos(w_1)) \qquad (32)$$

From this point forward, the system output vector will be:

$$\vec{y} = \begin{bmatrix} \dot{x}_{2;i} + gsin(w_{1;i}) \\ \dot{x}_{2;i+1} + gsin(w_{1;i+1}) \\ \vdots \\ \dot{x}_{2;N} + gsin(w_{1;N}) \end{bmatrix} \qquad (33)$$

or the entire left-hand side of (32).

The inherent limitations of linear least-squares estimation require that the algorithm estimates the parameters $\frac{1}{M}$, $\frac{C_d}{M}$, and C_r. As a result:

$$\vec{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{M} \\ \frac{C_d}{M} \\ C_r \end{bmatrix} \qquad (34)$$

Some simple post processing arithmetic, which is described at the end of this section, can be applied to calculate approximate estimates of M and C_d. However, this does introduce bias into the parameter estimates. For the analysis of the impact of terrain variability on chassis parameter identifiability, this post processing is not applied.

The model shown in (32) can be used to generate a regressor matrix for use by the estimation algorithm. This matrix is shown below:

$$[Reg] = \begin{bmatrix} F_{prop;i} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;i}^2 & -gcos\left(w_{1;i}\right) \\ F_{prop;i+1} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;i+1}^2 & -gcos\left(w_{1;i+1}\right) \\ \vdots & \vdots & \vdots \\ F_{prop;N} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;N}^2 & -gcos\left(w_{1;N}\right) \end{bmatrix} \qquad (35)$$

We can now write a complete matrix equation of the system described by (32), (33), (34), and (35):

$$[Reg]\vec{\theta} = \vec{y} \quad (36)$$

which in expanded form is:

$$\begin{bmatrix} F_{prop;i} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;i}^2 & -gcos(w_{1;i}) \\ F_{prop;i+1} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;i+1}^2 & -gcos(w_{1;i+1}) \\ \vdots & \vdots & \vdots \\ F_{prop;N} & -\frac{1}{2}\rho_{air}A_{ref}V_{veh;N}^2 & -gcos(w_{1;N}) \end{bmatrix} \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \end{bmatrix} =$$

$$\begin{bmatrix} \dot{x}_{2;i} + F_{grade;i} \\ \dot{x}_{2;i+1} + F_{grade;i+1} \\ \vdots \\ \dot{x}_{2;N} + F_{grade;N} \end{bmatrix} \quad (37)$$

The same relationships established in (19) and (20) can be used to calculate estimates of the model parameters $\vec{\theta}$.

Once such estimates, denoted $\hat{\vec{\theta}}$ or individually with a hat symbol, are obtained, additional post processing will be required in order to calculate approximations of the individual estimates for each parameter. This post processing is described by the following equations:

$$\widehat{M} = \frac{1}{\hat{\vec{\theta}}\,(1)} \quad (38)$$

$$\hat{C}_d = \widehat{M}(\hat{\vec{\theta}}\,(2)) \quad (39)$$

This overall procedure will yield estimates for the parameters $\widehat{M}$, $\hat{C}_d$, and $\hat{C}_r$ from time histories of relevant data.

5.4 Parameter Identifiability Algorithm Formulation

Utilizing the procedure established in (21), we can compute the system's Fisher information matrix with $[Reg]$ and an assumed measurement error σ matching the value for the noise added to the velocity data ($\sigma = 1\ \frac{m}{s}$). This Fisher information matrix can be inverted to obtain the following covariance matrix:

$$[Cov] = \begin{bmatrix} \sigma_{\frac{1}{M}}^2 & \sigma_{\frac{1}{M}}\sigma_{\frac{C_d}{M}} & \sigma_{\frac{1}{M}}\sigma_{C_r} \\ \sigma_{\frac{C_d}{M}}\sigma_{\frac{1}{M}} & \sigma_{\frac{C_d}{M}}^2 & \sigma_{\frac{C_d}{M}}\sigma_{C_r} \\ \sigma_{C_r}\sigma_{\frac{1}{M}} & \sigma_{C_r}\sigma_{\frac{C_d}{M}} & \sigma_{C_r}^2 \end{bmatrix} \tag{40}$$

