SCHREYER HONORS COLLEGE

DEPARTMENT OF CHEMICAL ENGINEERING

n-PLAYER TRAGEDY OF THE COMMONS AND CHEMICAL GAME THEORY

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SPRING 2019

A thesis
submitted in partial fulfillment
of the requirements
for a baccalaureate degree
in Chemical Engineering
with honors in Chemical Engineering

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ABSTRACT

The purpose of this thesis is to apply the methods of material balances, equilibrium thermodynamics, and reaction equilibrium from the disciplinary area of chemical engineering to model cooperation in the tragedy of the commons dilemma in the disciplinary area of game theory. Chemical Game Theory introduces a novel way to model and predict human behavior, however until this point only games with two players have been considered. Chemical Game Theory has largely been successful due to the inclusion of entropic effects and pre-bias, which have been demonstrated to be shortcomings of classical game theory models. This thesis will demonstrate Chemical Game Theory's ability to quantify games with up to 10 players, and the effect of the number of players on cooperation.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Velegol for welcoming me into the CGT group and allowing me to learn so much about game theory, programming, and myself. I would also like to thank my group mates Natalie Morrissey, Laura Cook, Sneha Srinivasan, and Miras Katenov for their insights and dedication to the group. Lastly I would like to thank my family for their support and allowing me to pursue my passions at Penn State.

Chapter 1

Introduction

This objective of this thesis is to quantify a tragedy of the commons game in the framework of Chemical Game Theory. Specifically, two questions will be answered in this thesis. First, the relationship between the number of players in a tragedy of the commons game and the cooperation rate will be identified. Second, a method for the computation of tragedy of the commons games with a large number of players will be developed. If successful, this research will provide a quantitative model for how humans play tragedy of the commons games. The current bottlenecks surrounding traditional game theoretic analyses is that they do not include entropic effects, or the idea of pre-bias in the players. Previously in chemical game theory, only games with two players were investigated. This thesis provides programs for solving n-player games, and analyses of the effect of group size as observed in Figures 3.1 and 3.4.

1.1 Defining The Prisoner's Dilemma and Tragedy of the Commons

In the classical rendition of the prisoner's dilemma, two robbers are caught in the act and taken into custody for separate questioning.[1] Each is faced with two options, to either blame his accomplice or stay quiet. For the sake of explanation and simplicity, values spanning 0 to 3 will be used to describe this game. If both prisoners stay quiet, there is enough evidence against each

[1] The "prisoner's dilemma got its name from Albert Tucker, but the game was originally framed in 1950 by Merrill Flood and Melvin Dresher working at RAND Corporation. This history is described in: Poundstone, William. "Prisoner's Dilemma." Anchor Books, New York 1992.

of them to merit 1 year in prison. If they both tell on each other, more evidence against each prisoner results in 2 years in prison for each of them. If one tells and the other stays quiet, the prisoner who tells receives 0 years and the one who stays quiet receives 3 years. These values are summarized in **Table 1.1**, with the payoff for the row player, or Player I, entered before the payoff for the column player, or Player II.

Table 1.1 Prisoner's Dilemma Game. This payoff matrix contains arbitrary values corresponding to years in prison. There is a Nash Equilibrium in cell 2,2 in which two "rational" players would be expected to play. The first value listed corresponds to player I and the second for player II.

		Player II	
		Quiet	Tell
Player I	Quiet	1,1	3,0
	Tell	0,3	2,2

More generally, the prisoner's dilemma can be defined using variable prison sentences. This is done using variables R which stands for reward, P which stands for punishment, S which is the suckers payoff, and T which stands for temptation. The generalized prisoner's dilemma can be observed in **Table 1.2**.

Table 1.2 Generalized Prisoner's Dilemma Game. The payoffs in this game are replaced by variables for demonstration of a qualitative definition of a prisoner's dilemma according to the inequality S<P<R<T in order of preference (Poundstone, William. "Prisoner's Dilemma." Anchor Books, New York 1992.)

		Player II	
		Quiet	Tell
Player I	Quiet	R,R	S,T
	Tell	T,S	P,P

The game will be defined as a prisoner's dilemma so long as S < P < R < T if the inequalities indicate order of preference. In **Table 1.1** the payoff most desirable for the player would be 0 years in prison, which corresponds to T in **Table 1.2**. The most characteristic aspect of the prisoner's dilemma is that it is always advantageous to play tell. No matter what player A chooses, if player B chooses tell it results in a lower sentence for him. This is what classical game theory says that players should play, that the P, P box is the Nash Equilibrium.

The prisoner's dilemma is perhaps the most famous and well studied game in the field of game theory, which was formulated by John von Neumann and Oskar Morganstern in 1944 in their book *Theory of Games and Economic behavior*.[2]

The tragedy of the commons is defined to be a multiplayer prisoner's dilemma. However, since each additional player adds another dimension to the payoff matrix, payoff qualifications for players who choose to cooperate (stay quiet) and those who defect (tell) are functionalized below.

1. D(m) > C(m+1) That is to say, each player is better off choosing D rather than C, regardless of how many players choose C on a particular play of the game.
2. C(N) > D(0) That is to say, if everyone cooperates, each player is better off than if everyone defects.
3. D(m+1) > D(m), C(m+1) > C(m) that is to say, the more players cooperate, the better off each player is, regardless of whether he chooses C or D.

[2] Von Neumann, J.; Morgenstern, O. Theory of Games and Economic Behavior. Princeton: Princeton UniversityPress, 2007.

4. $(m+1)*C(m+1) + (N-m-1)*D(m+1) > m*C(m) + (N-m)*D(m)$ that is to say, society as a whole is better off the more players who cooperate.[3]

Where N is the total number of players in the game, m is the number of people choosing to cooperate, or to stay quiet, D(m) is the payoff to each player who chose to tell as a function of cooperators, and C(m) is the payoff to each player who chose to cooperate as a function of cooperators.

The above qualifications are classically accepted qualifications for a tragedy of the commons game, although chemical game theorists hypothesize that they may not all be exactly accurate. Specifically, group utility may not be maximized at universal cooperation, and group pain may not be a simple linear relationship with number of cheaters. The above qualifications were revised using more precise scientific language rather than “better off” as this phrase seems arbitrary.

1. $D(m) < C(m+1)$ An individual receives less pain by defecting rather than cooperating, for any number of m cooperators. It is to be assumed that the more of the common resource a player has, the less pain he receives, or the marginal benefit to the individual is greater than the marginal cost to the group.
2. $C(N) < D(0)$ The individual’s pain if all players defect is greater than that if all players cooperate

[3] Definitions for a tragedy of the commons game were taken verbatim from Akimov et al, although definitions 1 and 2 first were formulated by Dawes in 1980. Akimov, V., & Soutchanski, M. (1994). Automata Simulation of N-Person Social Dilemma Games. Journal of Conflict Resolution, 38(1), 138-148.

3. $D(m+1) < D(m)$, $C(m+1) < C(m)$ As the number of players who cooperate increases, the less pain each player feels.
4. There may be a small number of cheaters that results in the minimum pain that is observed by the group.

1.2 Previous Analyses of Tragedy of the Commons

Qualitatively, a tragedy of the commons refers to a social situation in which members of a group are faced with choices where selfish or uncooperative decisions provide short term benefits to the individual but also provide undesirable consequences to the group.[4] Although the problem of common pool resource distribution has been discussed since the times of Aristotle,[4] modern formalizations of the problem have been documented by Gordon[5] in 1954 and Hardin[6] in 1968. The later has sparked much interest in the problem and ultimately provided a grim outlook on the fate of shared resources. Hardin asserts that people are inherently selfish and will eventually deplete all common pool resources unless there are breeding regulations set in place.

Examples of tragedy of the commons dilemmas today include: water as a resource during a drought,[4] overuse of antibiotics,[7] littering in the park, air pollution and climate change,[8]

[4] Shultz, Clifford J., and Morris B. Holbrook. “Marketing and the Tragedy of the Commons: A Synthesis, Commentary, and Analysis for Action.” *Journal of Public Policy & Marketing*, vol. 18, no. 2, 1999, pp. 218–229. *JSTOR*, JSTOR, www.jstor.org/stable/30000542.

[5] Gordon, H. Scott. “The Economic Theory of a Common-Property Resource: The Fishery.” *Journal of Political Economy*, vol. 62, no. 2, 1954, pp. 124–142. *JSTOR*, JSTOR, www.jstor.org/stable/1825571.

[6] Hardin, Garrett. "The Tragedy of the Commons." Science 162.3859 (1968): 1243-248. Web.

[7] Porco TC, Gao D, Scott JC, Shim E, Enanoria WT, Galvani AP, et al. (2012) When Does Overuse of Antibiotics Become a Tragedy of the Commons? PLoS ONE 7(12): e46505. https://doi.org/10.1371/journal.pone.0046505

[8] Ostrom, Elinor. *Governing the Commons: The Evolution of Institutions for Collective Action*. Cambridge, England: Cambridge UP, 2015. Print.

overfishing,[8] traffic congestion,[9] cutting down trees for lumber,[9] cars causing noise and air pollution,[9] and “wildlife crimes” where a species is overexploited for short term gain.[10] In each of these examples there is an individual benefit that incurs a cost on the group.

