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SCHREYER HONORS COLLEGE

DEPARTMENT OF AEROSPACE ENGINEERING

END OF LIFE DISPOSAL OF SATELLITES IN HIGHLY ELLIPTICAL ORBITS

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ABSTRACT

Highly elliptical orbits allow for coverage of large parts of the Earth through a single satellite, simplifying communications in the globe's northern reaches. These orbits are able to avoid drastic changes to the argument of periapse by using a critical inclination (63.4°) that cancels out the first level of the geopotential forces. However, this allows the next level of geopotential forces to take over, quickly de-orbiting satellites. Thus, a balance between the rate of change of the argument of periapse and the lifetime of the orbit is necessitated. This thesis sets out to find that balance. It is determined that an orbit with an inclination of 62.5° strikes that balance best.

While this orbit is optimal off of the critical inclination, it is still near enough that to allow for potential use of inclination changes as a deorbiting method. Satellites are deorbited when the propellant remaining is enough to perform such a maneuver, and nothing more; therefore, the less change in velocity necessary for to deorbit, the better. Following the determination of an ideal highly elliptical orbit, the different methods of inclination change is tested against the usual method for deorbiting a satellite, an apoapse burn to lower the periapse, to find the most propellant-efficient method. The normal apoapse burn is found to be slightly more propellant efficient that inclination changes, and thus is recommended for use with highly elliptical orbits.

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ACKNOWLEDGEMENTS

I’d like to thank the academy.

Chapter 1

INTRODUCTION AND BACKGROUND

As the space age moved past its infancy, the USSR ran into several problems with the capabilities of the satellites it was flying. Engineers found that it was hard to put satellites in low earth orbits with high enough inclinations that they were able to cover the most northern regions of their country. Additionally, due to the sheer size of the country, satellites in low earth orbit were only capable of covering a fraction of the area of the country at any given moment.

In response to these issues, Soviet engineers sought to create stable orbits, capable of covering large portions of the country at once, including the far northern regions. They began investigating the possibility of a highly elliptical orbit with apoapse high above the northern hemisphere. Such an orbit would spend the majority of its time above the northern hemisphere, at the peaks of the ground track shown in Figure 1, passing very quickly through its periapse at the southern half of the globe.

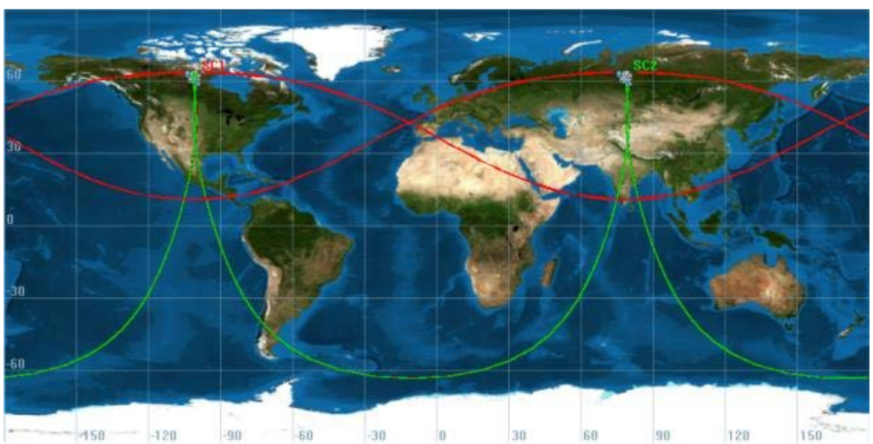

Figure 1. Ground Track and Coverage of a Constellation of Two Highly Elliptical Orbit (Dunham)

The high altitudes such satellites would reach would allow the satellites to cover most (if not all) of the USSR at once, with the northern regions included. One problem this orbit would face was the issue of keeping its periapse from drifting into the northern hemisphere due to perturbations. Should this occur, a satellite would spend less time over the northern hemisphere, and more time over the southern hemisphere, defeating the benefits of a highly elliptical orbit. Perturbations due to geopotential forces could cause the argument of periapse for such an orbit to change very quickly. Argument of periapse, labeled as ω in Figure 2, is the angle along the orbit between the equator and the periapse, or the point of the orbit closest to the Earth. Figure 2 also defines the inclination *i*, Right Ascension of the Ascending Node (RAAN) Ω and the true anomaly ν (usually referred to as θ).

