ACOUSTIC CHARACTERIZATION OF A MODEL GAS TURBINE COMBUSTOR

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ABSTRACT

Regulations in the aerospace and power generation industries continue to place increasingly stringent limitations on the amount of pollutants a gas turbine combustion system can emit. This is often achieved by lean fuel-air mixture systems that operate at much lower flame temperatures. Lower flame temperatures push these systems to the limit of instabilities that destroy hardware and decrease the lifetime of these systems. These instabilities are the result of a feedback cycle between acoustic pressure oscillations, perturbations in the flow and mixture of reactants, and consequent oscillations in heat release. Although each component of this feedback loop has been studied, acoustic pressure oscillations are highly dependent on the geometry of each combustor configuration. This inherent challenge is further driven by the fact that these studies must often be conducted experimentally after all components of the system have been assembled. Developing techniques capable of evaluating acoustic characteristics of a system early in the design of the system will help lower cost of development while improving detection and prevention of instabilities.

This study assesses inherent acoustic characteristics of a model gas turbine combustion system experimentally and through simulation. Resonant frequencies of the physical system are presented and impedances of boundary conditions at the nozzle exit are calculated. These boundary conditions serve as necessary inputs to numerical modeling methods that replicate experimental studies conducted prior. Comparison between experimental results and modeling results show that modeling techniques can be effective in analyzing a system at preliminary stages of the design process.
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Chapter 1

Introduction

As regulations governing pollutants have tightened across almost every industry, the aviation and power industries, which almost universally employ gas turbines, have been equally affected by the requirement to uphold these standards [1]. In order to achieve these improved levels of efficiency, gas turbine development has led to the production of engines that contain combustion systems in which the fuel-air mixture ratio is much leaner than traditional diffusion flame combustors. These leaner combustion systems effectively produce lower flame temperatures within the combustion chamber by using excess air in the combustion process, reducing the emissions of harmful products such as NOx [2]. The latent effect of these lower flame temperatures, however, is an increase in the vulnerability of these combustion systems to exceedingly destructive instabilities [2]. As advancements continue to be made to improve the efficiency of gas turbine engines and minimize emissions, these instabilities will continue to become more prevalent and continue to require a greater understanding of the fundamental physics responsible.

The most evident consequences of these combustion instabilities are exemplified by unstable fluctuations in acoustics and heat release rates that inflict damage on combustion chamber hardware [2]. The mechanism responsible for these effects involves a feedback cycle with three distinct, coupled phases [2]. The first phase begins with the combustion system under normal operation, in which it naturally exhibits oscillations in heat release rate as the flame interacts with the turbulent environment in the combustion chamber [2]. This inherently stable phenomenon introduces energy into the acoustic field, inciting the second phase in the feedback process in which acoustic pressure and velocity also begin to fluctuate throughout the combustor [2]. Variations in the acoustic pressure and velocity are then responsible for the third phase, fluctuations in the fuel and air mixture during the combustion process. Ultimately, this results in even greater heat release rate oscillations, completing the feedback loop and
Thus perpetuating the resulting acoustic pressure fluctuations [2]. Therefore, it is evident that a significant amount of emphasis must be placed on the relationship between heat release oscillations, $q'$, and pressure oscillations, $p'$. Particularly, the focus is on the relation of phase and frequency between these two oscillating system properties, which illustrate the coupling nature between each process described above [2]. However, a crucial factor governing the ability for this feedback loop to incite the harmful instabilities is the capacity for the system itself to dampen these intensifying oscillations and effectively dissipate the added energy [3].

In order to understand the entire picture of the pressure oscillation input affecting the feedback loop, it is necessary to define the entire acoustic field present in the combustion system. Lieuwen concluded that the net acoustic energy field develops from the competition of two processes: (a) acoustic energy addition driven by the non-linear nature of combustion processes, (b) acoustic energy damping. [3] The fundamental basis that defines the circumstances, under which an instability may arise and intensify, is whether the magnitude of the acoustic energy dissipated by the system is greater than the magnitude of acoustic energy added [3]. Therefore, if damping exceeds the amplitude of the perturbation, the system will never experience an instability and the system would completely suppress any perturbation. Standard operation of combustion systems however, exhibits an interesting relationship between acoustic driving and damping processes. As the magnitude of acoustic pressure oscillations increases, the feedback loop causing instabilities saturates while the magnitude of damping processes increases linearly. This relationship results in distinct operating conditions in which damping may potential be greater or less than driving, based on the magnitude of the acoustic pressure oscillations. Consequently, this establishes distinct operating conditions where the combustion system is stable or unstable. Damping mechanisms by which acoustic energy is dissipated will be discussed further. This discussion will provide context for understanding the entire acoustic field in both the experimental and numerical models used in this study. It remains evident, despite the presence of damping mechanisms dissipating energy in the system, that
defining acoustic modes in the system is necessary to identify the operating conditions under which instabilities occur. Determining these acoustic modes then will be the focus of this study.

As the development of gas turbine engines continues to push the combustion systems closer to the limits of instability, some solutions to prevent these instabilities have been developed. The conventional technique that has seen moderate levels of success over the years in combatting these particular instabilities involves detailed experimental effort conducted on a completed combustion system design [4]. Characterization of the acoustic energy field in the combustion system is performed to predict specific frequencies in which the combustion chamber has a resonant acoustic mode. Once the acoustic characteristics of the system have been identified, three conventional measures can be taken in order to prevent these issues as demonstrated by General Electric’s development of their ‘Aero’ and ‘Aeroderivative’ engines [4].

The first measure one can take is to completely redesign certain components such as the fuel-air mixing device in the combustor to suppress combustion dynamics. The changes to fuel and airflow paths have the ability to move combustion dynamics away from inherent acoustic characteristics and thus prevent these two system features from interacting. Associated with this method are an evident increase in both the cost and timeline of development. The second alternative to prevent these instabilities, as studied by Davis and Black [5], employs an unequal offset in fuel flow distribution known as staging. Although staging changes combustion dynamics in a relatively cheap and efficient manner, it limits the ability for the engine to achieve ideal emission performance since the flame temperature is not evenly distributed [5].

The third and final method currently used in industry is split into either passive or active control technologies. Passive control involves the application of Helmholtz resonators or quarter wave tubes to the combustion chamber. This mechanism increases acoustic damping at specific frequencies. However, these devices only work at a finite range of frequencies, limiting the extent to which the device can control a variety of instabilities. In active control, combustion dynamics are both monitored and altered
by actively pulsing fuel flow rates at a specific phase and frequency in response to the instability. Although this method remains promising, it is typically too expensive and complex for engines in the field. Similarly, control sensors and actuators would need to be improved enough to meet the resiliency requirements of operation under the conditions in a combustion chamber. Thus, the current methods of instability prevention are afflicted by factors like cost and timing of further development, ability to maintain equivalent levels of performance, and limited progress in development of active control technologies.

Based on the current state of research, the principal obstacle affecting development of gas turbine engines in industry is the difficulty for combustion dynamics to be analyzed until the entire system has been designed. Some factors affecting performance of the engine can be derived analytically beforehand or through individual component tests. The vulnerability of the entire combustion system however, cannot be adequately understood until it is completely assembled and available for testing. Thus, it becomes clear that more work needs to be conducted to develop techniques capable of evaluating these instability characteristics throughout the development and production of a combustion system. In order to develop these capabilities, the physics behind these combustion instability mechanisms need to be explored in detail as can be seen in the subsequent chapter. Together these concepts will work to more accurately characterize, predict, and effectively control the processes affecting combustion dynamics.
Chapter 2

Literature Review

Instability Mechanisms

The driving physics of combustion instabilities are defined by the interaction between heat release rate and pressure oscillations. Some of the first work in quantifying this relationship, published by Lord Rayleigh, established a criterion to determine when these oscillation-driven instabilities occur and persist [6]. The Rayleigh criterion determines that when the fluctuations of both pressure and heat release are ‘in phase’ with one another these oscillations will couple and potentially grow depending on the damping present in the system. This theoretical description is described mathematically in Eq. 1.

\[
\frac{1}{T} \int_{0}^{T} p'q' dt > 0
\]

This equation expresses the effect of the relative phase, of both the acoustic pressure oscillations and heat release rate oscillations, on determining the occurrence of an instability. The Rayleigh Criterion signals an instability when the integration of the product of heat release rate and pressure oscillations over an acoustic cycle is positive. This indicates that the two oscillating system properties are ‘in phase’. For instance, when the phase difference between the pressure and heat release rate oscillations is less than 90 degrees the product of the two properties will be greater than zero, indicating an instability is present [2]. The Rayleigh criterion method for identifying an instability further reinforces the importance of comprehensively characterizing the acoustic field within the combustion chamber. Specifically, it is necessary to define not only the amplitude of a specific acoustic pressure, but also to define the phase of the signal as well [2].
Although the Rayleigh criterion distinguishes acoustic pressure oscillations and heat release rate oscillations as the key parameters defining the onset of an instability it is important to recall that these two factors do not directly influence one another. The feedback cycle responsible for exciting instabilities in combustion processes involves three factors: heat release oscillations, acoustic pressure oscillations, and flow and mixture perturbations [2]. Therefore, understanding the physical processes through which acoustic pressure oscillations directly couple with flow and mixture perturbations provides insight into the effect acoustic pressure oscillations has on heat release oscillations. These two important coupling mechanisms are defined as velocity coupling and fuel to air mixture coupling.