Current methods to mitigate the effects of tragedy of the commons games include: regulation or intervention such as taxes or privatization, organization of players such as alliances or partnerships, moralistic approaches such as communicating a social responsibility, and communication such as negotiation amongst group members or educational campaigns.[4] In addition, it has been observed that decreasing the size of the group can help promote cooperation.

Many philosophical, psychological, and moral discussions of the dilemma have been discussed. Some experimental studies have been conducted to determine how people actually play the prisoner’s dilemma game, and it has been determined that people play around 50% cooperate, 50% defect.[11,12,13] No study has attempted to create a model for human behavior under such conditions, rather focusing on theoretical problems and mainly evolutionary games

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[9] Weber, J. Mark, et al. “A Conceptual Review of Decision Making in Social Dilemmas: Applying a Logic of Appropriateness.” *Personality and Social Psychology Review*, vol. 8, no. 3, 2004, pp. 281–307., doi:10.1207/s15327957pspr0803_4.

[10] Pires, Stephen F., and William D. Moreto. “Preventing Wildlife Crimes: Solutions That Can Overcome the Tragedy of the Commons.” *European Journal on Criminal Policy and Research*, vol. 17, no. 2, 2011, pp. 101–123., doi:10.1007/s10610-011-9141-3.

[11] Swope, K.; Cadigan, J.; Schmitt, P.; Shupp, R. “Personality preferences in laboratory economics experiments.” Journal of Socio-Economics 2008, 37, 998-1009.

[12] Horton, John, et al. “The Online Laboratory: Conducting Experiments in a Real Labor Market.” 2010, doi:10.3386/w15961.

[13] Sally, D. Conversation and Cooperation in Social Dilemmas: A Meta-Analysis of Experiments from 1958 to 1992. Ration. and Soc. 1995, 7, 58-92.

which occur over a period of time.[14,15,16,17,18] In addition, current models do not attempt to incorporate bias other than what they have learned through reputation, and do not include any sort of entropy or randomness factor. Chemical game theory is the only successfully demonstrated model to fit experimental results for human behavior in a tragedy of the commons game. It aims to predict probabilities for which choice humans will choose among their various alternatives. CGT does not dictate how players should play, as dictated in classical game theory. This thesis will focus on the tragedy of the commons game to and show the various ways in which the parameters of pain and bias affect cooperation rate.

1.3 Chemical Game Theory

Chemical Game Theory (CGT) is an alternative model for contested human decision making.[19] Decisions are represented by "knowlecules" in which player A and B are represented by chemical reactors. In each reactor molecules labeled "a1, a2, b1, b2, etc." represent each player's decision of choice 1 or choice 2. These molecules react according to their energetic favorability, which is assumed to be the payoffs or pains associated in the same game. **Table 1.3**

[14] Grant, W.e., and Paul B. Thompson. "Integrated Ecological Models: Simulation of Socio-Cultural Constraints on Ecological Dynamics." *Ecological Modelling*, vol. 100, no. 1-3, 1997, pp. 43–59., doi:10.1016/s0304-3800(97)00155-5.
[15] Rankin, Daniel J., et al. "Sexual Conflict and the Tragedy of the Commons." *The American Naturalist*, vol. 177, no. 6, 2011, pp. 780–791., doi:10.1086/659947.
[16] Chakra, Maria Abou, and Arne Traulsen. "Evolutionary Dynamics of Strategic Behavior in a Collective-Risk Dilemma." *PLoS Computational Biology*, vol. 8, no. 8, 2012, doi:10.1371/journal.pcbi.1002652.
[17] Almansa, Gemma Del Rey, et al. "A Mathematical Model for the TCP Tragedy of the Commons." *Theoretical Computer Science*, vol. 343, no. 1-2, 2005, pp. 4–26., doi:10.1016/j.tcs.2005.05.005.
[18] Deadman, P.j. "Modelling Individual Behaviour and Group Performance in an Intelligent Agent-Based Simulation of the Tragedy of the Commons." *Journal of Environmental Management*, vol. 56, no. 3, 1999, pp. 159–172., doi:10.1006/jema.1999.0272.
[19] Velegol, D. Physics of Community Course Notes for Fall 2015, 1st ed.; Amazon, 2015

illustrates the CGT matrix to model the prisoner's dilemma, using the same prison sentences as those in **Table 1.1**.

Table 1.3 Prisoner's Dilemma game according to CGT. In this game matrix the years in prison are converted to arbitrary, normalized units of pain to represent the energetic favorability of the reaction, or ΔG/RT. In CGT, a1, a2, b1, and b2 are "knowlecules" which interact to a certain extent to form products which represent the outcome of the dilemma.

		Player B	
		b1	b2
Player A	a1	1,1	3,0
	a2	0,3	2,2

In addition to these reactions, in each reactor is a solid catalyst "A" for reactor A. This catalyst helps to speed up the reaction, and ultimately produce the products labeled A11 for a1+b1, A12 for a1+b2 etc. in the A reactor. These products are then fed into another reactor, to a player called the decider, in which a similar set of reactions occur to produce the resulting products D11, D12, etc. The relative fractions of these final products are said to be the percentage of time people will receive the payouts in the corresponding boxes in **Table 1.3**. **Figure 1.1** below displays concisely these reactions and reactors with the relative energetics in brackets.

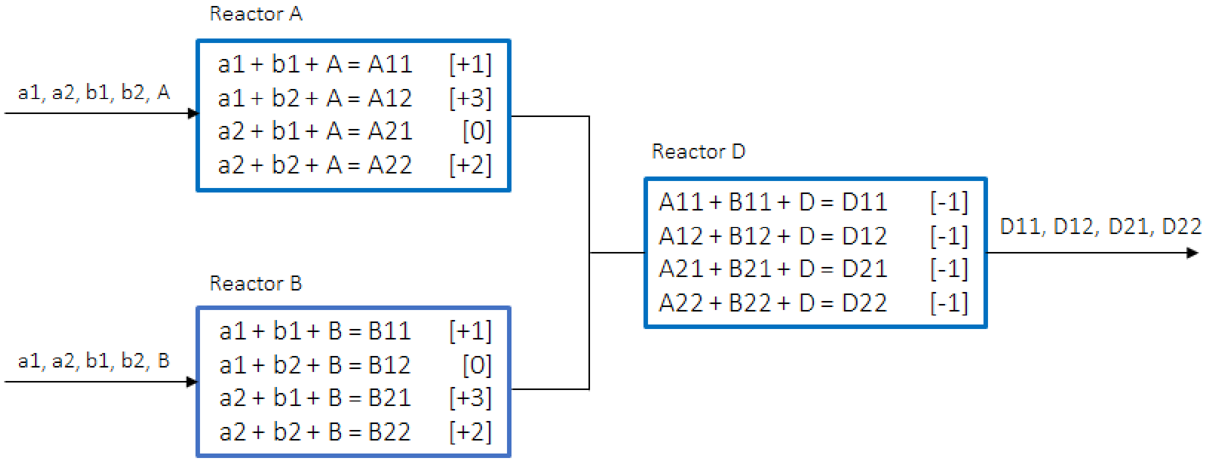

Figure 1.1 Process flow diagram for a 2 player prisoner's dilemma framed using CGT. Here, it is assumed that B has exactly the same perspective, so that there is no information asymmetry, After species exit each Reactor A, B, and D, a separation step removes unreacted reactants. For example, going into Reactor D, there is no a1, a2, b1, b2, A, or B. Separators are not shown for space considerations.[20]

This model overcomes some of the shortcomings of current tragedy of the commons modelling by the inclusion of pre-bias and entropic effects. Pre-bias, or reputation, is represented by the initial concentrations of each species in each reactor. For example, if player A is believed to be a fair, respectable person by player B, there might be a concentration of a1=0.5 in the B reactor. This is not necessarily how player A perceives himself, and may or may not be the same concentration in the A reactor. In addition, the inclusion of a decider can be thought of as the judge in the prisoner's dilemma story. The decider enforces the prison sentences, yet in real life this judge may not be unbiased, and could favor a lighter or heavier prison sentence.

In order to solve such a game, several parameters need to be known. Namely the initial concentrations of each species in both reactors, and the associated change in Gibbs free energy for each reaction. Given these parameters, the final concentrations of each species may be

[20] Taken from "Chemical Game Theory" submitted for publication in Industrial and Engineering Chemistry Research by Velegol, D., Suhey, P, Connolly, J., Morrissey, N. Cook, L.

calculated using Gibbs free energy and equilibrium equation, or **Equation 1.1**, combined with the equilibrium constant definition, or **Equation 1.2**

(1.1) $$\Delta G^0 = -RTlnK$$

(1.2) $$K = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

1.4 Representing Tragedy of the Commons with Chemical Game Theory

For each player added in a tragedy of the commons game, there necessarily is a corresponding reactor added in the chemical game theory representation. In addition, each reactor contains an additional two molecules, one per each possible decision. Generally speaking, if there are "n" number of players, there are n+1 reactors, when also considering the decider reactor. The number of molecules reacting in each reactor is 2n, and the number of possible reactions is 2^n. This information can be summarized in **Figure 1.2** found below.