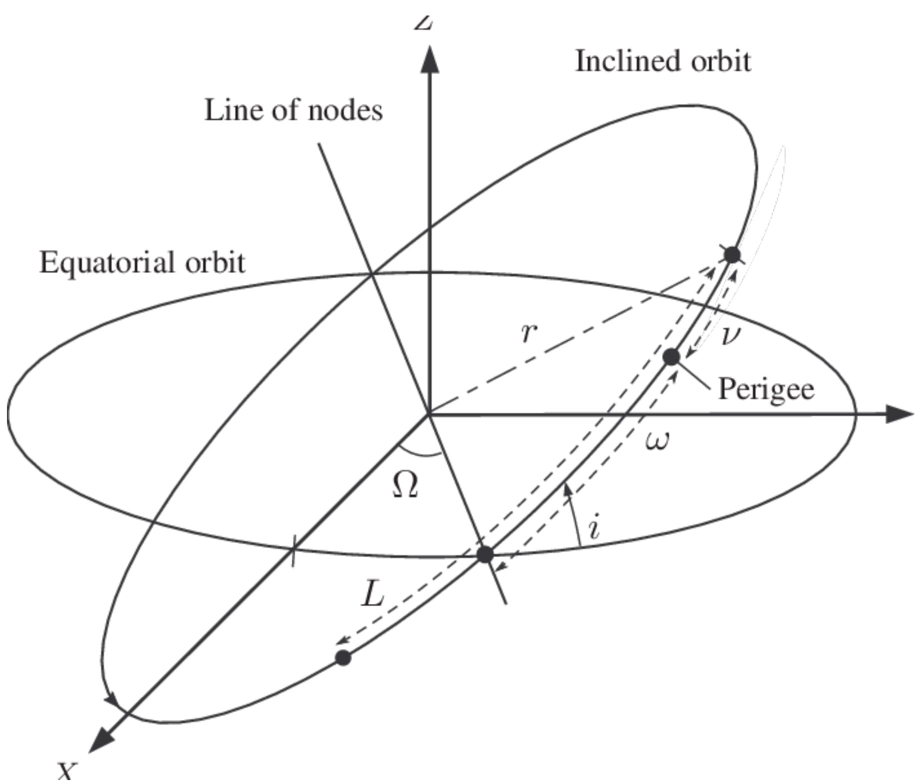

Figure 2. Classical Orbital Elements

Geopotential forces are perturbations caused by the non-uniformity of the Earth's mass. Due to the various distributions, there are several degrees of geopotential forces the can affect satellites around the Earth; the most prominent of the geopotential forces, referred to as J_2, is caused by the oblateness of the Earth, as illustrated in Figure 3. The left side of Figure 3 demonstrates the general shaping of the Earth that causes the first level of geopotential effects, J_2, where the shaded areas are more prominent that the non-shaded areas. Similarly, the right side shows the shaping that leads to the second level of geopotential effects, J_3. There are many more levels of geopotential effects, but these two are the only ones that will be discussed in this thesis.

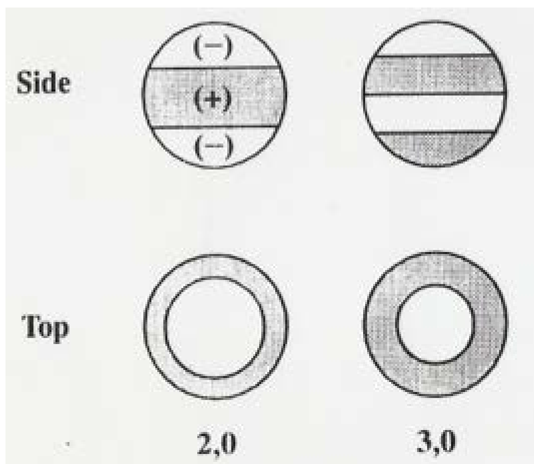

Figure 3. Mass Distribution in the Earth Causing Geopotential Effects (Vallado)

Geopotential forces have a wide array of effects upon satellites orbiting the Earth, but for the purposes of this thesis the most important is the direct correlation between J_2 and the rate of change of argument of periapse. Left unchecked, J_2 would cause the argument of periapse to change, moving the periapse of the highly inclined orbits the Soviets were working with constantly. This change would cause satellites in this orbit to spend more time over the southern hemisphere than the northern, as mentioned above. It's true that, eventually, the satellites would move back into a useful position, but too much time would be wasted by these satellites in a type of orbit essentially useless to the USSR.

The solution was to create an orbit that would negate the effects of J_2. After some study, it was found that the average change in argument of periapse due to J_2 is completely dependent upon the inclination of the orbit, shown in the equation (1).

$$\frac{d\omega}{d\theta} = \frac{3}{2} n J_2 (\frac{R_e}{p})^2 (2 - \frac{5}{2} sin^2 i) \tag{1}$$

Therefore, an inclination of approximately 63.4 degrees (a 'critical inclination') would completely negate the effects of J_2 by making the time rate of change for the argument of periapse in equation (1) equal to 0. The engineers at the time expected such an orbit to be capable of persisting in that same orbit, unperturbed, essentially forever. This orbit shape was named 'Molniya,' Russian for lightning. A satellite was launched into such an orbit to test this hypothesis. The engineers were surprised to see that the satellite crashed back into the atmosphere after a short time in orbit.

After thorough investigation, the problem was found. The engineers came to the realization that while geopotential effects of a higher order than J_2 can normally be neglected completely, this was not true for the orbit they had created. J_2 usually has the effect of 'covering up' all higher order tidal forces, but reduced to zero in this instance, that did not happen. When the satellite was sent into orbit, J_3 became the most dominant geopotential effect was able to take control and very quickly throw the satellite out of orbit by increasing the orbit's eccentricity, as in equation (2), ultimately raising the apoapse and lowering the periapse into the atmosphere. In equation (2), R is the radius of the Earth, and a is the (unchanging) semi-major axis of the orbit (Chobotov). J_3 is the most prominent geopotential force after J_2 caused by the Earth's slight pear shape, as shown in Figure 3.

$$\Delta e = -\frac{1}{2}\frac{J_3}{J_2}\left(\frac{R}{a}\right) sinisin\omega \tag{2}$$

The solution for this problem was quickly found. By putting satellites into an orbit very near to the critical inclination, but not quite there, J_2 effects could still impact the satellite enough that J_3 would be effectively negated. The effects of J_2 in this instance are small enough that, with some occasional orbit maintenance, such an orbit could be held by a satellite for a long time without the argument of periapse changing drastically. Such orbits are known as 'frozen orbits' due to the orbital elements essentially freezing once created, with satellites only leaving such orbits if boosters are fired to force them to do so or the necessary orbit maintenance is not performed.

In addition to geopotential forces, the nature of Molniya orbits allows for third body effects of the Sun and Moon to have a large impact upon them. Third body effects are gravitational forces from bodies other than the body being orbited that act upon the satellite. In the case of satellites orbiting the Earth, the most prominent (and only third body effects that will be considered for the purpose of this thesis) third body effects are those of the Sun and the Moon. Because highly elliptical orbits reach such high altitudes, third body effects can be especially impactful upon them as compared to other orbits.

This thesis will seek to find the inclination near the critical value that best strikes a balance between damping out J_3 effects to allow the satellite to remain in orbit for a longer period of time, while also minimizing J_2 effects to sufficiently minimize the rate of change of the argument of periapse. The search for this ideal inclination will be performed using the Systems Tool Kit (STK). Simulations will be run with the full range of geopotential forces active, as well as the third body effects of the sun and the moon. All tests will be run for the duration of a year starting on February 14 2019. The results will be used to find the duration of a satellite in such an orbit (without maintenance) and the annual change of the argument of periapse. These values will be used to find the optimal inclination that maximizes duration while minimizing the change of the argument of periapse.

