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OPTIMIZING OPERATIONAL DECISIONS IN A BEEKEEPING BUSINESS

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ABSTRACT

A beekeeping business involves decisions at specific times of the year that often have effects throughout the season. As beekeeping is a form of agricultural activity, these decisions are made in cycles from year-to-year where most beekeepers rely on experience to make them. The goal of this research is to create various models that would assist a beekeeper in making optimal decisions when running a business.

The first major decision a beekeeper has each year relates to replacing the colonies lost over winter. Two initial models focused on maximizing the total number of hives, and the optimal solution involves replacing the old hives with colonies before beginning to purchase new hives. A final model alteration examines maximizing productivity by replacing colonies with bee packages or nucleus colonies. This solution is highly dependent on the budget set by the beekeeper and the prices of the two colony types.

The second major decision examined involves the allocation of honey to various products. Using a given amount of raw product, a model allocates it out based on the profitability and demand of the individual products. The optimal solution is then dependent on the values of these two parameters.

A final analysis combined the models from both major decisions by using the optimal allocation of hives to determine total production output. Then, the optimal allocation of products was found from this production output. The given numerical scenario produces enough supply to fill all demand, but the total number of hives would decrease from this year to the next. Multiple courses of action were provided for the beekeeper to choose based on individual goals for the business.

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Chapter 1 : Introduction

This chapter discusses the basic aspects of a beekeeping business as well as the major events that occur throughout a beekeeper’s year. Beekeeping as an activity involves many decision factors where a beekeeper has to balance numerous aspects of a hive. When beekeeping becomes a business, these decisions usually involve balancing cost, production, and the hive health. A beekeeper balances the amount of product that can be taken from the hive with its strength to maintain both the health of his bees and the cash flow of the business. For many beekeepers these decisions are based on trial and error over many experienced years and often are not the optimal choice. This chapter will look at the basics of beekeeping to understand the complexity of maintaining and operating multiple hives.

1.1 Structure of a Bee Hive

In simple terms, beekeeping is the process of maintaining housing for the bees to live and grow. Honeybees fend for themselves and take care of finding food, building up the hive, and reproducing. Humans only provide the structure the bees build their hive inside. If the structure becomes inadequate over time, the entire colony will leave and find a new place to build a hive. In this way, the beekeeper’s responsibility is to maintain a structure that does not provide stresses to the colony or else he will lose his hive. Among hobbyists, farmers, and researchers, there are many different designs of man-made beehives serving different purposes. However, one design stands out among the rest for its flexibility in size changes and ease of management.

The Langstroth hive (Figure 1) consists of four-sided wooden boxes with an open top and bottom. These boxes, called “hive supers”, hold vertically hanging frames which can be easily removed out of the open top of each box. Typically, one hive super is sized to hold ten frames for maximum efficiency, but there are eight-frame and five-frame variations to suit various needs. The beekeeping industry has standardized to three different heights for hive supers colloquially known as shallow, medium, and deep with medium being most common. A complete hive is composed of a number of supers stacked on top of each other that rest on a bottom board with a cover laying across the top. The bottom board contains an opening for a single entrance into or out of the hive. The whole structure usually sits on a stand to provide distance from the ground and immediate elements and a level base. When fully assembled, a Langstroth hive effectively acts as a much larger hollow box with levels of frames that the bees can freely move around in between. The bees then build out honeycomb on each of the frames which are spaced in a way that they do not connect them. The stackable hive supers and individual frames provide the main benefits with this hive design. If the colony is growing and needs more space, a beekeeper can easily stack on an appropriate number of supers without much effort. When looking to harvest, a beekeeper can inspect, remove, and replace each frame individually to target honey and ensure the colony can continue with the least disturbance.

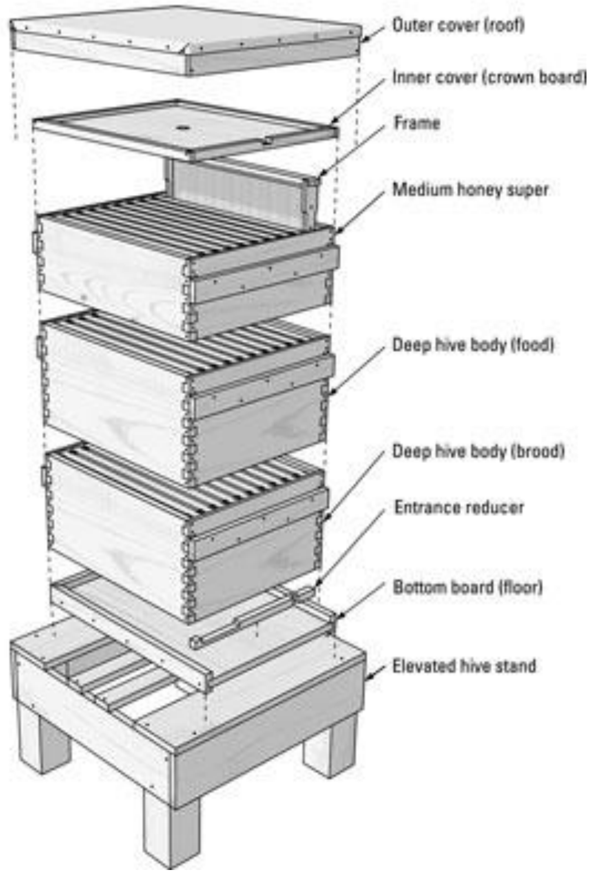

Figure 1. Diagram of a Langstroth Beehive (Blackiston, 2013)

1.2 Yearly Beekeeping Schedule

Bee colonies are fairly self-sufficient on their own and often only need weekly or monthly inspections to maintain. A beekeeping business runs similar to an agricultural farm that is segmented into seasons of varying work. For beekeeping, the cyclical time frame is generally a year with winter being the most harmful season where colonies die if they are too weak.

In spring, a beekeeper's main goal is to bring his colony count back up to normal operating levels, repair damaged hives, and let the colonies increase their populations. Coming out of winter, a beekeeper will have a number of empty hives due to death and all hives will be weakened. The queen begins laying eggs again towards the end of winter or beginning of spring and before long scout bees are out searching for food sources. Once flowers start to bloom, the hives becoming active and can grow rapidly under the proper conditions. During the spring, a beekeeper has to take inventory of all current hives, purchase and install new colonies up to the desired amount, and nurture the new hives to promote growth. This often involves supplemental feeding of sugar syrup or sugar patties called "fondant" especially if winter is long or there is a late bloom. Depending on the year, the end of spring may be a good time to start harvesting frames of honey in particularly strong hives.

Moving into summer, the focus is on growing the hives and harvesting honey. A colony will continue to collect flower nectar for honey provided it has the room to store it. The beekeeper must watch and add new supers to keep up with the growth of the hive or risk swarming. Swarming is the situation when a hive becomes too large for its enclosure and splits into two separate hives. The hive nurtures a few specific eggs so that they develop into new queens. Right before the new queens emerge from their cells, the old queen leaves with half the hive (called the swarm) and the strongest of the queens continues the colony. This is undesirable for the beekeeper as the remaining colony is only half of its original strength resulting in a slower production rate and higher chance of death in the coming winter. In order to help regulate the hive rate of growth, many beekeepers install queen excluders. A queen excluder is a thin plastic grate that has holes the workers bees can fit through, but not the queen. This is placed between two supers to stop the queen from traveling any higher in a hive. Queen excluders are multi-functional in that they limit the egg laying rate of the queen, and they segment the hive between brood (the term for cells filled with developing bees) and honey. The beekeeper then has an easier time inspecting and collecting honey. Oftentimes with this technique, beekeepers will replace entire supers at once as they can be sure they only contain honey.

In the fall, the main focus for beekeepers is to strengthen the hives as much as possible before winter. This involves limiting honey harvesting but also treating for diseases and pests. The most common pest affecting honeybee hives is the varroa mite. These mites reproduce in the brood and affect the development of new bees. They also carry a lot of viruses which they can transmit to the bees. A strong hive can withstand a varroa mite infestation but can become vulnerable when population growth slows in the fall. Many beekeepers use a chemical treatment to combat varroa mite populations in the hive. Additionally, many beekeepers provide supplemental food to their colonies to bolster their food stores as well.

During the winter, bee colonies work to preserve heat and reduce activity in an effort to survive as long as possible. The fate of a hive depends on its population and food stores entering winter compared to the length and severity of the season. A colony's population steadily decreases throughout the winter as its only goal is to keep the queen warm. A bee colony forms a cluster around the queen and vibrate to maintain a proper temperature at its center. During this time, the queen does not lay eggs as the hive cannot look after them sufficiently. A beekeeper's responsibility it to disturb the hive as little as possible during this time as opening the hive loses valuable heat. An entrance reduce is installed which prevents other pests from entering the hive such as mice or chipmunks as well as slows heat loss. Some beekeepers also wrap their hives in black paper as an additional measure to absorb heat from the sun. With a strong hive from fall and the proper heat retention measures in place, a beekeeper can do no more but hope his hives survive until spring where the entire process begins anew.

Chapter 2 : Selecting the Most Productive and Cost-Effective Hives

2.1 Problem description

In the spring of each year, a beekeeper must decide how many new hives to purchase in order to fulfill a given demand throughout the year. This demand is determined by the beekeeper beforehand based on the business's budget. As long as beekeeper is not limited by space, the goal is to maximize the total number of hives which will result in the maximum amount of product created. However, new hives produce less product within their first year as the colony is establishing itself and growing inside its new structure. A beekeeper must consider how much of the budget to allocate to purchasing new colonies versus equipment. Then, the type of colonies to purchase needs to be considered as hives can be bought in various stages of development. Additionally, the beekeeper needs to manage equipment by choosing how many hives to repair and reintroduce new bees or set out to catch drifting colonies called "swarms."

The first situation to model is the simplest decision a beekeeper needs to consider when exiting winter. After taking inventory of the colonies lost over winter and determining a budget, a beekeeper decides how many new colonies to purchase and how many total hives to purchase. For the colonies that died, the equipment still remains and only a package of bees needs to purchased and installed into the hive. Additionally, a beekeeper may look to expand and purchase new hives including equipment and bees.

2.2 Mathematical Formulation

The goal of this model is to maximize the total number of hives being added to the colony. *N* is the sum of the number of hives to replace, *x*, and number of new hives to purchase, *y*. The hives that are replaced (*x*) use previously purchased equipment, and consequently only require a new colony to be purchased and installed. The new hives (*y*) require a new colony and equipment. The model is constrained by the given total budget, *B*. The total cost is the sum of the cost to replace *x* hives and the cost to purchase *y* new hives. This is found multiplying *x* by c_1 and *y* by c_2 which are the costs to replace and purchase new hives respectively. The total cost then has to be less than or equal to the total budget.

$$\max N = x + y$$

Such that:

$$c_1 x + c_2 y \leq B$$

$$0 \leq x \leq x_{max}$$

$$0 \leq y$$

N = Total number of hives

x = Number of hives that are replaced

x_{max} = Total number of empty hives given from loss of the previous year

y = Number of hives that are new

c_i = Cost per hive of type i (1 = replaced hive, 2 = new hive)

B = Total budget

2.2.1 Numerical Example

In order to test the model, an example of a beekeeper with 500 hives and a yearly budget of $10,000 is used. If hive loss from the previous year is estimated at 35% (Döke, Frazier, & Grozinger, 2015), the beekeeper will have 325 colonies entering the spring and 175 empty hives. A bee package costs $110 and a new hive costs $500. From this information we can conclude the following:

$$x_{max} = 175\ hives$$

$$c_1 = \$110$$

$$c_2 = \$110 + \$500 = \$610$$

$$B = \$10{,}000$$

Given this data, the beekeeper should first buy up to x_{max} hives spending $c_1 x$. Then spend the remaining budget on *y* hives. With a budget of $10,000 this results in *x* = 90 hives and *y* = 0 hives with $100 of the budget remaining.

2.2.2 Sensitivity Analysis

The numerical analysis shows that all hives should be replaced at the current parameter values. The following sensitivity analysis views the effects of each parameter on the decision variables, *x* and *y*. All other parameters are held constant except for the parameter in question. Then, the model is solved for the new value of the parameter in question and all the values are graphed. This shows how sensitive the decision variables are to the parameter in question. Each parameter is presented with a table of values and a graph. The highlighted row shows the original values tested in the numerical analysis.

The first parameter tested is the cost to replace a hive, c_1. As seen in Figure 2, c_1 has an effect on how many hives a beekeeper can replace, but does not generally affect how many new hives to purchase. When the cost to replace a hive drops significantly, the solution includes buying new hives. This is only after all possible hives have been replaced and extra budget is left to expand the business. Overall, variance in c_1 changes the solution slightly and has a greater effect when reaching the extreme ends.

Table 1. Values of *x* and *y* at Various Values of c_1

c_1	x	y
50	175	2
60	166	0
70	142	0
80	125	0
90	111	0
100	100	0
110	90	0
120	83	0
130	76	0

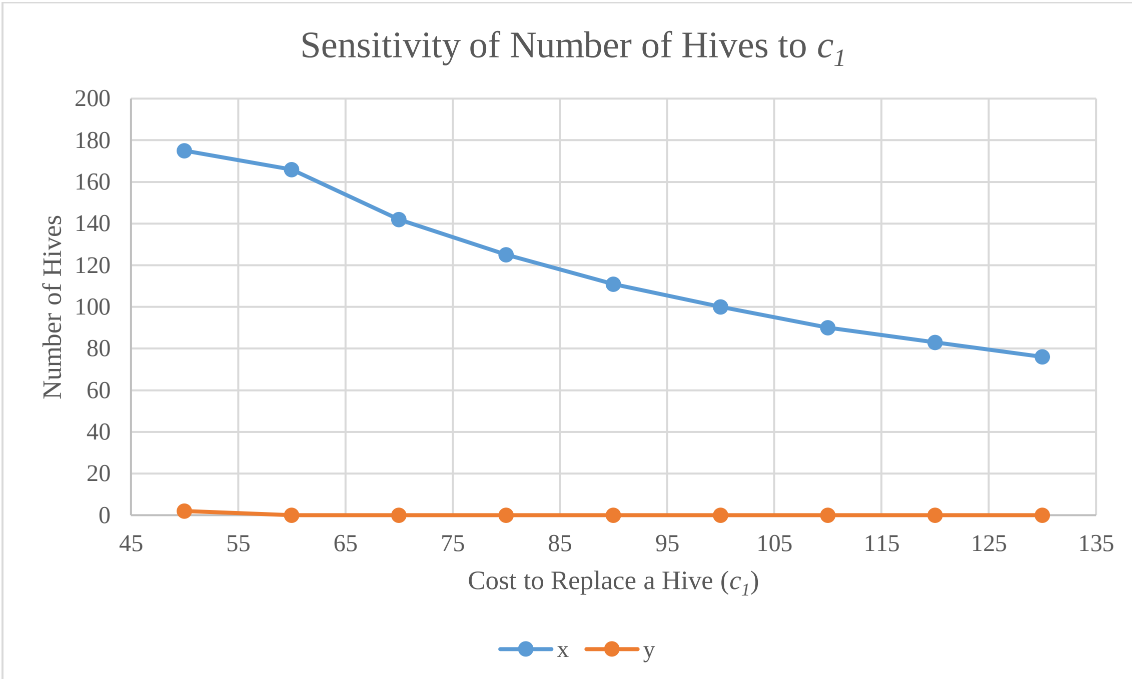

Figure 2. Graph of Number of Hives vs. Cost to Replace a Hive (c_1)

The second parameter tested is the cost to purchase a new hive, c_2. Figure 3 shows that the solution is very inelastic to changes in c_2 through a wide range of values. This is expected since c_2 is c_1 plus the cost of new equipment. Unless the cost of new equipment is negligible, the solution is going to remain dependent on c_1. For this reason, variance in c_2 shows no effect on the solution.

Table 2. Values of x and y at Various Values of c_2

c_2	x	y
510	90	0
530	90	0
550	90	0
570	90	0
590	90	0
610	90	0
630	90	0
650	90	0

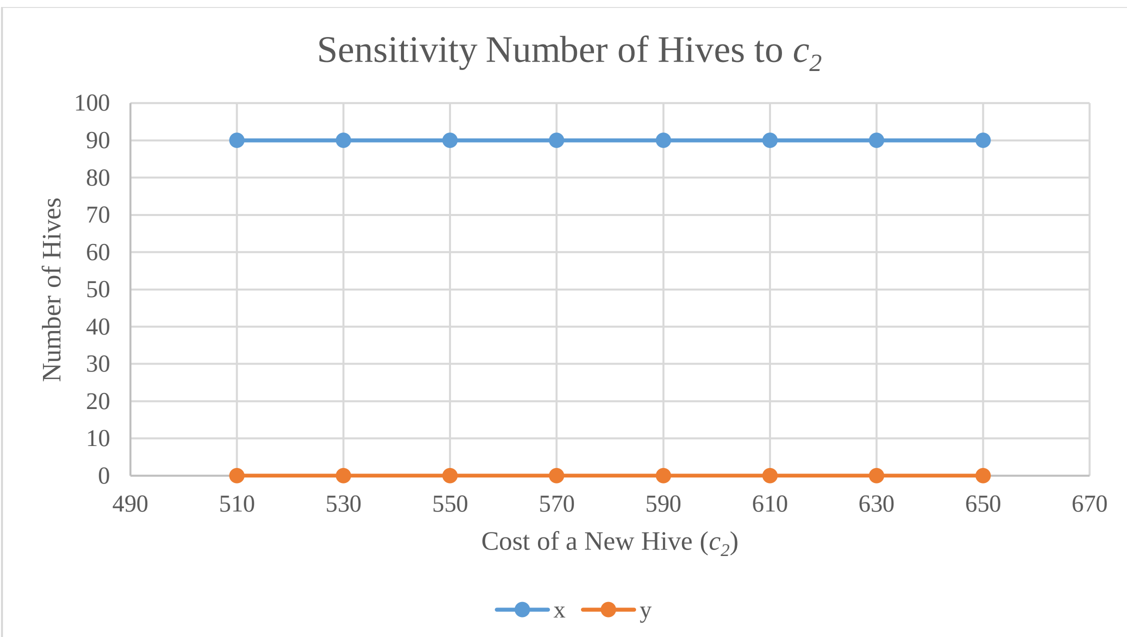

Figure 3. Graph of Number of Hives vs. Cost of a New Hive (c_2)

The last parameter tested is total budget, *B*. Figure 4 shows how the ordering strategy changes with an increasing budget. When the budget is low, the priority is shifted towards replacing as many hives as possible. Then, when all possible hives have been replaced, x_{max}, the extra budget can be allotted to purchasing new hives and equipment. As seen in Table 3, a beekeeper should budget around $20,000 at the current parameters in order to maintain a similar sized business from one year to the next. If a beekeeper allocates $5,000 more for a budget of $25,000, the total number of hives will grow by approximately 1.8%. When allocating $5,000 less at $15,000, the total number of hives will shrink by around 7.8% as seen in the numerical analysis example. The main limiting factor behind expansion is the significantly higher cost of new bee equipment. A beekeeper can possibly reduce this cost by building the equipment or purchasing it used which will increase the ability to grow the number of hives. Overall, variance in the total budget, B¸ has a significant effect on the solution since it acts as the limiting factor in the constraint.

Table 3. Values of *x* and *y* at Various Values of *B*

B	***x***	***y***
5,000	45	0
10,000	90	0
15,000	136	0
20,000	175	1
25,000	175	9

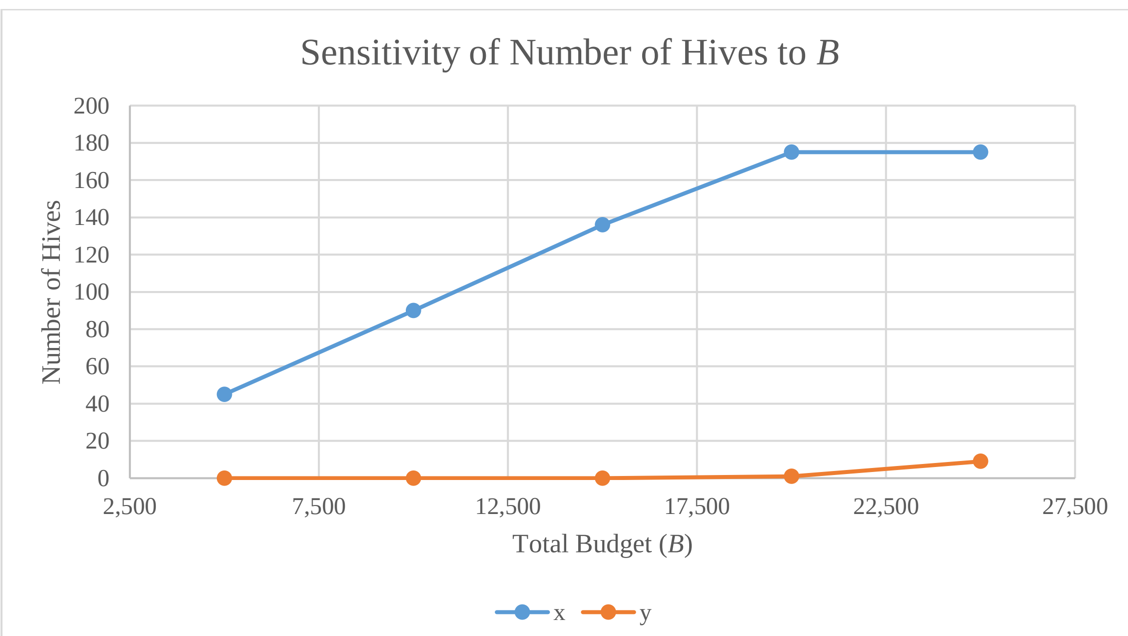

Figure 4. Graph of Number of Hives vs. Total Budget

Using the optimum budget of $20,000 per year to maintain the total number of colonies, the sensitivity analysis for c_1 and c_2 were repeated. As seem in Figure 5, x_{max} is reached at the original parameter of c_1 = $110. Increases in c_1 from that point result in a decrease in *x*. Decreases in c_1 from that point result in an increase of *y* as more new hives are then purchased. A budget of $20,000 shows that this is where the solution changes over to allocating from *x* to *y*.