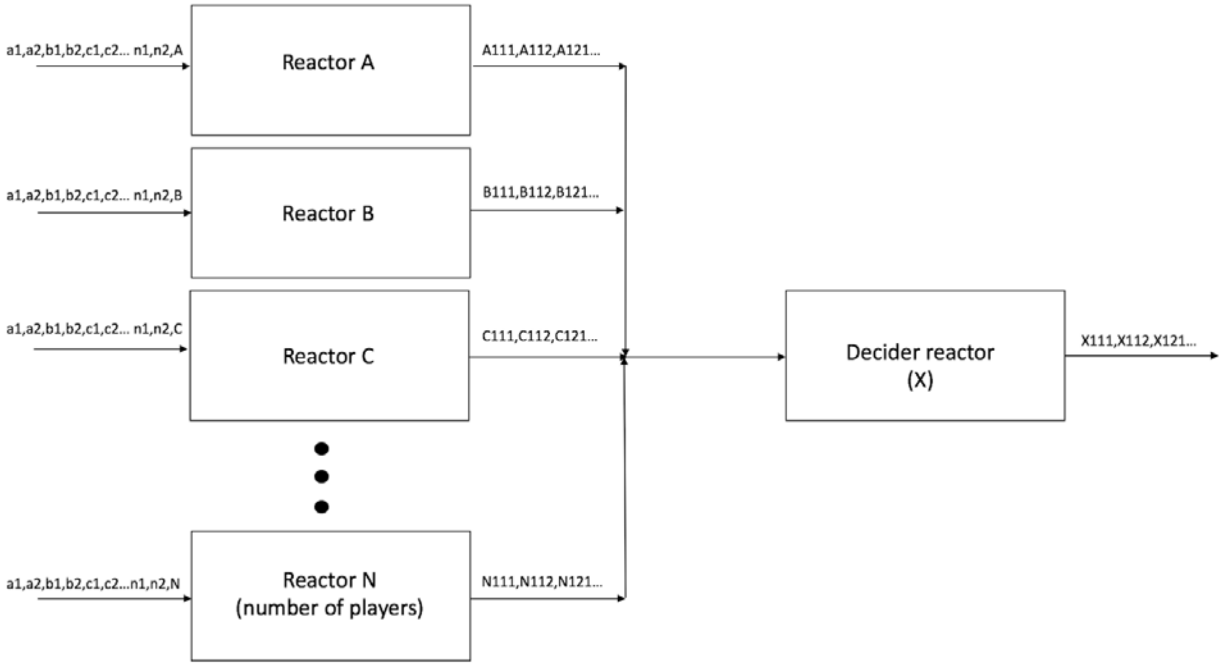

Figure 1.2 Representing n-player tragedy of the commons with chemical game theory. The design of multiplayer tragedy of the commons process flow diagram is similar to that for 2 players, but with each player added another reactor, and set of reactants are added. This corresponds to 2^n reactions per reactor.

In order to represent the payoffs, or pains, associated with a given decision set, pain functions were created to concisely communicate this information. Since each additional player adds another dimension to the payoff matrix, it would be difficult to show any game greater than three.[21] As such, functions for the payouts to a player who cooperates, determined by the number of cooperators or C(m), and for the payouts to a player who defects, D(m), were created much like those observed in the criteria for a commons game displayed earlier in this thesis. To clearly demonstrate the pain functions, an example of linearly increasing pain functions which pass all the qualifications listed in section 1.1 is provided. (**Table 1.4**)

[21] In game theoretic literature it is common to show a payoff matrix for three players are two matrices, one with the third player fixed on the first decision, and the other with the third player fixed on the second decision.
Li, Jiawei, and Graham Kendall. “On Nash Equilibrium and Evolutionarily Stable States That Are Not Characterised by the Folk Theorem.” Plos One, vol. 10, no. 8, 2015, doi:10.1371/journal.pone.0136032.

Table 1.4 Representation of pain functions as a function of the number of cooperators. For this table the pain units are arbitrary and kept simple for explanation purposes. These pains correspond to a tragedy of the commons game as defined by the qualifications in section 1.1

m "number of cooperators"	D(m) "Pain to a defector"	C(m) "Pain to a cooperator"
5	N/A	1.5
4	1	2.5
3	2	3.5
2	3	4.5
1	4	5.5
0	5	N/A

Chapter 2

Quantification

Previous work done by our team in chemical game theory was conducted on Excel using the Data Solver add in. The final concentrations of each species were obtained by solving a system of equations from known values of ΔG and initial concentrations through the common term of equilibrium constant found in **Equations 1.1** and **1.2**. Qualitatively, the reaction of each species combination is said to react to a certain extent dependent upon the energetic favorability of that reaction, given by the corresponding ΔG value. Each extent of reaction can be calculated as an independent variable, each of which contributes to the final concentration of each species. However, since this is a nonlinear system of equations, there are inherent limitations on the scope of the inputs and methods of calculation. **Tables 2.1** and **2.2** concisely show the relationship between extents of reaction and final concentrations in a SPICEY table, an acronym for SPecies, Initial concentration, Change, Equilibrium, and Y fraction. This chapter will be dedicated to the computational determination of said extents of reaction, with the goal of finding the final species' concentrations.

Table 2.1 SPICEY Table for Reactor A for a 2 player prisoner's dilemma game with equal pre-bias. This table serves as the basis for defining the extent of reaction, which is used in determining other other variables in the context of CGT.

species	initial	change	end	y mole fraction
a1	0.50	$-(\varepsilon_1 + \varepsilon_2)$	$0.50 - (\varepsilon_1 + \varepsilon_2)$	$[0.50 - (\varepsilon_1 + \varepsilon_2)] / \sum$
a2	0.50	$-(\varepsilon_3 + \varepsilon_4)$	$0.50 - (\varepsilon_3 + \varepsilon_4)$	$[0.50 - (\varepsilon_3 + \varepsilon_4)] / \sum$
b1	0.50	$-(\varepsilon_1 + \varepsilon_3)$	$0.50 - (\varepsilon_1 + \varepsilon_3)$	$[0.50 - (\varepsilon_1 + \varepsilon_3)] / \sum$
b2	0.50	$-(\varepsilon_2 + \varepsilon_4)$	$0.50 - (\varepsilon_2 + \varepsilon_4)$	$[0.50 - (\varepsilon_2 + \varepsilon_4)] / \sum$
A11	0	$+\varepsilon_1$	ε_1	$\varepsilon_1 / \sum$
A12	0	$+\varepsilon_2$	ε_2	$\varepsilon_2 / \sum$
A21	0	$+\varepsilon_3$	ε_3	$\varepsilon_3 / \sum$
A22	0	$+\varepsilon_4$	ε_4	$\varepsilon_4 / \sum$
inert	0	0	0	0
total	$\sum_0 = 2.00$	$-(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)$	$\sum = 2.00 - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)$	1.00

2.1 Nonlinear Optimization

As observed in **Table 2.1**, the mole fraction or final concentration of each species is a function of several extents of reactions. Since there are independent variables in both the numerator and denominator, it is necessary to solve this system of equations using non-linear programming methods. For this reason, GAMS (General Algebraic Modelling System) programming language, specifically its CONOPT solver was used to minimize the difference between the equilibrium constant as a function of Gibbs free energy, and that as a function of concentrations. GAMS is commonly used to solve linear, nonlinear, and mixed integer optimization problems for fields such as finance, economics, and chemical engineering. The software's website is extensive with example programs, explanations for error codes, and a support line.

The CONOPT solver uses an iterative method to achieve minimization, and can use second derivatives if the number of variables is much larger than the number of constraints. Since GAMS uses a floating point error system to determine the extents of reaction to a given accuracy, there are limitations on the size of the models used. For instance, the lower bound of mol fractions typically cannot be lower than 1E-09, and if so the algorithm has trouble converging. It measures no change in the objective value although the reduced gradient is greater than the tolerance. One way to avoid this is to shift ΔG values to become more negative so that the extents of reaction are larger. However, as each player is added to the model the extents of reaction become smaller so it becomes increasingly difficult for the algorithm to reach an optimal value of 0.

Specifically, for the program formulated to solve tragedy of the commons games, matrices were utilized to concisely represent the information needed to solve all calculations. The equations were formulated based on the **Equation 2.1**, and as such stoichiometric coefficient matrices v were inputted.