Velocity coupling drives heat release rate oscillations in two different ways: flame-boundary interactions or flame-vortex interactions. In flame-boundary interactions, a steady premixed flame stabilized on a cylindrical burner produces significant noise when interacting with a horizontal plate. This noise generation is a direct result of fluctuations in the surface area of the flame [7]. This rapid change in flame surface area introduces non-steady heat release in the system, ultimately introducing more acoustic pressure radiation into the system [7]. When acoustic pressure generation and resulting fluctuations in heat release rate caused by this mechanism are in phase the Rayleigh criterion is met and instabilities occur. In a slightly different situation the second source of velocity coupling results from the effects of vortex interactions with the flame. The formation of vertices in combustion processes readily occur in many different combustor configurations [8]. In a similar manner as flame-boundary interactions, the presence of a vortex has an immense effect on the surface area of the flame. As the flame rolls up in the formation of a vortex the surface area increases rapidly [9]. To analyze the coupling effects of vortices Poinset et al. [10] combined information on changes in local heat release in the combustor to compare with different physical images of the vortex structure at distinct times. These studies showed that the formation of vortices have an immense impact on flame structure and thus generate fluctuations in heat release rate in the system. However, vortices also offer another distinct mechanism in which the steady heat release of a stable flame can be disturbed. During the formation of a vortex, fresh reactants may be
contained within the vortex structure [7,11]. When this structure interacts with a wall or another separate body, the interaction stimulates the sudden ignition of the contained reactant. This coupling process also introduces non-steady heat release fluctuations, providing the driving force necessary to initiate the feedback cycle [7].

Mixture coupling, the other significant coupling mechanism prevalent in combustion systems, particularly the model can-combustor used in this study, effects the rate at which fuel mixes with air in the system. Fuel to air mixing can be affected in two ways, by acoustic pressure oscillations either altering the rate at which the fuel nozzle delivers fuel to the system, or altering the velocity and pressure of the flow [12]. In the first mixture coupling mechanism, fuel line acoustic coupling, acoustic pressure oscillations present in the combustor affect the pressure drop across the fuel nozzle. As the pressure drop across the fuel nozzle oscillates at the rate of the acoustic pressure fluctuation, the rate at which fuel is injected into the system mirrors that oscillation. Work conducted by C. E. Johnson et al. [13] explores how changes to externally forced oscillations in the acoustic driving of the fuel injection rate affect combustor stability. A siren was used to apply an external, acoustic driving force while a fuel injection actuator was used to control the rate at which fuel was injected into the system. In measuring pressure signals and changes to heat release fluctuations throughout the combustion chamber this study was able to determine the correlation and effects these changes had on instabilities in the system. In the second mixture coupling mechanism, acoustic pressure oscillations are transmitted into the premixer section of the combustion system, affecting the pressure and velocity of the mixing flow. As a result, the reactive mixture comprised of periodically, differing equivalence ratios [12]. Each of these mixture-coupling methods introduce perturbations in the fuel to air ratio in the mixed flow. Research conducted by both Lieuwen and Zinn [12] and J. G. Lee et al. [14] analyzed the effect of equivalence ratio fluctuation on heat release oscillations, which subsequently induce instabilities in the system. Each combined the simultaneous record of pressure fluctuation signals and measured heat release to show that perturbations in equivalence ratio are convected into the flame, producing significant fluctuations in heat release.
Acoustic Modes

Given the significance of acoustic pressure oscillations in each coupling mode, it is critical to define the physics governing these acoustic interactions. Frequencies of combustion processes often oscillate near the acoustic resonant frequencies of the combustion system [2,15]. Thus, it becomes necessary to conduct a more comprehensive analysis of the various acoustic modes possible within the combustion system. The entire acoustic pressure field in any geometry can be described by the following nonhomogeneous linear wave equation,

\[ \frac{\partial^2 p'}{\partial t^2} - c^2 \nabla^2 p' = \frac{\gamma - 1}{c^2} \frac{\partial \dot{q}'}{\partial t} \]

Here, \( p' \) represents the magnitude of the acoustic pressure, \( \dot{q}' \) represents the magnitude of the fluctuating heat release rate, \( c \) is the localized speed of sound, and \( \gamma \) is the ratio of specific heats. To ensure the mathematics effectively describe the geometry of a can-type combustion chamber, the acoustic wave equation can be rewritten in cylindrical coordinates as follows [2,15],

\[ \frac{\partial^2 p'}{\partial t^2} - c^2 \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p'}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p'}{\partial \theta^2} + \frac{\partial^2 p'}{\partial x^2} \right] = 0 \]

This equation neglects the oscillating heat release rate term on the right-hand side of the equation and thus provides an isolated picture of how the acoustic oscillations will behave in the system without the effects of a flame included. Next, it is necessary to assume the acoustic wave amplitude, \( p' \), can be described by the following complex function oscillating in both time and space [2,15],

\[ p' = \hat{p}(r) \exp[(kx + m\theta - \omega t)i] \]

Here, \( k \) is the longitudinal wavenumber, \( m \) is the azimuthal wavenumber, \( \omega \) is the frequency of the oscillation, and \( \hat{p}(r) \) is the amplitude of the acoustic pressure oscillation in the radial direction. By inserting this expression into Equation 3 the linear wave equation takes the following form,
Equation 5 – Wave Equation for Radial Acoustic Pressure
\[
\frac{\partial^2 \hat{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{p}}{\partial r} + \left( \frac{\omega^2}{c^2} - \frac{k^2}{r^2} - \frac{m^2}{r^2} \right) \hat{p} = 0
\]
This equation is of the form of Bessel’s equation and thus is easily solved to find the general solution for \( \hat{p} \), as follows,

Equation 6 – Radial Acoustic Pressure Bessel’s Equation
\[
\hat{p}(r) = A J_m(\lambda r) + B Y_m(\lambda r)
\]

Equation 7 – Eigenvalues of Radial Acoustic Pressure
\[
\lambda^2 = \frac{\omega^2}{c^2} - k^2
\]

Applying the effects of the hard-wall combustion liner on pressure and fluid particle velocity, as boundary conditions for these expressions, provides a description of all the potential azimuthal modes that can occur in a cylindrical geometry. These azimuthal modes are described by the expression: \( j_{m,n} = \lambda_{m,n} R \), where \( R \) is the distance from the axis of the combustor to the liner wall [2].

In can-type gas turbine combustors, like the model used in experiments in this study, however, the axial length is significantly large in comparison to the width of the combustion liner cross-section. As a result, the acoustic modes primarily responsible for driving instabilities in these systems oscillate along the length of the combustor. To resolve the longitudinal modes of the system it is necessary to describe both the longitudinal acoustic pressure in the system and longitudinal velocity of the flow as the superposition of waves moving both forwards and backwards along the axis of the system,

Equation 8 – Longitudinal Acoustic Pressure
\[
p'(r, \theta, x, t) = C_1 \hat{p}(r) \cdot \exp\left( (kx + m\theta - \omega t) i \right) + C_2 \hat{p}(r) \cdot \exp\left( (-kx + m\theta - \omega t) i \right)
\]

Equation 9 – Longitudinal Flow Velocity
\[
u'_x(r, \theta, x, t) = \frac{1}{\rho_0c} \left[ C_1 \hat{p}(r) \cdot \exp\left( (kx + m\theta - \omega t) i \right) - C_2 \hat{p}(r) \cdot \exp\left( (-kx + m\theta - \omega t) i \right) \right]
\]

Following the simplified formulation outlined by O’Connor et al. [2] the boundary conditions are established such that air enters from a large plenum into the combustion chamber at the inlet and all
acoustic waves are reflected at the exit. This indicates pressure is constant at the inlet and there is no
fluctuation in the axial flow velocity at the exit, written mathematically as follows [2,15],

**Equation 10 – Acoustic Pressure Boundary Condition at Inlet (x =0)**

\[ p'(r, \theta, 0, t) = 0 \]

**Equation 11 – Axial Flow Velocity Fluctuation Boundary Condition at Exit (x=L)**

\[ u'_x(r, \theta, L, t) = 0 \]

Applying these boundary conditions to Equations 8 and 9, cancels out the coefficients \( C_1 \) and \( C_2 \) and gives the following relationship [2,15],

**Equation 12 – Longitudinal Wavenumber Relationship**

\[ \cos(kL) = 0 \]

This then provides a simple expression that describes all of the longitudinal acoustic mode
solutions [2,15],

**Equation 13 – Longitudinal Wavenumber Solution**

\[ k_l = (2l + 1) \frac{\pi}{L} \]

Combining this expression with the expression for azimuthal eigenvalues shown in Equation 7
results in an expression for the natural frequencies corresponding to all azimuthal and longitudinal modes
in a cylindrical geometry [2,15],

**Equation 14 – Natural Frequencies of Cylindrical Duct**

\[ \omega_{l,m,n} = (2l + 1) \frac{\pi c}{2L} \sqrt{1 + \left[ \frac{2 jm n}{\pi (2l+1)} \right]^2 \left( \frac{L}{R} \right)^2} \]

Finally, the solution for the entire acoustic pressure field, a combination of all modes described
above, can be expressed by the following equation [2,15],

**Equation 15 – Acoustic Pressure Field in Cylindrical Duct**

\[ p'_{l,m,n}(r, \theta, x, t) = j_m \left( \frac{jm n R}{R} \right) \sin \left[ (2l + 1) \frac{\pi x}{L} \right] \cos(\omega_{l,m,n}t - m\theta) \]
From this equation, purely longitudinal modes are described when the azimuthal wavenumbers $m$ and $n$ both equal zero. This results in $j_{0,0} = 0$, indicating that the first order Bessel Function $J_0 = 1$.

Thus, using this equation a preliminary expectation of how the longitudinal acoustic pressure modes may occur in a cylindrical geometry can be visually demonstrated in Figure 1.