Table 4. Values of *x* and *y* at Various Values of c_1

c_1	*x*	*y*
50	175	20
60	175	16
70	175	13
80	175	10
90	175	7
100	175	4
110	175	1
120	166	0
130	153	0

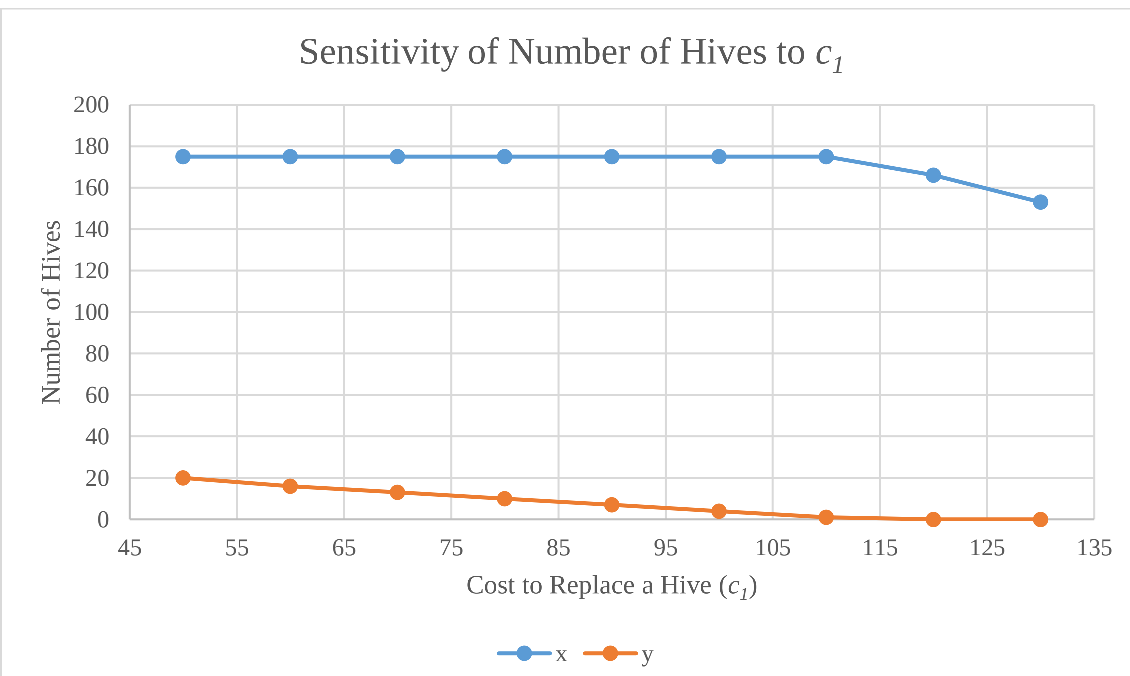

Figure 5. Graph of Number of Hives vs. Cost to Replace a Hive (c_1)

Even when the budget is increased to $20,000, changes in c_2 do not affect the optimal solution. The model still allocates all the budget to purchasing up to x_{max} colonies and is mostly depleted at $20,000. For that reason, there isn't any budget left for *y* colonies.

Table 5. Values of *x* and *y* at Various Values of c_2

c_2	*x*	*y*
510	175	1
530	175	1
550	175	1
570	175	1
590	175	1
610	175	1
630	175	1
650	175	1

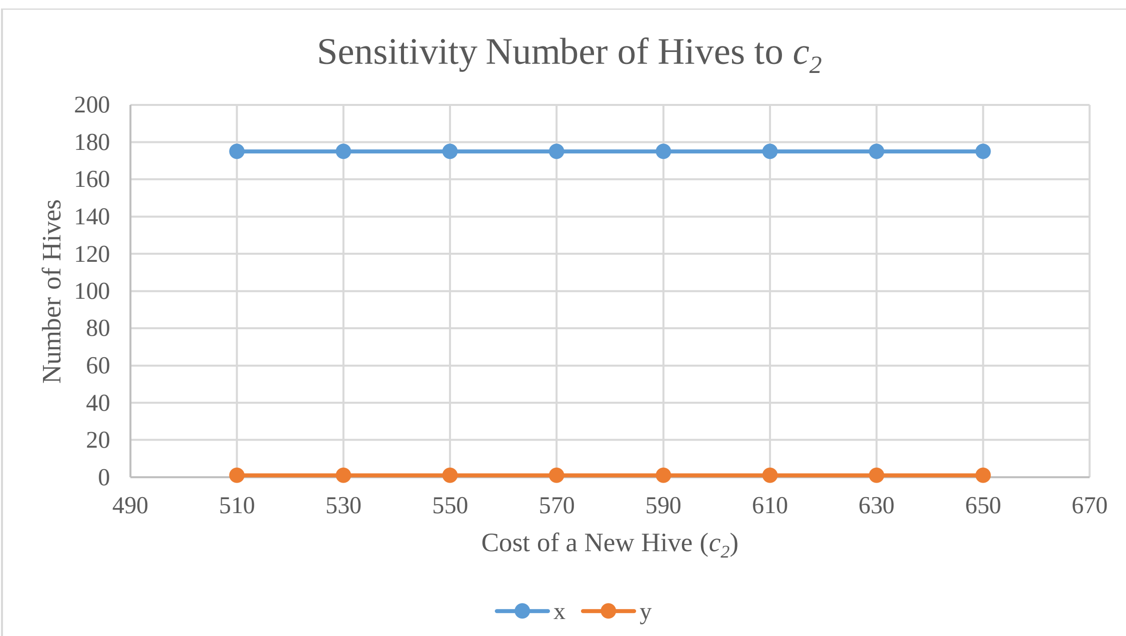

Figure 6. Graph of Number of Hives vs. Cost of a New Hive (c_2)

The sensitivity analysis reveals a simple solution to the problem which relies on c_1, *B*, and x_{max}. In general, the budget will be allocated to replacing hives until either the budget is completely used or there are no more hives to replace (x_{max} is reached). Then, the remaining budget will be allocated towards purchasing new hives and subsequently increasing the size of the business compared to the previous year. Due to this solution's simplicity, it can be modeled in the following way.

If:

$$B \leq x_{max} * c_1$$

Then:

$$x = \frac{B}{c_1}$$

$$y = 0$$

Otherwise if:

$$B > x_{max} * c_1$$

Then:

$$x = x_{max}$$

$$y = \frac{B - (x_{max} * c_1)}{c_2}$$

2.3 Model Alteration: Catching Swarms

Swarming is honeybee colonies' natural method of reproduction in which the colony produces a new queen and the old queen leaves with approximately 60% of the hive (DeBerry, Crowley, & Ellis, 2012). The bees that leave, called the swarm, settle temporarily in an open area until scout bees find a new permanent location. One of the primary reasons for swarming is lack of space in the original hive due to colony growth. Beekeepers are called by members of the community to capture a swarm that has found itself in an undesired location. These can be moved to a hive super or large bag for transportation and reintroduced to a more suitable location. Additionally, beekeepers may choose to have empty hives out for the purpose of catching swarms. Empty hives often are the most desirable location for swarm to move to due to their innate design. If scout bees find an empty hive and chooses it as a suitable location, the swarm will relocate itself there without any work from the beekeeper. Oftentimes, beekeepers choose to allocate a certain amount of their equipment for the purpose of capturing swarms since it is a free way to increase the total number of hives owned. Some even go a step farther and stop adding hive supers to naturally induce a swarm from their own hives. By targeting strong hives and inducing swarms, a beekeeper can estimate a general proportion of how many hives will swarm in a given year.

In order to model this, the probability of a hive generating a swarm, p, is introduced. This parameter is found from the beekeeper's own experience and knowledge of their strongest hives. Projected number of new hives via swarm, z, then has to be subtracted from x_{max} as a number of empty hives need to be allocated to capture swarms throughout the year. This creates an updated $\tilde{x}_{max}$ as the total number of available empty hives has decreased. The number of hives via swarm, z, equals the total number of hives from the previous year, N_{t-1}, times the probability of generating a swarm, p. The total cost formula remains unchanged since there is no extra cost in obtaining a swarm.

$$\max N = x + y$$

Such that:

$$c_1 x + c_2 y \leq B$$

$$0 \leq x \leq \tilde{x}_{max}$$

$$0 \leq y$$

Where:

$$\tilde{x}_{max} = x_{max} - z$$

$$z = N_{t-1}\, p$$

N_{t-1} = Total number of colonies from the previous year

p = Probability of a hive generating a swarm in the given year

z = Total number of expected hives from swarms that year

$\tilde{x}_{max}$ = New total number of empty hives available to fill after allocating some to capturing swarms

2.3.1 Numerical Example

To test this model, an example of a beekeeper with 500 hives and a yearly budget of \$10,000 is used. Hive loss is estimated at 35% from the previous year (Döke, Frazier, & Grozinger, 2015). A bee package costs \$110 and a new hive costs \$500. The probability of a hive generating a swarm in the given year is 25%.

$$x_{max} = 500 * 0.35 = 175\ hives$$

$$z = 325 * 0.25 = 81.25 \Rightarrow 81\ hives$$

$$\tilde{x}_{max} = 175 - 81 = 94\ hives$$

$$c_1 = \$110$$

$$c_2 = \$110 + \$500 = \$610$$

$$B = \$10{,}000$$

Given this data, a beekeeper should set aside 81 hives for swarming capture purposes which costs no additional expenses. Then, should buy up to $\tilde{x}_{max}$ hives spending $c_1 x$. The remaining budget should be spent on y hives. This results in x = 90 hives and y = 0 hives with $10 of the budget remaining. Assuming all allocated hives catch a swarm, the beekeeper will have a total of 171 new colonies.

2.3.2 Sensitivity Analysis

A similar analysis of the parameters seen in Chapter 2.2 will show how the inclusion of swarms affects the solution. The same three parameters, c_1, c_2, and B, will be analyzed to determine their effect on the solution. The highlighted row still signifies the original value for each parameter. The main difference for this model including swarms is that x_{max} is reduced which slightly changes how the parameters affect the solution.

The first parameter tested is the cost to replace a hive, c_1. As seen in Figure 7, c_1 only affects the number of hives to replace until the cost reduces and x_{max} is reached. Since the number of hives dedicated to swarms reduces the possible hives to replace, Figure 7 appears similar to Figure 2**Error! Reference source not found.** with a horizontal shift to the right. At a c_1 between $90 and

$100, all the empty hives can be replaced and the remaining budget can be allocated to new hives. Overall, a variance in c_1 has a greater effect than did in 2.2, but the solution is still fairly stable.

Table 6. Values of *x* and *y* at Various Values of c_1

c_1	x	y
65	94	7
70	94	6
80	94	4
90	94	2
100	94	0
110	90	0
120	83	0
130	76	0

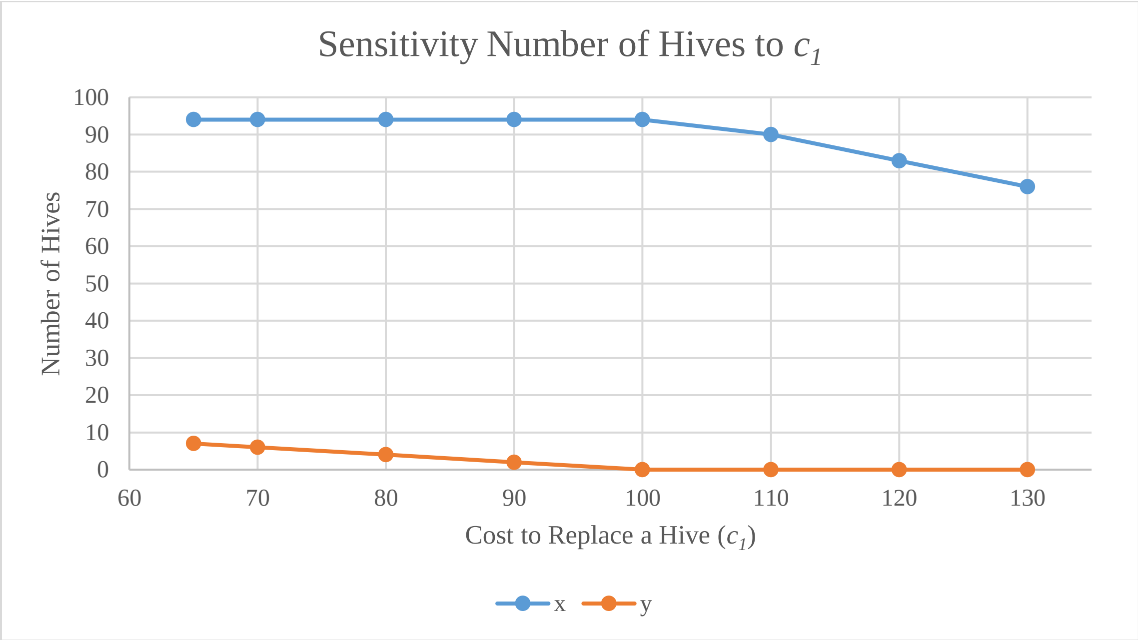

Figure 7. Graph of Number of Hives vs. Cost to Replace a Hive (c_1)

The second parameter tested is cost to purchase a new hive, c_2. Similar to the previous model, this has negligible effect on the solution as seen in Figure 8. Variance in c_2 will not change the solution due to the high cost of equipment compared to the cost of bees.

Table 7. Values of *x* and *y* at Various Values of c_2

c_2	x	y
510	90	0
530	90	0
550	90	0
570	90	0
590	90	0
610	90	0
630	90	0
650	90	0

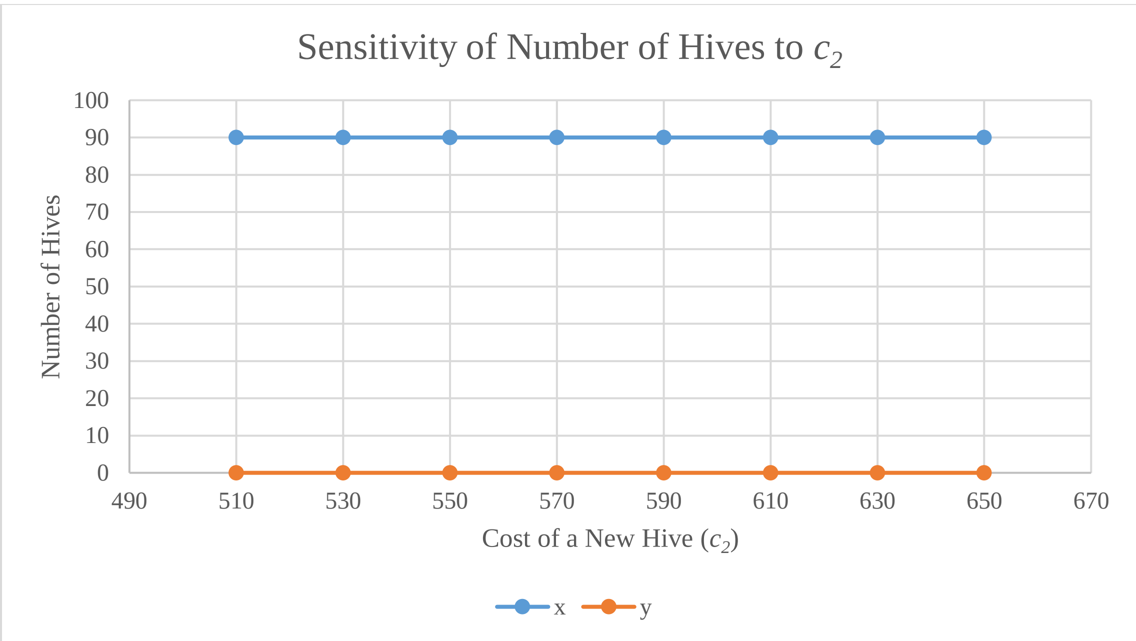

Figure 8. Graph of Number of Hives vs. Cost of a New Hive (c_2)

The last parameter tested was total budget, *B*. In this case, the cost needed to maintain total number of hives decreased compared to the first model seen in Figure 9. The new maintenance budget drops to around $12,500 each year since a number of hives can be acquired for free as swarms. At the previously found maintenance budget, $15,000, this model shows a 1.4% increase in total number of hives. At $20,000 the total hives will grow by 3%, and at $10,000 the total hives will shrink by 0.8%. The budget still has the strongest effect on the solution out of all the parameters.

Table 8. Value of *x* and *y* at Various Values of *B*

B	***x***	***y***
5,000	45	0
7,500	68	0
10,000	90	0
15,000	94	7
20,000	94	15
25,000	94	24

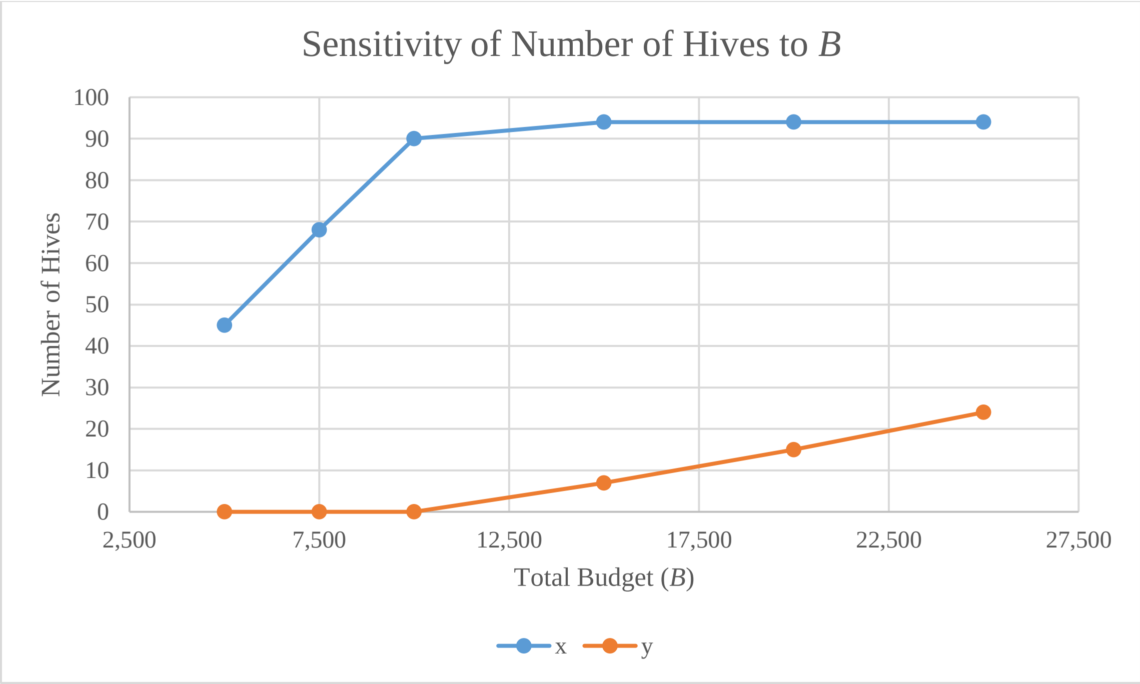

Figure 9. Graph of Number of Hives vs. Total Budget (*B*)

2.4 Model Alteration: Considering Nucleus Colonies

The most common form of purchasing bees comes from a bee package. This usually contains around three pounds of honeybees with a queen. The bees are then placed into a hive where they start with nothing and have to build out foundation before the queen can start laying eggs for the colony to grow. However, there is another form of purchasing bees where they are more developed and can begin producing product faster. A nucleus colony (nuc) is a small honey bee colony that contains all the parts of a normal hive. When purchased, a nuc comes with about 5 frames that include the queen, workers, some honey stores, and most importantly brood. Brood is the eggs, larvae, and pupae of honeybees that develops in the honeycomb cells. This small hive already has brood in all stages of development which is the main benefit over a bee package. Since a bee package starts in a hive with empty frames or frames that still need to be built, there is a startup time before the colony population will grow. A worker bee takes twenty-one days to develop from egg to hatching. Then after this twenty-one-day period, the colony can begin growing and start collecting nectar for honey. At the very least, a bee package will be three weeks behind a nucleus colony in population and development. This does not factor in the time for the colony to settle into the new hive or the weakening of the hive during that three-week period. Nucs can reach a healthy production size much quicker than bee packages, and the beekeeper can expect a larger harvest over the course of a year. However, nucs are much more expensive and the beekeeper must balance cost with desired production.

To include nucleus colonies into the model, the maximization equation must consider total productivity, P, rather than maximizing total number of hives. Instead of maximizing x and y, the model maximizes the number of hives times that hive type's productivity. Additionally, the variables x and y are split into x_1, x_2, y_1, and y_2 to distinguish between how many colonies are purchased as packages versus nucs for each type of hive. The productivity of package hives and nucleus hives are represented as a_1 and a_2, respectively. The constraint remains similar in that the total cost must be less than or equal to the total given budget, *B*. In this case, c_1 designates the cost of a single package colony and c_2 designates the cost of a single nuc colony. These costs with the cost of a hive are represented as c_3 and c_4. As mentioned previously, c_2 is expected to be greater than c_1, but a_1 is expected to be less than a_2.

$$\max P = a_1x_1 + a_2x_2 + a_1y_1 + a_2y_2$$

Such that:

$$c_1x_1 + c_2x_2 + c_3y_1 + c_4y_2 \leq B$$

$$x_1 + x_2 \leq x_{max}$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$y_1 \geq 0$$

$$y_2 \geq 0$$

2.4.1 Numerical Example

To test this model, an example of a beekeeper with 500 hives and a yearly budget of $10,000 is used. Hive loss is estimated at 35% from the previous year. A bee package costs $110, a nucleus colony costs $180, and a new hive costs $500. The productivity of a bee package is 0.5 times that of an established colony and the productivity of a nucleus colony is 0.75 times that of an established colony.

$$x_{max} = 500 * 0.35 = 175\ hives$$

$$a_1 = 0.5$$

$$a_2 = 0.75$$

$$c_1 = \$110$$

$$c_2 = \$180$$

$$c_3 = \$110 + \$500 = \$610$$

$$c_4 = \$180 + \$500 = \$680$$

$$B = \$10{,}000$$

Given this data, a beekeeper should buy 86 packages of bees and 3 nucleus colonies for a total of $10,000. This results in x_1 = 86 hives, x_2 = 3 hives, y_1 = 0 hives, and y_2 = 0 hives with a remaining budget of $0. The total productivity for this solution is 45.25 which equates to the productivity of 45.25 established hives.

2.4.2 Sensitivity Analysis

In the addition of nucleus colonies to the model, four new parameters are added. The following sensitivity analysis tests c_1, c_2, c_3, c_4, a_1, a_2, and B compared to the decision variables (Number of Hives) x_1, x_2, y_1, and y_2 as well as total productivity, P.

The first parameter test was the cost of a single package colony, c_1. As this increased, the number of hives replaced from packages, x_1, and the number of hives replaced from nucleus colonies, x_2, switched. This makes sense since the nucleus colonies are more productive and an increase in the package colonies makes them less desirable seen in Figure 10. This switch over occurs around a price of $125 for a package colony, c_1. Another interesting occurrence is that when c_1 drops, there is an increase in the number of nucleus colonies purchased. This signals the opportunity to increase productivity by allocating some of the budget to nucleus colonies rather than just packages. However, it is never profitable to purchase new hives of either type which aligns with the previously found trends in 2.2 and 2.3.