Highly elliptical orbits have been utilized widely since this time, branching out a few times from the basic Molniya orbit for a wide variety of reasons. Such orbits are very effective in connecting people from faraway places. Improvements are being made constantly; the longer such satellites can maintain their orbits, the more cost effective they become. However, with the ever-growing problem of space debris, it becomes essential to dispose of such satellites when they do eventually reach the end of their life span. This will lead to the defining question for this thesis - what is the optimal method for disposing of highly elliptical orbits following the end of useful lifespan. Three methods present themselves.

The first option is a direct inclination change. As these orbits are so near to the critical inclination that will quickly send the satellite back to Earth due to the effect of J_3, a small amount of propellant fired at the ascending node (where the satellite crosses the equatorial orbit) to create an inclination change would be enough to move the satellite directly into the critical inclination. This would send the satellite back into the atmosphere quickly with the J_2 effects negated. The second option arises from the proximity of the periapse to the top of the atmosphere. A small burn at the apoapse should be able to lower the periapse enough that the satellite will re-enter the atmosphere when it swings back around. The third options, which is more complicated, involves a change to both the inclination and the RAAN at the apoapse. Again, this would force the orbit into the critical inclination, allowing J_3 effects to take dominate and force the satellite to re-enter.

This thesis explores these three options to find the best method for disposing of a satellite from a highly elliptical orbit. The methods will be judged on their cost in propellant (in the form of Δv. The methods will be tested on an orbit with the determined ideal inclination for a highly elliptical orbit.

Chapter 2

METHODOLOGY

The Systems Tool Kit (AGI) is being used to analyze data for the purposes of this thesis. All cases run in STK are created using the High-Precision Orbit Propagator (HPOP) force model; drag, solar radiation pressure and tidal forces are always turned off. Throughout all tests, effects of the Sun and Moon and the full range of geopotential forces are active. All tests are run over the period of a year, with time histories of the classical orbital elements generated every 1200 seconds. These reports are then exported into Excel, from which the information can be graphed using MATLAB. The majority of the tests shown will include the radius of periapse, the difference in instantaneous inclination from initial inclination, and the difference in instantaneous argument of periapse from initial argument of periapse.

Before using STK to determine the optimal inclination to use for frozen orbits, it is critical to verify that the program is able to replicate real life scenarios. For this reason, the initial tests completed were done upon a simulation for a real-life satellite, and a satellite with all of the same orbit except the inclination, which was moved to exactly critical. The real-life satellite chosen is of the International Designator 1999-036A (named Molniya 3-50), which is listed on Celestrak as the most recently launched Molniya satellite (at this writing) that is still in orbit. The most recent update to the satellite's orbital Two Line Element (TLE) set have the orbital elements listed in Table 1.

Table 1. Molniya 3-50 Orbital Elements (Epoch MJD 19043.77) (*Celestrak)*

Inclination (*i*)	62.1107°
RAAN (Ω)	163.1681°
Eccentricity (e)	0.7194429
Argument of Periapse (ω)	271.1684°
Mean Motion	2.00635219143645 revs/day

The simulation of Molniya 3-50's orbit and the version with inclination changed to critical (63.4349°), the results are shown in Figure 4 (actual inclination) and Figure 5 (critical inclination). The radius of periapse during the year is calculated through the values given by STK, including the semi-major axis, a, at every time step through equation (3).

$$r_p = a(1 - e) \tag{3}$$

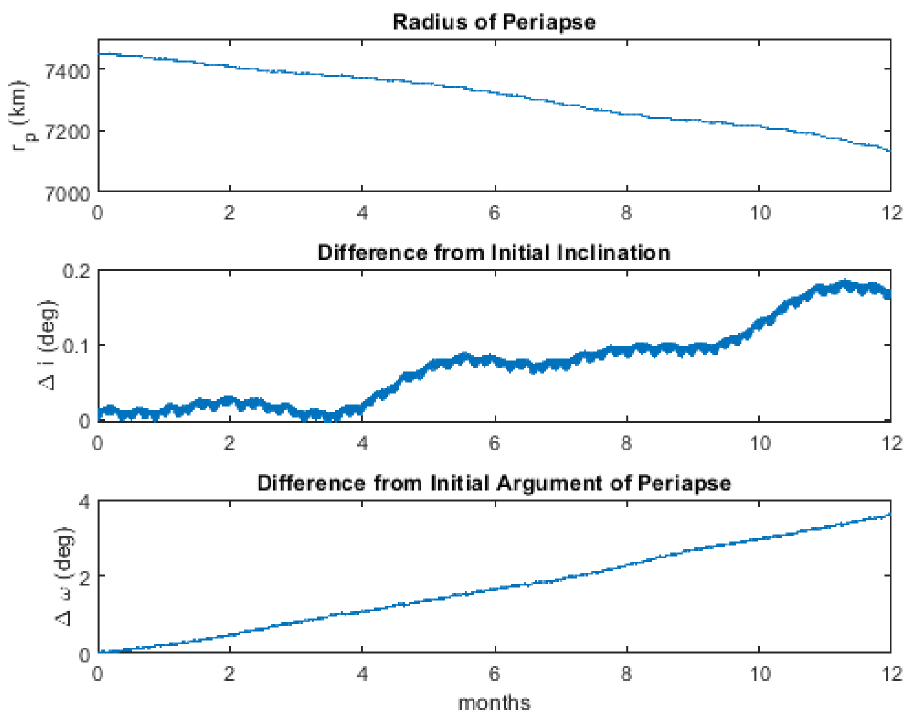

Figure 4. Molniya 3-50 Radius of Periapse, Inclination and Argument of Periapse (Actual Elements)