Figure 1 – Long. Mode Solutions for Acoustic Pressure where: left) $l = 0$ and right) $l = 1$

This visualization will help guide understanding of the results of computational models presented later in this study. These models will reflect the more complex geometry of the combustion system used in the experiments conducted in this study, and thus will feature different boundary conditions. However, the physics defining the longitudinal acoustic pressure modes in a simple system will provide a basis for explaining the results of this study.

Acoustic Damping

Acoustic damping mode involves interactions between the flow and boundary conditions in the system, leading to dissipation through viscous and thermal effects. For instance, due to the no-slip boundary condition at hard walls, acoustic energy may be transferred to vortical perturbations when the flow encounters the wall. Similarly, since the flow experiences a temperature gradient at the wall of the combustion chamber, some of the acoustic energy may be transferred to entropy perturbations. Work by Noiray et al. [17] and Howe [18] shows the role combustion chamber liners in acoustic damping.
Interaction between the main flow of the combustion chamber and the layer of cooling air on the surface of the liner results in the formation of vortices and the dissipation of acoustic energy in the flow. Furthermore, Gullaud and Nicoud [19] expanded on this work that the acoustic pressure gradient is important in analyzing damping by showing that certain locations of the combustor liner between pressure antinode and node improve damping. The second mechanism of damping involves the ability of mean fluid motion to carry acoustic energy out of the system. The reflection coefficient of an open-ended pipe decreases as mean flow Mach number increases, proving that acoustic energy is dissipated and not reflected back into the system as mean flow velocity increases [20]. The third mechanism is the tendency for acoustic energy to be transferred to other acoustic modes, at different frequencies, present in the system. This dissipates energy since these alternative acoustic modes are typically not amplified as significantly by fluctuations in combustion processes [20]. Understanding how to improve acoustic attenuation directly improves the ability to control and prevent instabilities in these combustion systems, reinforcing the need to understand the physics behind these damping processes [21]. Although damping is not the focus of this present research it is still important to understand as it relates to understanding how certain boundary conditions may influence acoustic oscillations in a combustion system.

**Experimental Techniques**

Finally, in order to proliferate the exploration of this issue of combustion instabilities, advanced experimental technologies and diagnostic techniques have been developed [22]. The most important method of measurement focuses on understanding the pressure fluctuations in the system. Specifically, these measurements focus on the combustor pressure as a function of time, the magnitude of the pressure, the phase, and the frequency of the pressure oscillations. Measuring the pressure at multiple locations in the combustor can sufficiently establish a construction of the instability mode in the combustor. A minimum of three pressure transducers are necessary to determine the longitudinal mode in a combustor if the transducers are positioned at the chamber entrance, exit, and between the first two [22]. Pressure
fluctuations in the fuel flow rate also couple to contribute to changes in heat release leading to the
generation of instabilities. Therefore, measuring pressure fluctuations in the fuel line and nozzle is also
necessary to understand the presence and onset of instabilities. However, in order to image the overall
heat release within the combustion chamber another technique is required. Chemiluminescence emission
techniques involve the measurement of the radiative emission from electronically excited species formed
from chemical reactions in the combustion process [22]. These measurements have the ability to estimate
the overall heat release, as well as the time-dependent and spatial progression of the flame’s heat release.
A third technique, Infrared Absorption can measure the equivalence ratio throughout the combustor,
specifically providing information on rate and magnitude of equivalence ratio oscillations [22]. This
provides a necessary description of how significant the equivalence ratio feedback mechanism is in
contributing to the system instabilities. Then a fourth technique, planar Laser-Induced Fluorescence
(PLIF) offers the unique advantage of depicting the structure of the flame, imaging fluorescence photons
in the plane of the laser [22]. PLIF provides quantification of the flame area, which helps quantify how
changing flame area throughout the instability distinctly affects a certain instability. When combining the
phase-synchronized measurements of each of these techniques a comprehensive understanding of the
significance of each contribution to the instability can be developed [23].

Broadening our understanding of the acoustics of combustion systems with complex geometries
is important for the study of combustion instability. Using computer software such as COMSOL could
improve the ability for industrial applications of these combustion systems to quickly, yet effectively
understand the acoustic modes of their systems. The purpose of this work is to provide validation through
similar experimentation as has been described throughout this review, in order to improve understanding
of how to accurately model acoustics in various systems. This work will also help feed into current
studies using this same experimental setup, which will need this understanding of the system’s acoustic
resonant modes to help define limits of stability in operation.
Chapter 3

Experimental Overview

Experimental Setup

As discussed in work by Szedlmayer et al. [23], the model test rig used in the acoustic characterization experiments conducted in this study is a Multi-Nozzle Can Combustor shown in Figure 2. The design and build of this test rig was done prior to the experiment conducted in this study.

Figure 2 – Multi-Nozzle Model Can Combustor [23]

It is characterized by five separate nozzles attached to the combustor dump plane, configured in the “four around one” layout shown in Figure 3.

Figure 3 – Five Nozzle Vertical Layout [23]
Before the flow eventually reaches this nozzle configuration, however it is initially supplied by an air compressor system capable of achieving a molar flow rate of 600 SCFM at a pressure of 300 psi. Pressure and mass flow rate are monitored and controlled at this initial location along the flow path to allow the user to establish desired input values for each. Downstream of this compressor system air flows through a 50 kW process air heater, preheating the air to the desired temperature before entering the inlet of the combustor. This preheating process serves to both provide control over the input temperature and properly imitate the conditions of the airflow exiting a compressor system in an industrial gas turbine system. After exiting the preheating system, natural gas fuel is transversely injected into the flow path, to rapidly and to thoroughly mix the fuel and air. The mixed flow then continues along its path and enters a 1.5” diameter pipe with an length-to-diameter ratio of 15. The end of this pipe forces the mixed flow through an orifice with a diameter of 0.5”. Flow through this orifice results in a pressure drop of a factor of two or greater, ensuring dynamic changes to the system downstream do not affect the fuel-air mixing process upstream. Similarly, the orifice serves to guarantee that the equivalence ratio of the mixture entering the combustion chamber is constant.

The next stage of the flow path, which is essential to the experiment conducted in this study, directs the mixed flow through a siren device. This siren rotates at a specific speed that produces periodic fluctuations, of a desired frequency, in the mixed flow’s velocity. However, this stage also has the ability to modulate how much of the flow upstream may bypass the siren. This mechanism enables the user to vary the amplitude of the fluctuations produced by the siren by varying the fraction of the flow that bypasses the siren. This section of the flow path provides the external, acoustic forcing necessary in this experiment to study and characterize the effects of various boundary conditions on the system’s acoustics.

As the fuel-air mixture leaves the siren module, and continues towards the combustion chamber, it then enters a manifold. This manifold separates the mixture into five distinct, geometrically identical flow paths leading into each of the five fuel nozzles. Within each of the five different flow paths is a perforated plate that serves to ensure each nozzle receives the same volume of the fuel-air mixture. As the
flow enters into the 2” diameter nozzle structure, it is separated further either into a 1” diameter center-body or through an axial swirler, surrounding the center-body. The flow then finally exits the nozzle where it is combusted and stabilized on the dump plane. This combustion chamber is enclosed by a 10” diameter, 12” long quartz cylinder that has a downstream end open to the atmosphere. Operating a can combustor with an open end like this ensures that the combustion process occurs at the present atmospheric pressure. Next, the methods employed to measure the system’s various conditions throughout its operation are discussed.

Figure 4 – Model Can Combustor Airflow Diagram [23]

Diagnostics Setup

The various techniques that have made the study of combustion instabilities possible were discussed in detail in the literature review. Thus, it is worth noting that thermocouples are used to measure temperature of the fuel-air mixture, electronic differential pressure gauges can help calculate velocity through the nozzle, and chemiluminescence emission measurements help characterize rates of heat release in this system. In this study, however, the most essential measurement instrumentation and
procedures are concentrated in gauging the pressure fluctuations present in the system. As such, there are
two locations within each of the five nozzles between the swirler and the end of the center-body where
fluctuations in pressure are measured. Each pressure measurement is conducted using water-cooled
piezoelectric pressure transducers (PCB Model 112A22). Data acquisition for these measurements is done
at a sampling rate of 8192 Hz, providing resolution of the oscillation frequency down to 1 Hz. Using
these different techniques in conjunction will help quantify the dynamics of the system, as it undergoes
the experimental procedure to be described in the next section.

**Procedure & Analysis**

The experiment was conducted by varying the speed at which the siren, upstream of the
combustion system, rotated. This introduced acoustic forcing on the combustion chamber at a range of
frequencies from 100 to 520 Hz. Pressure signals throughout the system were recorded in accordance with
the diagnostics techniques described above. In order to adequately analyze and address the two goals of
the study listed above, the data from these different pressure sensors was analyzed in two distinct ways.

With the focus of assessing inherent acoustic characteristics of the system, the first method of
analysis sought to evaluate the frequencies of changes in the pressure signals. Due to the noisy, multi-
periodic nature of the pressure fluctuation data taken from this system, it is imperative that analysis
techniques are able to distinguish the various frequency components of the signal. This will aid in
identifying the frequency producing oscillations of the greatest amplitude. Fourier Transforms offer the
unique ability to model a continuous signal as a function of its frequency components, effectively
deallocating and identifying the contribution of each frequency to the signal. In a discrete-time signal
where the signal is not only composed of a variety of inherent frequencies, but is also sampled at distinct
time intervals, the application of a Fourier Transform is different. This is often the case when analysis is
performed on a digital computer. Thus, the use of a Discrete Fourier Transform is required instead. Many
properties of the continuous Fourier Transform remain the same using this transform with subtle
differences outlined by Bergland [24]. Conveniently, however, the MATLAB function \texttt{fft()} implements
and algorithm that applies a Discrete Fourier Transform to the sampled data. This function is thus applied
to pressure data recorded in this experiment to identify the most significant frequency contributions to the
signal. This helps to not only understand the response of the system to forcing at a particular frequency,
but also provides quantification of the scale of that response. Therefore, when acoustic forcing is applied
across a range of frequencies, as in this study, the magnitude of each response can be compared and
examined.