Table 9. Values of x_1, x_2, y_1, and y_2 at Various Values of c_1

c_1	x_1	x_2	y_1	y_2
40	90	35	0	0
50	97	28	0	0
60	105	20	0	0
70	114	11	0	0
80	125	0	0	0
90	111	0	0	0
100	100	0	0	0
110	86	3	0	0
120	77	4	0	0
130	2	54	0	0
140	2	54	0	0
150	3	53	0	0
160	4	52	0	0

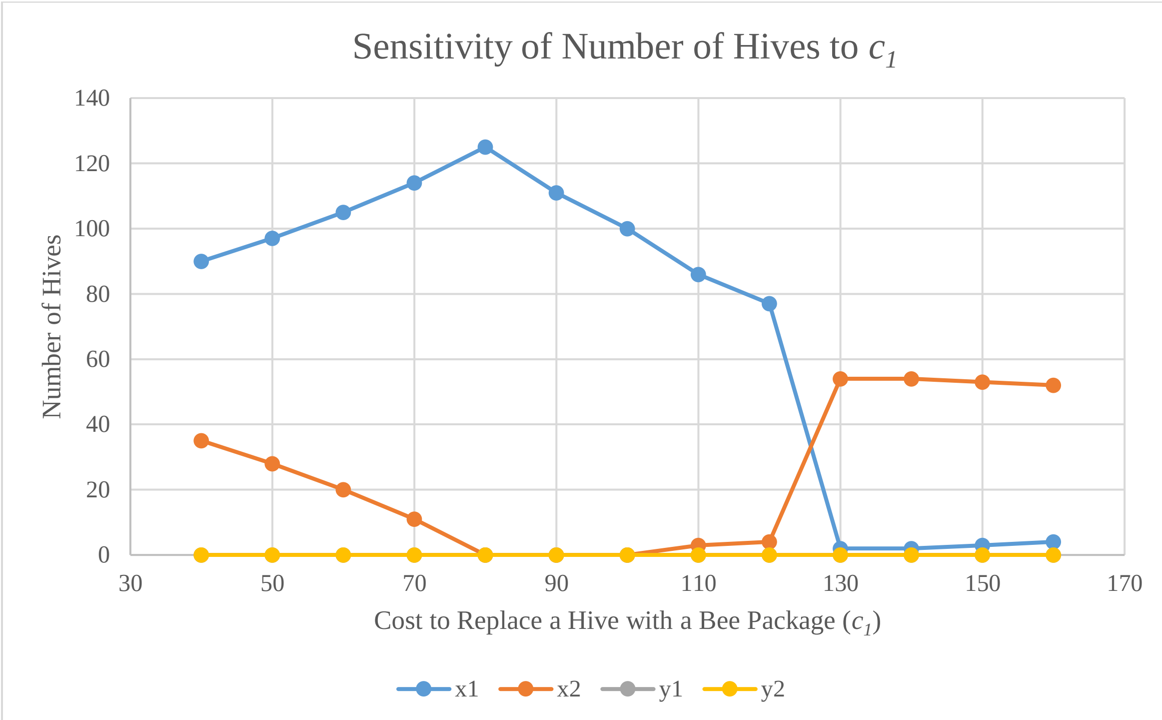

Figure 10. Graph of Number of Hives vs. Cost to Replace a Hive with a Bee Package (c_1)

As c_1 increases, the total productivity trends downwards until it hits around $125 as seen in Figure 11. This also makes sense since the productivity is mainly dependent on package hives since they are the majority of the hives purchased up until the $125 price point. At that point, nucleus hives are preferred and the productivity due does not change in relation to change of c_1.

Table 10. Values of *P* at Various Values c_1

c_1	*P*
40	71.25
50	69.5
60	67.5
70	65.25
80	62.5
90	55.5
100	50
110	45.25
120	41.25
130	41.5
140	41.5
150	41.25
160	41

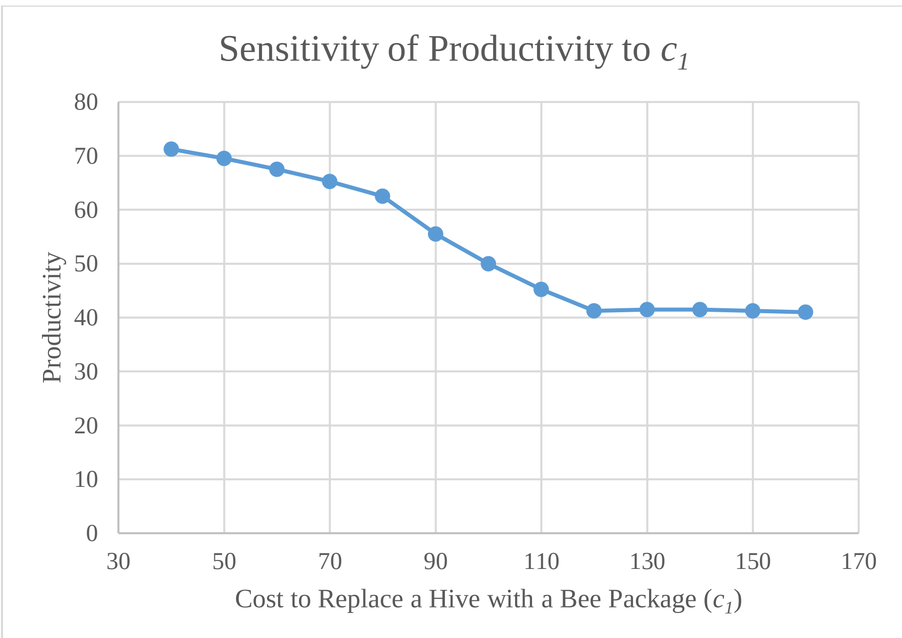

Figure 11. Graph of Productivity vs. Cost to Replace a Hive with a Bee Package (c_1)

The second parameter tested was the cost of a new hive with a package colony, c_3. There was no variation seen in the solution with changes in c_3 (Figure 12) since the price of new hives is much higher than replacing hives. As a result, the solution will not change unless c_3 drops a significant amount which could only happen if the new equipment cost becomes negligible.

Table 11. Values of x_1, x_2, y_1, and y_2 at Various Values of c_3

c_3	x_1	x_2	y_1	y_2
510	86	3	0	0
530	86	3	0	0
550	86	3	0	0
570	86	3	0	0
590	86	3	0	0
610	86	3	0	0
630	86	3	0	0
650	86	3	0	0

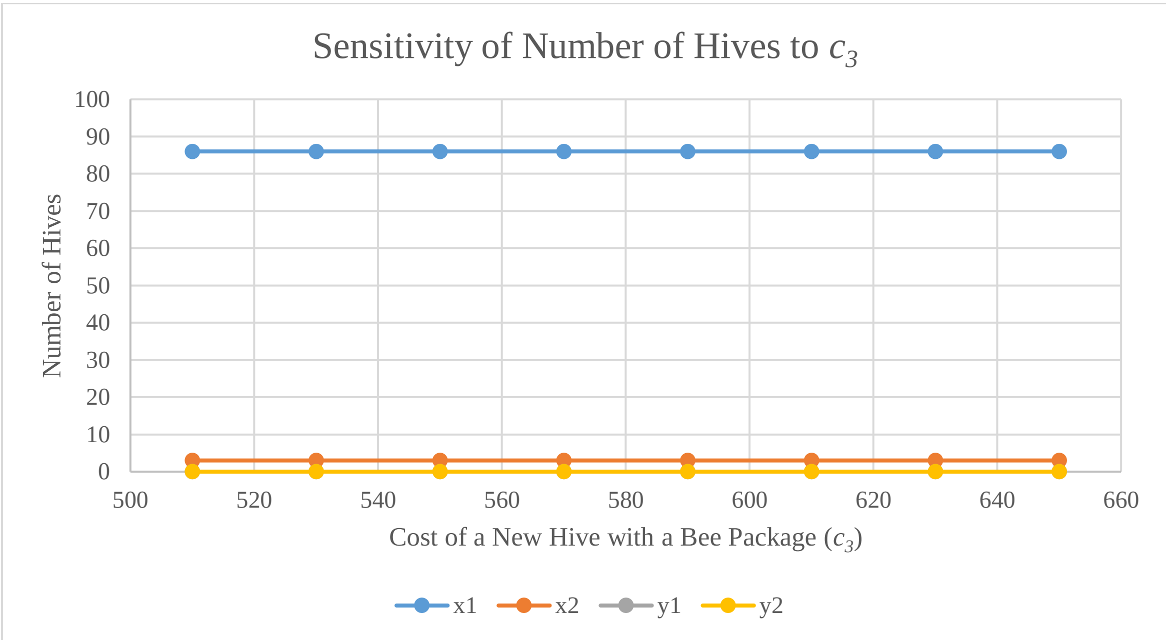

Figure 12. Graph of Number of Hives vs. Cost of a New Hive with a Bee Package (c_3)

Due to the lack of change in the solution, there is also no change in the productivity at various values of c_3 (Figure 13).

Table 12. Values of *P* at Various Values of c_3

c_3	*P*
510	45.25
530	45.25
550	45.25
570	45.25
590	45.25
610	45.25
630	45.25
650	45.25

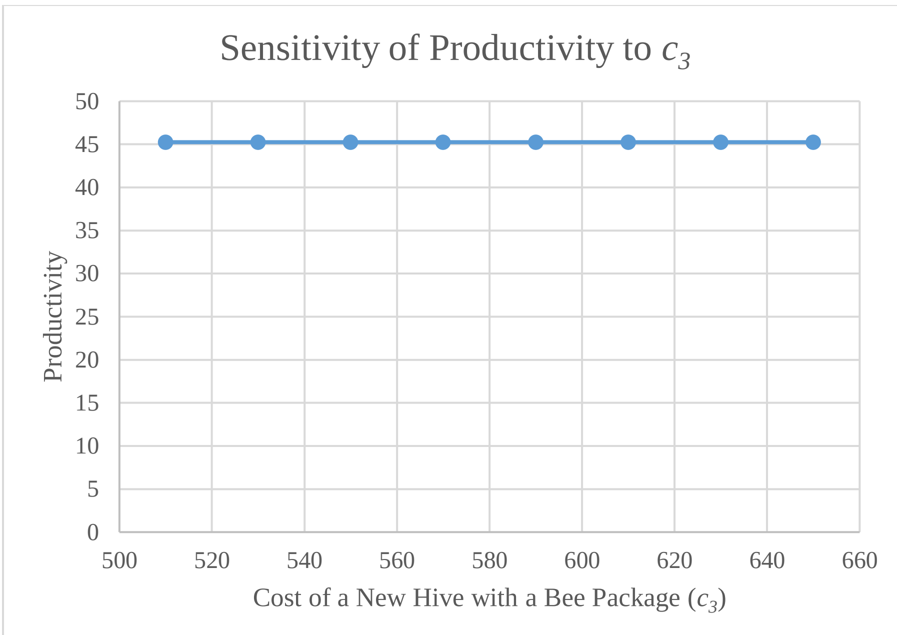

Figure 13. Graph of Productivity vs. Cost of a New Hive with a Bee Package (c_3)

The third parameter tested was the cost of replacing a hive with a nucleus colony, c_3. As seen in Figure 14, there is a similar switch between allocating the budget to packages hives versus nucleus hives. At around $165, it becomes more productive to spend the budget on nucleus hives as opposed to packaged hives even though the total number of hives decreases. Additionally, there are also no new hives purchased of either type which is similar to changes found in c_1.

Table 13. Values of x_1, x_2, y_1, and y_2 at Various Values of c_2

c_2	x_1	x_2	y_1	y_2
110	0	90	0	0
120	0	83	0	0
130	1	76	0	0
140	0	71	0	0
150	5	63	0	0
160	2	61	0	0
170	89	1	0	0
180	86	3	0	0
190	89	1	0	0
200	89	1	0	0

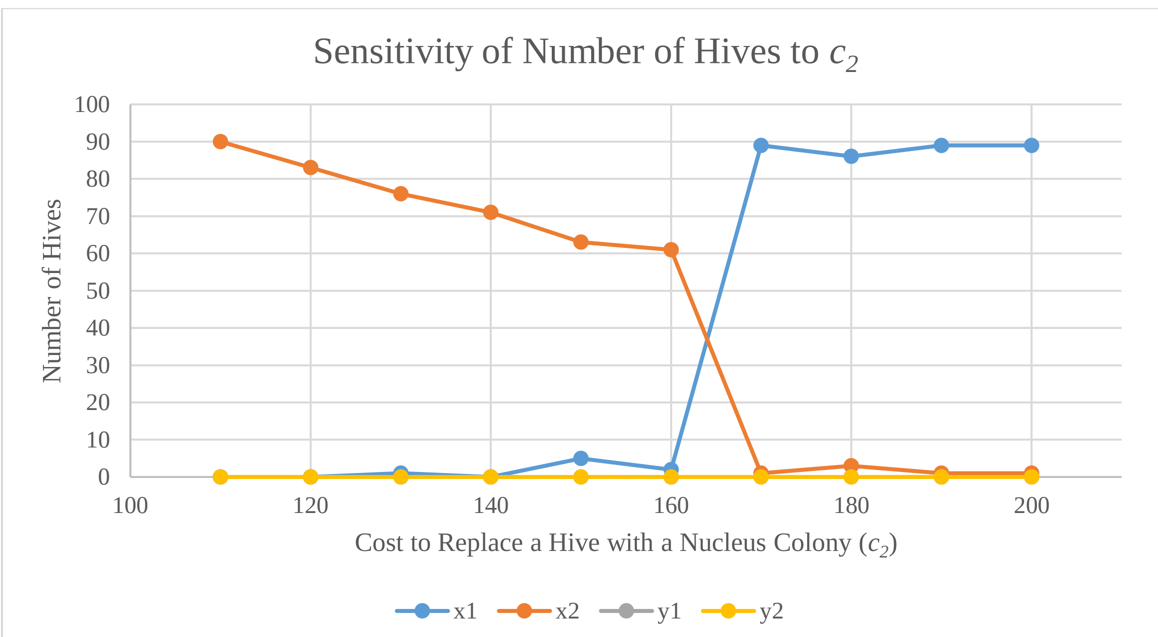

Figure 14. Graph of Number of Hives vs. Cost to Replace a Hive with a Nucleus Colony (c_2)

Figure 15 shows a similar trend to Figure 11 in that the productivity decreases as c_2 increases until it hits a point where it flattens off. This is around $165 where the model flips to allocating budget to package colonies as opposed to nucleus colonies. Above $165 for c_2, any changes do not matter as the productivity depends on number of packaged hives, x_1, not nucleus hives, x_2.

Table 14. Values of *P* at Various Values of c_2

c_2	*P*
110	67.5
120	62.25
130	57.5
140	53.25
150	49.75
160	46.75
170	45.25
180	45.25
190	45.25
200	45.25

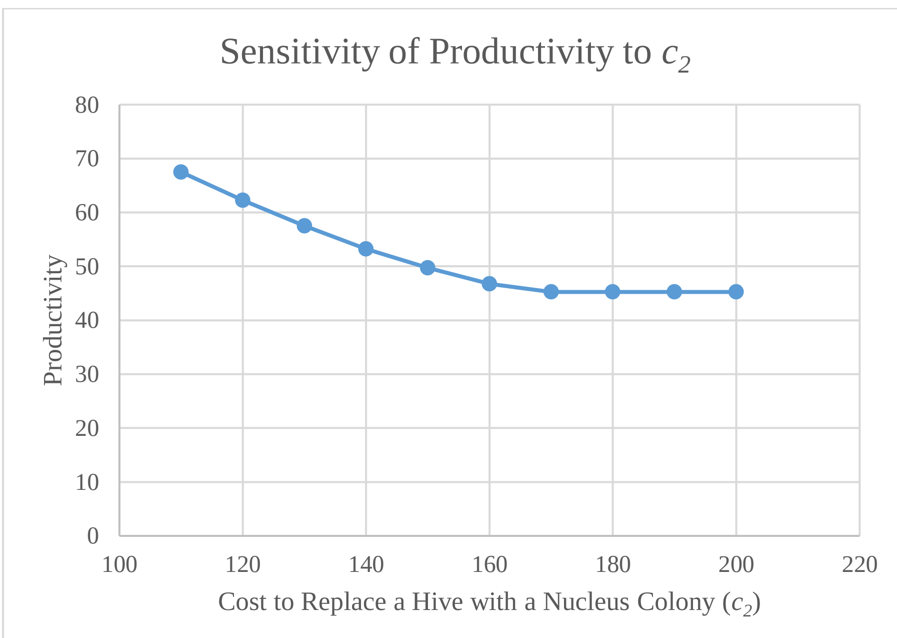

Figure 15. Graph of Productivity vs. Cost to Replace a Hive with a Nucleus Colony (c_2)

The fourth parameter tested is cost of a new hive with a nucleus colony, c_4. As seen in Figure 16, c_4 has no effect on the solution since the cost of new equipment is high. Similar to the situation with c_3, the solution remains the same with through changes of c_4.

Table 15. Values of x_1, x_2, y_1, and y_2 at Various Values of c_4

c_4	x_1	x_2	y_1	y_2
580	86	3	0	0
600	86	3	0	0
620	86	3	0	0
640	86	3	0	0
660	86	3	0	0
680	86	3	0	0
700	86	3	0	0
720	86	3	0	0

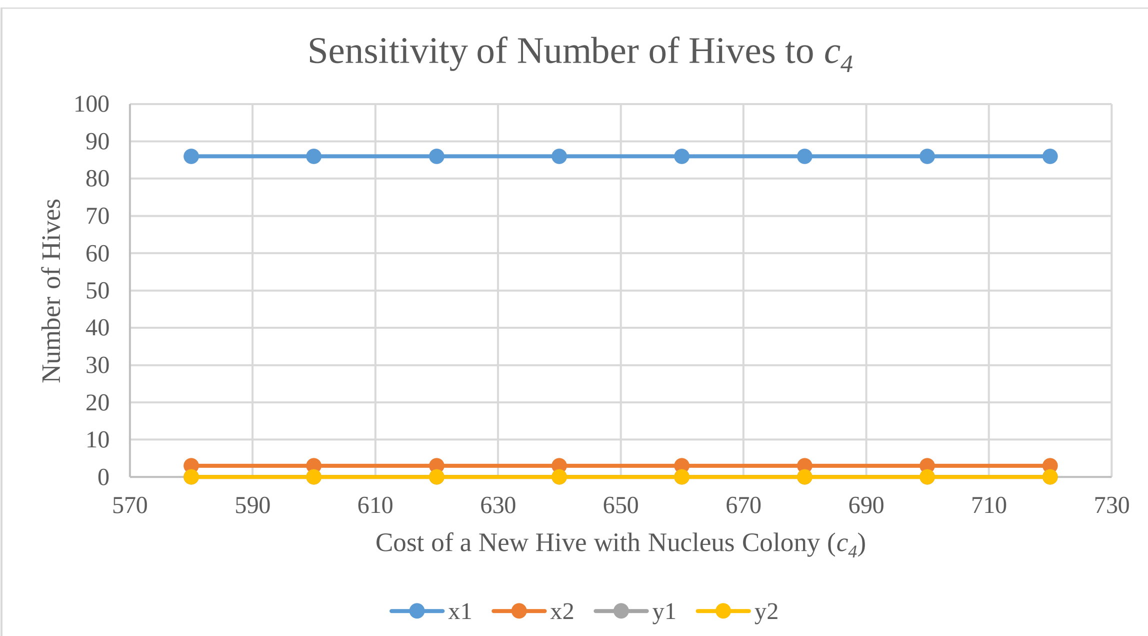

Figure 16. Graph of Number of Hives vs. Cost of a New Hive with Nucleus Colony (c_4)

Due to no change in the solution, the productivity will not change at various values of c_4 (Figure 17).

Table 16. Values of P at Various Values of c_4

c_4	P
580	45.25
600	45.25
620	45.25
640	45.25
660	45.25
680	45.25
700	45.25
720	45.25

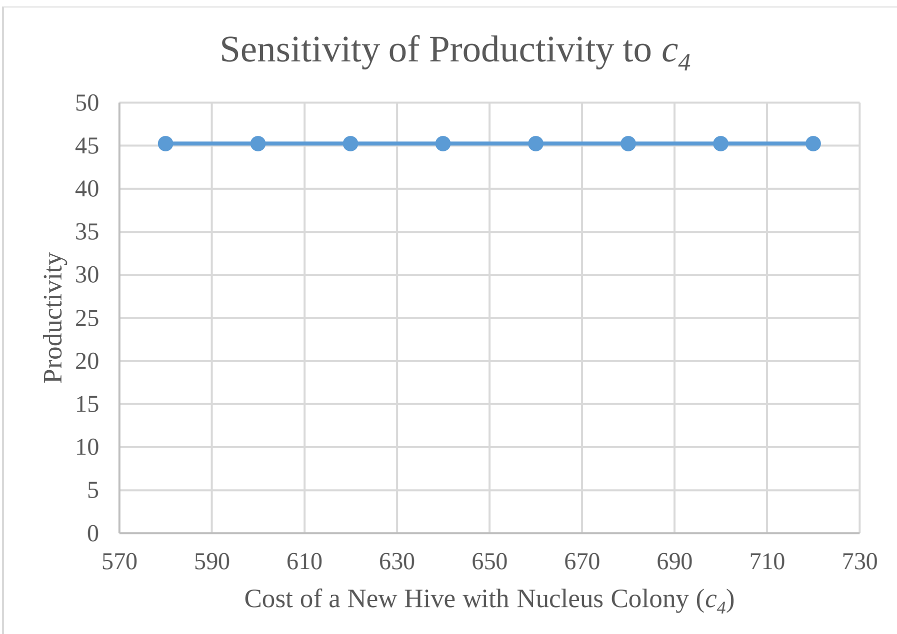

Figure 17. Graph of Productivity vs. Cost of a New Hive with Nucleus Colony (c_4)

The fifth parameter tested is total budget, B. Figure 18 shows a unique trend where number of hives replaced with a nucleus, x_2, continues to increase as budget increases. At the lower end of the budget, number of hives replaced with a package, x_1, dominates. The number of package hives increases until it reaches a peak at a budget of $15,000, then decreases to 0 at a budget of $25,000. Then, new hives start to receive budget allocation once all the empty hives are replaced with nucleus colonies. This trend doesn't show a particular location to set the budget because once a budget of $15,000 is reached, the total number of hives is maintained. Only at a budget of $25,000 does the total number of hives increase since the ratio of package versus nucleus colonies changes between those two budgets. If a beekeeper only wishes to maintain the same number of hives, a budget of $15,000 is required. If a beekeeper wants to increase the number of hives without jumping to a budget of $25,000, a sacrifice of potential productivity will have to be made.