The square roots of the diagonal terms of (40) will yield theoretical lower bounds for the errors of the relevant parameters $\frac{1}{M}$, $\frac{C_d}{M}$, and C_r. The additional post processing described by (38) and (39) will have implications for converting the errors of $\frac{1}{M}$ and $\frac{C_d}{M}$ from the system covariance matrix into meaningful error bounds for M and C_d. This can similarly introduce bias in the results. The following equations define this post processing:

$$\sigma_M = \frac{1}{\hat{\vec{\theta}}\,(1) - \sigma_{\frac{1}{M}}} - \widehat{M} \tag{41}$$

$$\sigma_{C_d} = \left(\hat{\vec{\theta}}\,(2) + \sigma_{\frac{C_d}{M}}\right)\left(\hat{\vec{\theta}}\,(1) + \sigma_{\frac{1}{M}}\right) - \hat{C}_d \tag{42}$$

The fact that the post processing described by (38), (39), (41), and (42) introduces bias into the estimation and identifiability results, coupled with other limitations of the linear least-squares method, means that the analysis described in the following sections will strictly evaluate the unprocessed parameters. This post processing can, however, be used periodically to provide a relatable sanity check of the efficacy of the estimation algorithm.

6 Results and Discussion

Now that the parameter estimation and identifiability algorithm has been formulated, it can be incorporated into Matlab code which utilizes data from the simulated drive cycles established in Section 4.2.2. This Matlab code is included in Appendix C. The following sections describe the results of this algorithm evaluated based on the terrain profile shown in Figure 3 which is progressively scaled up to a factor of 2.

6.1 Terrain Variability Metric

Theoretically, estimates obtained from drive cycles with more terrain excitation should yield parameter estimates with lower associated error values. As a result, a terrain variability metric is defined allows the comparison of estimates obtained from routes over different terrain profiles.

$$\varsigma = \frac{\sum_{i=1}^{N} \theta_{road;i}^{2}}{N} \tag{43}$$

This metric sums up the road grade across an entire sample and normalizes that value by the size of the sample. A higher value for ς indicates a higher degree of mean-square terrain variability relative to zero in a data sample.

6.2 Parameter Estimation Results

Table 3 shows results from the parameter estimation algorithm described in Chapter 5 for the terrain profile shown in Figure 3 scaled up to a factor of 2.

Table 3. Parameter Estimation Results

Scaling Factor	*Mass Estimate* ($\frac{1}{M} = \mathbf{0.000555}\ kg^{-1}$)	*Drag Estimate* ($\frac{C_d}{M} = \mathbf{0.0001944}\ kg^{-1}$)	*Rolling Resistance Estimate* ($C_r = \mathbf{0.0125}$)
1	0.00055479	0.000182927	0.018010557
1.1	0.000557093	0.000183416	0.01823568
1.2	0.000555393	0.00018401	0.017581247
1.3	0.000555327	0.000184525	0.01731914
1.4	0.00055618	0.000184976	0.017232048
1.5	0.000557523	0.000185593	0.017242714
1.6	0.000553248	0.000185952	0.016142849
1.7	0.000557133	0.000186381	0.016797372
1.8	0.000555928	0.000187029	0.016186674
1.9	0.000555913	0.000187111	0.016150069
2.0	0.000553493	0.000187759	0.015312255

These results indicate that the addition of the noise has created some estimation bias, particularly regarding the drag and rolling resistance parameter estimates. When the artificial noise is not added to the velocity data signal, the parameter estimation results are within the order of 10^{-11} of the nominal parameter values.

6.3 Parameter Identifiability Results

Figures 4-6 show the simulated relationships between the parameter estimation error values and the mean-square terrain variability calculated from the terrain profile shown in Figure 3 as it is scaled up to a factor of 2.