Using the same pressure data that informed analysis on the frequency of fluctuations in the
system, the second method of data analysis helped in quantifying the effect of the system’s boundary
conditions on impeding or reflecting acoustic energy. Boden and Abom [25,26] discussed the method of
analysis used for impedance calculations titled, ‘the Two-Microphone Method’.

![Figure 5 – Two-Microphone Method Setup [25]](image)

This analysis technique compares the data from two different pressure signals, at a known
distance of separation, to develop a transfer function between each signal. In a straight, cylindrical duct,
the sound field will consist of a planar acoustic wave in the frequency domain that can be described as the
linear sum of incident and reflective components at any location within the duct,
Equation 16 – Decomposed Pressure Signal

\[ \hat{p}(x, f) = \hat{p}_+(f) \exp(-j k x) + \hat{p}_-(f) \exp(j k x) \]

To understand important acoustic characteristics of a system it is important to define a specific parameter known as the reflection coefficient,

Equation 17 – Reflection Coefficient

\[ R(f) = \frac{\hat{p}_-(f)}{\hat{p}_+(f)} \]

The reflection coefficient at a certain location within the duct can be defined as the ratio of the reflective component over the incident component of a propagating wave. Data recorded in this experiment by two sensors separated by a known distance along the flow path contain meaningful information about the incident and reflected component of the acoustic wave propagating in the system.

Equation 18 – Pressure signals at a.) \( l = 0 \); and b.) \( l = s \)

a.) \( \hat{p}_1(f) = \hat{p}_+(f) + \hat{p}_-(f) \)

b.) \( \hat{p}_2(f) = \hat{p}_+(f) \exp(-j k s) + \hat{p}_-(f) \exp(j k s) \)

Thus, a relationship between data recorded by each microphone can be defined by a transfer function given as the ratio of the FFT of the second sensor pressure data over the first [27].

Equation 19 – Transfer Function between First and Second Microphone

\[ H_{12} = \frac{\hat{p}_2}{\hat{p}_1} \]

This transfer function can be used to define the reflection coefficient as a function of data explicitly recorded in this experiment. Using the reflection coefficient, more relevant system acoustic characteristics such as impedance can be defined and calculated as a function of pressure data recorded in the experiment.

Equation 20 – Impedance Equation

\[ \frac{Z}{\rho c} = \frac{1 + R}{1 - R} \]

\[ \frac{Z}{\rho c} = j \frac{H_{12} \sin(k l) - \sin[k(l - s)]}{\cos[k(l - s)] - H_{12} \cos(k l)} \]
Here, the variables, \( l \) and \( s \), are defined as the distance from Microphone #1 to the desired boundary condition and the distance between microphones, respectively. Notice, the result of the impedance calculation has the potential to be complex. To understand the physical consequences of this possibility both the real and imaginary parts of the complex result have important functions. The real part of the impedance serves as the resistance to an acoustic wave. If the real part is positive the surface will absorb energy, while if the real part is negative the surface will produce energy in the system. The imaginary part of the impedance is defined as the reactance and functions to alter the phase of the incident wave [28].

Another important aspect of the impedance to consider, resulting from this derivation, is its dependence on the frequency of the acoustic pressure oscillations in the system. As a result, at each frequency in which the acoustic pressure oscillates the impedance of a certain boundary condition will take on a unique value. Impedances at each acoustic forcing, test condition will be calculated from experimental pressure data. Although this impedance information was derived from experimental data, it serves as a critical input into numerical modeling techniques that will be described in more detail in the next chapter. The sensitivity of the system’s response to changes in acoustic impedance at the inlet boundary conditions will be a key focus of analysis conducted in this study.
Chapter 4
Modeling Methods

Identifying the intrinsic acoustic characteristics of a system is often only possible once the entire system has been produced and assembled. An emphasis throughout present research efforts has thus been placed on developing effective acoustic modeling techniques during the design and development phase of combustion system production. In an effort to complement the experimental studies detailed above, COMSOL modeling software was utilized to provide insight into acoustic characteristics of the exact system used in the physical experiment conducted in this study. COMSOL offers the ability to compute and determine robust solutions to a variety of engineering problems. By simply importing the geometry of the system intended to be analyzed and defining parameters that accurately reflect the environment the system is in, different studies including acoustic modeling studies can be conducted. This chapter will outline how this acoustic model was developed and how it can be replicated with different systems or geometries.

Model Architecture

Geometry

Using COMSOL enables the user to either design the geometry they wish to study within the program interface or import a preconceived CAD representation of the system’s geometry. Since a SolidWorks model of the system used in the physical experiment conducted in this study was already made, it facilitated the production of a conforming airflow model also made in SolidWorks. The focus of the experimental study centers more on the inherent acoustic nature of airflow contained by a specific geometry than acoustic interactions with the system geometry itself. This thus necessitated the creation of
a model of the internal airflow enclosed by the SolidWorks model already present. Due to the immensely complicated geometry of many internal components of the combustion system, a more simplified representation of the internal airflow was constructed and used in these modeling studies. The simplified model of internal airflow can be seen in Figure 6.

![Figure 6 – Internal Airflow Geometry](image)

**Boundary Conditions**

In establishing the initial, inherent characteristics of the system the predefined material ‘Air’ was selected to define the entire geometry imported in the model. This designation establishes the pertinent speed of sound and density throughout the model. Furthermore, a temperature of 366 K was set as the global temperature in the system. This temperature corresponds with the temperature of the airflow exiting the heater section in the physical experiment.

The next step in defining the model’s architecture is establishing fixed boundary conditions that remain unchanged regardless of the study being conducted. The first boundary condition applied to the present study, Hard Wall Boundaries, defines the majority of surfaces of the system’s geometry, as shown in Figure 7.
Hard Wall boundary conditions effect the system by affecting the acceleration of the fluid normal to the surface. Acceleration of flow in the system at a Hard Wall boundary condition is zero [29]. This has a unique effect on the change in pressure at this location. Specifically, the normal derivative of pressure is zero at a Hard Wall boundary, or, $\frac{\delta p}{\delta n} = 0$. The effect of this boundary condition on changes in pressure at the specific location propagate throughout the rest of the system and allow the software to effectively resolve the entire pressure field in the system. The other key boundary condition that remains the same throughout each different acoustics study conducted is defined at the combustion chamber exit, shown in Figure 8. The boundary condition applied at this location is known as the ‘Unflanged, Circular Waveguide End Impedance’.

Figure 7 – Hard Wall Boundary Condition Surfaces

Figure 8 – Unflanged, Circular Waveguide End Boundary Condition
This boundary condition models acoustics at the end of a circular waveguide without a flange. If the diameter is small compared to the wavelength of an acoustic pressure mode, the wave is nearly entirely reflected [28]. In COMSOL, the computation for this boundary condition is conducted using impedance value correlations to define the effect this boundary condition has an acoustic wave of any wavelength. These correlations were derived experimentally in the work by Levine and Schwinger [30]. The unique acoustic analysis techniques and corresponding boundary conditions changes applied in each study are discussed next.

**Acoustics Studies**

In COMSOL, the study of acoustics conducted on a particular geometry focuses on defining the pressure field throughout the geometry. Many different kinds of acoustics studies can be conducted within the software, the two of interest within this study will be the frequency domain study and the eigenfrequency study. The frequency domain study will provide insight into the geometry of the system’s pressure response to acoustic forcing across a range of frequencies. The eigenfrequency study will provide insight into the pressure field throughout the geometry at specific frequencies corresponding to natural modes inherent to the geometry.

**Frequency Domain Modeling Study**

The focus of this type of study is on understanding the pressure response within the system’s geometry when an acoustic force is applied to the system at a specific frequency. Thus, in implementing the mathematics behind the software’s acoustic calculations, the equations being solved are almost identical to the ones derived in the literature review. In this study however, the key difference is the need
to include an acoustic forcing term capable of prompting the acoustic pressure response in the system. The equation the software uses is presented as follows,

\[
\nabla \cdot \left( -\frac{1}{\rho_0} \nabla p \right) + \frac{\omega^2 p}{\rho c^2} = Q
\]

This equation appears similar, in structure, to Equation 5 presented in the Literature Review chapter only that this equation remains in Cartesian coordinates. Furthermore, on the right-hand side of the equation is the acoustic driving force from a monopole source [29]. The driving force is applied to the system at a chosen frequency, enabling the study to define a range of frequencies in which the system’s acoustic pressure response can be analyzed. In order for this study to be conducted accurately on a particular geometry however, certain boundary conditions need to be adjusted. In the present study conducted, a normal acceleration was applied at the inlet locations to the combustion chamber, as shown in Figure 9.

![Figure 9 – Combustion Chamber Inlet Boundary Conditions](image)

This establishes the correct orientation and direction of the monopole acoustic driving source.

The following equation describes the effect this new boundary condition has on the system mathematically.
Equation 22 – Normal Acceleration Boundary Condition [29]

\[-\mathbf{n} \cdot \left( -\frac{1}{\rho_0} \nabla p \right) = a_n\]

Here, \( \mathbf{n} \), is the unit vector normal to the surface of the boundary condition and, \( a_n \), is the magnitude of the flow’s acceleration entering through the selected surface [29]. This equation shows that at the inlet to the system, where this boundary condition is applied, the term on the left-hand side of Equation 21 is equal to the chosen acceleration. The present study sets the magnitude of the acceleration set at the default value of one.