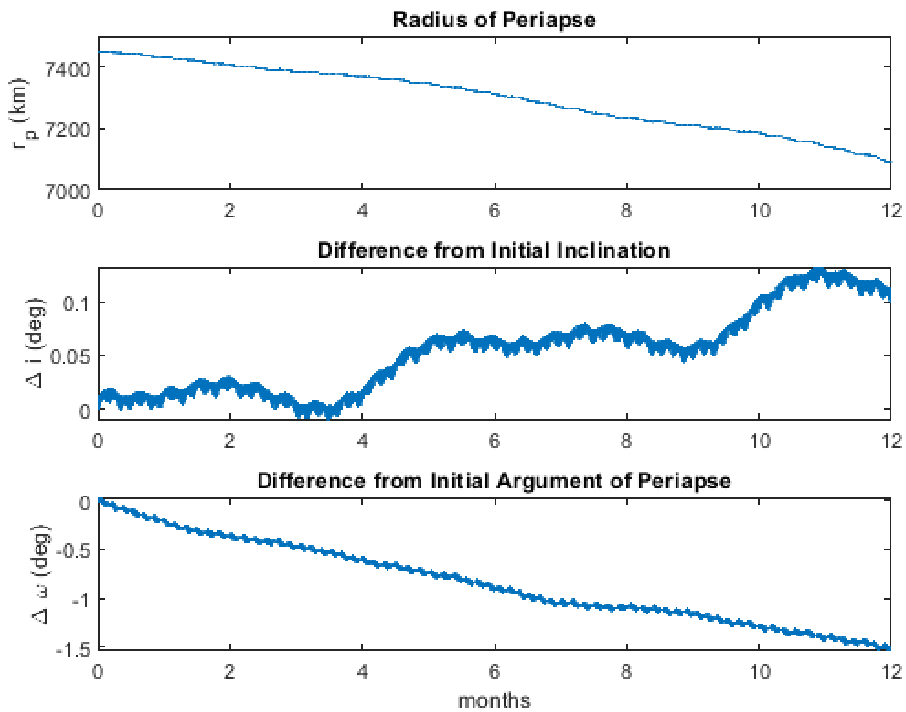

Figure 5. Molniya 3-50 (Modified to Critical Inclination) Radius of Periapse, Inclination and Argument of Periapse

It is important to note that in reality, satellites such as Molniya 3-50 have their orbits maintained regularly with small thrusting maneuvers to avoid decay. However, the results shown in the image above show what would happen to the satellite's orbit if it was left for a year without being maintained.

Both of the above tests were performed including the effects of both the Sun and the Moon, and the full range of geopotential forces. The most striking difference between the two tests is how the argument of periapse changes over the course of the year. For Molniya 3-50, it increases nearly 4°, while the critically inclined satellite shows a decrease of approximately 1.5°. The smaller change for the critically inclined satellite goes with expected results, as critically inclined orbits are expected to have a rate of change for argument of periapse of 0. The amount ω does change for the critically inclined orbit is likely due to third body effects, as well as the slow drift from critical inclination resulting from those effects.

Importantly, the critically inclined orbit shows a faster decay in the radius of periapse. For the purposes of this thesis, satellites that come within 100 kilometers of the surface of the Earth are considered to have re-entered the atmosphere. At the end of the year, the critically inclined orbit has reached r_p=7091.2 km, while Molniya 3-50 is at r_p=7135.7 km. This corresponds to the satellites being 613.2 km and 657.7 km above the top of the atmosphere at periapse, respectively. Taking these trends to be approximately linear, the critically inclined satellite would re-enter the atmosphere approximately 2.71 years, while Molniya 3-50 would last approximately 3.09 years without maintenance. This shows that Molniya 3-50, by having an inclination only 1.3242° away from critical, is able to decrease the impact of J_3 that causes Molniya orbits to quickly re-enter the atmosphere. These results collectively show that STK can accurately recreate real life results.

Previous results display that, while Molniya 3-50 will go longer than a critically inclined orbit before re-entering the atmosphere, it still requires orbit maintenance to last longer than just a few years. As the inclination varies further from critical, it should take longer for re-entry to occur; however, the effect of J_2 forces will emerge with this change in inclination, causing the rate of change of the argument of periapse to grow. Simulations are performed with the inclination changing incrementally. This data is used to search for an inclination that strikes a balance - far enough from critical that J_3 can't take over and crash de-orbit the satellite very quickly, but close enough to critical that J_2 does not cause a large rate of change in the argument of peraipse. Because Molniya 3-50 has an inclination lower than critical, the tests are run with the inclination first moving lower away from critical. After the inclination has gone low enough that the J_2 term is determined to be causing the argument of periapse to change too quickly, similar tests will be run on orbits with inclinations higher than critical.

As tests are run in search of an optimal inclination, radius of periapse will be the most important orbital characteristic. Assuming r_p stays generally linear, as with the simulation for Molniya 3-50 and the critically inclined Molniya orbit above, it can be used to project how long satellites will be able to remain in orbit before re-entering the atmosphere. Inclination and argument of periapse, as above, is observed as well. Inclination is also observed to watch for movement towards critical. Molniya 3-50 shows the inclination increasing over the course of the year. This change was relatively insignificant, but larger movements could potentially cause orbit’s inclination to become critical. The argument of periapse is observed for rate of change. If the rate of change becomes high enough, the inclination will be determined to be too far from critical so that J_2 is taking over too much.

Chapter 3

RESULTS

In an effort to stay as close to reality as possible, tests on hypothetical orbits are done while retaining many of the features of the orbit described in the TLE for Molniya 3-50. The mean motion, argument of periapse and eccentricity from this orbit are not changed throughout all of the tests conducted.

As mentioned in the previous section, the first step was to search for an optimal inclination for a highly inclined orbit that allows for a satellite to last a long time without maintenance while limiting rate at which the argument of periapse changes. Inclinations up to two degrees above and below the critical inclination were tested at intervals of 0.25°. As the Sun and the Moon were active in all tests, each tested inclination angle was done at three different RAANs - 0°, 120° and 240°. These values were selected for their even distribution around the Earth. All tests were run between February 14, 2019 and February 14, 2020, so the different RAAN values test the effects of the Sun and Moon from different points around the Earth.

For each simulation, as with the test of the Molniya 3-50 orbit and the critically inclined orbit in the previous chapter, the radius of periapse, inclination and argument of periapse are tracked and shown in the resulting graphs. Note that the range of the radius of periapse graph, the only one held to a constant y-axis, has changed to include up to 8000 km, as several of the results that will be shown have the radius of periapse increase over time.