(2.1) $$n = n_o + v\varepsilon$$

Where "n" number of moles of each species is equivalent to an initial amount of moles plus a vector of stoichiometric coefficients multiplied by a vector of extents of reactions. **Equation 2.1** may be written explicitly for a two player game found below.

$$\begin{bmatrix} n_{a1,0} \\ n_{a2,0} \\ n_{b1,0} \\ n_{b2,0} \\ n_{A11,0} \\ n_{A12,0} \\ n_{A21,0} \\ n_{A22,0} \end{bmatrix} + \begin{bmatrix} \nu_{a1}^{1} & \nu_{a1}^{2} & \nu_{a1}^{3} & \nu_{a1}^{4} \\ \nu_{a2}^{1} & \nu_{a2}^{2} & \nu_{a2}^{3} & \nu_{a2}^{4} \\ \nu_{b1}^{1} & \nu_{b1}^{2} & \nu_{b1}^{3} & \nu_{b1}^{4} \\ \nu_{b2}^{1} & \nu_{b2}^{2} & \nu_{b2}^{3} & \nu_{b2}^{4} \\ \nu_{A11}^{1} & \nu_{A11}^{2} & \nu_{A11}^{3} & \nu_{A11}^{4} \\ \nu_{A12}^{1} & \nu_{A12}^{2} & \nu_{A12}^{3} & \nu_{A12}^{4} \\ \nu_{A21}^{1} & \nu_{A21}^{2} & \nu_{A21}^{3} & \nu_{A21}^{4} \\ \nu_{A22}^{1} & \nu_{A22}^{2} & \nu_{A22}^{3} & \nu_{A22}^{4} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix} = \begin{bmatrix} n_{a1} \\ n_{a2} \\ n_{b1} \\ n_{b2} \\ n_{A11} \\ n_{A12} \\ n_{A21} \\ n_{A22} \end{bmatrix}$$

This method allows for easier input of variables and equations for solving n-player games. In order to organize equations and create a consistent pattern of coefficient matrices, a scheme was developed according to **Figure 2.1** below for a four player game.

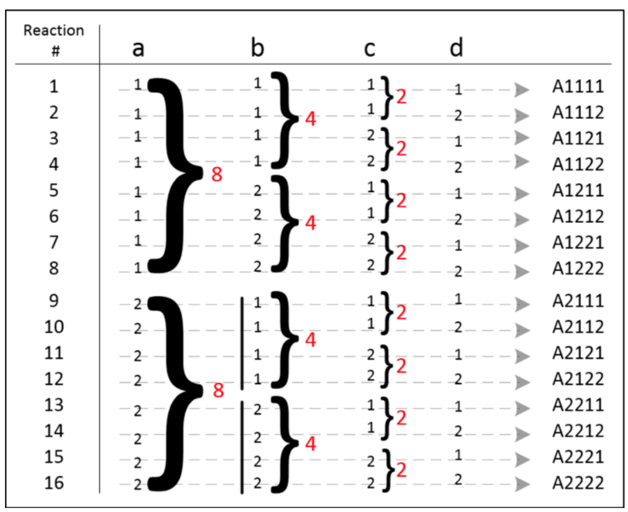

Figure 2.1 Organization scheme for reactions in a 4 player ToC game. The repeating pattern allows the use of for loops and if statements in a concise and convenient manner for the program.[22]

[22] This figure was created by Miras Katenov in his thesis "Representing N-player tragedy of the commons problem in chemical game theory" (2017)

Due to this emerging pattern, for loops and if statements were utilized to develop coefficient matrices and pain matrices in a concise way. See appendix for full length program for a 2 and 5 player game.

2.2 Linear approximation

Because the GAMS CONOPT solver uses a floating point system with a limited capacity for precision, the solver may not always converge to a solution if extents are small enough. Such is the case for games with players greater than 5. Ways to ensure convergence include implementing stricter bounds, a better starting point, and better scaling. The last suggestion is not of much use to chemical game theory models because relative pains are used, and a game that is inherently high stakes will have larger magnitudes than a lower stakes game.

A way to avoid nonlinear optimization altogether is to assume that all terms of order 2 or higher are approximately equal to zero. Once this assumption is made, the calculation of extents of reaction becomes linear, and results in a simple expression (**Equation 2.2**) that requires just the number of players and input parameters.

(2.2)
$$\frac{\Sigma_0^{(n-1)} e_R^t}{a_0^R b_0^R c_0^R \ldots} = Exp(-dG)$$

In this equation, Σ_0 is the sum of the initial amount of the species in the reactor, e is the extent of reaction t, in reactor R, and a_0^R is the initial amount of species a in reactor R. The full derivation for **Equation 2.2** may be found in the appendix. Extents of reaction can then be solved using any programming language, and the preferred language for this group was Mathematica.

In order to determine the accuracy of this approximation, a tragedy of the commons game was devised with linearly increasing pains and a uniform pre-bias of 0.5 stay quiet, 0.5 tell. The pains for the 5 player game spanned from 1 to 5.5, and the percent difference in cooperation rate was determined between the accepted nonlinear optimization solver, and the linear approximation method. (**Figure 2.2**) As observed, the percent error decreases as the numbers of players increases. This is both convenient for solving games with larger players, and logical because as the extents become smaller with each player, the assumption that terms involving an extent to the order 2 or higher is zero becomes stronger.

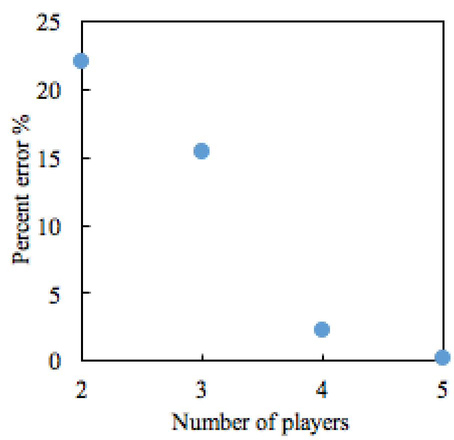

Figure 2.2 Percent error in cooperation rate vs number of players for linear approximation method. As observed the linear approximation becomes viable when the number of players in the game is larger than 5. Data was not collected for players greater than 5 due to difficulties in convergence.

In light of this error, the linear approximation method was used for calculations of 5 or greater players. This will be the case for any graph henceforth in this thesis. However, there are still constraints on how many players this program can handle. Due to the reliance on matrices for stoichiometric coefficients and pain values, some of the largest matrices have $2^n(n+1)$ rows and 2^n columns. For a 10 player game, this amounts to 11,264 rows by 1024 columns giving

11,534,336 entries. Given current memory and run time of standard personal or desktop computing, this was the practical limit of the size of the program. Even larger games could be computed using more powerful computers, but these games were not considered in this thesis. As n grows, the run time of the program grows exponentially. (Figure 2.3)

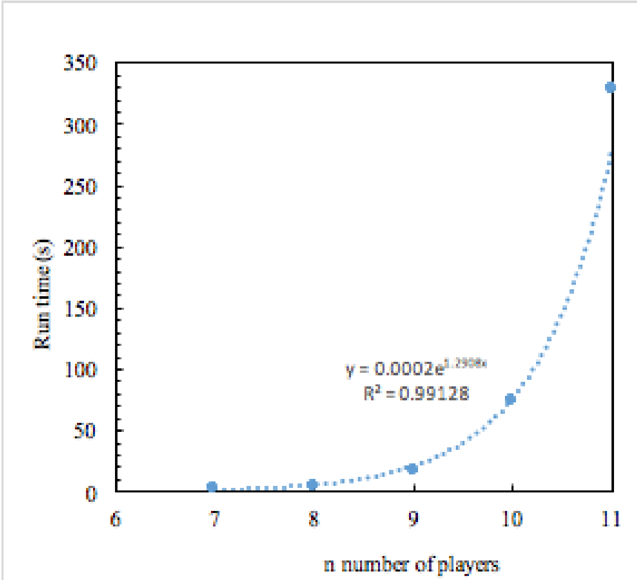

Figure 2.3 Run time vs number of players for a tragedy of the commons linear approximation with linearly increasing pains and 0.5/0.5 universal pre-bias. The exponential run time is due to the reliance on exponential growth of operations as the number of players increases.

2.3 Validity of method to literature

One of the shortcomings of classical analyses of tragedy of the commons games is the lack of experimental data and quantification techniques. In order to test the validity of chemical game theory on how humans will actually play a tragedy of the commons game, parameters were estimated for an experimental study conducted by Isaac, Walker, and Williams.[23]

[23] Isaac, R.mark, et al. “Group Size and the Voluntary Provision of Public Goods.” *Journal of Public Economics*, vol. 54, no. 1, 1994, pp. 1–36., doi:10.1016/0047-2727(94)90068-x.