Eigenfrequency Modeling Study

In contrast to the frequency domain study, the eigenfrequency study does not focus on the system’s response when an external acoustic pressure source is applied. Instead, this study calculates an eigenvalue corresponding to a frequency of acoustic pressure oscillations where the system is at an inherent eigenmode. Then, the software provides insight into the acoustic pressure field throughout the system at these inherent modes. To solve for these eigenfrequencies the software implements a modified version of the equation solved in the frequency domain study. The equation that the eigenfrequency study solves is presented below.

Equation 23 – Eigenfrequency Study Dynamics [28]

\[ \nabla \cdot \left( -\frac{1}{\rho_0} \nabla p \right) + \frac{\lambda^2 p}{\rho c^2} = 0 \]

Since the study does not involve a driving force, the monopole source term is removed. As the study analyzes the inherent nature of an acoustic pressure wave within the specified geometry, the software seeks to solve for the eigenfrequency, \( \lambda \) [29]. Given this eigenfrequency, the model then presents a solution for the acoustic pressure at every location within the combustor. With the focus of this study different from that of the frequency domain study, the boundary conditions must also change. In the
present study, the normal acceleration boundary condition is no longer necessary; however, another boundary condition must be applied at the same inlet location as shown in Figure 8. This study requires a definition instead, for the acoustic impedance at the inlet. This provides insight into the effect the unique geometry of the inlet has on reflecting acoustic pressure waves back into the combustion system. The effect this impedance boundary condition has on the calculation conducted by the software is described by the following equation,

\[ -n \cdot \left( -\frac{1}{\rho_0} \nabla p \right) = -\frac{i\omega p}{Z} \]

Conceptually, the impedance of any system boundary provides an expression relating the pressure at the surface of the boundary condition and the normal velocity of the flow normal to the surface [29]. Thus, inputting a known impedance of the inlet surfaces to the combustion chamber in this study provides the model with a value for the velocity of the flow interacting with the inlet surface.

Results of both the frequency domain study and the eigenfrequency study conducted using COMSOL modeling software may be found in Chapter 6 and Chapter 7.
Chapter 5

Experimental Results

This section will discuss two unique features of the acoustics of a model can-type combustion system. It will first prepare a basis of expectation for results attained through numerical analysis techniques conducted in subsequent sections, and then provide inputs necessary to refine those techniques. The combination of experimental results and numerical results will provide an understanding of the ability to comprehensively and accurately define acoustic characteristics of a system.

The experimental study was personally conducted in the Reacting Flow Dynamics Laboratory on campus in accordance with the specifications of the experimental overview chapter. The experiment involved applying acoustic forcing, upstream of the combustion chamber, across a range of frequencies from 120 Hz to 520 Hz. The acoustic pressure response in the combustor was recorded at three locations of interest: at the dump-plane, and at two locations along the length of the center nozzle, downstream of the swirler exit. A model of the dimensions of the two-microphone locations is presented in Figure 10,

![Figure 10 – Dimensions of Two-Microphone Locations](image-url)
A sample of the pressure response recorded at each of these locations is presented on the left of Figure 11,

![Figure 11 – FFT of Pressure Signal at 320 Hz Forcing Frequency](image)

The sample pressure response presented here was recorded at a forcing frequency of 320 Hz and expresses the inherently complicated nature of the pressure fluctuations within the system. As such, the significance of the Fourier transform as an analysis technique can be distinctly realized. The right image of Figure 11 presents the result of the Fourier transform applied to the same pressure signal. The peak in the figure occurs at 320 Hz, the exact frequency in which the system was acoustically driven upstream of the combustion chamber. This figure, thus, conveys the ability of this analysis technique to accurately withdraw information about the frequencies of the most significant, driving acoustic pressure oscillations in the system. The Fourier transform applied to these pressure signals, however, also provides a normalized means of comparing the intensity of the system’s response across the range of forcing frequencies. The pressure signals at each location were thus analyzed at each forcing frequency to discern the most significant oscillation present.

A comparative analysis of the intensity of acoustic pressure responses at each driving frequency is presented in Figure 12. This figure presents the comparative pressure response at the dump plane in the combustion chamber. From this figure, it can be concluded that the most intense acoustic pressure response in this combustion system at the dump plane occurs at a resonant frequency of 320 Hz,
The same comparative analysis was conducted on the acoustic pressure signals from each of the pressure transducers positioned within the column, downstream of the swirler. The analysis for the first pressure signal is presented in Figure 13. The analysis for the second pressure signal, recorded sequentially downstream of the first pressure transducer, is presented in Figure 14.
An interesting result of the comparison conducted at each location within the swirler exit column is the recurring presence of a dominant, resonant frequency at 320 Hz. There appears, however, to be an additional resonant frequency present within this smaller cylindrical column at 160 Hz. The fact that this other modal frequency is present at a value exactly half of the frequency present at the dump plane in the combustor is supported by the understanding of the physics of acoustics presented in Chapter 2. Acoustic modes are always separated by distinct frequency intervals, governed by integer wavenumbers. Of greater significance however, is the location in which these lower frequency modes are observed. This result would suggest it should be expected that different eigenfrequencies and unique acoustic pressure responses might be present in numerical computations conducted in subsequent sections.

From the analysis of the pressure signals recorded within the fuel nozzle column another important result can be developed. Using the ‘Two-Microphone Method’, differences in each pressure signal, because of the spacing between each respective measurement location, can be characterized by a transfer function, $H_{12}$. Although this transfer function is merely a ratio between the two pressure signals, because the pressure signals are complex in nature, it offers unique insight into the phase difference between each pressure signal. The phase of the transfer function and corresponding difference between each pressure signal, is presented in Figure 15,
This phase difference is present at a unique location in the frequency domain. Occurring directly after the most significant acoustic pressure response observed in the system may indicate a unique shift in the mode shape present at these operating conditions. Modeling techniques will seek to identify the effect phase differences between pressure signals may have on acoustic pressure response in the system.

As outlined in the ‘Two Microphone Method’, the more significant purpose of this transfer function is the ability to calculate the impedance, $Z$, of different acoustic boundary conditions in the
system. By applying Equation 20, the acoustic impedance of the exit of the middle-nozzle swirler was calculated across the range of forcing frequencies using the range of transfer function values. The amplitude of the acoustic impedance of the exit of the middle-nozzle swirler is presented in Figure 16, and the phase information of the calculated impedances is presented in Figure 17.

**Figure 16 – Amplitude of Acoustic Impedance at Middle Nozzle Exit**

**Figure 17 – Phase of Acoustic Impedance at Middle Nozzle Exit**
In assessing the impedance, understanding the effect an impedance has on the phase change of an incident acoustic wave is critical to understanding how a reflected wave may influence its environment. As a wave of particular phase and frequency interacts with these boundary conditions, the impedance will impart its own phase on the wave. Thus, in preparation for modeling techniques that will seek to accurately recreate the pressure response recorded in the physical experiment, the impedance derived here will help to improve those models and enhance the understanding of the results.

Finally, the effect of frequency on the real and imaginary components of the complex impedance results are presented in Figure 17,

![Graph showing real vs. imaginary parts of acoustic impedance](image)

Figure 18 – Real vs. Imaginary Parts of Acoustic Impedance, Z

Recall from the discussion of acoustic impedance in the Experimental Overview, the impedance of a boundary condition describes how the boundary condition reflects energy of an incident wave back into the system. Of particular interest then, is not only that the maximum amplitude acoustic impedance occurs at the frequency of the maximum acoustic pressure response, but also that the impedance at that value has a significantly large negative value. When the real part of the acoustic impedance is negative, the surface actively reflects energy back into the system. This can be seen as a key indicator that the acoustic mode at 320 Hz has the strong potential of initiating a combustion instability in this system.
Chapter 6

Frequency Domain Modeling Results

The first model, that will begin to accurately reflect the pressure response of a system undergoing acoustic forcing, seeks to simulate the experiment conducted in Chapter 5. The frequency domain modeling analysis was applied over the same range of acoustic forcing frequencies as conducted in the physical experiment, from 120 Hz to 520 Hz. Recall, in order to apply this modeling technique, the boundary condition at the exit of the swirler is required to be set at a constant acceleration. This enables the model to orient the incident acoustic forcing applied to the system and build the full response resulting from acoustic interaction with this boundary condition. Furthermore, two different tests will be applied, incorporating two different boundary conditions at the exit of the combustion chamber: i.) An open-ended, unflanged cylindrical exit; and ii.) A hardwall, closed cylindrical exit.

As a preliminary analysis, this modeling method seeks to identify three unique aspects of the acoustic pressure response in the system. First, a key focus will be placed on understanding the shape of various acoustic modes that develop within the geometry. By looking at the distribution and location of the maximum and minimum pressure values throughout the geometry, the effect of frequency on acoustic mode shapes will be discussed. Second, as a corollary to the first focus, identifying the locations of maximum and minimum pressure values will be essential in comparing the model to the pressure response recorded in the physical experiment. Third, this model will serve as an important foundation for comparing the acoustic pressure response in this study with the eigenmodal response presented in Chapter 7. The foundation of this comparison will be understanding the effect of including impedance boundary conditions in the numerical model to improve accuracy of the results. This will follow the logic of defining the effect on the development of eigenmodes of reflecting more energy back into the system. The location of the maximum and minimum pressure values distributed throughout the system will again be
an important consideration, only now, in comparing the two models. The absolute value of the differences between these pressure responses in each model will also be an important comparison tool. Lastly, the phase difference caused by these impedances will also examined and compared between these two studies. Using different boundary conditions at the exit of the combustion chamber will help more accurately describe how the impedance boundary condition influences phase changes in the system.