Figures 6, 7 and 8 show that the radius of periapse for satellites at 61.4° inclination increases over the course of a year when the satellite starts at a RAAN of either 0° or 240°. Initial RAAN of 120° is the only one that produces the expected result of a steadily decreasing radius of periapse. This is due to the position of the orbit relative to the position of the Sun and the Moon, which play a large part in forcing satellites out of orbits that get as far from the Earth at apoapse as the highly elliptical ones being tested here. Such results remain constant for every inclination tested through 65.4°. Take, for example, Figure 9, which shows that the radius of periapse for an orbit inclined at 65.15° with an initial RAAN of 240° increases (albeit not as much as orbits inclined at 61.4°) over a year. This essentially renders all of the tests run with initial RAAN of 0° or 240° useless in determining the lifetime of a satellite in an orbit before re-entry.

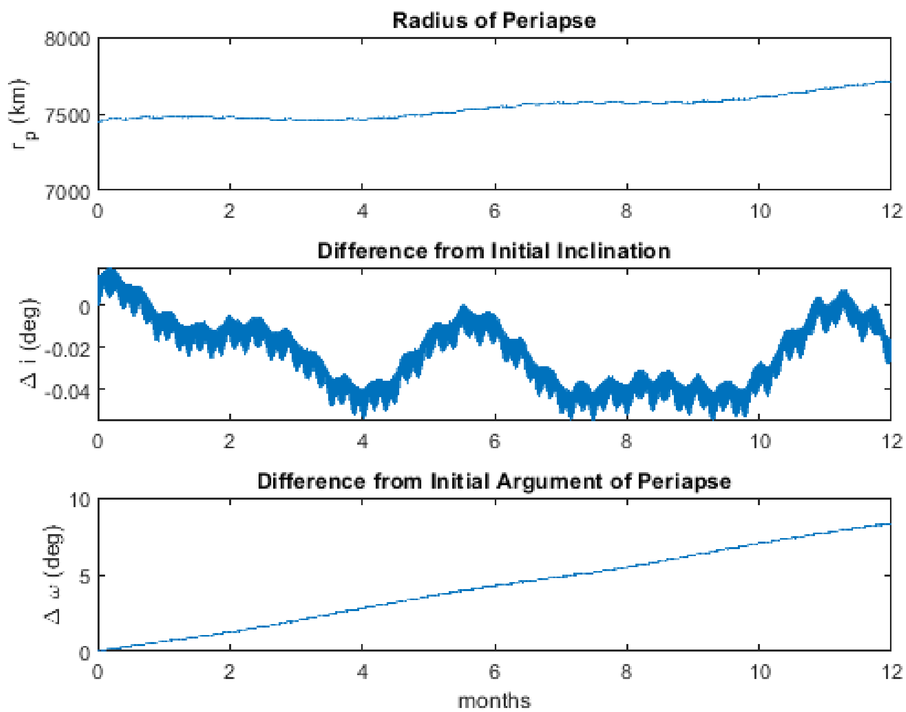

Figure 6. Initial Inclination 61.4°, Initial RAAN 0°

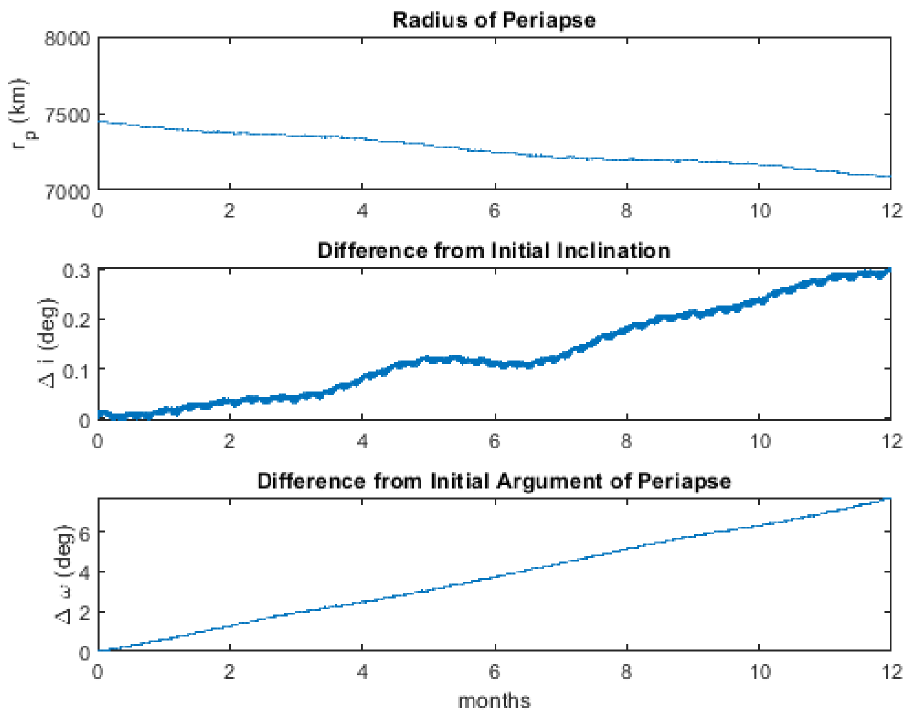

Figure 7.Initial Inclination 61.4°, Initial RAAN 120°

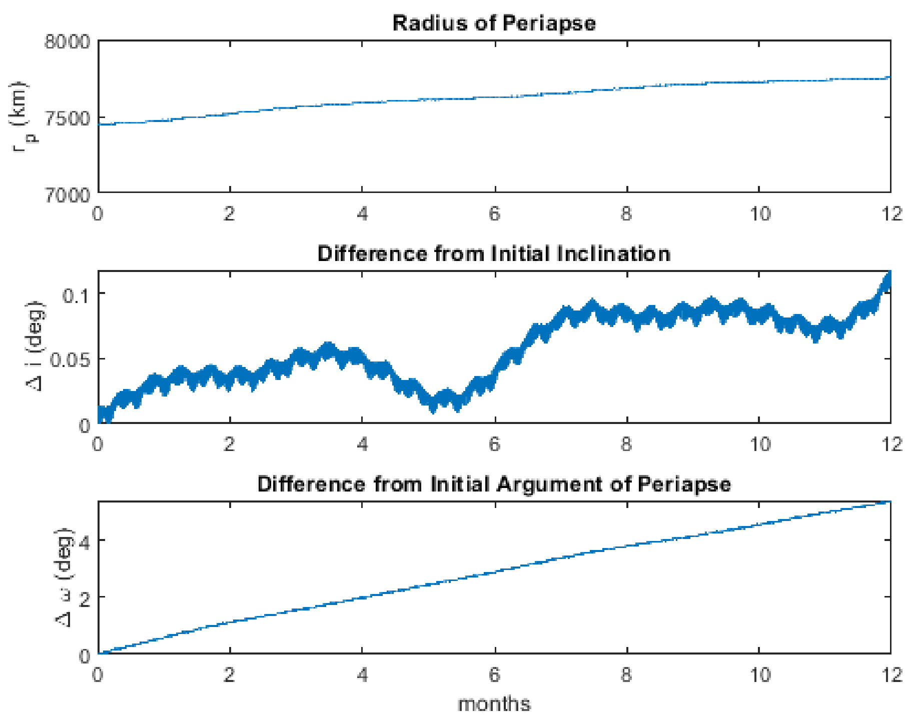