In the experiment conducted by Isaac et al, various sized groups of college students played a tragedy of the commons game for a combination of money and extra credit points. This article was a follow up study conducted by the same group, and results were compared to their previous study which just gave out cash payoffs. Each player was given an initial endowment of tokens, and could choose to contribute to the group account, which earned interest based on how many tokens were contributed, or to keep their tokens and earn a private interest on them. The MPCR, or marginal per capita return from the group account was either 0.30 or 0.75 for each game played, and the group size varied to groups of 4, 10, 40 and 100. Each player received 3 dollars for showing up on time plus half of his earnings during the experiment. Since the payouts were not exactly the same for each group size, extra credit earnings were based on the how well each player, i, did relative to how well he could have done. (**Equation 2.3)**

(2.3) $$\frac{i's\ actual\ earnings - i's\ minimum\ possible\ earnings}{i's\ maximum\ possible\ earnings - i's\ actual\ earnings}$$

In order to compare this experiment to chemical game theory, a formulation for the pain values as a function of dollars and extra credit was need. Previous work done in CGT yielded an equation that transforms dollar amounts into an amount of pain, where m is the amount of money received (**Equation 2.4**).[24] This type of equation is called a perception function, and has a negative domain for positive monetary values, which gives a negative pain, or pleasure, from money.

(2.4) $$pain = -0.98 \ln \frac{m}{\$3.06}$$

[24] Equation 2.4 was formulated by Laura Cook and can be found in her thesis set for submission spring 2018. It was determined based on experimental data from members of the CGT group

Equation 2.4 is empirical, with data taken from authors of a chemical game theory manuscript, in order to fit how much money would be needed to feel an arbitrary level of pain. The same process was conducted but with extra credit, and yielded the following equation. (**Equation 2.5**) Since it was theoretically possible to earn 0 units of extra credit, that is if everyone else defected and 1 person cooperated, the pain value was assumed to be 0 for such a scenario. Given the low probability of such an event, altering this value had little effect on the outcome of the game according to uncertainties in the game.

(2.5) $$pain = -1.2257 \ln \frac{xc}{0.4728}$$

Once these perception functions were determined, the unknown parameters were reduced to only the pre-bias for each player. Since there were 2n unknown pre-bias parameters, it was assumed that each pre-bias, or initial concentration, could be classified as either what the player thinks his pre-bias is, and what he thinks every other player's bias is. Commonly players in a tragedy of the commons game see large groups as groups of cheaters, and so this was a place to start to determine the pre-bias of the players.

For a MPCR of 0.3 in a four player game, literature values give a cooperation rate of roughly 31%. Using this information, the pre-bias for what a player perceives himself to be was determined to be 0.30 cooperate and 0.70 defect, and the pre-bias for how a player perceives his opponents was 0.23 cooperate, 0.77 defect. Taking these values and applying to a 10 player game, chemical game theory's method of predicted human behavior gave a value of 0.51. The observed value for an MPCR of 0.30 and a 10 player game is about 0.43. This results in a relative error of 18%. Although the predicted value from CGT isn't exactly that as observed, the result is encouraging because qualitatively it predicted an increase in cooperation with group size. In addition, there were still many assumptions such as the simplification that each player perceived himself the exact same way, and there were many potential combinations of perceived self bias and perceived others bias that could have resulted in a 31% cooperation rate for a four player game.

The same process was repeated for a 0.75 MPCR, and the pre-bias for how each player perceives himself was 0.40 cooperate, 0.60 defect, the pre-bias for perception of the opponent was 0.32 cooperate, 0.68 defect for a four player game to match the cooperation rate of about 49% of experiments. When the same biases were used in a 10 player game, a cooperation rate of 52% was predicted by CGT. Although the experimental value observed was 48%, this result was still encouraging because the qualitative result that with an increase in MPCR led to a decrease in distance between the cooperation rates of four and ten players.

Again, it is important to consider qualitative results here, and given the large uncertainties associated with each parameter in the calculation of cooperation rate, it is not surprising that relative errors as high as 18% were observed. Given more information, a more accurate modeling system could be established, and human behavior could further be predicted, or parameters could be established. In addition, for several perception function values which included both monetary and extra credit gains, the game being played was not a tragedy of the commons. Experimenters may not have been aware of this and assumed that players only cared about the extra credit as small monetary payouts were given, or improvements for to each perception function may be necessary. Inflation from the time of publication to today was considered in the perception function calculation.

Chapter 3

Manipulation of tragedy of the commons game

Typically for tragedy of the commons games, the total amount of pain increases as the number of players increases. This makes logical sense as the more people involved in a game, the more importance it typically has on society. Take for example climate change. Many people everyday emit CO_2 not just by exhaling, but through product consumption, energy consumption, or transportation. Almost everyone is playing this game of CO_2 emittance, usually with the more CO_2 you emit the more utility you feel. Yet there is a fair amount everyone can emit in order to ensure a relatively stable climate. There are 7 billion people playing this game, and although the pain is small for everyone they sum to a large value. Compare this example with littering in a park, in which everyone enjoys a clean park yet people incur a cost of throwing away their trash, the pain values are much different. The question becomes not do the pains increase with n players, but how? In this thesis, logarithmically, linearly, and quadratically increasing pains are considered, and each affect on cooperation rate is displayed. For ease of computation relatively simple pain functions were used with small values.

3.1 Effect of number of players on cooperation rate

A tragedy of the commons game was investigated for a set of logarithmically increasing pains. It was Bernoulli who analyzed that humans perceive stimuli on a logarithmic scale, such examples include sound loudness for the decibel scale, sound tone for the octave scale, and geological richter scale. For the given game, arbitrarily set pain functions were:

$$C(m) = \ln(1.5m + 2.25)$$

$$D(m) = \ln(1.5m)$$

Where m is the number of cheaters, C(m) is the payout to those who cooperate and D(m) is the payout to those who defect. An outstanding observation for such a tragedy of the commons is that there is a maximum cooperation rate attained. (**Figure 3.1**)

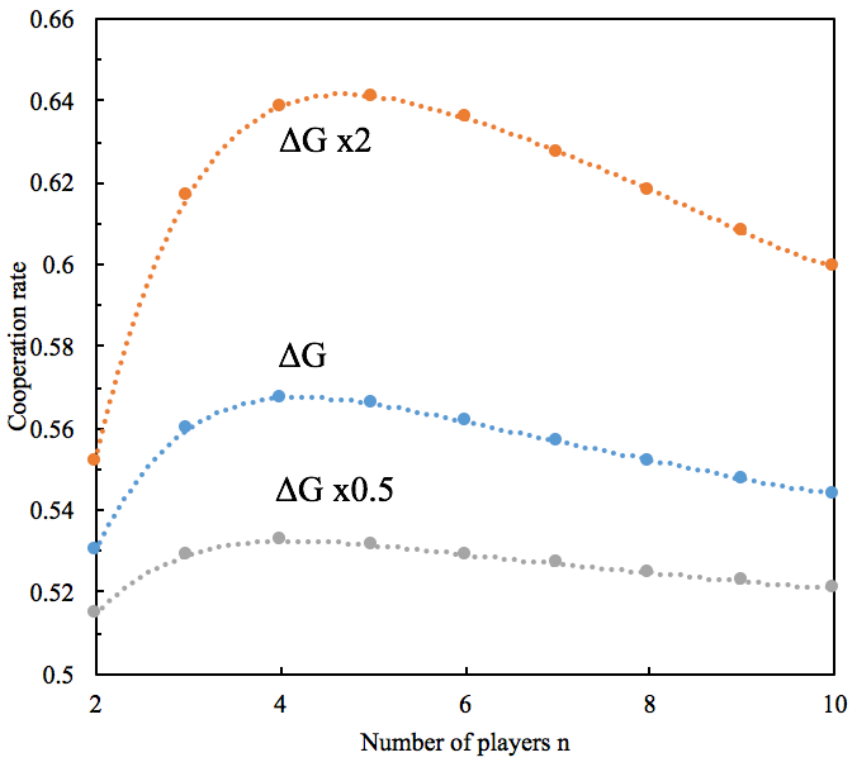

Figure 3.1 Logarithmically increasing pains for 0.5/0.5 uniform bias in a ToC game. In this figure a maximum cooperation rate is observed at n=4. The maximum cooperation rate does change however based on the magnitude of the pain values, although the same general trend is observed throughout.

A naturally arising question is: do all tragedy of the commons games with logarithmically increasing pains have a maximum cooperation rate? The answer unfortunately is unknown. This may be the continued work of the CGT community to investigate this question, however it is important to note that tragedy of the commons games can have a maximum cooperation, and group size can be manipulated to promote cooperation. Unlike classical remedies to a tragedy of the commons problem which rely on avoiding the problem, instituting new rules, or changing human perception, this solution relies solely on changing the size of the group.[4]

Another type of increase in pain that was considered is linearly increasing pains. Although the maximum level of cooperation is attained at the largest number of players (**Figure 3.2**), another qualitative insight could be observed. Similar to the log scale pains, there is an increase in cooperation when pains are increased. Although each pain is multiplied by two, this increase in space between pains drastically changes the cooperation rate. Logically this may be because people in general are risk averse, so if there are higher stakes at hand they do not play as risky. The opposite is true when the pains are multiplied by 0.5 as well. Pain functions for this game were:

$$C(m) = 0.5m + 0.75$$

$$D(m) = 0.5m$$

Collectively these pain functions create the ΔG/RT for each reaction, and are symmetric for each type of molecule. That is, the ΔG/RT of reaction for a1+b1+c2 = A112 is equal to that of a1+b2+c1 = A121.