Overall, the comprehensive analysis of each of these acoustic features modeled in this experiment will serve to determine how frequency of acoustic oscillations effects the acoustic pressure response of the system. Then it will serve as a basis for improving modeling techniques to accurately predict acoustics within the combustor and therefore, accurately predict when a system is vulnerable to combustion instability. The results for the frequency domain modeling analysis are presented in Table 1. Each capture of a distinct acoustic pressure response has a unique color bar that describes the corresponding range of pressure values present in the system.

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Acoustic Pressure Response</th>
<th>Acoustic Pressure Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Open Ended Exit Boundary Condition</td>
<td>Hardwall Exit Boundary Condition</td>
</tr>
<tr>
<td>120 Hz</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Table 1 – Frequency Sweep Modeling Results
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Image 1</th>
<th>Image 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>380 Hz</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td>400 Hz</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
<tr>
<td>420 Hz</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
</tbody>
</table>
In looking at the entire body of results presented in Table 1, in each test, it is clear that changing the frequency has a distinct effect on the distribution of pressure throughout the system. Assessing the open-ended exit boundary condition first, between 120 Hz and 240 Hz, the distribution of pressure throughout the system is almost constant for each frequency. There are high-pressure amplitudes within the column at the swirler exit, while at the exit to the combustion chamber the pressure is approximately zero. A particularly interesting observation can be seen between 120 Hz and 180 Hz, where the center of the acoustic pressure oscillation of about 0.11 kPa can be seen at the end of the shorter, swirler exit columns. This indicates that an entire peak of the acoustic pressure oscillation occurs along the short, narrow length of these five columns, while the other peak is spread across the entire length of the
combustion chamber. Between 200 Hz and 240 Hz, however, the node between the peaks shifts axially downstream towards the center of the combustion chamber.

At 260 Hz, a significant phase shift occurs. The locations of the maximum and minimum of the pressure oscillations completely flip. The most positive amplitude is now located towards the combustion chamber exit. Amplitudes of the acoustic pressure oscillation continue to move axially towards the exit of the combustion chamber between 260 Hz and 320 Hz. At 320 Hz, a node between acoustic pressure amplitudes is present at the center of the swirler exit column and at the combustion chamber exit. Beyond this frequency, the pressure distribution remains approximately constant throughout the rest of the test cases. Between 340 Hz and 520 Hz, the shape of the distribution is constant with a peak at the swirler exit, a node approximately halfway downstream of the swirler exit column, and a minimum located along the center of the combustion chamber length. Although the shape remains almost constant, it is of note that the amplitude of the pressure oscillation at the swirler exit continually increases as the frequency of acoustic forcing increases. These pressure oscillation amplitudes increase between 340 Hz and 520 Hz. The transition of these shapes will be an important position of comparison between the modeling technique described in Chapter 7.

Now in assessing the acoustic pressure response of the system when a hard-wall boundary condition is applied at the combustion chamber exit, there are differences present from the open-ended exit test. A constant mode shape is present in the system from 120 Hz to around 420 Hz. This mode shape is very similar to the shape present in the open-ended exit test for frequencies between 340 Hz and 520 Hz. It is characterized by a peak amplitude at the exit of the swirler, a node located about halfway along the length of the swirler exit column, and then an acoustic pressure oscillation minimum located along the length of the combustion chamber. At 440 Hz, the node between peaks of the pressure oscillation begins to move out of the swirler exit column and into the combustion chamber as frequency of acoustic forcing increases. At 520 Hz, the node is distinctly located at the dump plane of the combustor. The final pressure distribution at 520 Hz appears similar to the 140 Hz distribution for the open-ended exit test. The pressure
response due to this hard-wall boundary condition appears to follow the same transition and movement of acoustic pressure nodes throughout the geometry. The difference however, is that the transition out of the constant, low-amplitude mode shape occurs at higher frequencies in the hard-wall exit case. Meanwhile, the open-ended exit case exhibits this transition at lower frequencies. As such, it will be useful to identify how these transitions each behave differently due to the application of an impedance at the inlet in the next model.

In comparing the acoustic pressure response, at different frequencies, with the pressure response recorded in the physical experiment, a few unique aspects of each must be analyzed. Three pressure measurements were recorded at different locations in the physical experiment, each providing a comparison of the experimental pressure response to acoustic forcing at different frequencies. It is important to note that since the experimental pressure response charts (Figures 12-14) were developed using a Fourier transform, the values presented are not actual pressure values. As such, all comparisons between experimental pressure response and modeled pressure responses should remain qualitative.

In assessing the pressure response at the dump plane of the combustion chamber, refer to Figure 11 for the experimental pressure response. At this location, pressure recorded showed a significant peak in response to acoustic forcing at a frequency of 320 Hz. At frequencies less than 320 Hz, the pressure response was random, but on average, it was greater than the pressure response to frequencies greater than 320 Hz. The pressure responses at frequencies of 400 Hz and greater were approximately zero at the dump plane. In first investigating the comparison between the dump plane pressure response and the model results for the open-ended exit case, these three areas will be the focus. At frequencies lower than 320 Hz, pressure at the dump plane is low between acoustic forcing frequencies of 120 Hz to 200 Hz. However, the model begins to deviate from the experimental results, in that the modeled pressure at the dump plane increases significantly at around 220 Hz. Then, after the phase shift in the pressure response between 240 Hz and 260 Hz, the pressure at the dump plane switches to a negative value. Beyond this point, across the remaining ranges of frequencies tested, the pressure at the dump plane decreases to
approximately zero. A summary of the modeled and experimental acoustic pressure response at each significant frequency is presented in Table 2.

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Exp. $P$</th>
<th>FD Model $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.366667</td>
<td>0.288462</td>
</tr>
<tr>
<td>220</td>
<td>0.333333</td>
<td>1.000000</td>
</tr>
<tr>
<td>320</td>
<td>1.000000</td>
<td>0.192308</td>
</tr>
<tr>
<td>420</td>
<td>0.100000</td>
<td>0.038462</td>
</tr>
<tr>
<td>520</td>
<td>0.033333</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Since, the results from COMSOL and the Fourier Transform cannot be directly compared the results presented in the table above have been normalized by the maximum value measured. In this summary, it is clear that the general shape of the experimental pressure response was followed. However, the necessary peaks do not correspond with the correct forcing frequency observed in the experiment. Thus, it would appear that the pressure response at each frequency in the model does reflect the same pattern as the experimental results. Each response remains almost constant before a large peak is observed, and then the response effectively decreases down to zero. The difference, however, resides in the frequency at which this peak in dump plane pressure response occurs.

In assessing the pressure response from each pressure transducer located along the length of the swirler exit column, refer to Figure 10. At this location, due to the narrow geometry of the airflow path, the experimentally measured pressure response differs from the response measured at the dump plane. Although, each measurement recorded in the swirler exit column also demonstrates a significant response at a forcing frequency of 320 Hz, another significant response occurs in both measurements at 160 Hz. Furthermore, although at frequencies greater than 320 Hz the pressure response similarly reduces to zero in all experimental measurements, the swirler exit column measurements begin to show a gradual increase in the acoustic pressure response as the acoustic forcing frequency approaches 520 Hz. Refer to Figure 10 in Chapter 5 for the relative locations along the length of the swirler exit column, of $P_1$ and $P_2$. In
evaluating the model’s pressure response, $P_1$ is located immediately after the swirler exit, while $P_2$ is located about halfway along the swirler exit column. By comparing the pressure values at each of these locations, with the model results, it is apparent that at low forcing frequencies the model does a decent job at reflecting reality. The pressure at the locations where $P_1$ and $P_2$ were measured show a steady increase as the acoustic forcing frequency increases below 320 Hz. However, again the peak of these increasing pressure amplitudes is reached at a forcing frequency of 260 Hz, much earlier than the experimental peak. At frequencies from 280 Hz to 420 Hz, the model shows a steady decrease in the pressure oscillation amplitude, at $P_1$ and $P_2$, to zero. However, from 420 Hz to 520 Hz an increase is observed in each measurement. A summary of the comparison between modelled and experimental pressure responses in each measured location are presented in Table 3.

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Exp. $P_1$</th>
<th>FD Model $P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.892857</td>
<td>0.428571</td>
</tr>
<tr>
<td>220</td>
<td>0.821429</td>
<td>1.000000</td>
</tr>
<tr>
<td>320</td>
<td>1.000000</td>
<td>0.057143</td>
</tr>
<tr>
<td>420</td>
<td>0.142857</td>
<td>0.285714</td>
</tr>
<tr>
<td>520</td>
<td>0.464286</td>
<td>0.514286</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Exp. $P_2$</th>
<th>FD Model $P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.892857</td>
<td>0.393939</td>
</tr>
<tr>
<td>220</td>
<td>0.821429</td>
<td>1.000000</td>
</tr>
<tr>
<td>320</td>
<td>1.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>420</td>
<td>0.142857</td>
<td>0.181818</td>
</tr>
<tr>
<td>520</td>
<td>0.464286</td>
<td>0.454545</td>
</tr>
</tbody>
</table>

Results presented in the table above were similarly normalized by the maximum measured value for a more effective qualitative comparison. In this analysis of the swirler exit column measurements, it is again shown that the frequency domain modeling technique incorrectly estimates the most significant pressure response in the system. While the experimental pressure distribution shows a clear peak at a forcing frequency of 320 Hz, this model predicts the peak will occur at a forcing frequency of 220 Hz. At
both the dump plane, and within the swirler exit column, the model prematurely identifies the frequency at which the most significant pressure response will occur in reality.