Table 17. Values of x_1, x_2, y_1, and y_2 at Various Values of B

B	x_1	x_2	y_1	y_2
5,000	45	0	0	0
7,500	68	0	0	0
10,000	86	3	0	0
15,000	108	17	0	0
20,000	36	89	0	0
25,000	0	125	0	3

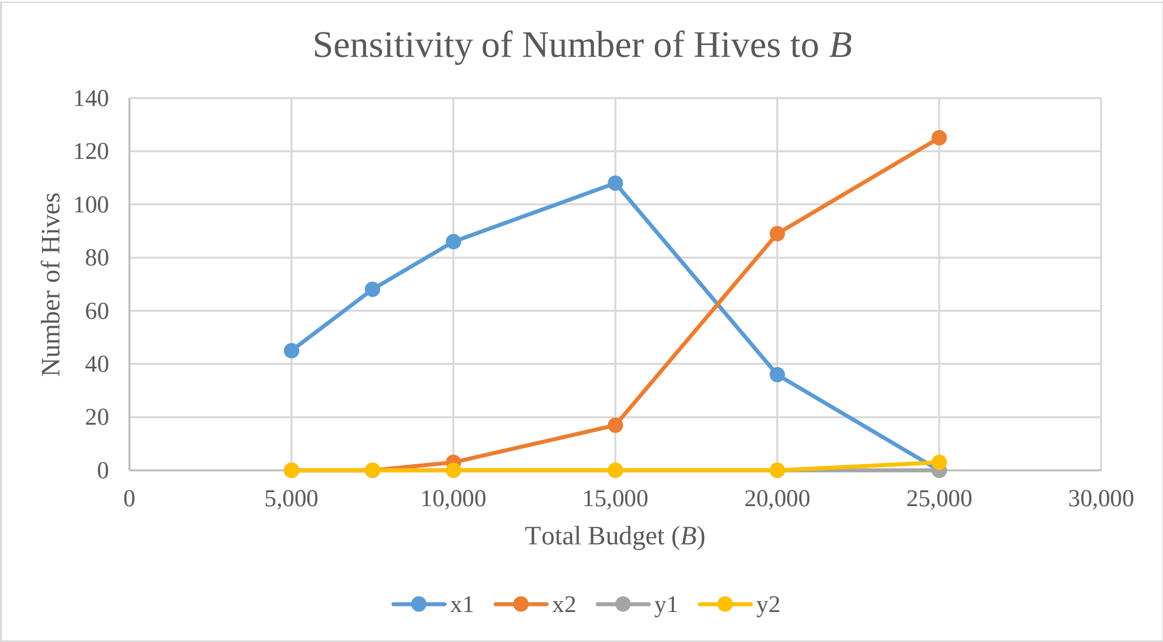

Figure 18. Graph of Number of Hives vs. Total Budget (B)

The positive relationship between budget and productivity can be seen in Figure 19. This is expected since a beekeeper can purchase more hives with higher productivity with a higher budget. However, the figure does show a point of diminishing marginal returns after a budget of $20,000 where the slope of the line decreases. This is due to the purchasing of new hives which have a higher cost than strictly replacing the hives.

Table 18. Values of *P* at Various Values of *B*

B	***P***
5,000	22.5
7,500	33.75
10,000	45.25
15,000	66.75
20,000	84.75
25,000	96

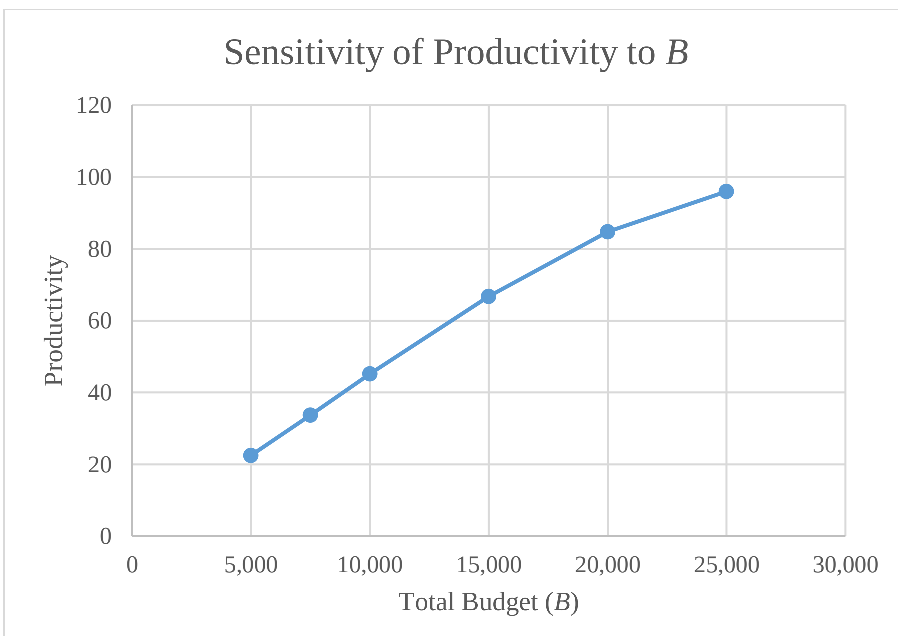

Figure 19. Graph of Productivity vs. Total Budget (*B*)

The sixth parameter tested is the productivity of package colonies compared to an established hive, a_1. As seen in Figure 20, there is a change in allocation between package hives and nucleus hives when a_1 falls below 0.5. Between a productivity of 0.45 and 0.5, there is a point where the solution changes to favor nucleus hives due to the resulting total productivity. This is an expected phenomenon since a decreasing a_1 shows a less desirable package colony.

Table 19. Values of x_1, x_2, y_1, and y_2 at Various Values of a_1

a_1	x_1	x_2	y_1	y_2
0.30	0	55	0	0
0.35	0	55	0	0
0.40	4	53	0	0
0.45	4	53	0	0
0.50	86	3	0	0
0.55	89	1	0	0
0.60	89	1	0	0
0.65	89	1	0	0

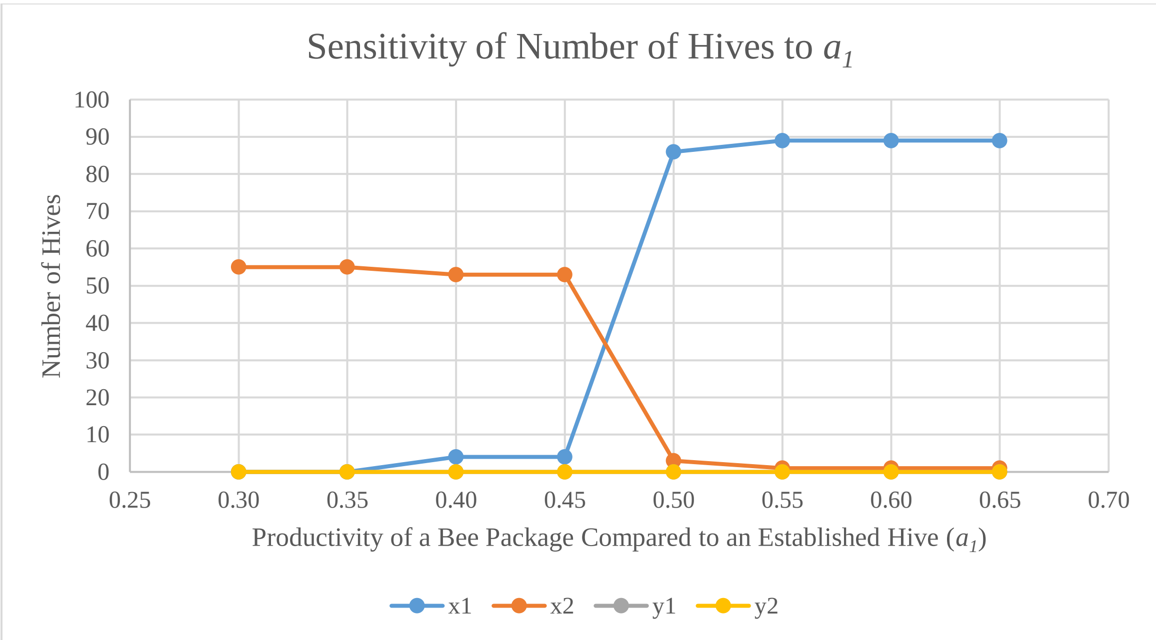

Figure 20. Graph of Number of Hives vs. Productivity of a Bee Package Compared to an Established Hive (a_1)

The switch from package hives to nucleus hives can be seen in Figure 21 since the productivity flattens under a value of 0.45 for a_1. This makes sense since under this point, the solution is dependent on nucleus colonies and a_1 has little to no effect.

Table 20. Values of *P* at Various Values of a_2

a_2	P
0.30	41.25
0.35	41.25
0.40	41.35
0.45	41.55
0.50	45.25
0.55	49.7
0.60	54.15
0.65	58.6

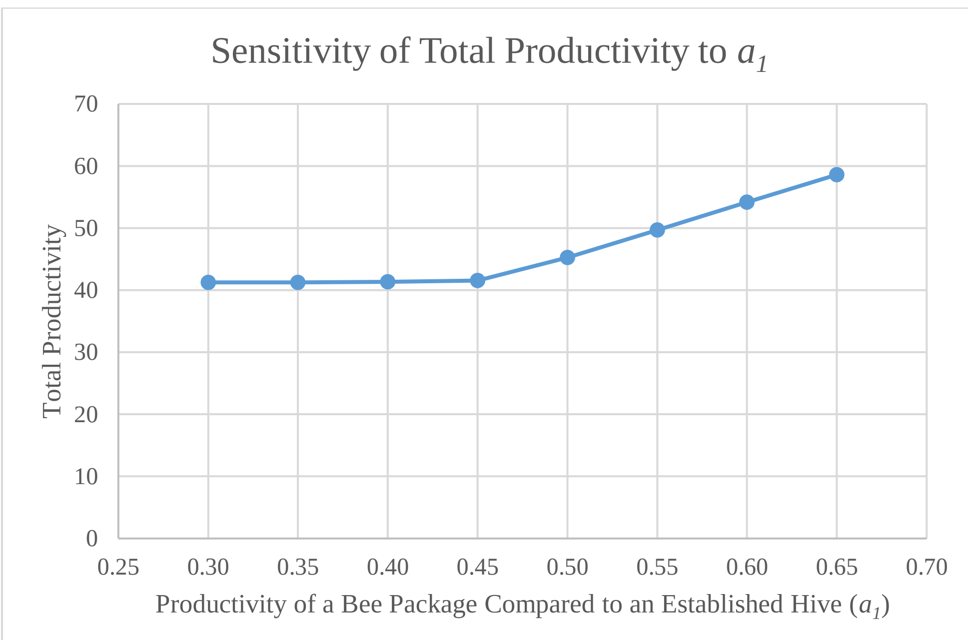

Figure 21. Graph of Total Productivity vs. Productivity of a Bee Package Compared to an Established Hive (a_1)

The sixth and last parameter tested is the productivity of nucleus colonies compared to an established hive, a_2. Figure 22 shows a switch from package colonies to nucleus colonies between an a_2 value of 0.8 and 0.85. This is expected since an increasing productivity of the nucleus colonies makes them more appealing than package colonies. It is interesting to note that above an a_2 value of 0.8, the total number of hives is less than that below 0.8. This is due to the higher cost of nucleus colonies compared to package colonies.

Table 21. Values of x_1, x_2, y_1, and y_2 at Various Values of a_2

a_2	x_1	x_2	y_1	y_2
0.55	90	0	0	0
0.60	89	1	0	0
0.65	89	1	0	0
0.70	89	1	0	0
0.75	86	3	0	0
0.80	86	3	0	0
0.85	4	53	0	0
0.90	4	53	0	0

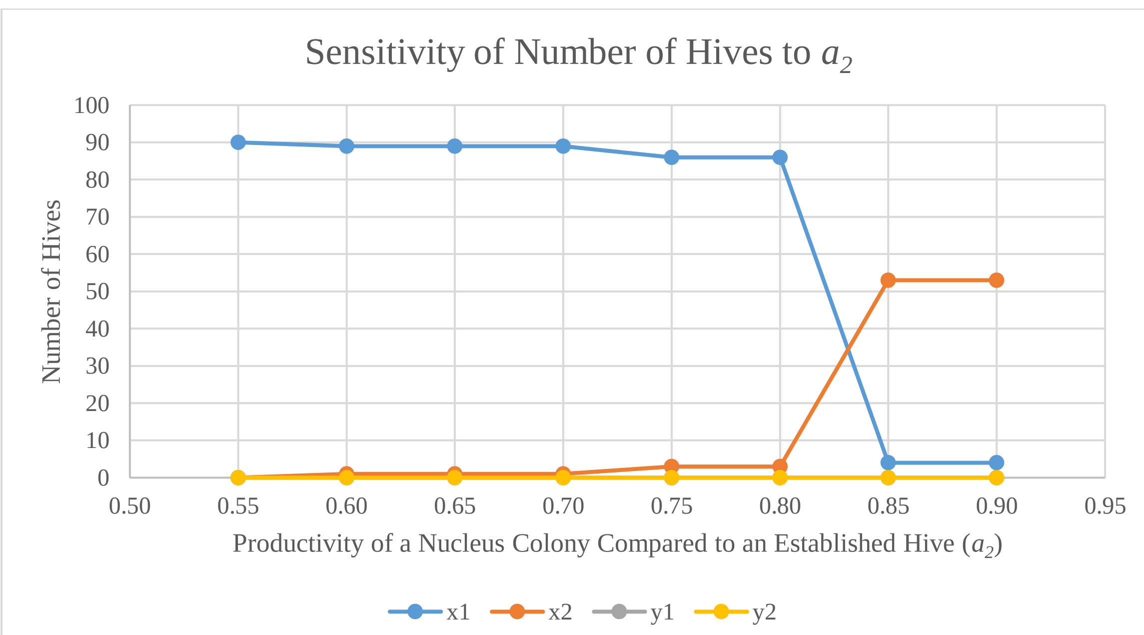

Figure 22. Graph of Number of Hives vs. Productivity of a Nucleus Colony Compared to an Established Hive (a_2)

Total productivity increases as nucleus colony productivity increases throughout multiple values of a_2 (Figure 23). Total productivity then takes a large jump upward after an a_2 of 0.8 due to the preference of nucleus colonies over package colonies at this point. The gradual trend upward before this is also expected since the model almost always has at least one nucleus colony except at the point where a_1 equal a_2.

Table 22. Values of *P* at Various Values of a_3

a_2	*P*
0.55	45
0.60	45.1
0.65	45.15
0.70	45.2
0.75	45.25
0.80	45.4
0.85	47.05
0.90	49.7

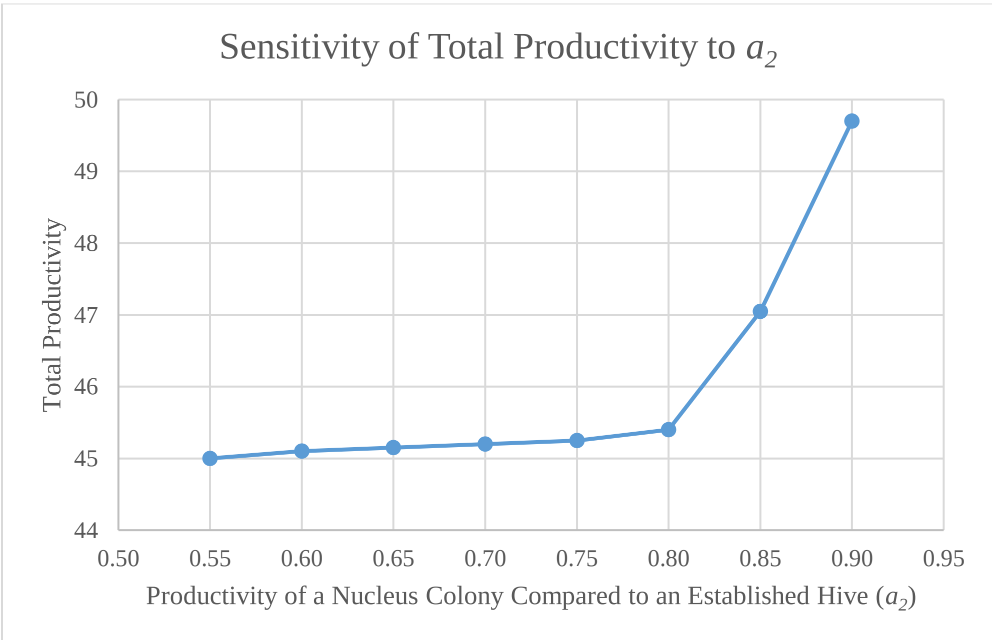

Figure 23. Graph of Total Productivity vs. Productivity of a Nucleus Colony Compared to an Established Hive (a_2)

2.5 Combining the Models

To incorporate the possibility of swarms into the nucleus colony model, a simple change in the maximum number of empty hives available, x_{max}, occurs. As seen in section 2.3, a new $\tilde{x}_{max}$ is found by subtracting the hives dedicated to catching swarms, z, from the previous x_{max}. The new model appears as the following:

$$\max P = a_1x_1 + a_2x_2 + a_1y_1 + a_2y_2$$

$$c_1x_1 + c_2x_2 + c_3y_1 + c_4y_2 \ \leq B$$

Such that:

$$x_1 + x_2 \leq \ \tilde{x}_{max}$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$y_1 \geq 0$$

$$y_2 \geq 0$$

Where:

$$\tilde{x}_{max} = \ x_{max} - z$$

$$z = N_{t-1}\, p$$

When using the same numerical example as previously, the values are:

$$x_{max} = 500 * 0.35 = 175\ hives$$

$$z = 325 * 0.25 = 81.25 \ \Rightarrow 81\ hives$$

$$\tilde{x}_{max} = \ 175 - 81 = 94\ hives$$

$$a_1 = 0.5$$

$$a_2 = 0.75$$

$$c_1 = \$110$$

$$c_2 = \$180$$

$$c_3 = \$110 + \$500 = \$610$$

$$c_4 = \$180 + \$500 = \$680$$

$$B = \$10{,}000$$

After solving this numerical analysis, the optimal solution becomes the following:

$$x_1 = 66\ hives$$

$$x_2 = 15\ hives$$

$$y_1 = 0\ hives$$

$$y = 0\ hives$$

$$P = 44.25\ effective\ hives$$

Each parameter has the same effect on the solution as it does in section 2.4. However, the graphs are shifted slightly due to the new $\tilde{x}_{max}$ reducing the total number of empty hives available to fill as seen in Figure 24 through Figure 37.

Table 23. Values of x_1, x_2, y_1, and y_2 at Various Values of c_1

c_1	x_1	x_2	y_1	y_2
40	33	48	0	0
50	36	45	0	0
60	39	42	0	0
70	42	39	0	0
80	46	35	0	0
90	51	30	0	0
100	55	25	0	0
110	66	15	0	0
120	75	5	0	0
130	2	54	0	0
140	2	54	0	0
150	3	53	0	0
160	4	52	0	0

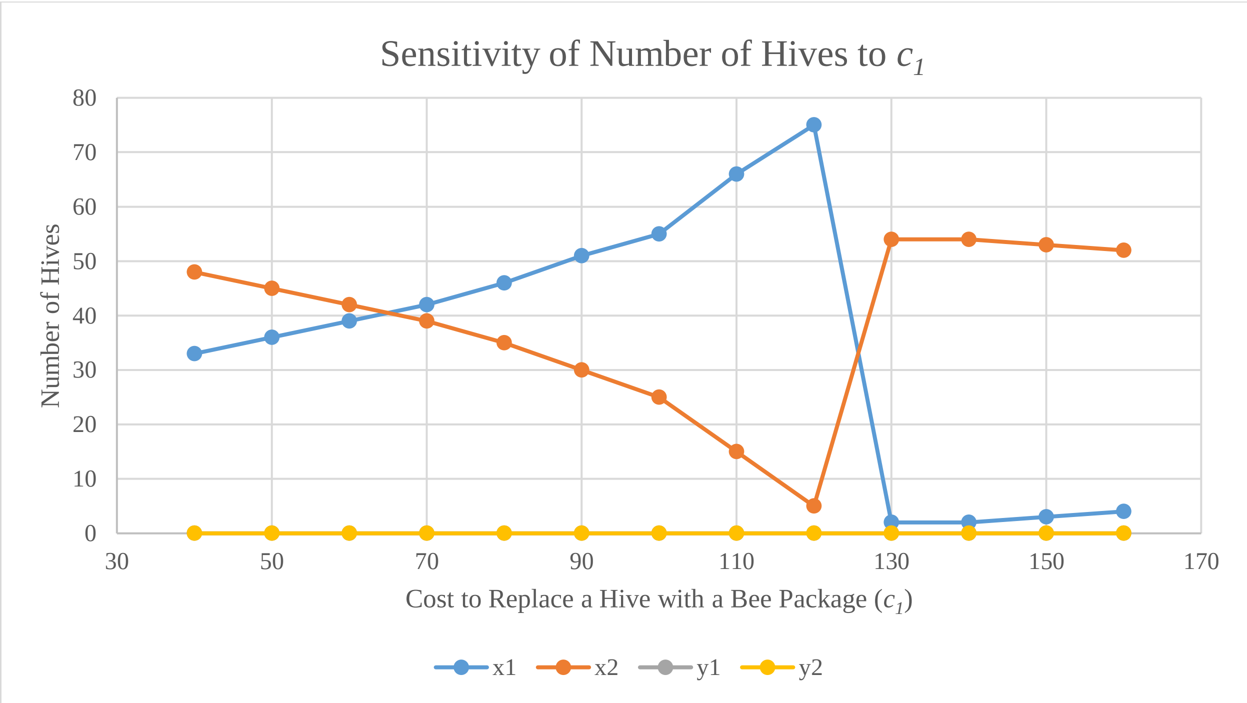

Figure 24. Graph of Number of Hives vs. Cost to Replace a Hive with a Bee Package (c_1)

Table 24. Values of *P* at Various Values of c_1

c_1	*P*
40	52.5
50	51.75
60	51
70	50.25
80	49.25
90	48
100	46.25
110	44.25
120	41.25
130	41.5
140	41.5
150	41.25
160	41

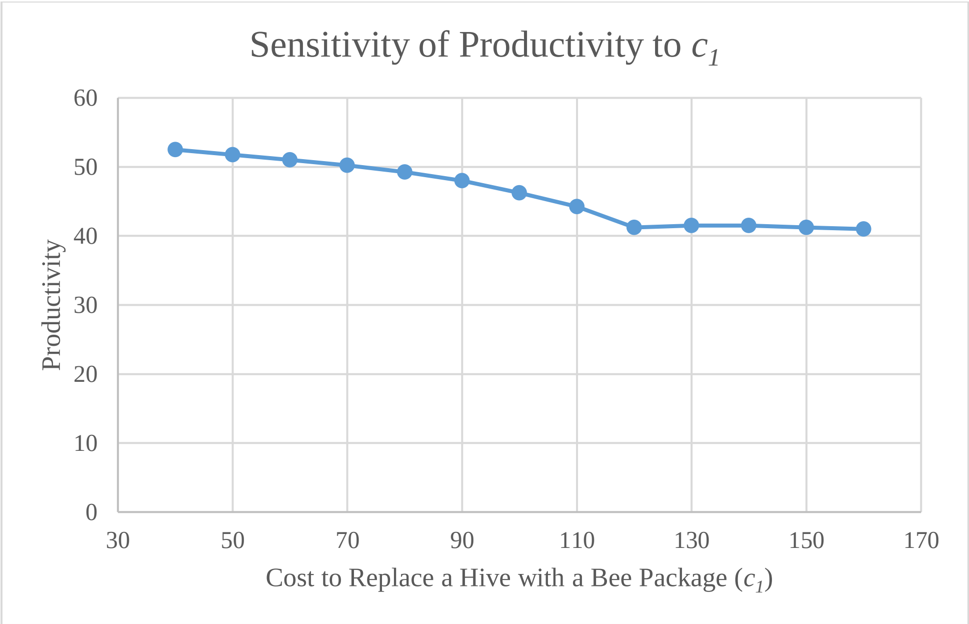

Figure 25. Graph of Productivity vs. Cost to Replace a Hive with a Bee Package (c_1)

Table 25. Values of x_1, x_2, y_1, and y_2 at Various Values of c_3

c_3	x_1	x_2	y_1	y_2
510	66	15	0	0
530	66	15	0	0
550	66	15	0	0
570	66	15	0	0
590	66	15	0	0
610	66	15	0	0
630	66	15	0	0
650	66	15	0	0