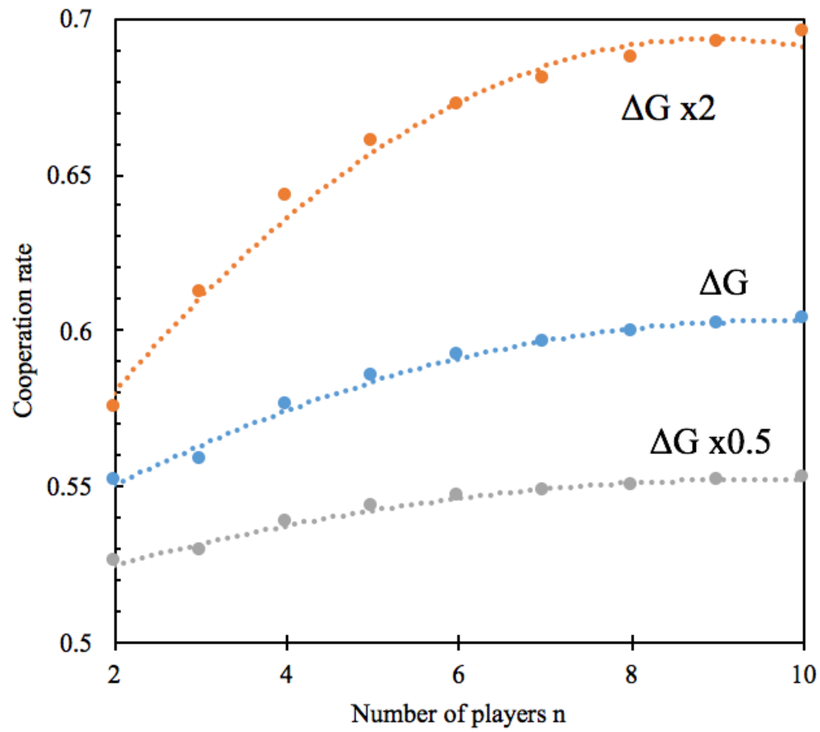

Figure 3.2 Linearly increasing pains for 0.5/0.5 uniform bias in a ToC game. In this graph there is an increasing cooperation rate with group size with decreasing returns to scale. The way that pains increase is important as compared to Figure 3.1 as the magnitudes of pains are roughly similar.

Lastly, a game in which pains increased quadratically with the number of cheaters was investigated. This game graphically looks similar to linearly increasing pains (**Figure 3.3**) but the slope in which the cooperation rate increased with number of players is larger. This again is intuitive given people will generally not risk defecting and will try to minimize pain. Pain functions for this game are as follows, and although they take the form of those quadratic pains hypothesized by CGT, they meet all the requirements for a classical ToC game, including a minimum pain occurring at universal cooperation.

$$C(m) = 0.75m^2 - 0.6m + 0.4$$

$$D(m) = 0.25m^2 - 0.6m + 0.4$$

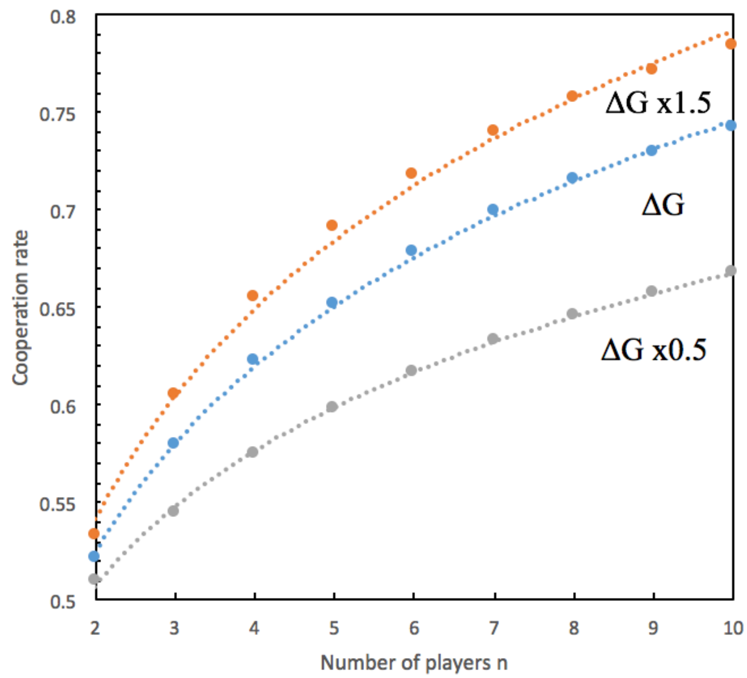

Figure 3.3 Quadratically increasing pains for 0.5/0.5 uniform bias in a ToC game. Again an increasing cooperation rate with group size is observed, although the slope is larger than that in Figure 3.2 as the pains are increasing more dramatically.

3.2 Effect of playing against similar players

It is a common ideal of the tragedy of the commons that group size can be decreased to help mitigate the effects of the tragedy. This is observed because classically players perceive large groups as filled with cheaters, and that it can be easier to free ride rather than contribute their fair share if there are many people. However, this is not always the case as observed in the article by Isaac et al. To investigate this observation, several games were played and graphed. (**Figure 3.4**) A pain function was held constant given by the following equations:

$$C(m) = m + 1.5$$

$$D(m) = m$$

Equations were chosen for simplicity and ease of computation. Then, initial biases were varied for three games. First, a uniform bias of 0.8 for cooperation, 0.2 for defection and 0.2 for cooperation and 0.8 for defection were plotted. In both instances the cooperation rate increased with the number of players. So it did not matter whether each player was playing with a group of known cooperators or a group of known cheaters, as long as they were all the same in pre-bias the cooperation rate increased. Then the classical notion that large groups are perceived as cheaters was tested. Each player had a pre-bias of 0.50/0.50, which was chosen as a baseline value that seemed reasonable for the average person encountering a new situation, but he perceived everyone else's pre-bias to be 0.2 cooperate, 0.8 defect. When these pre-biases were used in the same game, the cooperation rate decreased with increasing number of players. This can be used to explain the philosophy, and show that in fact cooperation rate can be manipulated by changing pre-bias. Although decreasing group size can be effective if people perceive the group differently than themselves, it may be a more worthwhile strategy to show a common connection between players before playing the game.

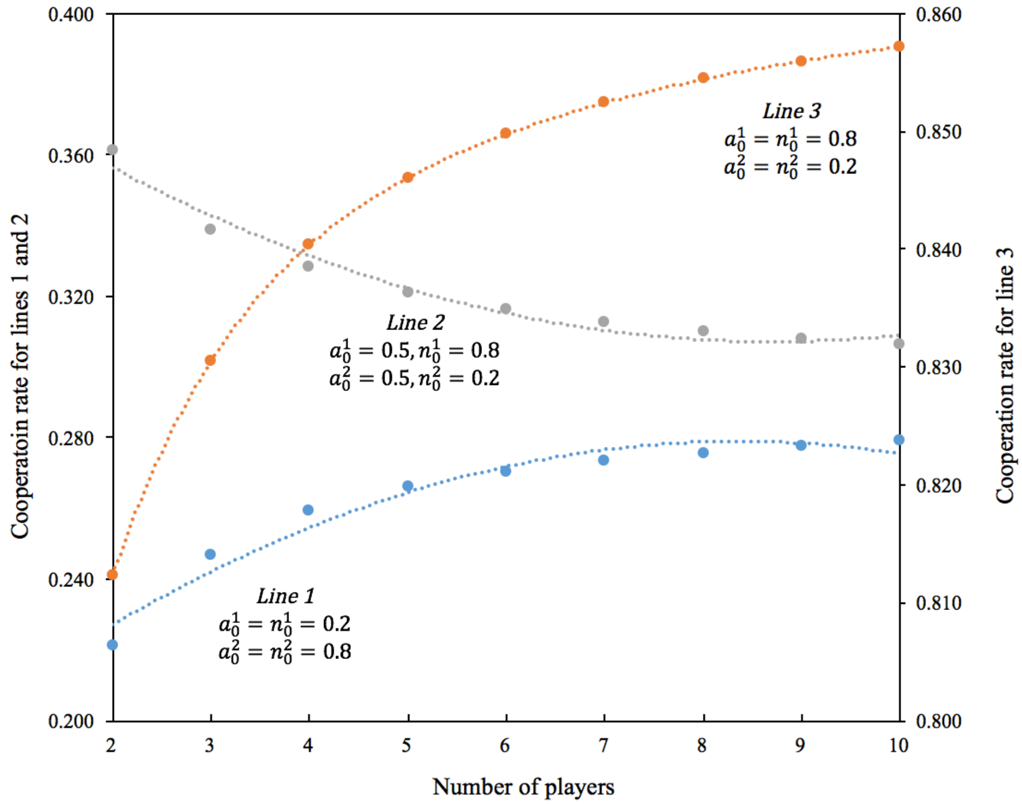

Figure 3.4 Effect of pre-bias and known vs. perceived pre-bias on a ToC game. This graph illustrates that if an individual player perceives that he is playing with a group of cheaters, which is a commonly held philosophy in tragedy of the commons games, then by decreasing group size cooperation is promoted. However, it also shows that if the individual believes he is playing against like-minded individuals, even if they are cheaters, cooperation is promoted with increasing group size.