Although this modeling technique fails to accurately reflect the experimental results, it does provide important information on the nature of how acoustics change inside this type of combustion chamber geometry. The two tests conducted using the frequency domain model demonstrated that a unique system response is dependent on the boundary condition present at the exit of the airflow geometry. As the two tests will be repeated using a different model, their comparison will shed light on the effect of an impedance boundary condition at the inlet to the system. Furthermore, in identifying the relationship between the frequency of acoustic forcing, and the resulting amplitude and mode shape of the pressure response using this model, the path towards the most significant pressure response was developed. Through this, an association could be made between a distinct shape corresponding with frequencies that produce a relatively minimal system response, and frequencies that produce the maximum response in the system. For example, the minimum response in the system is observed at frequencies greater than 320 Hz, while the maximum response is observed at frequencies around 260 Hz. Thus, in anticipation for the results of more efficient models, the expectation is that a similar progression of mode shapes in the system will serve as an indicator of the maximum pressure response in the system. Upon further comparison with the observed experimental response, a robust analysis on the accuracy of each model will be understood.
Chapter 7

Eigenfrequency Modeling Results

In an eigenfrequency analysis, the solution depicts the inherent, most energetic response to certain forcing conditions. In this analysis, the impedance values calculated at each frequency in the experimental study serve as a boundary condition at the swirler exit. With these inputs, the model effectively simulates the boundary conditions in reality, as they change in response to changes in the frequency of the oscillating disturbance. The frequencies studied will again range from 120 Hz to 520 Hz to compare with the most relevant responses observed experimentally. Since a frequency is one of the desired outputs of this study, the model does not include any actual acoustic forcing. However, in copying the boundary conditions that would be present at each respective forcing frequency the model must accurately predict the frequency responsible for those boundary conditions. At that frequency, the model can then build the pressure response in the system resulting from those boundary conditions. In assessing the accuracy of this type of model then, it must not only accurately predict the correct eigenfrequency, but it must also provide a corresponding pressure response that reflects reality.

Before the model can be run and tested, it is important to understand the effect impedance has on the system by comparing the response of different boundary conditions at the exit. By comparing the eigenmode response of different exit boundary conditions, the effect of an impedance boundary condition at the inlet can be evaluated. Specifically, the focus is on how the impedance affects the phase of the pressure oscillation response to more accurately predict reality. The results for the first longitudinal mode of the system with both an open-ended, and hard wall combustion chamber exit, are presented in Table 4. In comparing the differing response of each exit boundary condition in the two different modeling study results shown in Table 1 and Table 4, two important observations stand out. In the open-ended exit case in Table 4 there is an increase in the pressure response at frequencies between 240-320 Hz when compared
with the results in Table 1. Similarly, we begin to see a transition occur in the mode shapes of the hardwall exit case at these frequencies, whereas in Table 1 this transition does not begin to occur until much higher frequencies. Recall, in Figure 18 of the experimental results section though, that these frequencies exhibited the most negative real part of the calculated complex impedances. As such, it makes sense that a greater pressure response, in each case, can be observed at these frequencies. Furthermore, analysis into the relative comparison between modeled and experimental results will begin to reveal the effectiveness of the eigenfrequency modeling technique,

### Table 4 – Eigenmodal Modeling Results

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Acoustic Pressure Response</th>
<th>Acoustic Pressure Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Open Ended Exit Boundary Condition</td>
<td>Hardwall Exit Boundary Condition</td>
</tr>
<tr>
<td></td>
<td><img src="image1" alt="Graph 120 Hz" /></td>
<td><img src="image2" alt="Graph 120 Hz" /></td>
</tr>
<tr>
<td>120 Hz</td>
<td>130.08+45.34i Hz</td>
<td>243.15+49.298i Hz</td>
</tr>
<tr>
<td></td>
<td><img src="image3" alt="Graph 140 Hz" /></td>
<td><img src="image4" alt="Graph 140 Hz" /></td>
</tr>
<tr>
<td>140 Hz</td>
<td>157.94+30.587i Hz</td>
<td>279.5+39.396i Hz</td>
</tr>
<tr>
<td>Frequency</td>
<td>160 Hz</td>
<td>180 Hz</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td></td>
<td>192.4+27.499i Hz</td>
<td>194.49+14.908i Hz</td>
</tr>
<tr>
<td></td>
<td>329.35+45.521i Hz</td>
<td>336.57+25.663i Hz</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>Image 1</td>
<td>Image 2</td>
</tr>
<tr>
<td>---------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>240 Hz</td>
<td>![Image](265.4+11.76i Hz)</td>
<td>![Image](512.74+37.801i Hz)</td>
</tr>
<tr>
<td>260 Hz</td>
<td>![Image](273.03+1.4734i Hz)</td>
<td>![Image](536.39+4.0592i Hz)</td>
</tr>
<tr>
<td>280 Hz</td>
<td>![Image](283.93-12.646i Hz)</td>
<td>![Image](572.08-32.491i Hz)</td>
</tr>
<tr>
<td>300 Hz</td>
<td>![Image](276.64-7.108i Hz)</td>
<td>![Image](545.26-21.511i Hz)</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>298.39-8.5112i Hz</td>
<td>598.96-14.244i Hz</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>320 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>340 Hz</td>
<td>329.49-8.1545i Hz</td>
<td>631.61-5.7911i Hz</td>
</tr>
<tr>
<td>360 Hz</td>
<td>368.05-0.50496i Hz</td>
<td>650.98-0.22177i Hz</td>
</tr>
<tr>
<td>380 Hz</td>
<td>410.61+5.0573i Hz</td>
<td>665.61+1.5775i Hz</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>Imaginary Component</td>
<td>Real Component</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>400</td>
<td>450.92 + 5.3472i</td>
<td>678.11 + 1.6724i</td>
</tr>
<tr>
<td>420</td>
<td>487.48 + 23.852i</td>
<td>689.77 + 8.196i</td>
</tr>
<tr>
<td>440</td>
<td>526.02 + 13.361i</td>
<td>704.27 + 5.358i</td>
</tr>
<tr>
<td>460</td>
<td>541.27 - 2.358i</td>
<td>710.73 - 1.018i</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>Proposed Model Results</td>
<td>Experimental Results</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>480 Hz</td>
<td>542.21 + 25.549i Hz</td>
<td>710.53 + 10.992i Hz</td>
</tr>
<tr>
<td>500 Hz</td>
<td>588.66 + 23.687i Hz</td>
<td>732.85 + 12.669i Hz</td>
</tr>
<tr>
<td>520 Hz</td>
<td>609.1 + 15.146i Hz</td>
<td>744.95 + 8.8883i Hz</td>
</tr>
</tbody>
</table>

Analysis of the results from the eigenfrequency model will be conducted similarly to the analysis of the frequency domain results. In order to assess the accuracy of the model, the ability for the model to predict the frequencies of oscillation will be discussed first. Consideration of the shapes, locations and movement of the peaks throughout the geometry will guide the subsequent comparison between the proposed model’s results, and both the experimental results and the frequency domain modeled results.
The ability for the model to predict the correct frequency of the acoustic pressure oscillation, based on the impedance of the inlet boundary condition, will establish the analysis of the model’s accuracy. Conducting a simple error calculation, between the frequency at which the impedance was derived and the absolute value of the model’s resulting eigenfrequency, produces an average error of only $e = 9.14763\%$. While the average error of predicted eigenfrequencies at 420-520 Hz is higher, $e = 16.9244\%$, it is still evident that the model is effective at predicting the correct eigenfrequency.

Examining the eigenmode shapes present in the geometry at each frequency next, initially suggests that differences exist between the eigenfrequency model and the frequency domain model results. In the eigenfrequency model, mode shapes at frequencies ranging from 120-200 Hz are similar. In each of these acoustic pressure responses, there is a peak at the swirler exit, a node located towards the end of the swirler exit column, and then a minimum located at the exit of the combustion chamber. In each of these eigenmode shapes, the peak-to-peak range of the acoustic pressure oscillation remains almost constant. In the range of frequencies from 220-320 Hz the node between peaks moves further axially towards the exit of the combustion chamber. Furthermore, the maximum and minimum values of the acoustic pressure oscillation are located at the exit of the swirler and the exit of the combustion chamber, respectively. Across this range of frequencies, although the minimum acoustic pressure remains constant, the maximum decreases. Between 340 Hz and 380 Hz, the maximum peak is located at the combustion dump plane. In the 340 Hz response, a node between peaks of the acoustic pressure oscillation becomes visible at the swirler exit, while the original node moves further towards the combustion chamber exit. Eigenmode shapes at frequencies of 360 Hz and 380 Hz have a node at the center of the swirler exit column, with the original node reaching the exit of the combustion chamber in the 380 Hz eigenmode.

At 400 Hz, there is a significant phase shift as the location of the maximum pressure amplitude in the 380 Hz eigenmode flips to the location of the minimum pressure. The eigenmode shape of the 400 Hz response is repeated at every remaining eigenmode tested. From 400-520 Hz, the shape is characterized
by a maximum pressure amplitude at the swirler exit, a node located approximately halfway between the swirler exit column, and the minimum pressure located in the middle of the combustion chamber. The only difference in this case however, is that across the range of frequencies from 400-520 Hz the maximum pressure amplitude increases. The complete cycle of this progression thus appears similar to the progression modeled using the frequency domain techniques. The difference again is seen in the location where the pressure response experiences a significant shift. In the eigenfrequency model, this frequency occurs after 380Hz.