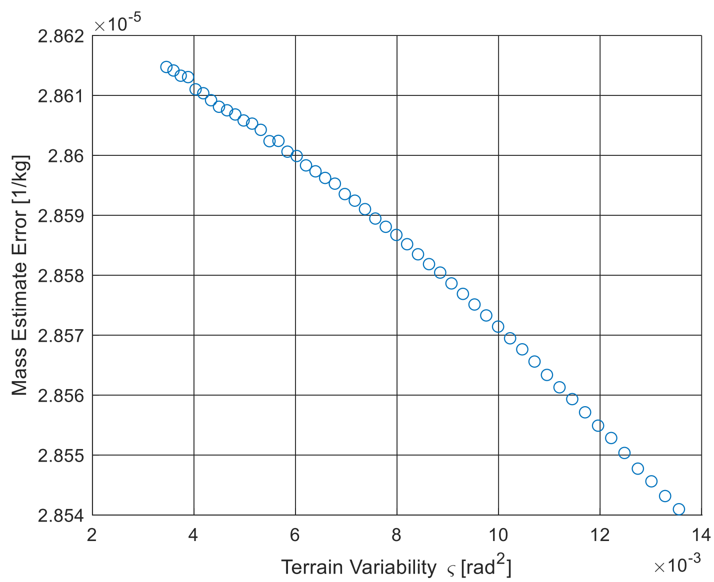


Figure 4. Relationship between $\frac{1}{M}$ Estimation Error and Terrain Metric ς

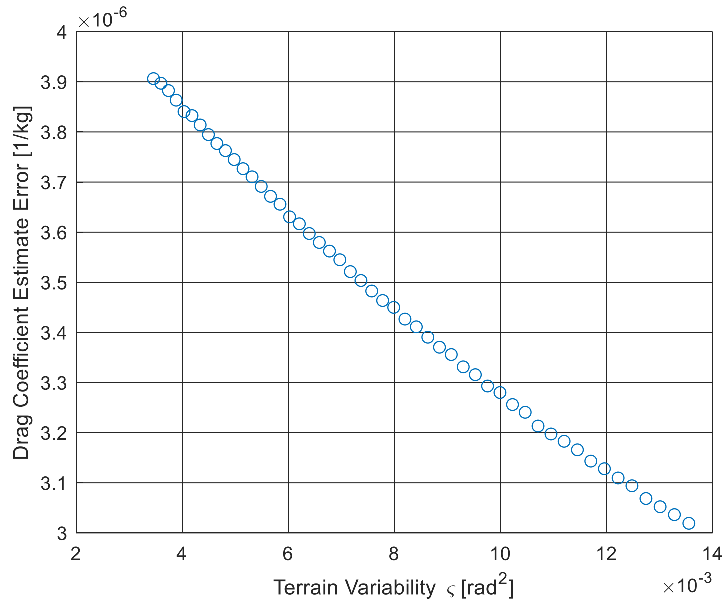


Figure 5. Relationship between $\frac{C_d}{M}$ Estimation Error and Terrain Metric ς