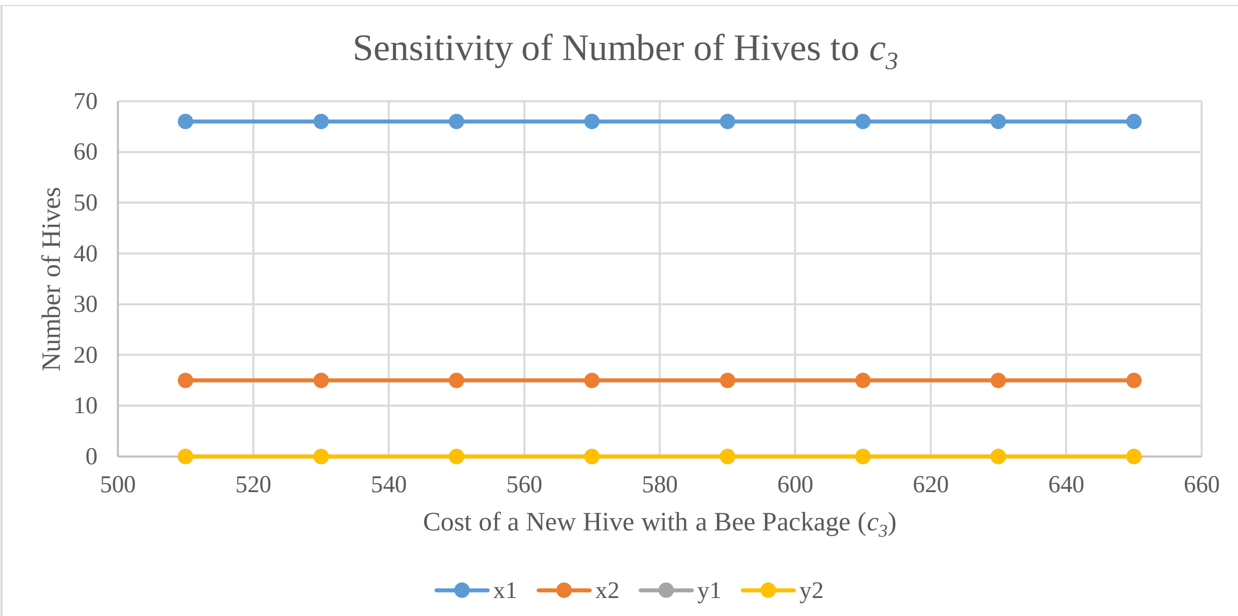

Figure 26. Graph of Number of Hives vs. Cost of a New Hive with a Bee Package (c_3)

Table 26. Values of *P* at Various Values of c_3

c_3	P
510	44.25
530	44.25
550	44.25
570	44.25
590	44.25
610	44.25
630	44.25
650	44.25

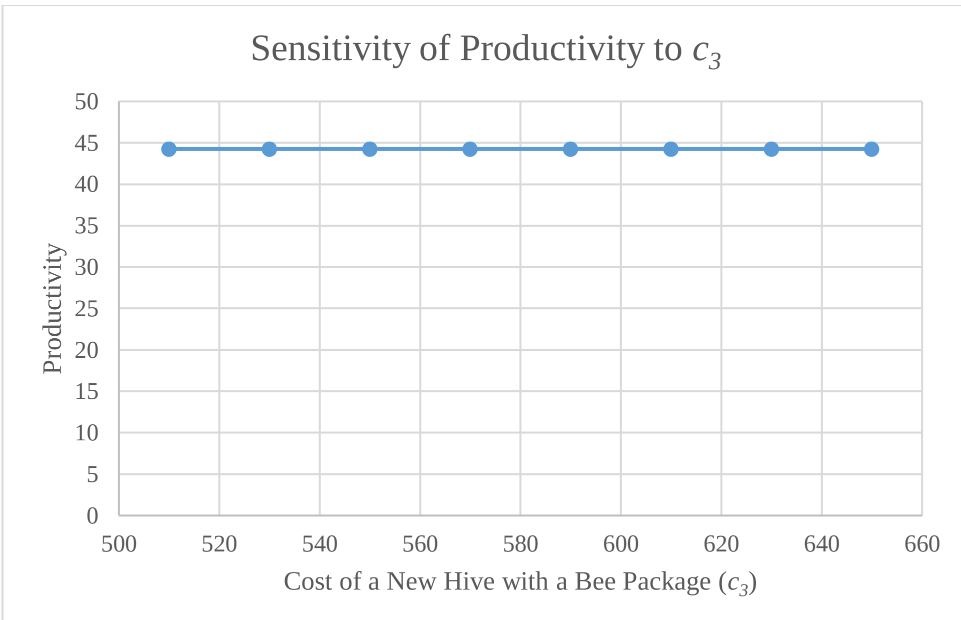

Figure 27. Graph of Productivity vs. Cost of a New Hive with a Bee Package (c_3)

Table 27. Values of x_1, x_2, y_1, and y_2 at Various Values of c_2

c_2	x_1	x_2	y_1	y_2
110	0	81	0	1
120	0	81	0	0
130	1	76	0	0
140	0	71	0	0
150	5	63	0	0
160	2	61	0	0
170	60	20	0	0
180	66	15	0	0
190	65	15	0	0
200	69	12	0	0

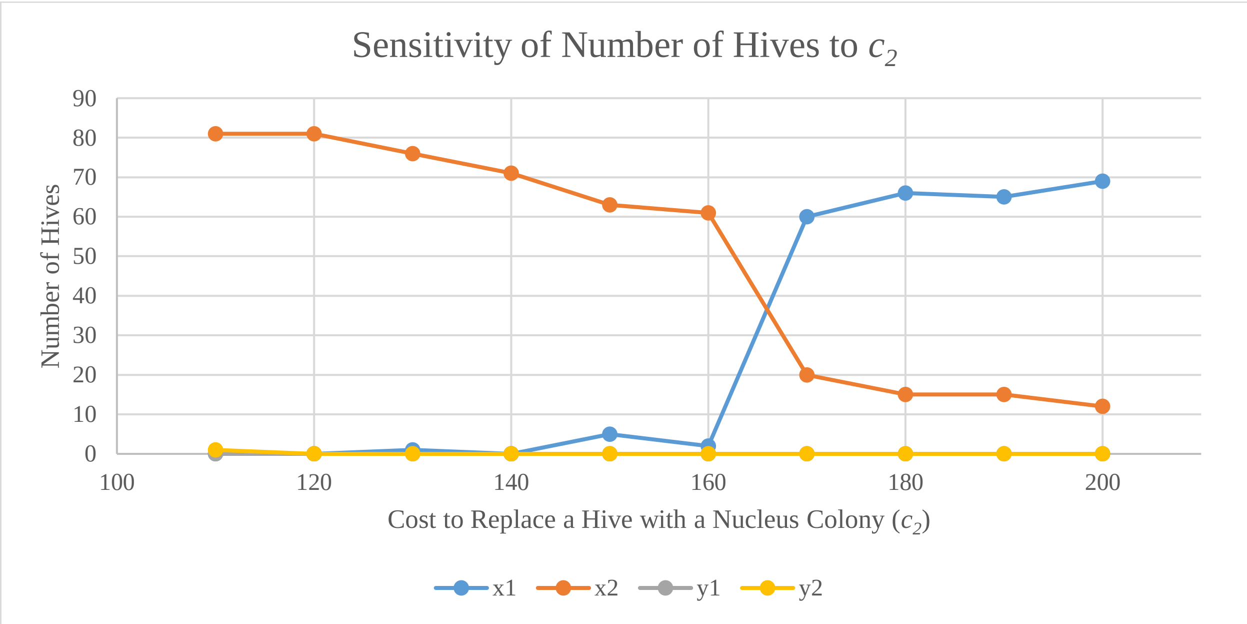

Figure 28. Graph of Number of Hives vs. Cost to Replace a Hive with a Nucleus Colony (c_2)

Table 28. Values of P at Various Values of c_2

c_2	P
110	61.5
120	60.75
130	57.5
140	53.25
150	49.75
160	46.75
170	45
180	44.25
190	43.75
200	43.5

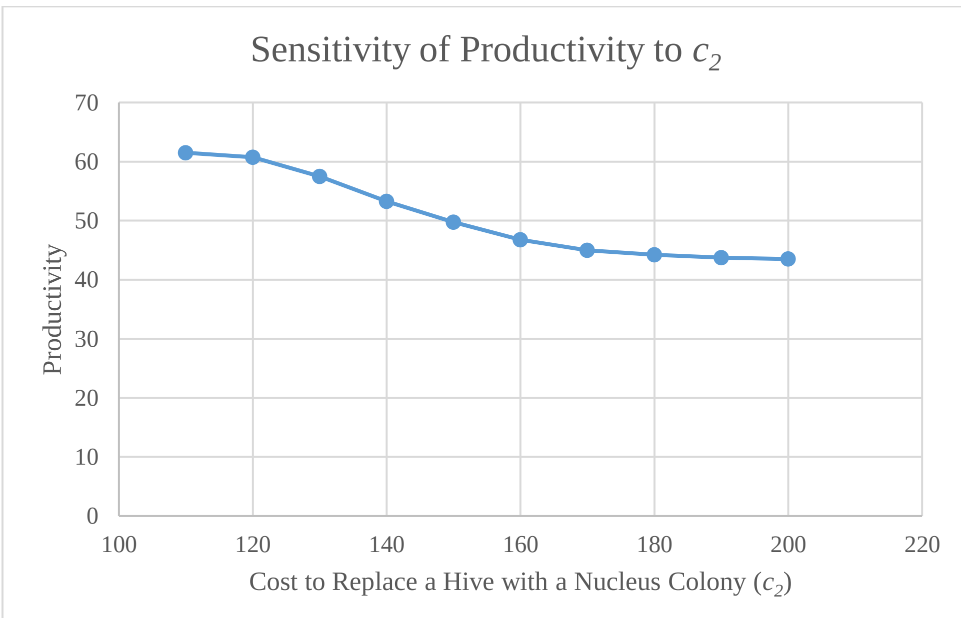

Figure 29. Graph of Productivity vs. Cost to Replace a Hive with a Nucleus Colony (c_2)

Table 29. Values of x_1, x_2, y_1, and y_2 at Various Values of c_4

c_4	x_1	x_2	y_1	y_2
580	66	15	0	0
600	66	15	0	0
620	66	15	0	0
640	66	15	0	0
660	66	15	0	0
680	66	15	0	0
700	66	15	0	0
720	66	15	0	0

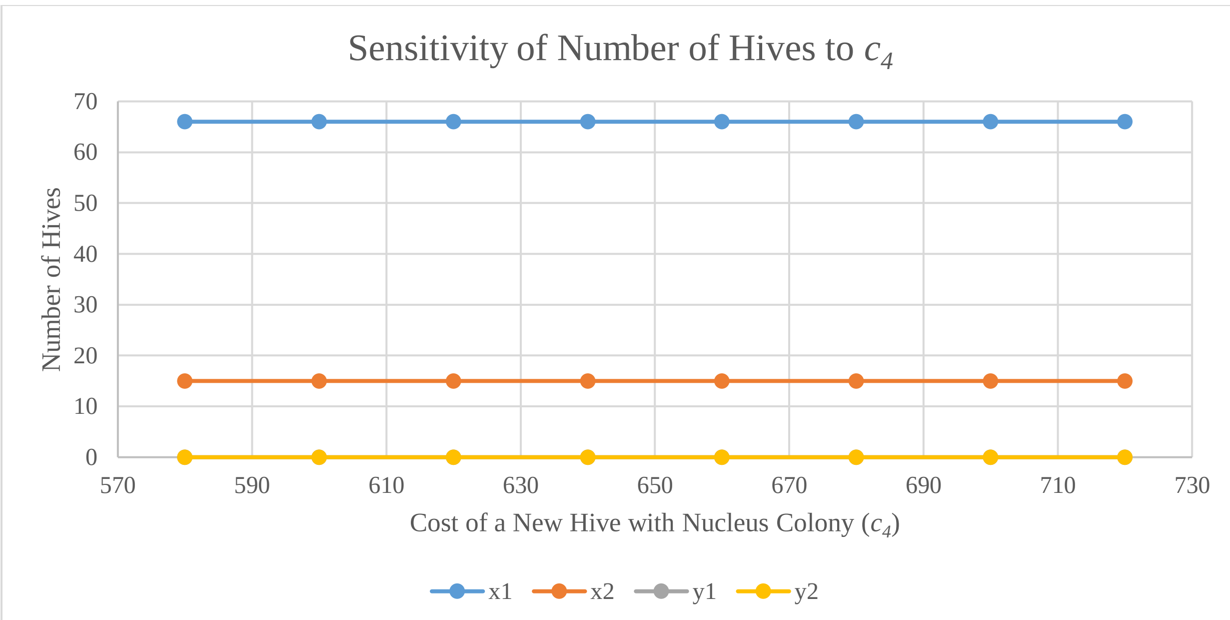

Figure 30. Graph of Number of Hives vs. Cost to of a New Hive with Nucleus Colony (c_4)

Table 30. Values of *P* at Various Values of c_4

c_4	P
580	44.25
600	44.25
620	44.25
640	44.25
660	44.25
680	44.25
700	44.25
720	44.25

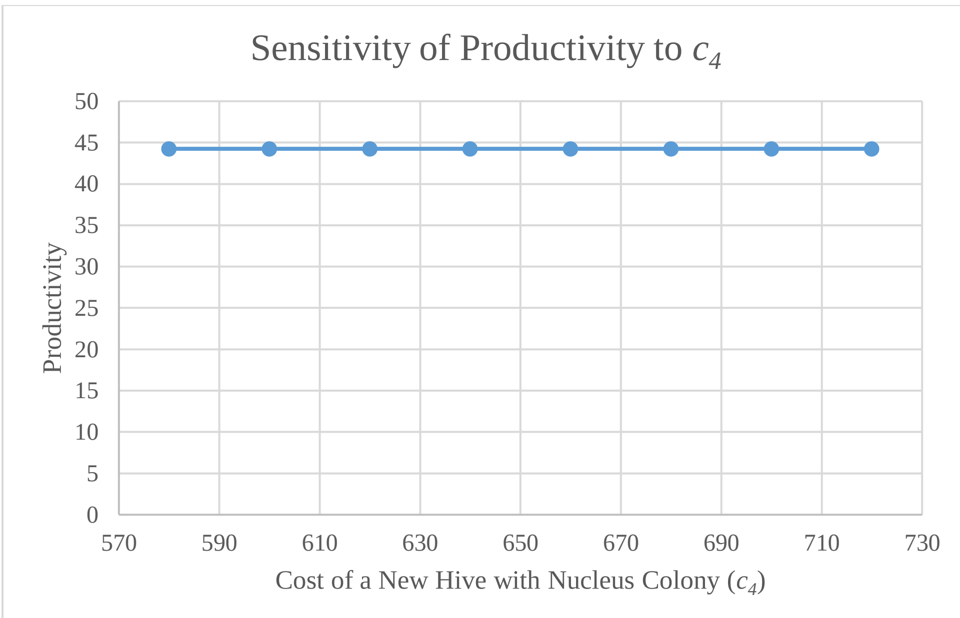

Figure 31. Graph of Productivity vs. Cost of a New Hive with Nucleus Colony (c_4)

Table 31. Values of x_1, x_2, y_1, and y_2 at Various Values of B

B	x_1	x_2	y_1	y_2
5,000	45	0	0	0
7,500	60	5	0	0
10,000	66	15	0	0
15,000	0	81	0	0
20,000	0	81	1	7
25,000	0	81	0	15

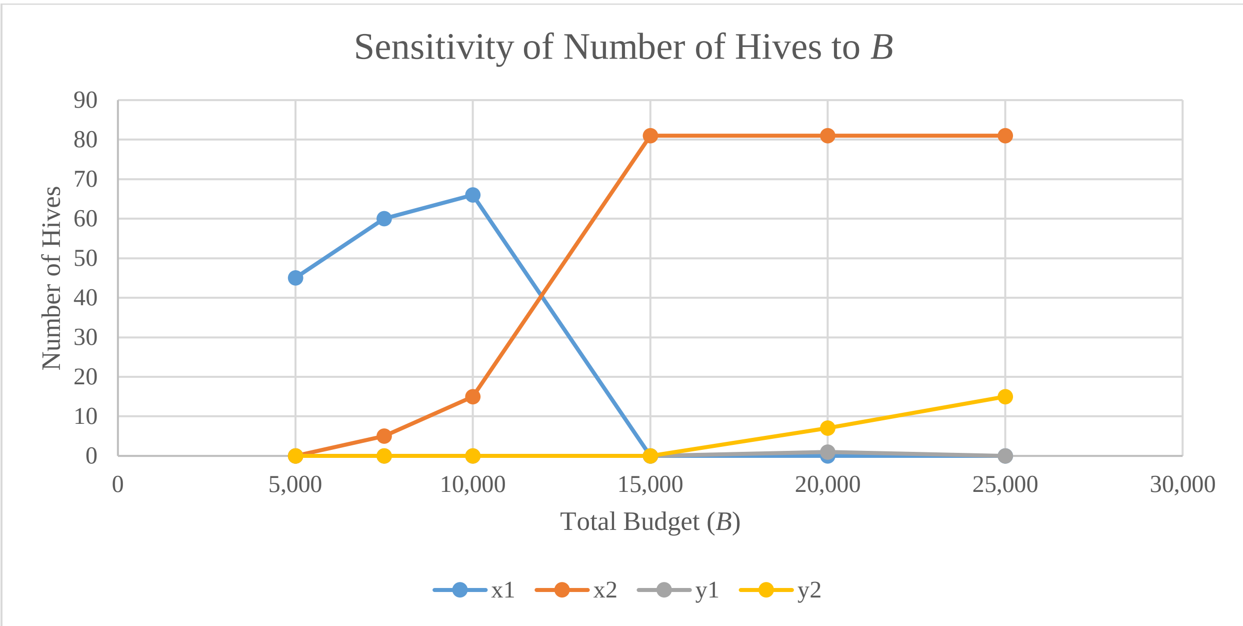

Figure 32. Graph of Number of Hives vs. Total Budget (B)

Table 32. Values of P at Various Values of B

B	P
5,000	22.5
7,500	33.75
10,000	44.25
15,000	60.75
20,000	66.5
25,000	72

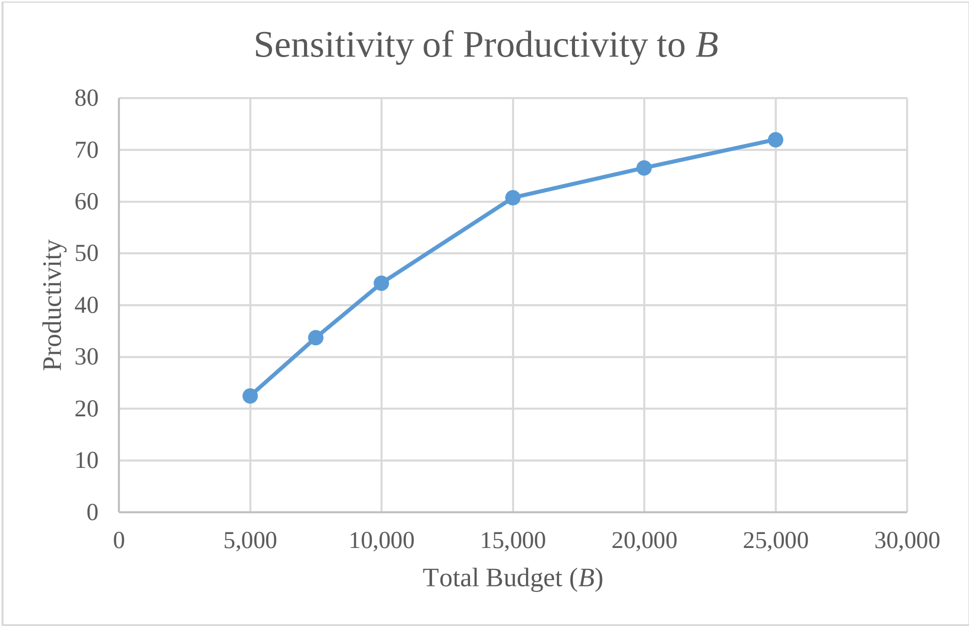

Figure 33. Graph of Productivity vs. Total Budget (*B*)

Table 33. Values of x_1, x_2, y_1, and y_2 at Various Values of a_1

a_1	x_1	x_2	y_1	y_2
0.30	0	55	0	0
0.35	0	55	0	0
0.40	4	53	0	0
0.45	14	47	0	0
0.50	66	15	0	0
0.55	66	15	0	0
0.60	66	15	0	0
0.65	66	15	0	0

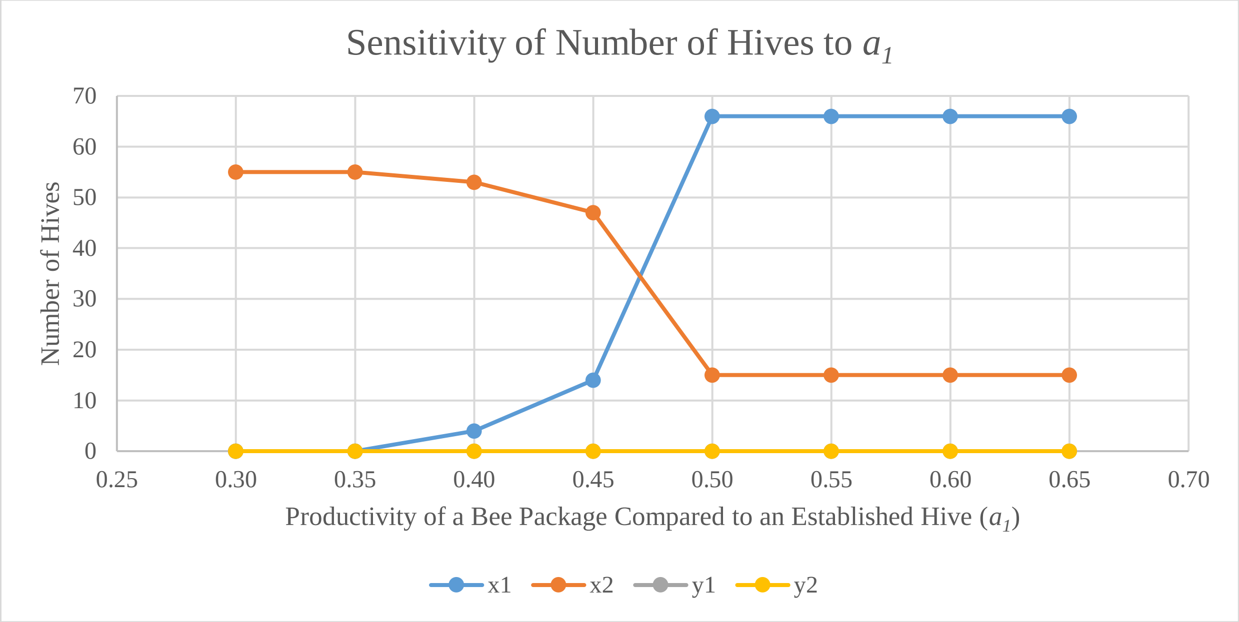

Figure 34. Graph of Number of Hives vs. Productivity of a Bee Package Compared to an Established Hive (a_1)