After addressing the progression of eigenmode shapes, distinct locations in which the modeled pressure values can be compared with both experimental data and frequency domain modeled data can now be analyzed. Three key areas of the pressure distribution will again be considered in comparing model results to the experimental results. Recall, the pressure distribution recorded at the dump plane in the experimental test exhibited a peak at 320 Hz, as presented in Figure 12. At frequencies greater and less than 320 Hz, the acoustic pressure response was significantly less pronounced. Frequencies less than 320 Hz showed a marginally larger response than frequencies greater than 320 Hz, however. In the eigenfrequency model, at frequencies between 120 Hz and 300 Hz the recorded pressure values at the dump plane do remain fairly small, oscillating between values of 0.8-0.9 kPa. Between the frequencies of 320-380 Hz, the acoustic pressure at the dump plane is larger than at lower frequencies. At its largest, the acoustic pressure in the model is approximately 1.5 kPa at a frequency of 360 Hz. Then, from frequencies of 400 Hz and greater the acoustic pressure in the model drops to values ranging from -0.1 to -0.3 kPa at the dump plane. A summary of the comparison between experimental and the two modeled pressure responses at the dump plane is presented in Table 5,
Table 5 – Eigenfrequency Modeled Pressure Response at the Dump Plane

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Exp. P</th>
<th>FD Model P</th>
<th>EF Model P</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.366667</td>
<td>0.288462</td>
<td>0.625000</td>
</tr>
<tr>
<td>220</td>
<td>0.333333</td>
<td>1.000000</td>
<td>0.666667</td>
</tr>
<tr>
<td>320</td>
<td>1.000000</td>
<td>0.192308</td>
<td>1.000000</td>
</tr>
<tr>
<td>420</td>
<td>0.100000</td>
<td>0.038462</td>
<td>-0.020833</td>
</tr>
<tr>
<td>520</td>
<td>0.033333</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Results presented in the table above were similarly normalized by the maximum measured value for a more effective qualitative comparison. The purpose of this table is to present a comparison of the acoustic oscillation frequency in which a significant pressure response occurs. As can be seen here, the experimental pressure has insignificant pressure response at frequencies greater and less than 320 Hz. The frequency domain model has a similar distribution, however instead the distribution is centered on a forcing frequency of 220 Hz. Finally, the eigenfrequency model shows a more accurate depiction of the true pressure distribution at the dump plane. Although the true peak is observed around 340 Hz, the shape of the distribution closely resembles that of the experimental pressure response.

Next, in evaluating the observed, experimental pressure response within the swirler exit column, refer to Figure 14. At each measured pressure response, three distinct areas will be important in defining the accuracy of the model. The model must correctly identify peaks at both 160 Hz and 320 Hz, and predict an increase in pressure response as the frequency of oscillations increases from 400 to 540 Hz. Refer to Figure 10 for the locations at which $P_1$ and $P_2$ are evaluated in the experiment as important locations for comparison with the eigenfrequency model results.

At these locations, across frequencies of oscillation less than 320Hz the model presents a maximum $P_1$ and $P_2$ at 140 Hz. As the frequency of oscillation progressively increases to 300Hz, the acoustic pressures $P_1$ and $P_2$ steadily decrease. At 320 Hz, a slight increase in the acoustic pressure at both locations compared to the acoustic pressure at 300 Hz. From 340 Hz to 380 Hz, as the amplitude of the acoustic pressure oscillation moves out of the swirler exit column, into the combustion chamber, the
pressure response, $P_1$ and $P_2$, both fall to almost zero. Although, at frequencies greater than 400 Hz a significant shift occurs in the acoustic pressure response, $P_1$ and $P_2$ remain small. As frequencies increase from 420 Hz to 520 Hz, each acoustic pressure result increases significantly as well. A summary of the comparison between the experimental and modeled pressure responses within the swirler exit column is presented in Table 6.

Table 6 – Eigenfrequency Modeled Pressure Response in the Swirler Exit Column

<table>
<thead>
<tr>
<th>Forcing Frequency</th>
<th>Exp. $P_1$</th>
<th>FD Model $P_1$</th>
<th>EF Model $P_1$</th>
<th>Exp. $P_2$</th>
<th>FD Model $P_2$</th>
<th>EF Model $P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.642857</td>
<td>0.428571</td>
<td>1.000000</td>
<td>0.892857</td>
<td>0.393939</td>
<td>1.000000</td>
</tr>
<tr>
<td>220</td>
<td>0.607143</td>
<td>1.000000</td>
<td>0.812865</td>
<td>0.821429</td>
<td>1.000000</td>
<td>0.880282</td>
</tr>
<tr>
<td>300</td>
<td>0.535714</td>
<td>-0.028571</td>
<td>0.614035</td>
<td>0.535714</td>
<td>-0.106061</td>
<td>0.704225</td>
</tr>
<tr>
<td>320</td>
<td>1.000000</td>
<td>0.057143</td>
<td>0.748538</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.880282</td>
</tr>
<tr>
<td>420</td>
<td>0.071429</td>
<td>0.285714</td>
<td>0.847953</td>
<td>0.142857</td>
<td>0.181818</td>
<td>0.915493</td>
</tr>
<tr>
<td>520</td>
<td>0.285714</td>
<td>0.514286</td>
<td>0.818713</td>
<td>0.464286</td>
<td>0.454545</td>
<td>0.897887</td>
</tr>
</tbody>
</table>

Results presented in the table above were similarly normalized by the maximum measured value for a more effective qualitative comparison. From this table the key elements of the analysis can be drawn. First, the eigenfrequency model is able to accurately predict the lower frequency peak present around 160 Hz. This model’s distribution showed a significant pressure response between 120 Hz and 160 Hz that gradually decline until about 300 Hz. Second, the model accurately predicted a subtle peak at 320 Hz where another significant pressure response occurred. Finally, in each pressure reading, the model predicted an increase in pressure response as the frequency increased from 420 Hz to 520 Hz, as observed in the experimental results. It is worth noting, that based on the normalized data, the eigenfrequency
model does appear to overestimate the relative pressure response at higher frequencies. The trends however, do appear to accurately reflect the nature of the acoustic pressure response progression observed in the experimental model.

Beginning with experimental data that quantified the real boundary conditions present in the system, the eigenfrequency model was able to more accurately simulate the acoustic environment in a combustion chamber with the same airflow geometry. Similar results in both open-ended and hardwall exit conditions expressed the expected effects of impedance acoustic boundary conditions on the model. Next, in comparison between a similar modeling technique, the same progression of acoustic mode shapes occurred. This comparison indicated a transition from a position of minimal acoustic pressure response towards a peak at a frequency of around 340 Hz. Upon further analysis, through comparing modeled pressure response values at specific locations with the measured values from the experimental results, trends in modeled pressure response values closely reflected the trends in the experimental values. The improvement seen between the two modeling techniques and the correlation between trends observed in the experimental data suggests that the eigenfrequency technique can be an effective method for modeling acoustics in a system. The implications of this advancement are strong. The results of this novel modeling technique offer the ability to understand distinct locations of a maximum acoustic pressure response in a system depending on the frequency of an oscillating disturbance. Combustion designs can thus be tailored to controlling and adapting to these conditions to prevent instabilities. However, the most important aspect of this modeling technique is the proven ability to apply a simple experimental test to achieve accurate modeled predictions. By applying accurate acoustic boundary conditions, which can be experimentally determined through simple tests, improved accuracy and effectiveness of modeling techniques can be realized.
Chapter 8

Conclusions & Future Work

While the emission of harmful combustion products, from the operation of gas turbine systems, continues to be reduced, the conditions required to achieve these targets increases the susceptibility of the systems to instability. The coupling between acoustic and heat release rate oscillations throughout the environment in a combustion chamber directly causes these instabilities. Although these driving mechanisms are widely understood, the ability to effectively model the acoustics of an entire system without physically testing it remains elusive. As such, it was the goal of this research to explore acoustic interactions in a combustion system through experimental analysis, then preparing an accurate modeling technique to replicate the results.

Experimentation was conducted by driving a model combustion system acoustically, across a range of frequencies, to evaluate inherent conditions that cause significant pressure responses. Using analysis techniques like the two-microphone method, the boundary conditions present in the physical experiment were modeled and quantified. The boundary conditions offered insight into why significant reactions occurred at each forcing frequency and why other responses were suppressed. However, the impedances calculated to quantify the boundary conditions served to improve and evaluate the accuracy of different modeling techniques.

The first modeling technique employed, attempted to replicate the experimental study by applying acoustic forcing in the model. This frequency domain technique expressed potential in the ability to understand distinct locations of maximum pressure response in the combustor geometry. Similarly, this technique helped show how changing frequencies affects the movement of an acoustic pressure oscillation throughout a system. However, the model lacked the ability to accurately reflect the true relationship between the most significant pressure distribution and the attributed frequency of
oscillation. Therefore, another model was applied that incorporated applying experimental results as inputs to improve the previous model. Using the boundary condition values derived in the experiment the model was able to more accurately reflect the effect of specific surfaces on acoustics in the system. This resulted in a more accurate portrayal of the forcing frequencies responsible for exciting inherent modes in a combustion system.

Overall, the experiment showed that by conducting a simple study to evaluate boundary conditions present in a physical system could be applied to numerical methods effectively. The model will help provide accurate information on the location, frequency and pressure of an inherent acoustic response in the system. This will provide an opportunity to improve design of combustion systems by efficiently analyzing acoustics without the need for large-scale testing operations.

In evaluating the results presented in this study though, a few areas could have been explored further given the time and the resources. Although an informal mesh-sensitivity study was conducted to ensure the effectiveness of the mesh used it was not presented in this work. As such, a mesh sensitivity analysis could be pursued in future studies to further explore the effectiveness of the model. Future work could also potentially be conducted in evaluating higher frequency modes present to the system. Limitations in the lab equipment, capable of applying the acoustic forcing, prevented the study of higher frequency modes. Furthermore, applying conditions to the model to more accurately define the temperature field in the combustor, when a flame is present, would help distinctly identify modes that may cause instabilities.
BIBLIOGRAPHY


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GE Aviation – Cincinnati, OH

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Penn State Men’s Club Lacrosse
Schreyer Consulting Group
Students Organizing the Multiple Arts