Figure 8. Initial Inclination 61.4°, Initial RAAN 240°

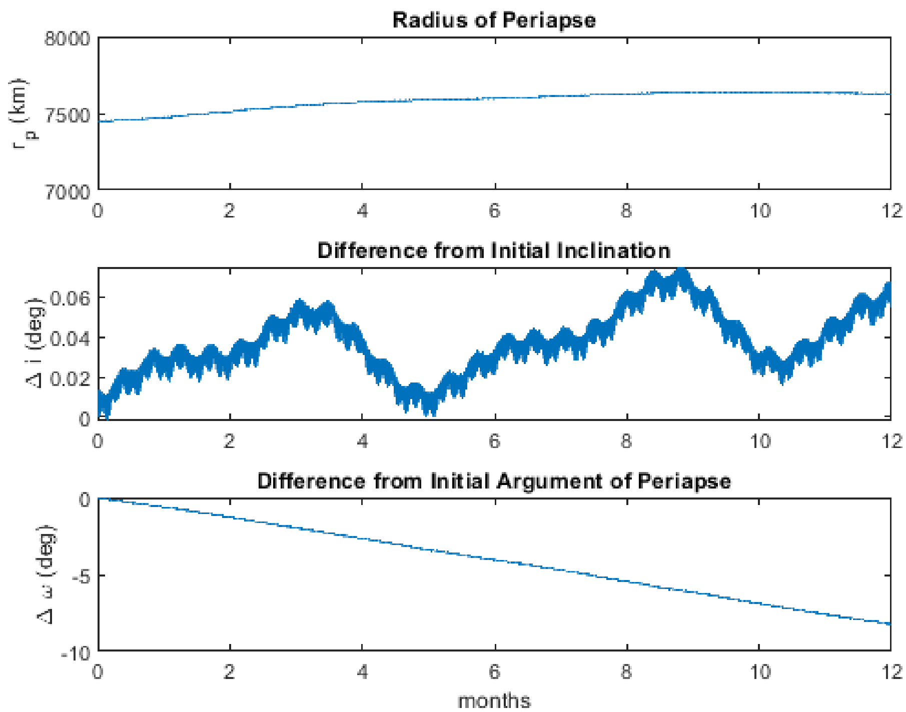

Figure 9. Initial Inclination 65.15°, Initial RAAN 240°

There is still some use in these tests though. Orbits with inclinations far from critical (such as those referred to in Figures 6, 7, 8 and 9) show a higher rate of change for the argument of periapse, which follows expectations established in previous chapters. This follows no matter the initial RAAN. Therefore, these tests can be utilized to search for a trend in the relation between the rate of change of argument of periapse and the inclination of an orbit.

Figure 10 shows the change in the argument of periapse after a year for all tests done on orbits with inclinations between and including 61.4° and 63.4°. Figure 11 shows the same for all tests done at inclinations between and including 63.4° and 65.4°. Values are shown are absolute, as the goal is to keep the argument of periapse as close to the original value of possible, no matter whether it is moving in a positive or negative direction.

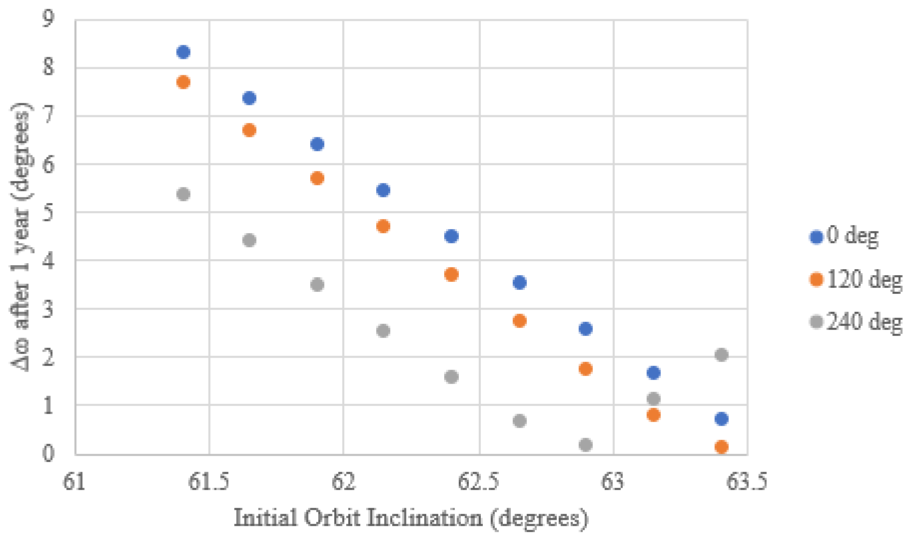

Figure 10. Change in Argument of Periapse for Inclinations under Critical, RAANs 0°, 120° and 240°