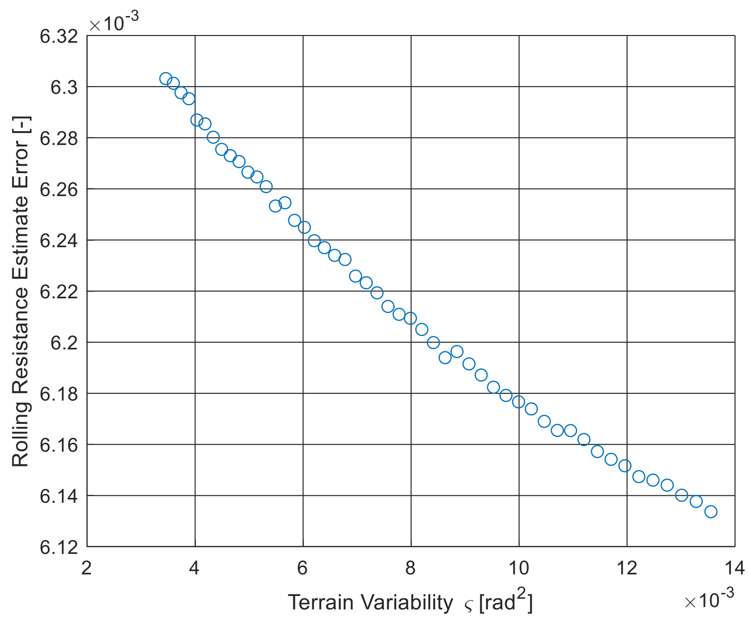


Figure 6. Relationship between C_r Estimation Error and Terrain Metric ς

These plots demonstrate the monotonic and decreasing relationship between estimation error and mean-square terrain variability for a sinusoidal terrain profile. An important note to make is that the observed monotonic relationship is most consistently obtained when the data represents a large section of terrain which exhibits near-zero *mean* road grade.

Overall, the results from the parameter estimation and Fisher identifiability analyses show how increasing the mean-square terrain variability of a sinusoidal drive cycle can increase chassis parameter identifiability. This insight can inform the design of on-road experiments for longitudinal chassis parameter identifiability.

7 Conclusion

This thesis has shown through simulation that as the mean-square terrain variability of an experiment increases, the accuracy of longitudinal vehicle chassis parameter estimates obtained from that experiment increases. This relationship follows a monotonic and decreasing shape as the terrain variability of a long sinusoidal road profile increases. In addition to yielding meaningful results, this project has allowed me to develop new skillsets which will be immediately applicable to my graduate studies and future career.