Chapter 4

Conclusions

The main goal of this thesis was to quantify a tragedy of the commons game in the context of GCT. As one of the only studies that attempts to model experimental data for a tragedy of the commons game, CGT has proven to be a viable method of predicting probabilities for human choices. This thesis has also demonstrated the relationship between cooperation rate and number of players in a tragedy of the commons game. This relationship was determined to be dependent upon how the pains increased as the number of players increased, and also on the pre-bias for each player. Generally, if players are playing against similar players and they know these perceptions to be true, cooperation increases with group size. However, if the players are playing against each other with the same pre-bias, but perceive all the other players to be cheaters, cooperation decreases with group size.

In addition, the question of how to solve games with large amounts of players was answered. A series of nonlinear programs were created to solve games with five or fewer players, which were accurate and true to CGT, but were inconvenient and limited in scope of pain values due to the small nature of the extents of reaction for positive pain values. A linear approximation was used to create a program to solve games between 5 and 10 players, and this could be done easily and quickly.

An important attribute of games considered in this thesis is that each player is the same. This is very much not the same in real life, as product manufacturers may have larger pain values or more influence in the game of climate change rather than those in other industries. An advancement of the current work would be to include those differences, to get a more accurate picture of how cooperation rate may be affected, rather than simply a more theoretical approach as considered in this thesis.

In addition, the definition of a tragedy of the needs to be further developed. One of the axioms of classical tragedy of the commons games is that a minimum pain observed by the group is located at universal cooperation. It may in fact be the case that group pain is minimized when a small amount of people cheat. Enforcement costs can be included in the calculation of pain for each individual, and if this is the case it might positively impact group pain to ensure that all players cooperate rather than defect.

A definition for the tragedy of the commons was attempted to be defined, and although there are mathematical guidelines and qualitative pointers to what a tragedy of the commons is, there needs to be improvement on a precise definition. This thesis and future work hope to establish this definition, and provide insights on how to manipulate a tragedy of the commons game.

Appendix A

Derivation of Linear Approximation for N-Player TOC games

$$K_1^A = Exp\left(-\frac{\Delta G}{RT}\right) = \frac{y_{A11}}{y_{a1}y_{b1}} = \frac{[A11]^1}{[a1]^1[b1]^1}$$

$$K_1^A = \frac{\left(\frac{A11_0 + e_1}{\Sigma_0 - (e_1+e_2+e_3+e_4)}\right)}{\left(\frac{a1_0 - e_1 - e_2}{\Sigma_0 - (e_1+e_2+e_3+e_4)}\right)\left(\frac{b1_0 - e_1 - e_3}{\Sigma_0 - (e_1+e_2+e_3+e_4)}\right)}$$

Let $\Sigma = \Sigma_0 - (e_1 + e_2 + e_3 + e_4)$

$$K_1^A = \frac{e_1\Sigma}{(a1_0 - e_1 - e_2)(b1_0 - e_1 - e_3)}$$

$$K_1^A = \frac{e_1\Sigma}{a1_0b1_0 - a1_0e_1 - a1_0e_3 - b1_0e_1 + e_1^2 + e_1e_3 - b1_0e_2 + e_2e_1 + e_2e_3}$$

Take the linear approximation for x=e_1 about 0 with x_0=y_0=0

$$T(x,y) = f(x_0,y_0) + f_x(x_0+y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$K_1^A = \frac{0 * \Sigma}{a1_0b1_0 - a1_0(0) - a1_0(0) - b1_0(0) + (0)^2 + (0)(0) - b1_0(0) + (0)(0) + (0)(0)} + \left(\frac{\Sigma * (-a1_0 - b1_0 + 2e_1 + e_2 + e_3) * e_1}{(a1_0b1_0)^2} + \frac{\Sigma}{a1_0b1_0}\right) * (e_1 - 0) + \cdots$$

Where the other partial derivatives reduce to zero and $\Sigma = \Sigma_0$

$$K_1^A = \frac{\Sigma_0 e_1}{a1_0 b1_0}$$

Note that this derivation is for 2 players, for a more general case Σ_0 is raised to the n-1 power

Appendix B

GAMS Code for 2 Player Game

```
Sets
M Reactors K /A,B,D/
r reactions in each reactor K /r1,r2,r3,r4/
i species in reactor A /a1,a2,b1,b2,A11,A12,A21,A22/
j species in reactor B /a1,a2,b1,b2,B11,B12,B21,B22/
k species in reactor D /A11, A12, A21, A22, B11, B12, B21, B22, D11, D12, D21, D22/ ;

Parameter   n0A(i)    initial concentration in reactor A
/     a1    0.5
      a2    0.5
      b1    0.5
      b2    0.5
      A11    0
      A12    0
      A21    0
      A22    0     /  ;
Parameter   n0B(j)    initial concentration in reactor B
/     a1    1.5
      a2    1.5
      b1    0.5
```

b2 0.5

B11 0

B12 0

B21 0

B22 0 / ;

Table dG(r,M) pains of rxn r in reactor K

	A	B	D	
r1	0.8	0.8	-1	
r2	1.15	0.65	-1	
r3	0.65	1.15	-1	
r4	1.00	1.00	-1	;

Table va(i,r) stoichiometric coefficients in reactor A

	r1	r2	r3	r4
a1	-1	-1	0	0
a2	0	0	-1	-1
b1	-1	0	-1	0
b2	0	-1	0	-1
A11	1	0	0	0
A12	0	1	0	0
A21	0	0	1	0

A22 0 0 0 1 ;

Table vb(j,r) stoichiometric coefficients in reactor B

	r1	r2	r3	r4
a1	-1	-1	0	0
a2	0	0	-1	-1
b1	-1	0	-1	0
b2	0	-1	0	-1
B11	1	0	0	0
B12	0	1	0	0
B21	0	0	1	0
B22	0	0	0	1 ;

Table vd(k,r) stoichiometeric coefficients in reactor D

	r1	r2	r3	r4
A11	-1	0	0	0
A12	0	-1	0	0
A21	0	0	-1	0
A22	0	0	0	-1
B11	-1	0	0	0
B12	0	-1	0	0
B21	0	0	-1	0
B22	0	0	0	-1

D11	1	0	0	0	
D12	0	1	0	0	
D21	0	0	1	0	
D22	0	0	0	1	;

Variables

e(r,M)

n1(i)

n2(j)

n3(k)

changeA(i)

changeB(j)

changeD(k)

sum1

sum2

sum3

y1(i)

y2(j)

y3(k)

ga(r)

gb(r)

gd(r)

```
G       ;

sum1.lo=1;
sum2.lo=1;
sum3.lo=1.e-2;
y1.lo(i)=1.e-7;
y2.lo(j)=1.e-7;
y3.lo(k)=1.e-7;

Positive Variable ga, gb, gd;

Equations
echangeA(i)
echangeB(j)
echangeD(k)
enA(i)
enB(j)
en3A11
en3A12
en3A21
en3A22
en3B11
en3B12
```

```
en3B21

en3B22

en3D11

en3D12

en3D21

en3D22

esum1

esum2

esum3

ey1

ey2

ey3

eobj

ega(r)

egb(r)

egd(r) ;


echangeA(i)..  changeA(i)=e=sum(r, va(i,r)*e(r,'A'));

echangeB(j)..  changeB(j)=e=sum(r, vb(j,r)*e(r,'B'));

echangeD(k)..  changeD(k)=e=sum(r, vd(k,r)*e(r,'D'));


enA(i)..       n1(i)=e=n0A(i)+changeA(i);

enB(j)..       n2(j)=e=n0B(j)+changeB(j);
```

```
en3A11..     n3('A11')=e=e('r1','A')-e('r1','D') ;

en3A22..     n3('A12')=e=e('r2','A')-e('r2','D') ;

en3A12..     n3('A21')=e=e('r3','A')-e('r3','D') ;

en3A21..     n3('A22')=e=e('r4','A')-e('r4','D') ;

en3B11..     n3('B11')=e=e('r1','B')-e('r1','D') ;

en3B22..     n3('B12')=e=e('r2','B')-e('r2','D') ;

en3B12..     n3('B21')=e=e('r3','B')-e('r3','D') ;

en3B21..     n3('B22')=e=e('r4','B')-e('r4','D') ;

en3D11..     n3('D11')=e=e('r1','D') ;

en3D22..     n3('D12')=e=e('r2','D') ;

en3D12..     n3('D21')=e=e('r3','D') ;

en3D21..     n3('D22')=e=e('r4','D') ;


esum1..   sum1=e=sum(i,n1(i));

esum2..   sum2=e=sum(j,n2(j));

esum3..   sum3=e=sum(k,n3(k));


ey1(i)..    y1(i)=e=n1(i)/sum1;

ey2(j)..    y2(j)=e=n2(j)/sum2;

ey3(k)..     y3(k)=e=n3(k)/sum3;


ega(r)..   ga(r)=e=0.5*dG(r,'A')+sum(i,va(i,r)*log(y1(i)));

egb(r)..   gb(r)=e=0.5*dG(r,'B')+sum(j,vb(j,r)*log(y2(j)));
```

```
egd(r)..   gd(r)=e=dG(r,'D')+sum(k,vd(k,r)*log(y3(k)));



eobj..   G=e=sum(r,ga(r))+sum(r,gb(r))+sum(r,gd(r));


Model PD123 /all/;

PD123.ScaleOpt=1;

Solve PD123 using NLP minimizing G;

option changeA:8;

option changeB:8;

option changeD:8;

option n3:8;


Display e.l,changeA.l,changeB.l,changeD.l,n3.l,dG;
```