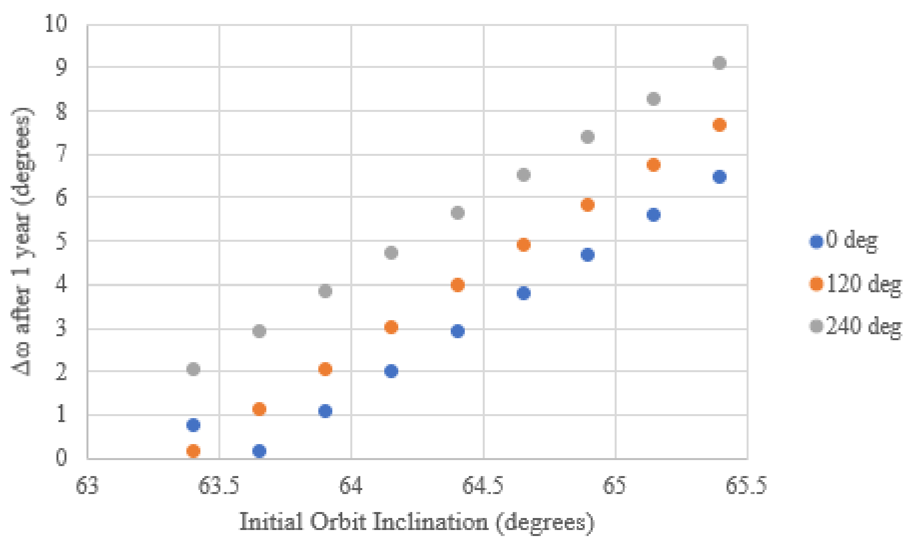

Figure 11.Change in Argument of Periapse for Inclinations above Critical, RAANs 0°, 120° and 240°

When slopes of the sets of points in these two figures are calculated, an interesting difference is revealed. For the orbits with inclination below and up to critical in Figure 10, the slope is -3.2442°/year. For orbits with inclinations including and above critical in Figure 11, the slope is 3.5011°/year. Therefore, orbits with an inclination below critical are preferable to those with inclinations higher than critical. Given two orbits with inclinations equally distant from critical, one above and one below, and all other factors remaining the same, the one with the smaller inclination will have a smaller absolute deviation in argument of periapse from the initial value.

Given the decision that an inclination below critical is preferable, and the trend for the absolute deviation of argument of periapse given the inclination, only a method for determining an expected lifespan based on the inclination is needed to find an optimal inclination. The previous tests showed that, given the orientation of the Sun and the Moon relative to the Earth during the tests, orbits with an initial RAAN of 120° show a steady decrease in radius of periapse as expected. It is then reasonable to assume that the worst-case scenario - the fastest an orbit of any given inclination could re-enter the atmosphere - must have an initial RAAN in the range of 120°. For this reason, further tests were done on orbits with inclinations below critical. At first, tests were run on orbits with inclinations between 62.85° and 63.15° at intervals of 0.1°; each inclination was paired with initial RAANs of 90°, 110°, 130° and 150° to cover the area around the RAAN of interest found in the initial round of tests, 120°.

Figures 12, 13, 14 and 15 show the results of an orbit with inclination 62.95° being tested with all of the above listed initial RAAN. All four results show a gradual decrease from the initial radius of periapse over the course of the year, with the sharpest decline at 130°. This pattern will stay true throughout this round of testing, meaning that 130° RAAN is closest to the worst-case scenario - that is, given the orientation of the Moon and Sun with respect to the Earth over the course of the test year, 130° is the closest of the RAAN tested to the fasting decaying.