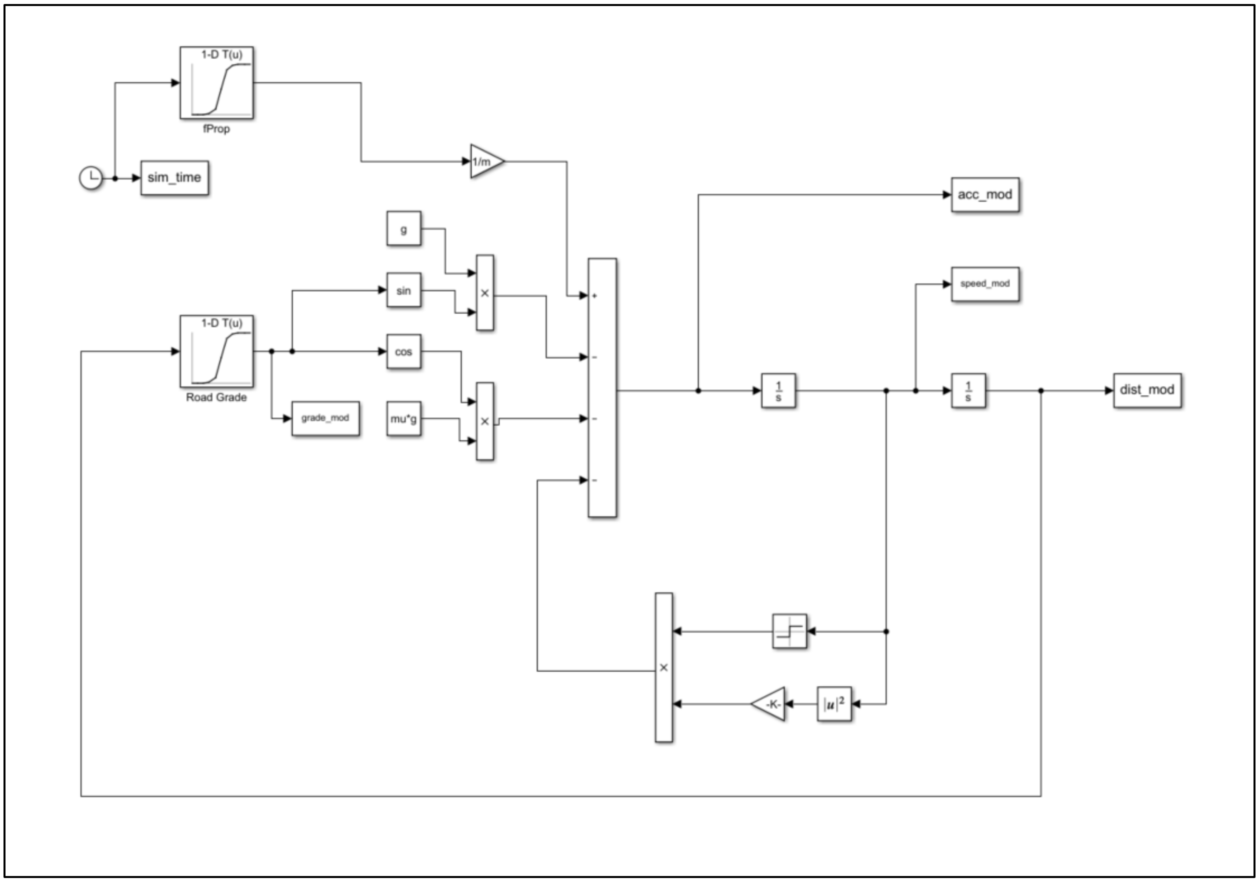
Appendix A

Parameter Estimation Algorithm Example Matlab Code

```
% Aaron Kandel
% Schreyer Honors Thesis
% Parameter Estimation Example (Section 4.2.2)
% 01/07/2018

clc
clear all

u = (0:0.5:10)';  % system inputs (x axis)
y = 15 +((4.*u) + 18*(rand(length(u),1)-0.5)); % system output (with random
measurement noise)

% Plot Hypothetical Experimental Measurements:
scatter(u,y)
ylim([0 70])
grid on
xlabel('System Input u')
ylabel('System Output y')

% Define Matrices in [9] on page 5:
ymat = [sum(y);sum(y.*u)];
umat = [length(u), sum(u); sum(u), sum(u.^2)];

% Calculate Parameter Estimates
thetaStar = inv(umat)*ymat;

% Calculate Errors with Fisher Information
sigma = 7.5; % assume measurement error of 7.5
F = umat./sigma; % system Fisher information matrix
C = inv(F); % system covariance matrix

% Print Parameter Estimates
fprintf('The parameter estimates for this system are: \n')
fprintf('theta1 = %0.2f +/- %0.5f\n',thetaStar(1),3*C(1,1))
fprintf('theta2 = %0.2f +/- %0.5f\n',thetaStar(2),3*C(2,2))
```