Appendix C

Mathematica Code for n-Player game (5 to 10)

"n"-Player Tragedy of the Commons linear approximation using CGT

Inputs

Number of Players "n"

```
n = 9;
```

Biases

```
myquiet = 0.5; (* This is how each player percieves his own bias to staying \
quiet. Or the amount of a1 initially in reactor A, b1 in reactor B, and so on*) \
mytell = 0.5;
quiet = 0.5;(*This is how each player percieves opponents bias to staying \
quiet. Or the amount of b1=c1=d1 in reactor A, the amount of a1=c1=d1 in \
reactor B and so on*)
tell = 0.5;
self = {myquiet, mytell};
other = {quiet, tell};
```

Formulations

```
coop = Table[
If[Mod[i, 2^(n + 1 - j)] > 2^(n - j), 1, 0],
{i, 2^n}, {j, n}];
coop2 = Table[
```

```
If[Mod[i, 2^(n + 1 - j)] == 0, 1, 0], {i, 2^n}, {j, n}];

coop3 = coop + coop2;

coop4 = Transpose[coop3];

count = Total[coop4, 1];

dG = coop3;
```

Pain functions for cooperators and defectors

```
funcDef = 2*Log[1.5*count];

funcCoop = 2*Log[1.5*count + 2.25];
```

The functions above describe the pains to people who choose to cooperate or stay quiet, dependent upon the number of people who tell. The number of people who tell is described by the variable "count." So for example, the pain for a cooperator is at a maximum if everyone else tells, or defects. This occurs when count= n-1.

```
For[i = 1, i <= 2^n, i++,

 For[j = 1, j <= n, j++,

{If[coop3[[i, j]] > 0, dG[[i, j]] = funcDef[[i]], 0],

If[coop3[[i, j]] < 1, dG[[i, j]] = funcCoop[[i]], 0]}
```

```
]

]
```

More formulations

```
v = Table[0, {i, 2^n + 2*n}, {j, 2^n}];

v1 = v;

For[i = 1, i <= 2^n + 2*n, i++,

For[j = 1, j <= 2^n, j++,

{If[i <= 2*n && Mod[i, 2] == 1 &&

0 < Mod[j, 2^(n + 1 - (i + 1)/2)] <= 2^(n - (i + 1)/2), v1[[i, j]] = -1,

0],

If[i <= 2*n && Mod[i, 2] == 0 && Mod[j, 2^(n + 1 - i/2)] > 2^(n - i/2),

v1[[i, j]] = -1, 0],

If[i <= 2*n && Mod[i, 2] == 0 && Mod[j, 2^(n + 1 - i/2)] == 0,

v1[[i, j]] = -1, 0],

If[i > 2*n && i - 2*n == j, v1[[i, j]] = 1, 0]}

]

]
```

```
v3 = Table[0, {i, 2^n*(n + 1)}, {j, 2^n}];
v2 = v3;
For[i = 1, i <= 2^n*(n + 1), i++,
For[j = 1, j <= 2^n, j++,
{If[Mod[i, 2^n] == j && i <= 2^n*n, v2[[i, j]] = -1],
If[Mod[i, 2^n] == 0 && i <= 2^n*n && j == 2^n, v2[[i, j]] = -1],
If[Mod[i, 2^n] == j && i > 2^n*n, v2[[i, j]] = 1],
If[Mod[i, 2^n] == 0 && i > 2^n*n && j == 2^n, v2[[i, j]] = 1]}
]
]

MatrixForm[v2];

bias1 = Table[0, {i, 2^n}, {j, n}];

bias2 = bias1;

For[i = 1, i <= 2^n, i++,
For[j = 1, j <= n, j++,
{If[Mod[i, 2^(n + 1 - j)] > 2^(n - j), bias2[[i, j]] = mytell,
bias2[[i, j]] = myquiet],
```

```
If[Mod[i, 2^(n + 1 - j)] == 0, bias2[[i, j]] = mytell]}

]

]


bias3 = bias1;

For[i = 1, i <= 2^n, i++,

For[j = 1, j <= n, j++,

{If[Mod[i, 2^(n + 1 - j)] > 2^(n - j), bias3[[i, j]] = tell,

bias3[[i, j]] = quiet],

If[Mod[i, 2^(n + 1 - j)] == 0, bias3[[i, j]] = tell]}

]

]


MatrixForm[bias3];


bias = self + other*(n - 1);


bias4 = Total[bias]
```

9.

```
ex = bias1;


For[i = 1, i <= 2^n, i++,
 For[j = 1, j <= n, j++,
  ex[[i, j]] =
   Exp[-dG[[i, j]]]/(bias4^(n - 1))*
    Product[bias3[[i, j]], {j, 1, n}]/bias3[[i, j]]*bias2[[i, j]]
  ]
 ]


sum1 = Total[ex];
sum2 = Total[sum1];


exx = bias1;
For[i = 1, i <= 2^n, i++,
 For[j = 1, j <= n, j++,
  exx[[i]] = Exp[1]/sum2^(n - 1)*Product[ex[[i, j]], {j, 1, n}]
  ]
 ]
MatrixForm[exx];


change = v;
```

change = v1.ex;

change3 = change[[All, 1]];

Results

MatrixForm[

dG]; (*Remove the semicolons here if you want to see your table of pains. \

Rows are reaction numbers and columns are the players*)

MatrixForm[change];

MatrixForm[ex];(*This is a table of extents of reactions*)

MatrixForm[exx] ; (*This table is for the extents of reaction of the decider*)

sum3 = Total[exx];

yDecider = exx/sum3;

cooperationrate =

Total[change[[1]]]/(Total[change[[2]]] + Total[change[[1]]])

0.608199

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24. Laura Cook, SHC thesis

ACADEMIC VITA

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EDUCATION

The Pennsylvania State University, Schreyer Honors College **University Park, PA**

- *College of Engineering* | B.S. in Chemical Engineering, Energy and Fuels Option
- *College of Liberal Arts* | Minor in Economics
- *Eberly College of Science* | Minor in Mathematics

RELEVANT EXPERIENCE

Penn State Department of a Engineering **University Park, PA**
Member, Independent-based Research Lab "Chemical Game Theory" *Aug 2016 – Present*

- Created a nonlinear program to accurately model human behavior using chemical engineering and game theory principles in the GAMS programming languages
- Modeled and analyzed "Tragedy of the Commons" games relating to game theory

Penn State Department of Chemical Engineering **University Park, PA**
Teaching Assistant *May 2017 – July 2017*

- Assisted Professor Velegol in CHE 210 Material Balances, and CHE 320 Thermodynamics II
- Led review sessions, graded homework, and answered students' questions for a class size of 30 and 50 students

Penn State Hershey Medical Center **Hershey, PA**
Intern *Jun 2013-Aug 2013*

- Assisted with experiments to test the artificial heart/lung device (ECLS system)
- Published research paper titled: "Potential Danger of Pre-pump Clamping on Negative Pressure-associated Gaseous Microemboli Generation during ECLS - an In-Vitro Study."

LEADERSHIP AND INVOLVEMENT

Penn State THON **University Park, PA**
Hospitality Committee Member / Rules and Regulations Committee Member *September 2016 - Present*

- Served meals to dancers at the 46-hour dance marathon to raise money for pediatric cancer
- Check-in specialist for spectators entering onto the floor during the marathon

ServeState: Students for Philanthropy **University Park, PA**
Member *August 2017- Present*

- Completed 25 hours of community service per semester
- Favorite activities include visits to the retirement home, helping reconstruct a farm café, and THON involvement