Table 34. Values of P at Various Values of a_1

a_1	P
0.30	41.25
0.35	41.25
0.40	41.35
0.45	41.55
0.50	44.25
0.55	47.55
0.60	50.85
0.65	54.15

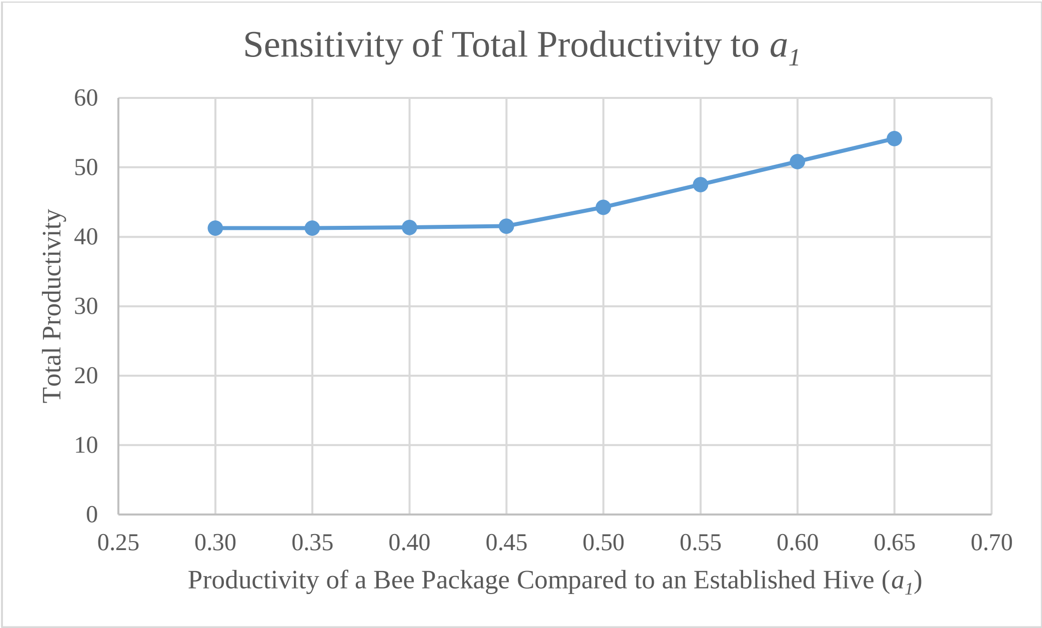

Figure 35. Graph of Total Productivity vs. Productivity of a Bee Package Compared to an Established Hive (a_1)

Table 35. Values of x_1, x_2, y_1, and y_2 at Various Values of a_2

a_2	x_1	x_2	y_1	y_2
0.55	76	5	0	1
0.60	66	15	0	0
0.65	66	15	0	0
0.70	66	15	0	0
0.75	66	15	0	0
0.80	63	17	0	0
0.85	4	53	0	0
0.90	4	53	0	0

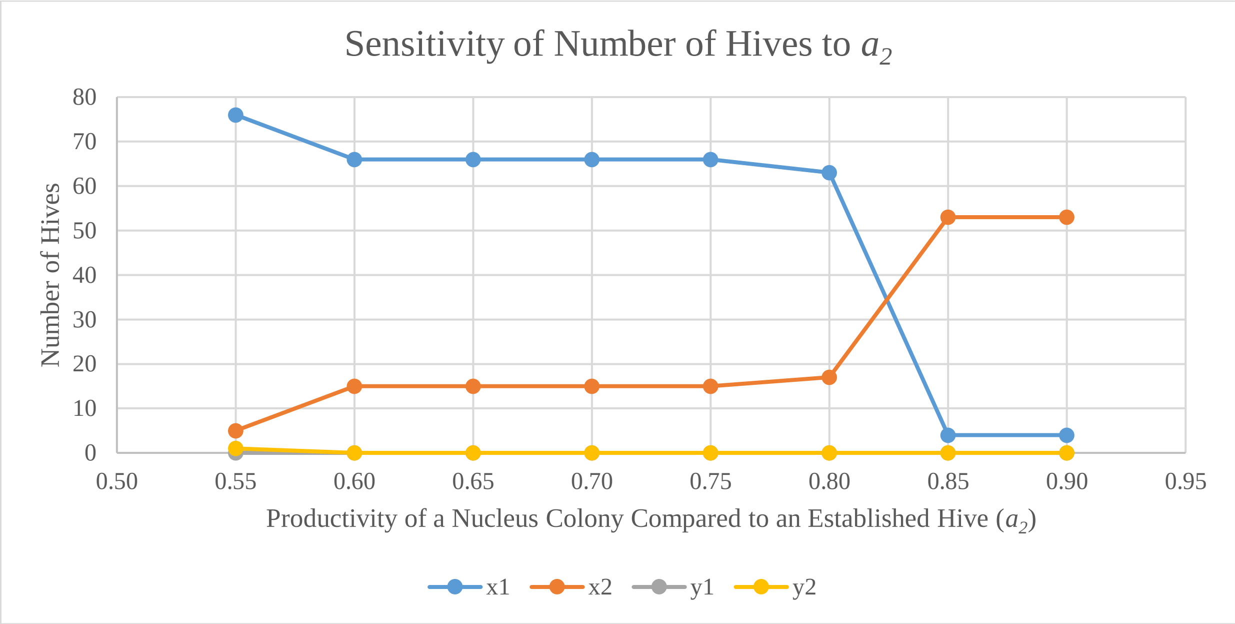

Figure 36. Graph of Number of Hives vs. Productivity of a Nucleus Colony Compared to an Established Hive (a_2)

Table 36. Values of P at Various Values of a_2

a_2	P
0.55	41.3
0.60	42
0.65	42.75
0.70	43.5
0.75	44.25
0.80	45.1
0.85	47.05
0.90	49.7

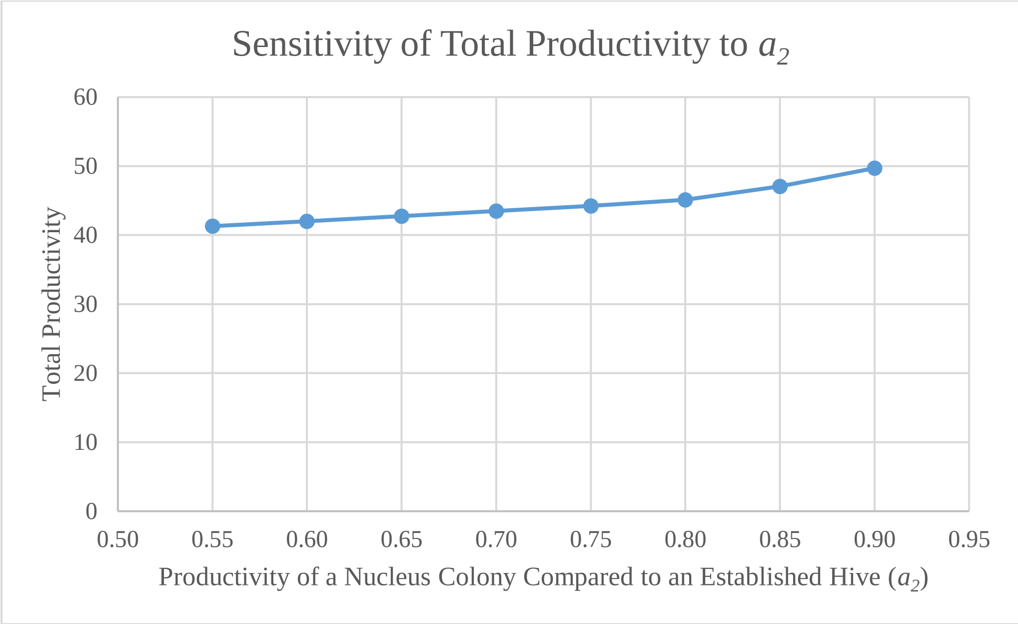

Figure 37. Graph of Total Productivity vs. Productivity of a Nucleus Colony Compared to an Established Hive (a_2)

Chapter 3 : Optimal Allocation of Honey to Products

3.1 Problem Description

After a beekeeper harvests honey, a decision must be made to determine how to divide the raw material into finished products. Honey can be sold in various different quantities and containers depending on the market the beekeeper is trying to target. In general, customers can be separated into two categories: end consumers and businesses. End consumers consist of anyone who purchases the honey for their own consumption and usually prefer smaller quantities. Businesses purchase honey in bulk to be used in manufacturing or in restaurants. Each of these customer types prefers a differently packaged product even if the raw material inside remains the same.

End consumers generally purchase honey from a store, farmer’s market, or similar location where the honey is on display. For a product that will be sitting out on display, aesthetics are much more important than when selling in bulk. The material and shape of the container are considered more thoroughly since they act as a marketing tool for the beekeeper. The most common materials used for containers are plastic and glass as these are both easily purchased in bulk from a supplier. Glass is usually seen as more desirable by the end consumer as it offers a quality of luxury, but costs the beekeeper more to source. Additionally, novelty container shapes such as bears, hexagons, or honey comb often attract end consumers but these more complex shapes also increase costs. The beekeeper can charge a premium for honey sold in smaller quantities as long as the container is higher quality or more luxurious.

Customers interested in purchasing honey in bulk tend to care about a better deal on honey than the aesthetics of the package it arrives in. Plastic is most common for bulk purchases as it is cheaper and also easier to transport. However, as a result the beekeeper cannot charge as much per unit weight for the honey. This results in a balancing formula incorporating the demand of various products with the supply of each.

The problem is modeled by trying to maximize total profit, P. Total profit equals the sum of profit per unit i, p_i, times the supply of product i, x_i, less the holding cost of producing anything above the demand. The supply of product i, x_i, is the same as the amount of honey dedicated to product i, z_i, divided by the amount of honey per product i, b_i. The holding cost equals the hold cost per product i, q_i, times the difference of unit demand, d_i, and supply of unit i, x_i. The main constraint is that sum of z_i for all products i has to be less than or equal to the total amount of honey produced, Z.

3.2 Mathematical Formulation

$$\max P = \sum_{i=1}^{I}\left(p_i * \frac{z_i}{b_i} - q_i\left(\frac{z_i}{b_i} - d_i\right)\right)$$

Constraints:

$$\sum_{i} z_i \leq Z$$

$$0 \leq z_i \leq \frac{d_i}{b_i}$$

P = Total profit

p_i = Profit per unit of product i

q_i = Hold cost per unit of product i

d_i = Demand of product i

x_i = Supply of product i

z_i = Amount of honey dedicated to product i

Z = Total amount of honey produced

b_i = Amount of honey per product *i*

Where:

$$x_i = \frac{z_i}{b_i}$$

3.2.1 Numerical Example

To test this model, an example of three different honey products is created. Honey is sold by weight, and the beekeeper sells an 8-ounce jar, a 16-ounce jar, and an 80-ounce tub. The beekeeper produced 20,000 lbs. of honey over the course of a year and is deciding how to divide it into the three products. The profit per unit is the following: \$2.50 for an 8-ounce jar, \$4.00 for a 16-ounce jar, and \$9.00 for an 80-ounce tub. The demand for the three different product sizes for the year is as follows: 4,000 8-ounce jars, 5,000 16-ounce jars, and 3,000 80-ounce tubs.

$$Z = 20{,}000\ lbs$$

$$p_1 = \$2.50$$

$$p_2 = \$4.00$$

$$p_3 = \$9.50$$

$$d_1 = 4{,}000\ units$$

$$d_2 = 5{,}000\ units$$

$$d_3 = 3{,}000\ units$$

$$b_1 = 8\ ounces$$

$$b_2 = 16\ ounces$$

$$b_3 = 80\ ounces$$

Given this data, the beekeeper should produce up to the demand of the highest profit per unit weight first. This is found by dividing profit per unit, p_i, by amount of honey per product, b_i. Product 1 has a profit of $0.3125/ounce, product 2 has a profit of $0.25/ounce, and product 3 has a profit of $0.1125/ounce. As a result, the beekeeper should produce 4,000 units of x_1, 5,000 units of x_2, and 2600 units of x_3 with 0 ounces of honey remaining. This results in a total profit, P, of $54,700.

3.2.2 Sensitivity Analysis

The following sensitivity analysis tests *p1*, *p2*, *p3*, *d1*, *d2*, and *d3* compared to the decision variables (Units of Supply) *x1*, *x2*, *x3*, as well as total Profit, *P*.

The first parameter tested was the profit per unit of product 1, p_1. As seen in Figure 38, the solution remains fairly stable until p_1 decreases below $1.00 per unit. At this point, the profit per ounce of product 1 falls below the profit per ounce of product 3 and the solution switches over to allocating honey to product 3. The solution for product 2 remains at 5000 units throughout any change in p_1.

Table 37. Values of x_1, x_2, and x_3 at Various Values of p_1

p_1	x_1	x_2	x_3
0.50	0	5000	3000
0.75	0	5000	3000
1.00	4000	5000	2600
1.25	4000	5000	2600
1.50	4000	5000	2600
1.75	4000	5000	2600
2.00	4000	5000	2600
2.25	4000	5000	2600
2.50	4000	5000	2600

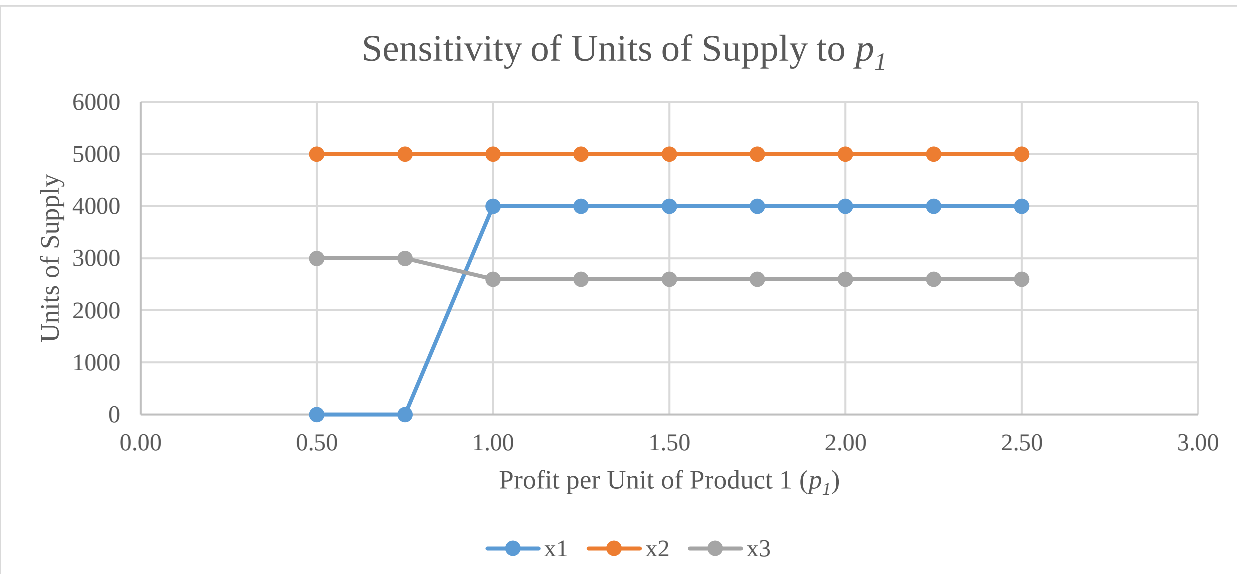

Figure 38. Graph of Units of Supply vs. Profit per Unit of Product 1 (p_1)

As p_1 decreases, the total profit decreases until it levels out Figure 39. This leveling out occurs when the model switches allocation of honey to product 3 from product 1.

Table 38. Values of P at Various Values of p_1

p_1	P
0.50	48500
0.75	48500
1.00	48700
1.25	49700
1.50	50700
1.75	51700
2.00	52700
2.25	53700
2.50	54700

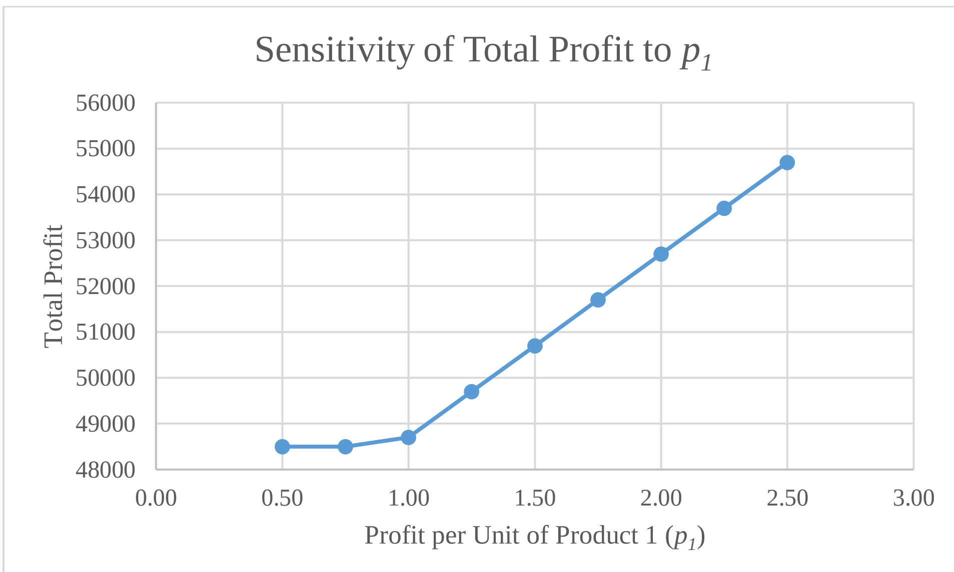

Figure 39. Graph of Total Profit vs. Profit per Unit of Product (p_1)

The second parameter tested was the profit per unit of product 2, p_2. Similar to the solution changes due to p_1, the solution remains stable until p_2 drops significantly. As seen in Figure 40, the model allocates more, but not all honey from product 2 to product 3 between a p_2 value of \$1.75 and \$2.00 per unit. After that point, the solution remains stable as the value of p_2 decreases further. Throughout any change in the profit per unit of product 2, the amount of honey allocated to product 1 does not change. Both the analyses for p_1 and p_2 show that honey allocation switches between the product whose profit is changing and product 3 but does not affect the other respective product.

Table 39. Values of x_1, x_2, and x_3 at Various Values of p_2

p_2	x_1	x_2	x_3
1.00	4000	3000	3000
1.25	4000	3000	3000
1.50	4000	3000	3000
1.75	4000	3000	3000
2.00	4000	5000	2600
2.50	4000	5000	2600
3.00	4000	5000	2600
3.50	4000	5000	2600
4.00	4000	5000	2600

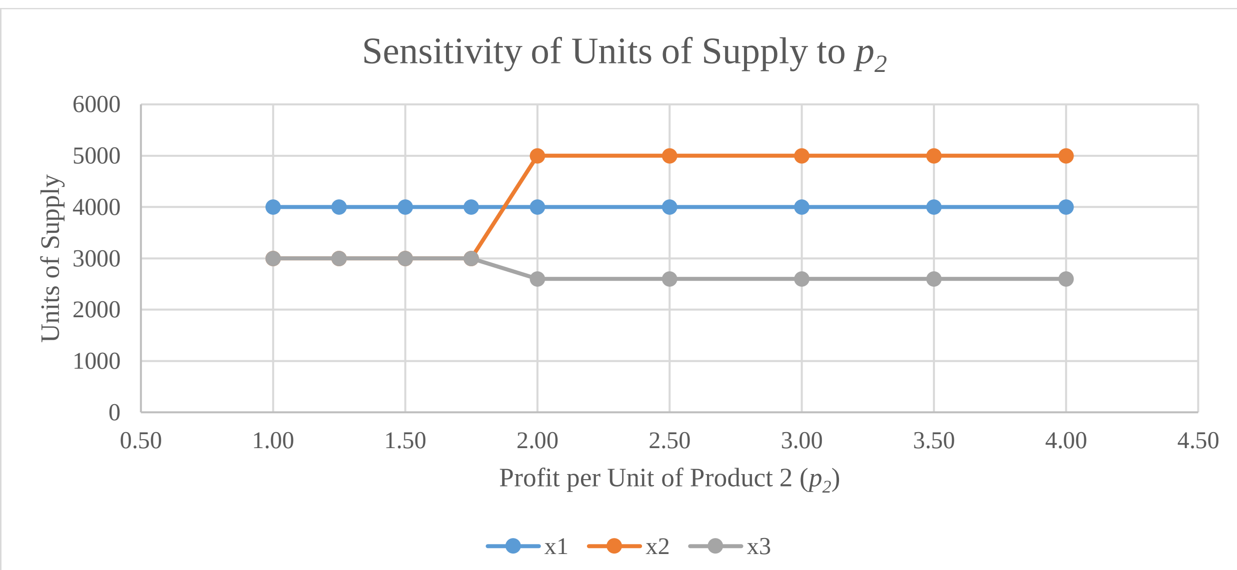

Figure 40. Graph of Units of Supply vs. Profit per Unit of Product 2 (p_2)

As the profit per unit of product 2, p_2, decreases, so does the total profit, P (Figure 41). There is a decrease in slope when the model allocates honey from product 2 to product 3 but not a complete leveling out. This results from a portion or honey being reallocated and not the whole initial amount for product 2.

Table 40. Values of *P* at Various Values of p_2

p_2	*P*
1.00	41500
1.25	42250
1.50	43000
1.75	43750
2.00	44700
2.50	47200
3.00	49700
3.50	52200
4.00	54700

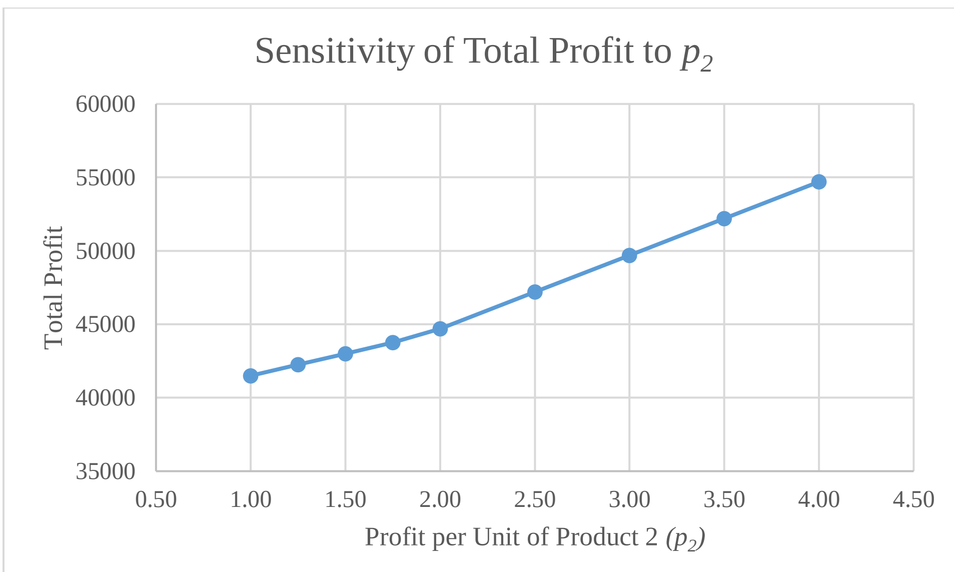

Figure 41. Graph of Total Profit vs. Profit per Unit of Product (p_2)

The third parameter tested was the profit per unit of product 3, p_3. Since product 3 received the honey left over after the demands for product 1 and 2 were filled, the profit per unit had to increase in order for a change in the solution to occur. Similar to the analyses for p_1 and p_2, the solution remained stable for a large range until p_3 reaches a specific point. As seen in Figure 42, this point is when p_3 doubles at a value between $20.00 and $20.50 profit per unit. At this point, honey is reallocated from product 2 to product 3. The solution for product 1 remains the same throughout any change in p_3.