Appendix B

Simulink Block Diagram of Longitudinal Chassis Model Simulation



Appendix C

Parameter Estimation and Identifiability Analysis Matlab Code

```
% Aaron Kandel
% Schreyer Honors Thesis
% Generating Drive Cycles (Section 4.2)
% 01/23/2018

clc
clear all
close all

%% Input Trajectory

 % Throughout this code, y is the elevation of the drive cycle, NOT the
 % output defined in Chapter 5.

%% Low Terrain-Variability Route

x = (0:50000)'; % [m] longitudinal direction
y_1 = 25.*cos((1/300).*x).^2; % [m]

% Generate Road Grade Data
for i = 1:(length(x)-1)
    w1_1(i,1) = atan((y_1(i+1)-y_1(i))); % [rad]
end

% Post-Processing
x(end) = []; % make array sizes consistent
y_1(end) = []; % make array sizes consistent

% Plot Terrain
% subplot(1,2,1)
% plot(x,y_1)
% grid on
% xlabel('x-Distance [m]')
% ylabel('Elevation [m]')
% xlim([0,5000])
% subplot(1,2,2)
% plot(x,w1_1)
% grid on
% xlabel('x-Distance [m]')
% ylabel('Road Grade [rad]')
% xlim([0,5000])

%% Generate Data for Par Est.
format long g

mu = 0.0125; % [-]
C_d = 0.35;% [-]
```

```
A_f = 5.5;% [m^2]
rho = 1.2;% [kg/m^3]
g = 9.81;% [m/s^2]
m = 1800; % [kg]
timeStep = 0.01; % [s]
timeVector = (0:timeStep:600)'; % [s] 300s sample time
rFD = 2.7; % [-] final drive gear ratio
rWheel = 0.3; % [m] wheel radius
tauFD = (225.*(1+0.1.*sin(0.1*timeVector)));% [N-m] final drive torque
fProp = (tauFD.*rFD)./rWheel; % [N] propulsion force
sigma = 1;% [m/s] error of velocity measurement
a = 1;
for i=1:11 % 44
    w1 = w1_1.*a;
    y = y_1.*a;
    velNoise = 1*randn(length(timeVector),1);
    sim('model_model2.slx')
    output = acc_mod + g.*sin(grade_mod);
    speed_mod = speed_mod + velNoise;
    reg = [fProp, -0.5.*rho.*A_f.*speed_mod.^2, -g.*cos(grade_mod)];
    theta = (inv(reg'*reg))*(reg')*output;
    mEstimate(i,1) = theta(1); % 1/
    dragEstimate(i,1) = theta(2); % *mEstimate(i,1)
    muEstimate(i,1) = theta(3);
    FI = (reg'*reg)./(sigma^2);
    cov = inv(FI);
    sd_m(i,1) = sqrt(cov(1,1)); %abs(1/((1/mEstimate(i,1))-(sd_theta1))-
mEstimate(i,1));
    sd_crr(i,1) = sqrt(cov(3,3));
    sd_cd(i,1) = sqrt(cov(2,2)); %abs((theta(2,1)-
sqrt(cov(2,2)))*(mEstimate(i,1)-sd_m(i,1))-dragEstimate(i,1));
    disp(i);
    a = a + 0.1; %0.02;
    k(i,1) = cond(FI);
    rgm(i,1) = mean((w1.^2)); %/length(w1);
    fprintf('massEstimate = %.6f +/- %.8f [kg]\n',mEstimate(i),sd_m(i))     %
1/mEstimate(i)
    fprintf('dragEstimate = %.6f +/- %.8f [-]\n',dragEstimate(i),sd_cd(i)) %
./mEstimate(i)
    fprintf('muEstimate = %.6f +/- %.8f [-]\n',muEstimate(i),sd_crr(i))
    disp('  ')
end


figure
scatter(rgm, sd_m)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Mass Estimate Error [1/kg]')

figure
scatter(rgm, sd_cd)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Drag Coefficient Estimate Error [1/kg]')
```

```
figure
scatter(rgm, sd_crr)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Rolling Resistance Estimate Error [-]')




figure
hold on
scatter(rgm, mEstimate)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Mass Parameter Estimate [1/kg]')
plot(rgm,(1/m).*ones(length(rgm),1))
set(findall(gca, 'Type', 'Line'),'LineWidth',2);
legend('Mass Parameter Estimates','Nominal Mass Value')


figure
hold on
scatter(rgm, dragEstimate)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Drag Coefficient Estimate Error [1/kg]')
plot(rgm,(C_d/m).*ones(length(rgm),1))
set(findall(gca, 'Type', 'Line'),'LineWidth',2);
legend('Drag Parameter Estimates','Nominal Drag Parameter Value')



figure
scatter(rgm, sd_crr)
grid on
xlabel('Terrain Variability \varsigma [rad^2]')
ylabel('Rolling Resistance Estimate Error [-]')
```

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Research Experience

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- Utilized nonlinear least-squares estimation to evaluate hydraulic fracturing well performance
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- Participated in efforts to develop a miniature, modular, hybridized RC car for use in K-12 educational outreach relating to hybrid vehicles and environmental issues in engineering

The Pennsylvania State University, Department of Physics **Jan. '15 – Mar. '17**
- Applied a physically motivated function from the literature on the atmospheric attenuation of gamma rays to model the local directionality of ultra-high-energy cosmic rays

Publications

[1, C] Kandel, A. I., Wahba, M., Geyer, S., and Fathy, H. K., "Impact of Terrain Variability on Chassis Parameter Identifiability for a Heavy-Duty Vehicle," *accepted* to the 2018 European Control Conference.

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- "Modeling the Distribution of Arrival Directions of Ultra-High-Energy Cosmic Rays with Physically Motivated Functions," 2017 Annual Meeting of the APS Mid-Atlantic Section, Nov. 4, Newark, NJ.
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