Table 41. Values of x_1, x_2, and x_3 at Various Values of p_3

p_3	x_1	x_2	x_3
9.50	4000	5000	2600
11.00	4000	5000	2600
12.50	4000	5000	2600
14.00	4000	5000	2600
15.50	4000	5000	2600
17.00	4000	5000	2600
18.50	4000	5000	2600
20.00	4000	5000	2600
20.50	4000	3000	3000
21.00	4000	3000	3000

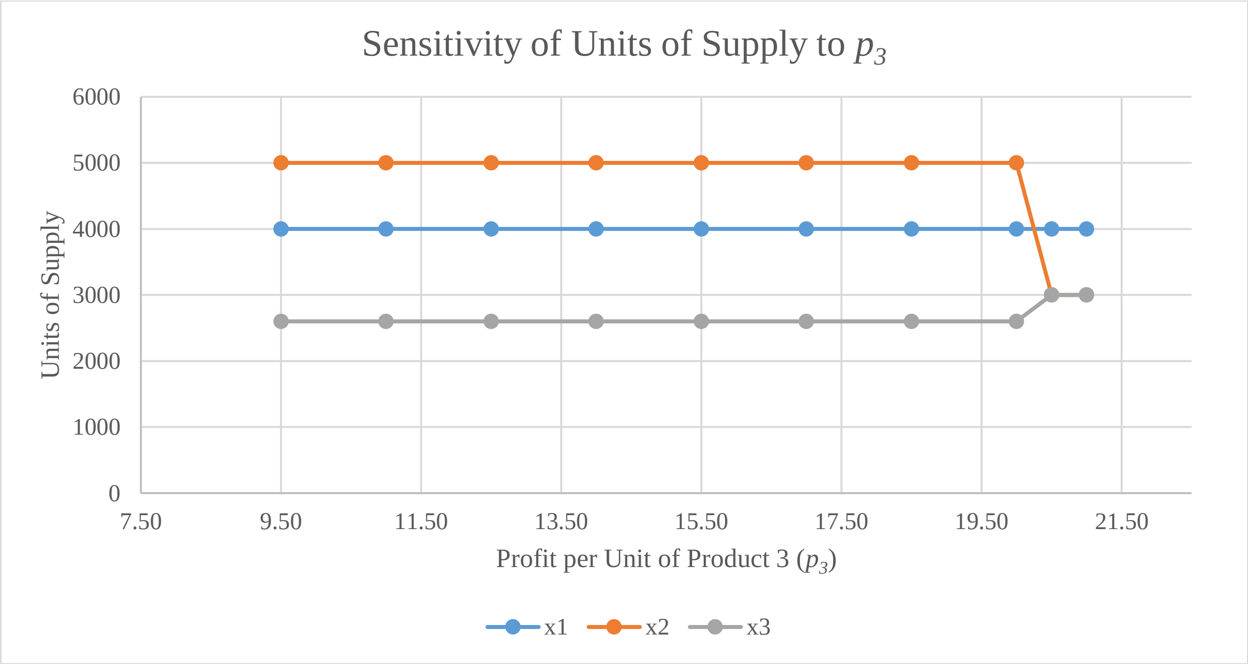

Figure 42. Graph of Units of Supply vs. Profit per Unit of Product 2 (p_3)

As the value of p_3 increases, the total profit also increases. At the reallocation point between \$20.00 and \$20.50 per unit, the total profit still increases at nearly the same amount.

Table 42. Values of *P* at Various Values of p_3

p_3	*P*
9.50	54700
11.00	58600
12.50	62500
14.00	66400
15.50	70300
17.00	74200
18.50	78100
20.00	82000
20.50	83500
21.00	85000

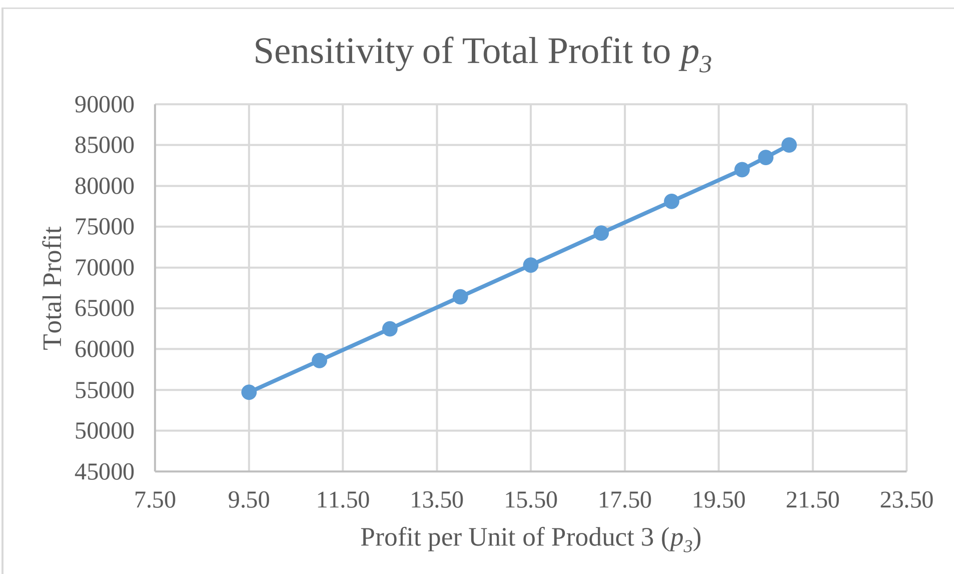

Figure 43. Graph of Total Profit vs. Profit per Unit of Product (p_3)

The fourth parameter tested was the demand of product 1, d_1. As d_1 changes, the solution changes to fulfil the demand of product 1 by reallocating honey from product 3 (Figure 44). The amount of honey allocated to product 2 remains the same for any change in d_1. This is due to the profits per unit weight of product 1 and product 2 are higher than that of product 3. Product 3 receives the honey left over after the demands for products 1 and 2 have been filled.

Table 43. Values of x_1, x_2, and x_3 at Various Values of d_1

d_1	x_1	x_2	x_3
2500	2500	5000	2750
3000	3000	5000	2700
3500	3500	5000	2650
4000	4000	5000	2600
4500	4500	5000	2550
5000	5000	5000	2500
5500	5500	5000	2450

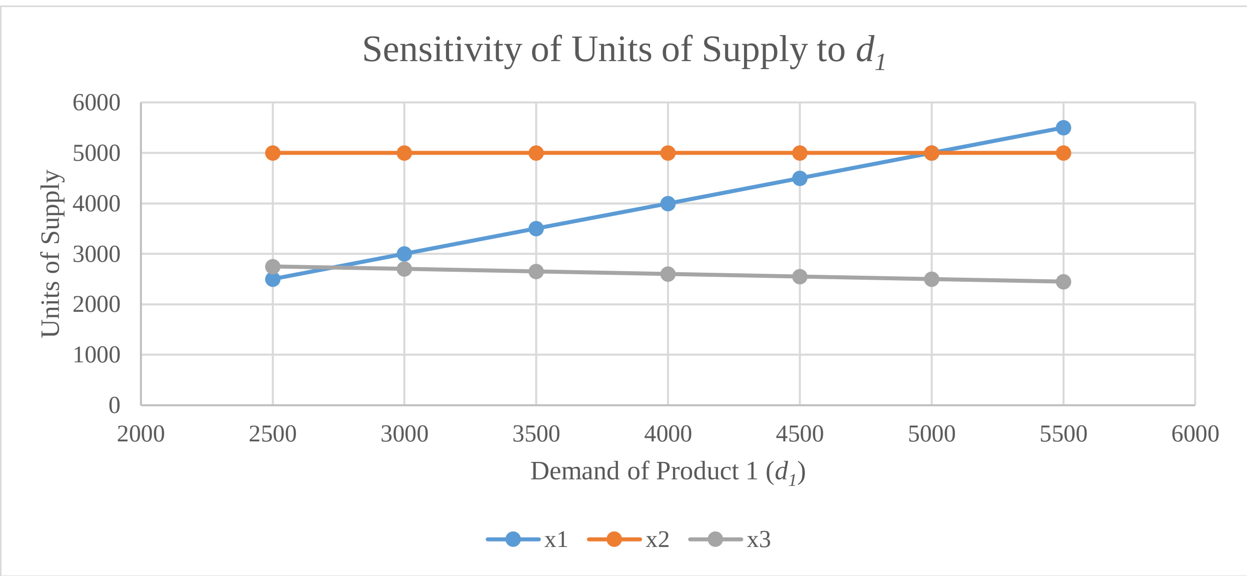

Figure 44. Graph of Units of Supply vs. Profit per Unit of Product 2 (d_1)

The relationship between total profit, *P*, and demand for product 1, d_1, is perfectly linear as seen in Figure 45. This results from the fact the model fills demand for product 1 at all values since product 1 has the best profit per unit weight of the three products.

Table 44. Values of *P* at Various Values of d_1

d_1	P
2500	52375
3000	53150
3500	53925
4000	54700
4500	55475
5000	56250
5500	57025

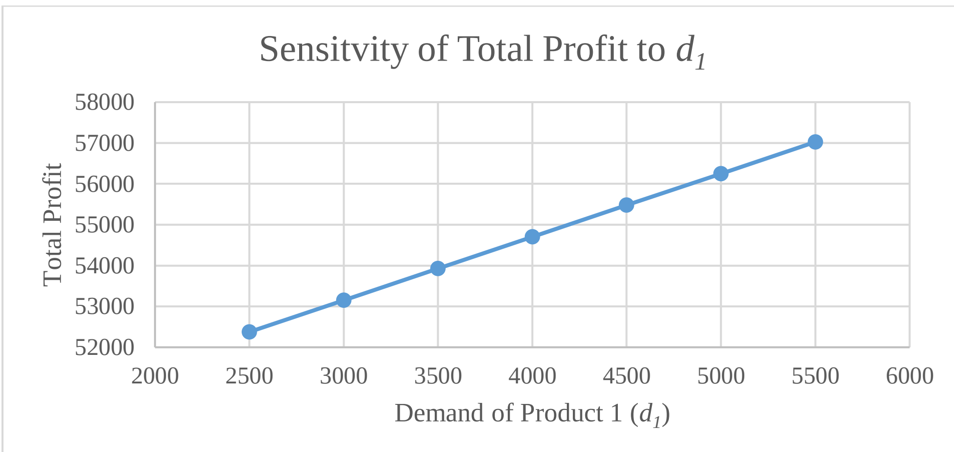

Figure 45. Graph of Total Profit vs. Profit per Unit of Product (d_1)

The fifth parameter tested was the demand for product 2, d_2. Similar to d_1, as d_2 changes, the solution changes to fill the entire demand of product 2 by reallocating honey from product 3 (Figure 46). Product 2 has a higher profit per unit weight than product 3 which is why it is preferred. The solution for product 1 remains the same since its profit per unit weight is the highest of the three.

Table 45. Values of x_1, x_2, and x_3 at Various Values of d_2

d_2	x_1	x_2	x_3
3500	4000	3500	2900
4000	4000	4000	2800
4500	4000	4500	2700
5000	4000	5000	2600
5500	4000	5500	2500
6000	4000	6000	2400
6500	4000	6500	2300

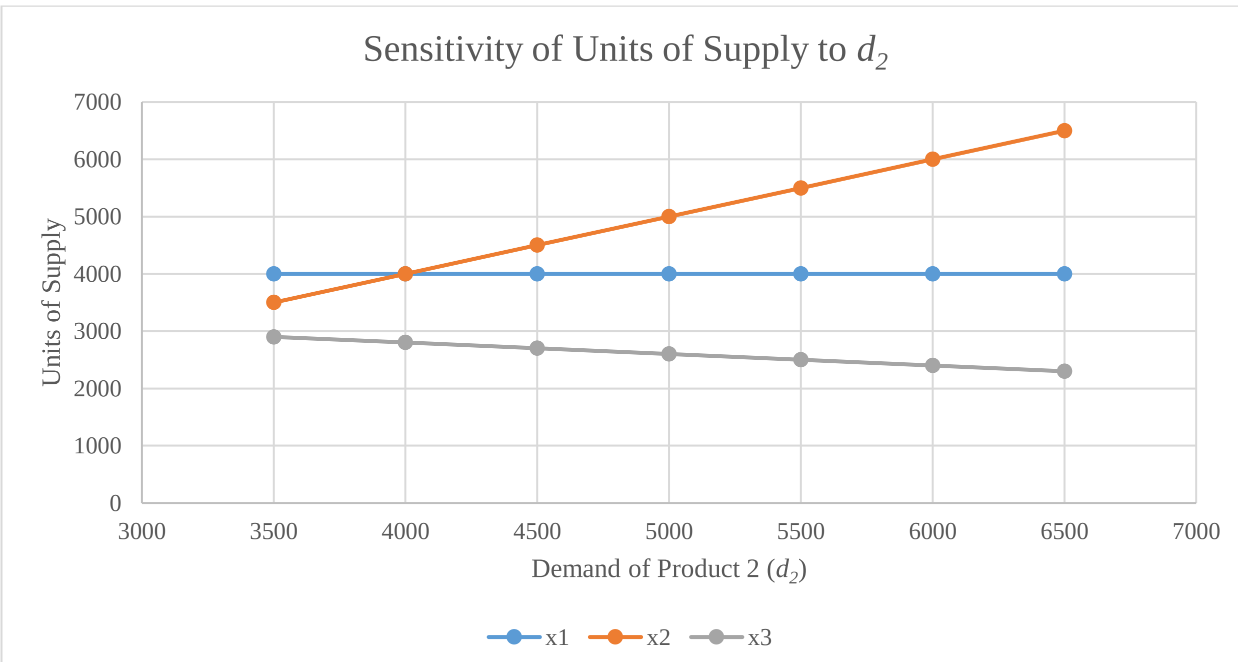

Figure 46. Graph of Units of Supply vs. Profit per Unit of Product 2 (d_2)

Similar to the relation between d_1 and total profit, the relation between d_2 and total profit is perfectly linear (Figure 47). As d_2 changes, the model fits the solution to fill demand for product 2. It does this by reallocating honey from a less profitable product to a more profitable product without wasting any honey through over allocation.

Table 46. Values of *P* at Various Values of d_2

d_2	P
3500	51550
4000	52600
4500	53650
5000	54700
5500	55750
6000	56800
6500	57850

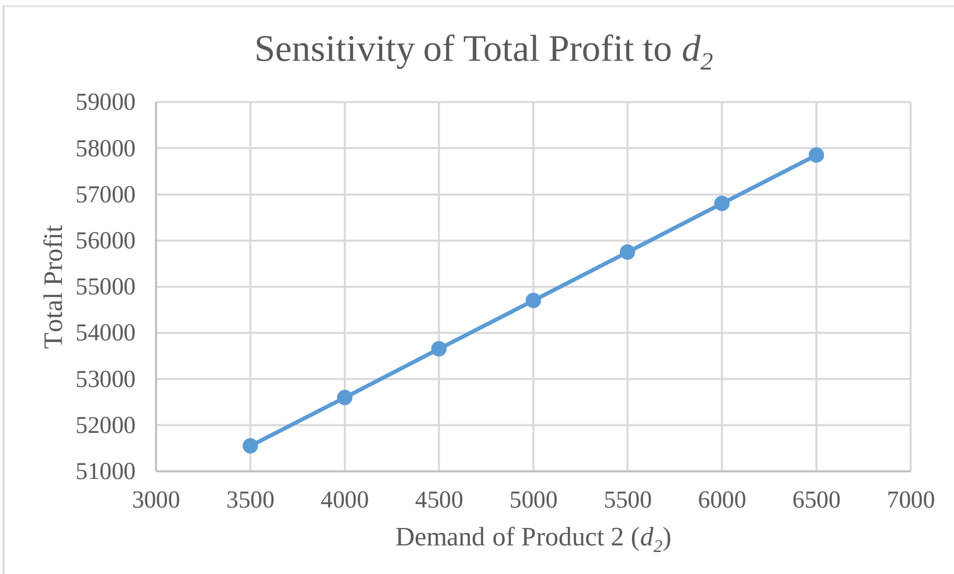

Figure 47. Graph of Total Profit vs. Profit per Unit of Product (d_2)

The sixth and last parameter tested was demand for product 3, d_3. Since product three is the least profitable per unit weight, it does not fill demand in the initial solution. As a result, any increase in demand for product 3 will not result in a change of the solution. There is never any reallocation of honey from another product to product 3. When the demand decreases, the new solution decreases with it (Figure 48). The resulting solutions have left over raw product (honey) since the demands for the three products are filled.

Table 47. Values of x_1, x_2, and x_3 at Various Values of d_3

d_3	x_1	x_2	x_3
1500	4000	5000	1500
2000	4000	5000	2000
2500	4000	5000	2500
3000	4000	5000	2600
3500	4000	5000	2600
4000	4000	5000	2600
4500	4000	5000	2600

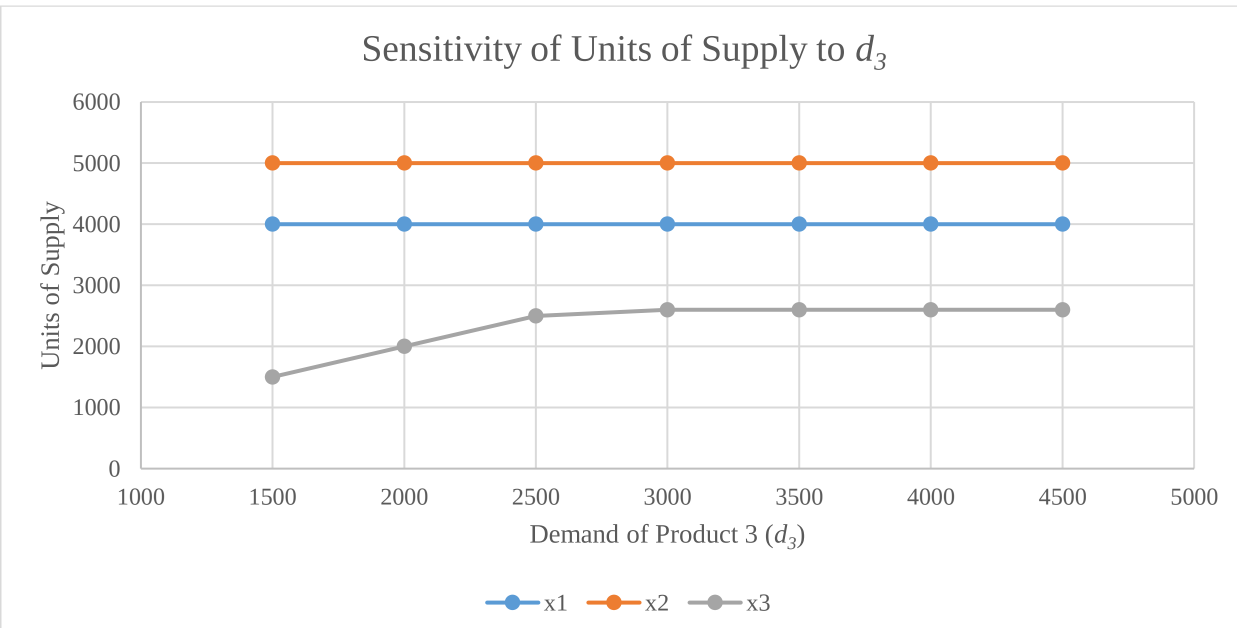

Figure 48. Graph of Units of Supply vs. Profit per Unit of Product 2 (d_3)

As demand for product 3, d_3, increases from the initial solution, there is no change to the total profit. This is assumed since the solution does not change for this situation. When the demand decreases, the total profit steeply declines since less total product is sold (Figure 49). This points to an optimum solution which first tries to sell off the raw product (honey) through allocating to the most profitable products first up to their respective demands.

Table 48. Values of *P* at Various Values of d_3

d_3	P
1500	44250
2000	49000
2500	53750
3000	54700
3500	54700
4000	54700
4500	54700

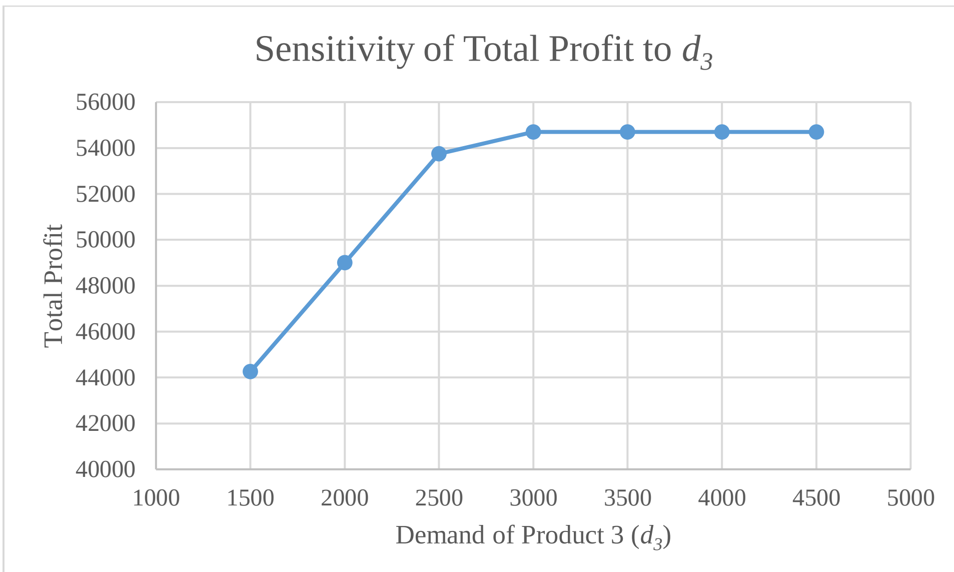

Figure 49. Graph of Total Profit vs. Profit per Unit of Product (d_3)

Chapter 4 : A Comprehensive Study from Optimal Hive Selection to Product Allocation

Using the solution found in section 2.5 and the model from section 3.2, the profitability of a beekeeping business with the initial parameters can be found. The numerical analyses tested a beekeeping business with 500 hives and a hive lost estimate of 35%. This left 325 established colonies entering the spring and 175 empty hives. Using a probability of swarming at 0.25, 81 empty hives are set aside to catch swarms. A bee package costs $110, a nucleus colony costs $180, and a new hive costs $500. Every bee package is estimated to have an efficiency of 0.5 compared to an established colony, and every nucleus colony is estimated to have an efficiency of 0.75. With a budget of $10,000, the optimal solution to maximize productivity is 66 bee packages and 15 nucleus colonies for the effective output of 44.25 hives. Since the swarms captured have to start a colony without any brood, their efficiency is estimated to be similar to that of a bee package. The 81 swarms captured then have an effective output of 40.5 hives. The established colonies from the previous year adds 325 effective hives to the total. Under these conditions, the beekeeper has an effective number of 409.75 hives. Using an estimate of 55 pounds of honey per hive per year (National Agricultural Statistics Service, 2018), the total estimated production is 22,536.25 pounds of honey.

This estimate can then be used with the parameters for the numerical analysis of section 3.2.1 to show profitability of the business. With three products offered with different profit margins and demands, the model in 3.2 allocates the honey in the most profitable manner. Product 1 has a size of 8 ounces, a profit of $2.50 per unit, and a demand of 4,000 units. Product 2 has a size of 16 ounces, a profit of $4.00 per unit, and a demand of 5,000 units. Product 3 has a size of 80 ounces, a profit of $9.50 per unit, and a demand of 3,000 units. The optimal solution for these parameters fills demand entirely at 4,000 units of product 1, 5,000 units of product 2, and 3,000 units of product 3 with 536.25 pounds left over. The total profit for this solution is $58,500 which is limited by the demand.

This solution provides insight into the beekeeping business and suggests different courses of action. The total number of new colonies introduced to the business is 162 (81 from swarms, 66 from bee packages, and 15 from nucleus colonies). Since the number of hives lost from the previous year was 175, the total number of hives decreased by 13. If the beekeeper wants to maintain the total number of hives at 500, a change has to be made. Either the budget can be increased or more effort can be put towards promoting swarms in the colonies.

That being said, demands for all the products were met. If the beekeeper were to put more effort into maintaining a larger number of hives, there should also be consideration as to where the extra honey can be sold. The beekeeper may need to spend time and money marketing or possibly expand into different types of products. Beeswax, pollen, propolis, and royal jelly can all be harvested from beehives at varying degrees of effort. Any of these products can add supplemental income for the beekeeper even when demands for honey are met. A beekeeper could also look to providing products for other beekeepers such as raising nucleus colonies. Not only would this decrease the cost filling empty colonies each year, it would also provide an income source not tied to production.

If a beekeeper decides to keep all aspects of the business the same, there may be a new optimal number of hives for the business to meet demand at the current budget. Even though the total number of hives decreases, the demand for honey is still met with a surplus. The beekeeper could naturally let the total number of hives decrease from year to year until it reaches an equilibrium point where the budget is able to fully replenish the hives lost each year.

A beekeeper could also look to find the optimal budget to spend this year in order to still maintain maximum profit. As seen in Figure 50, total profit doesn't begin to decrease until the budget drops below $8,000. This is because the total production needed to fill demand is 22,000 pounds. Table 49 shows that at a budget of $8,000, there is still a surplus of 96.25 pounds of honey after all product demands are filled. Using this information, a beekeeper could decrease budget by $2,000 and still meet demand for the year. However, the number of hives going into next year will be lower. A budget of $8,000 may not be sustainable to always meet the product demands, but for this one year it can.

Table 49. Values of Production Output (*Z*) and Profit at Decreasing Budgets (*B*)

B	*Z*	*Profit*
6000	21587.5	57715.3
7000	21848.75	58211.2
8000	22096.25	58500
9000	22343.75	58500
10000	22536.25	58500

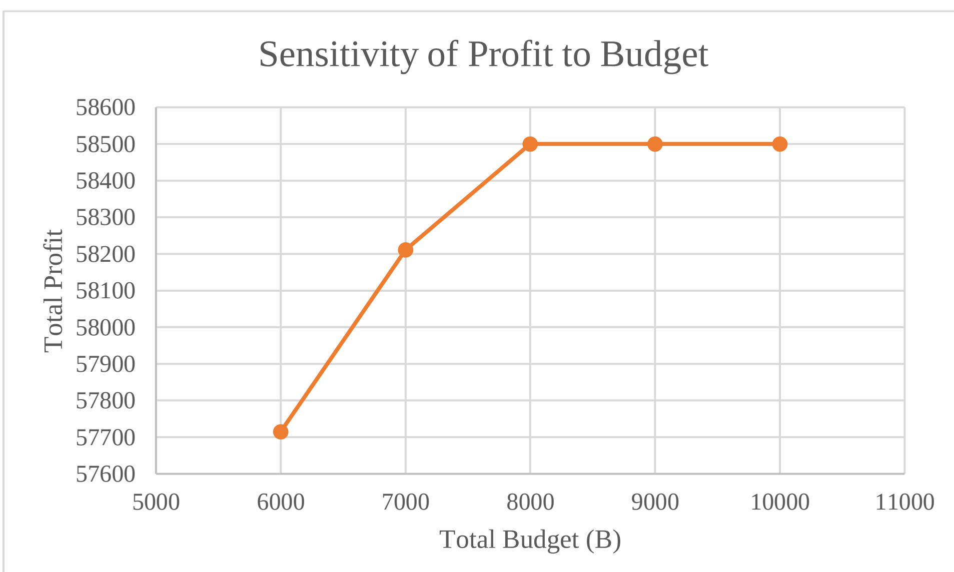

Figure 50. Graph of Total Profit vs. Total Budget (*B*)

Since the solution is limited by the demands of each product, the only way to increase profit is to find a larger market or increase price. Assuming all the costs are accounted for, an increase in price should directly relate to an increase in profit for each unit. As seen in Figure 51 through Figure 53, an increase in price for any one product is directly linear to an increase in profit. However, the slopes of these three graphs are different due to their varying prices. Table 52 shows that profit is most sensitive to changes in the price of product 3. This is due to product 3 being the largest and consequently having the largest profit per unit. If the beekeeper wishes to increase profits, the price of product 3 should be given priority, then the price of product 2, and finally the price of product 1. In this way, a beekeeper can achieve the most efficient increase in profit.

Table 50. Values of Total Profit and Profit Increase at Increased Values of p_1

Price Increase	p_1	p_2	p_3	***Profit***	***Profit Increase***
0%	2.50	4.00	9.50	58500	0
5%	2.63	4.00	9.50	59020	0.89%
10%	2.75	4.00	9.50	59500	1.71%
15%	2.88	4.00	9.50	60020	2.60%
20%	3.00	4.00	9.50	60500	3.42%
25%	3.13	4.00	9.50	61020	4.31%

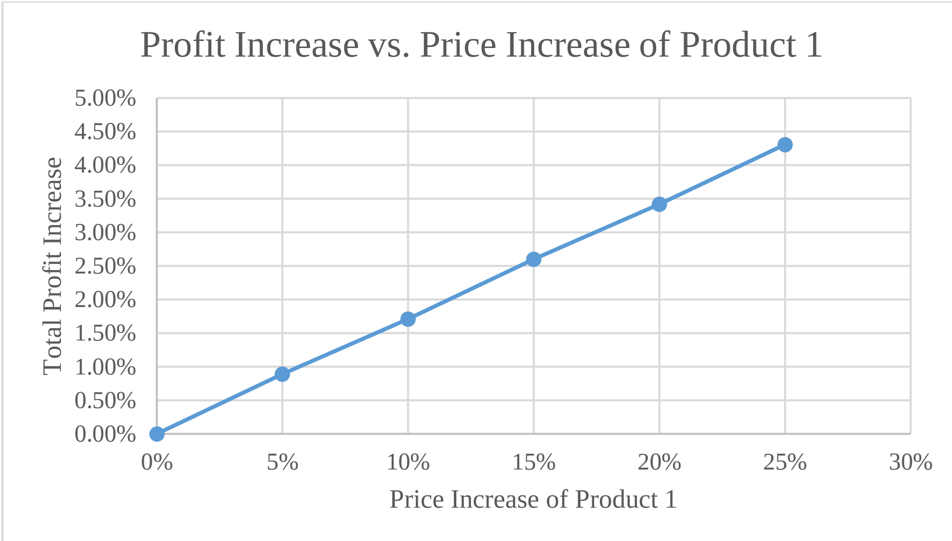

Figure 51. Graph of Total Profit Increase vs. Price Increase for Product 1

Table 51. Values of Total Profit and Profit Increase at Increased Values of p_2

Price Increase	p_1	p_2	p_3	*Profit*	*Profit Increase*
0%	2.50	4.00	9.50	58500	0.00%
5%	2.50	4.20	9.50	59500	1.71%
10%	2.50	4.40	9.50	60500	3.42%
15%	2.50	4.60	9.50	61500	5.13%
20%	2.50	4.80	9.50	62500	6.84%
25%	2.50	5.00	9.50	63500	8.55%

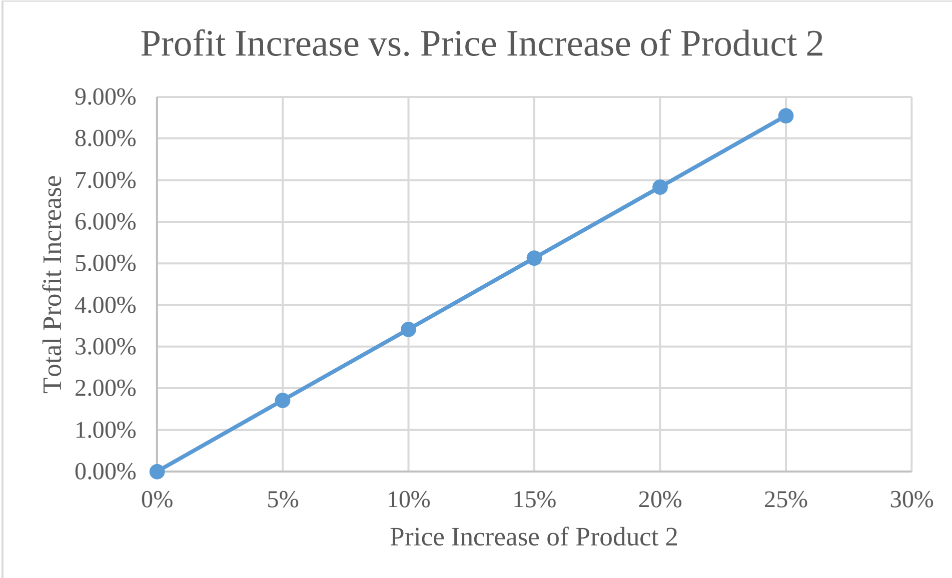

Figure 52. Graph of Total Profit Increase vs. Price Increase for Product 2

Table 52. Values of Total Profit and Profit Increase at Increased Values of p_3

Price Increase	*p1*	*p2*	*p3*	*Profit*	*Profit Increase*
0%	2.50	4.00	9.50	58500	0.00%
5%	2.50	4.00	9.98	59940	2.46%
10%	2.50	4.00	10.45	61350	4.87%
15%	2.50	4.00	10.93	62775	7.31%
20%	2.50	4.00	11.40	64200	9.74%
25%	2.50	4.00	11.88	65625	12.18%

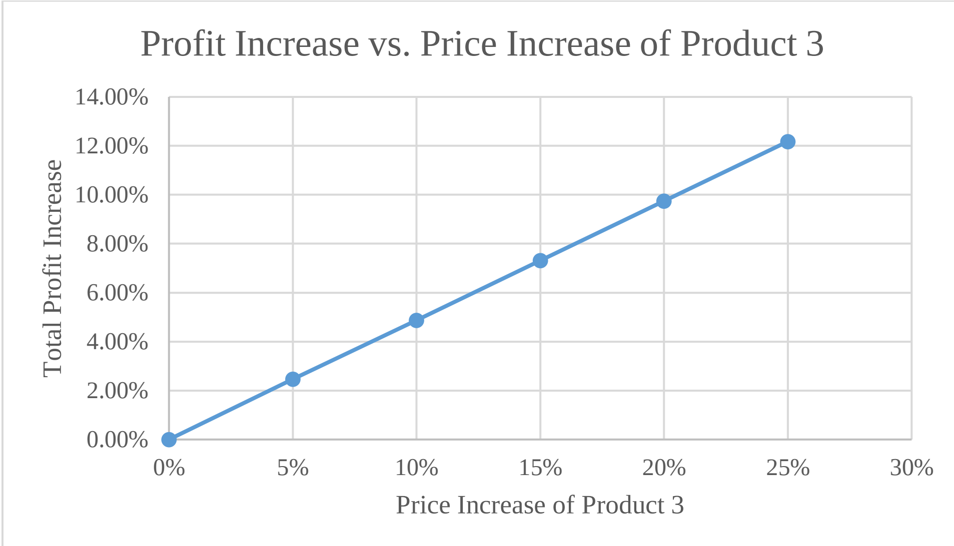

Figure 53. Graph of Total Profit Increase vs. Price Increase for Product 3

Chapter 5 : Conclusion and Future Work

A beekeeping business is cyclical and contains many similar decisions year after year. Most beekeepers rely on trial and error or past experience to decide some of the most influential business decisions. This research looked to understand a beekeeping business and create models that would find the optimal solutions to situations with multiple parameters. By inputting their own parameters, beekeepers can modify these models to fit their own unique situations. In this way, little to no experience is needed to make decisions that affect the bottom line of the business.

One of the most pivotal times for a beekeeper is the beginning of spring when the hives become more active. It is during this time that a beekeeper must take inventory the number of colonies lost over winter and replace them to ensure a stable business moving forward. A lot of money will be spent repairing and restoring the hives as hive loss over winter is not a nominal issue. Hives losses of 35% are not uncommon which causes beekeepers to make large financial decisions to restore their businesses. Additionally, this is the time for beekeepers to expand their business if they have the means of doing so. The springtime includes some of the largest operating costs for the whole year which can create anxiety for beekeepers trying to set a budget. Chapter 2 discusses this issue and delves into finding the optimal number of colonies to purchase during the spring.

Chapter 2 includes three separate models of increasing complexity to decide the best way to spend a given budget. The first section model (2.2) is simple and looks to maximize the total number of hives introduced to the business. The two decision variables are the number of hives that are replaced and the number of hives that are new. The main constraint is budget with the additional constraint that the number of hives replaced cannot exceed the number of hives lost from the previous year. This is a relatively simple model since new hives include the cost of equipment and the cost of bees; whereas, replaced hives only include the cost of bees. As a result, the optimal solution first allocates budget to replacing hives. Then, once all hives have been replaced, the remaining budget is used for new hives. Using the parameters found in the initial numerical analysis, a budget of $20,000 will maintain the total number of hives a beekeeper has from year to year. Although simple, this model can roughly provide an idea of how much a beekeeper needs to spend in order to maintain the same total number of hives from one year to the next.

The second model in Chapter 2 (2.3) adds in the concept of swarms. Swarming is the natural method that honeybees use to reproduce. When a beehive becomes too large for its container, new queen eggs will be laid and the old queen will leave with a little over half of the hive. The new mobile hive called a “swarm” searches for a new location to establish a new hive. Beekeepers can take advantage of this by leaving out empty hives and effectively “capturing” them. This method is desirable for beekeepers as it is a free way to gain another working colony. While swarming is generally not preferred for one’s own hives, it can be induced as a method for expansion. Since bees will swarm when they overgrow their container, beekeepers can promote swarming by refusing to add space to their stronger hives. This was added to the model by incorporating a percentage relating to the number of hives a beekeeper expects to swarm. Then, the beekeeper would find the number of their hives they expect to swarm and set aside that many empty hives to capture them. In the model, this effectively reduces the one constraint relating to total number of hives replaced. Capturing swarms is preferred in this model since swarms are free, leaving more budget to expand the business. The numerical analysis showed that 81 empty hives should be allocated to capturing swarms. Then, 90 hives can be replaced for a total of 171 new hives at a budget of $10,000.

The third model in Chapter 2 (2.4) considers the different types of colonies that can be purchased. Instead of maximizing number of hives, this model maximizes productivity. The most common form of purchasing a bee colony is a bee package. A bee package is a ventilated box containing about three pounds of bees and a queen. These bees are poured into the empty hive and have to build it from nothing. One more expensive alternative is a nucleus colony. A nucleus colony is essentially a miniature, fully functioning bee colony containing around five frames. These frames contain brood and the queen's egg laying cycle is underway. As a result, a nucleus colony already has momentum when starting a new hive. Comparing the two options, a bee package is less productive over the first season than a nucleus colony but costs less. The model in this section then looked at how many of each type should be purchased to provide the most productive set of hives throughout the year. After a numerical analysis and a sensitivity analysis, a balance was found between the two colony types where the solution depends on the beekeeper's budget and the cost of each colony type. At lower budgets, bee packages are preferred since more can be purchased. Then, as the budget increases, the ratio of bee packages to nucleus colonies shifts towards nucleus colonies due to their higher productivity. At high budgets, nucleus colonies are preferred before buying new hives. The solution does change a fair amount, but the model allows for unique parameters for a truly optimal solution. At the parameters provided, 86 bee packages and 3 nucleus colonies should be purchased to replace 89 hives with a budget of $10,000.

The last section of Chapter 2 (2.5) combined the three models to form the most complex version. It looked to maximize productivity by balancing bee packages and nucleus colonies while also including the aspects of capturing swarms. The optimal solution of this model looks fairly similar to that of the model in section 2.4, but it is shifted to the left. This is a result of some empty hives being allocated to capturing swarms meaning there is a slight budget allocation shift.

Depending on budget size, either there will be a higher ratio of nucleus colonies to bee packages in the optimal solution, or there will be more new hives purchased. The trends in the optimal solution remain fairly similar between sections 2.4 and 2.5. The optimal solution for the numerical analysis results in 81 empty hives allocated to capturing swarms, 66 hives replaced with bee packages, and 15 hives replaced with nucleus colonies for a total of 162 colonies added.

The second large decision tackled in this research is the allocation of honey to various products. After a beekeeper harvests and collects honey, a decision must be made how to package it in the most efficient way. Chapter 3 includes a model that looks to maximize total profit by examining the profits of each individual products. It also considers the demand for each product with an added holding cost of overproducing any particular product. The solution first allocates honey to the product with the highest profit per unit weight until it reaches demand for that product. Then, it allocates honey to the second highest profit per unit weight until it reaches that product's demand. This continues until all the honey is allocated or all the demand is filled. A beekeeper can enter as many possible products into the model as desired to examine different business scenarios. The optimal solution will then allocate honey to the most profitable options and ignore the others. In the numerical analysis, the optimal solution allocated 4,000 units of product 1, 5,000 units of product 2, and 2,600 units of product 3 with no honey remaining. The total profit for this solution was $54,700.

Chapter 4 explores a combination of the models from Chapters 2 and 3 based on the numerical example that has been used throughout this paper. It shows the relationships between the various parameters and recommends a course of actions for the example beekeeper to make. The initial optimal solution to the numerical analysis took the solution to section 2.5 to find a production quantity of 22,536.25 pounds of honey. The beekeeper in the example is meeting all the demand available with 536.25 pounds left over, and the optimal solution results in a decrease in the total number of hives. However, the beekeeper can make the decision to continue spending the budget at the current level which will cause a gradual decline in the total number of hives until an equilibrium point it reached. The beekeeper could also decide to grow the business by increasing the budget, but would have to find more ways to sell product. The last option would be to decrease budget as it is not needed to meet demand, but this does not guarantee the ability to meet demand at a lower budget in the future. The chapter also considers expanding into products other than honey which leads this paper to possible future work.

There are many possible avenues of research from what this paper has started. One aspect of a beekeeping business that has not been considered here are the different types of products that can be harvest from a beehive. Beeswax, pollen, propolis, and royal jelly are all additional products that can be collected from honeybees, albeit with varying degrees of effort. For example, beeswax is a highly desired product used from cosmetics and soaps to woodworking applications. A normal beekeeper generally acquires a good amount of beeswax over time due to cutting off the wax cappings when harvesting honey. However, a beekeeper can make a few small changes that will greatly increase the production of beeswax at the cost of honey production. This balance between producing for honey versus producing for beeswax can be modeled to determine number of hives for each that will maximize profit. Another possible area of research is hive layout on both a micro and macro level. The orientation of hives in relation to each other can be studied to determine the best layout on a smaller scale. Oftentimes, beekeepers orient their hives in a manner that makes them easiest to perform maintenance; however, this may not be the best method for productivity. On a larger scale, modeling the optimal number of hives in a region would be very helpful. Beekeepers may have a few locations in mind where they are able to place their hives and have to make a decision as to the number of hives at each location. A model could be formed examining the distance of each location to large sources of food (namely flowers) to optimize the total amount of production from all the hives.

There are still many areas of study within beekeeping that can benefit from optimization models. This paper touches the most superficial parts of a beekeeping operation, but many decisions that occur on a regular basis have been left untouched. Further research will provide better decision-making models tailored to more specific situations within beekeeping.

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ACADEMIC VITA
PATRICK M. GOLDEN

EDUCATION

Bachelor of Science, Industrial Engineering
The Pennsylvania State University, University Park, PA **Graduation**: December 2018
Schreyer Honors College

EXPERIENCE

Johnson & Johnson Consumer Inc., Fort Washington, PA December 2017 – July 2018
External Manufacturing Operations Co-op

- Developed weekly and monthly external manufacturing site performance metrics and communications to leadership.
- Maintained department data stored in SharePoint and Reliability Room and update work instructions as needed.
- Performed data analysis to support the business with cost improvement opportunities and risk mitigation assessments.
- Coordinated events including monthly staff meetings, weekly process execution meetings, and Credo activities.

Bell Helicopter, Fort Worth, TX May 2017 – August 2017
Industrial Engineering Intern

- Redesigned a 12,000 square foot area using lean fundamentals resulting in $86000 yearly savings, an increase in same day shipping from 73% to 96%, and 22% decrease in motion.
- Formulated a process in a unionized environment to complete work orders in an amicable manner.
- Analyzed sets of data to find standard cut sizes in crate production.
- Examined and reduced areas of waste in the storage process.

Bridge Gap Engineering, LLC, Northampton, PA May 2015 – August 2015
Intern

- Created weekly and monthly financial reports for the controller to present at company meetings.
- Wrote drafts for cost estimate letters sent to prospective customers.
- Compiled equipment specification sheets for various engineering projects.
- Communicated with equipment suppliers determining the price and quantity of orders to be placed.

LEADERSHIP & INVOLVEMENT

Johnson & Johnson Intern and Co-op Association, Fort Washington, PA March 2018 - July 2018
Professional Development Chair and *Lean Yellow Belt Lead*

- Coordinated events for 30 members practicing soft skills (resume reviews, mock interviews)
- Mentored 5 individuals throughout the completion of their yellow belt projects

Boys Scouts of America, Schnecksville, PA
Eagle Scout Service Project Fall 2012

- Planned and managed the creation of a Zoo Garden with a paver walkway at the Lehigh Valley Zoo
- Organized a team of 30 volunteers throughout the construction process to ensure timely completion of the project

Member, Nittany Data Labs, Data Science Club (August 2017 - Present)

SOFTWARE SKILLS

Minitab SAP NetWeaver SolidWorks MATLAB C++

HONORS & AWARDS

William A. Schreyer Scholarship University College Freiburg Grant