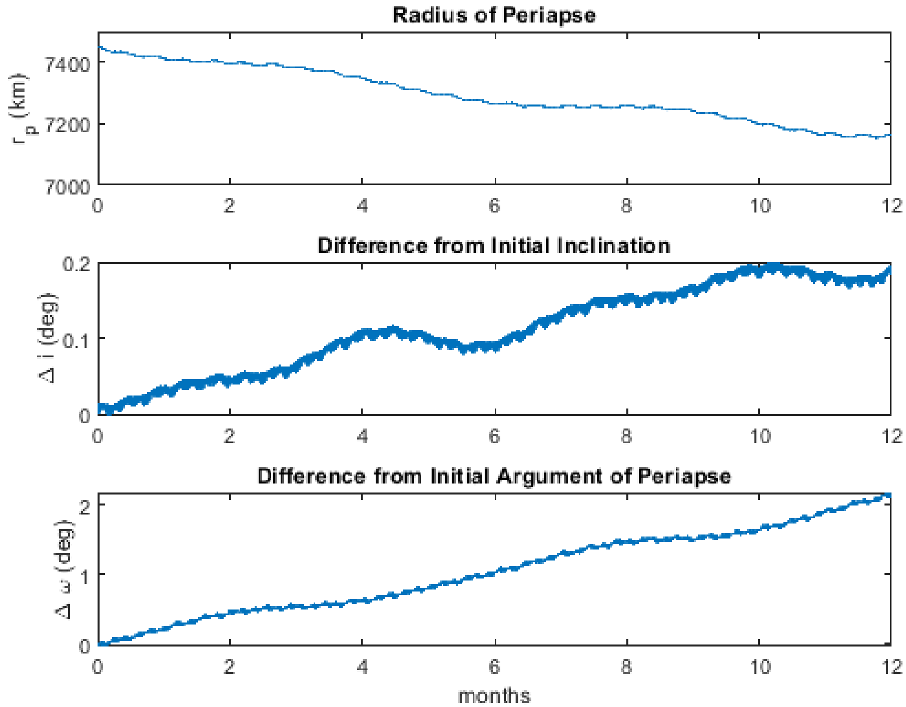

Figure 12. Initial Inclination 62.95°, Initial RAAN 90°

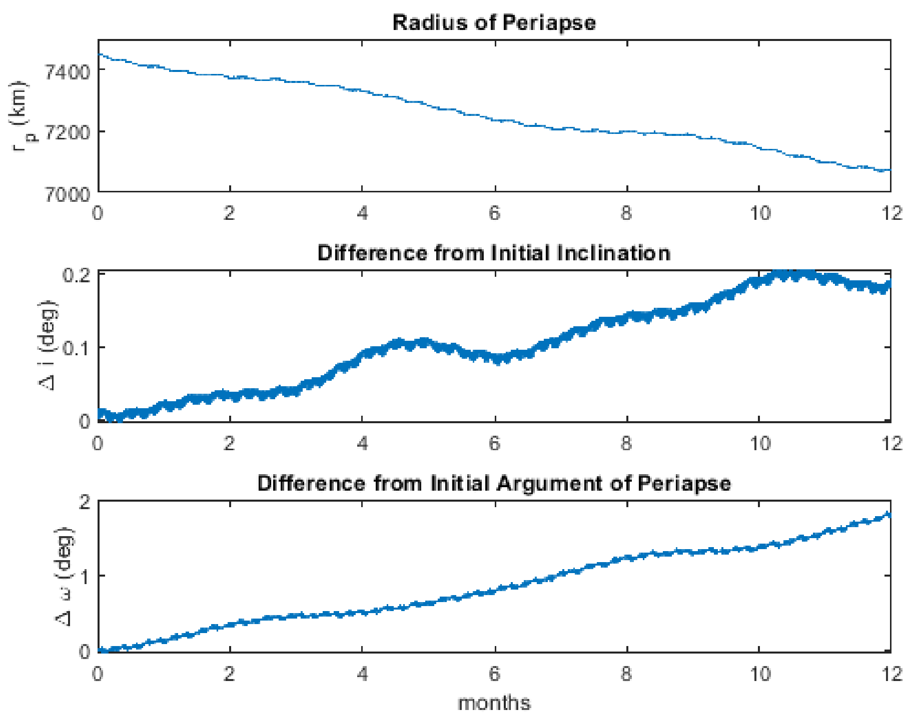

Figure 13. Initial Inclination 62.95°, Initial RAAN 110°

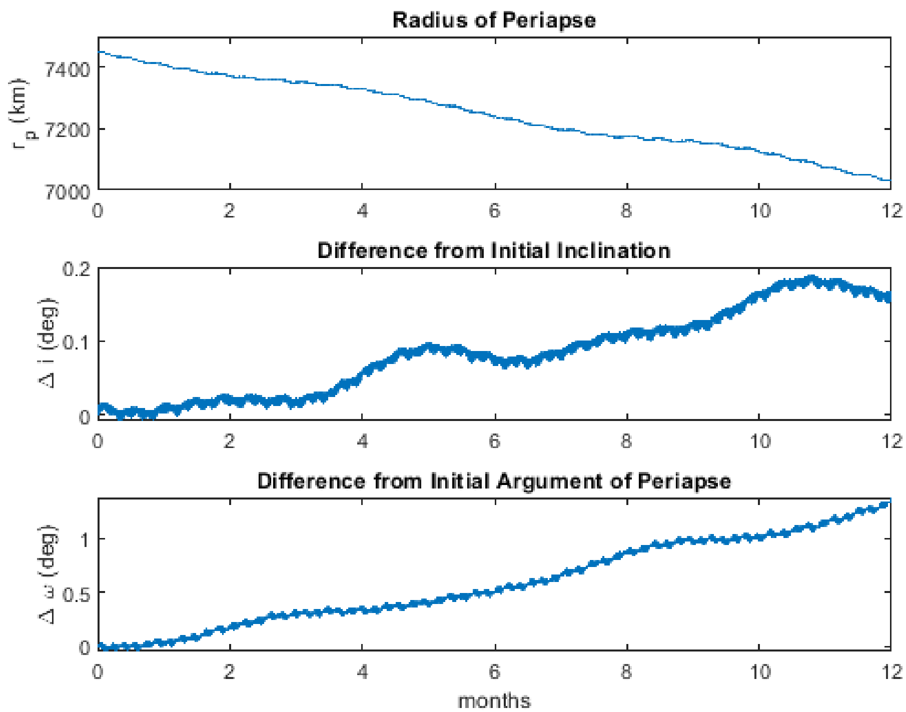

Figure 14. Initial Inclination 62.95°, Initial RAAN 130°

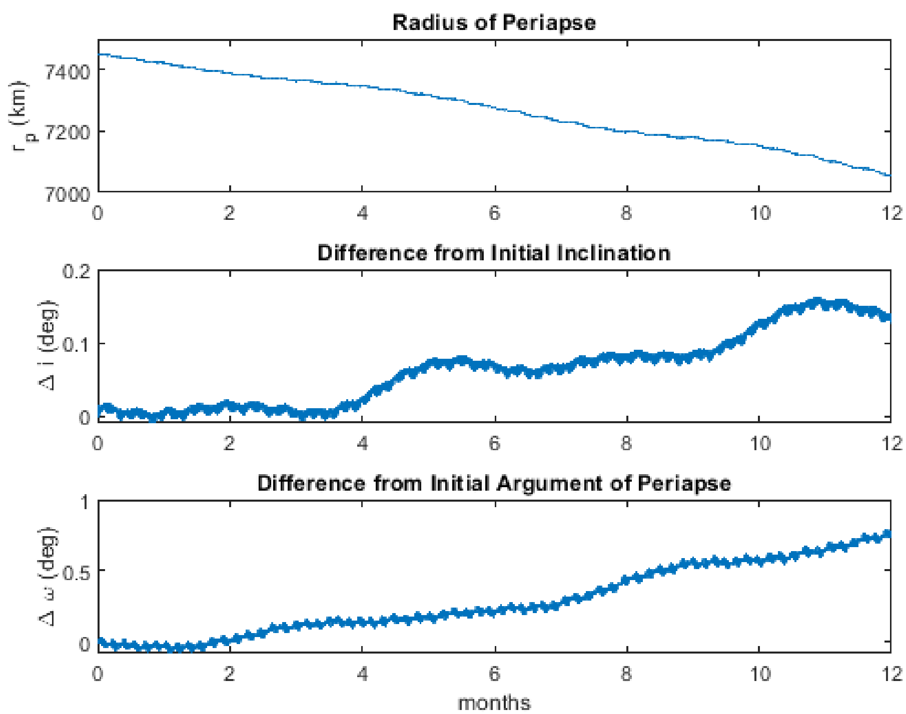

Figure 15. Initial Inclination 62.95°, Initial RAAN 150°

As with orbit exactly in critical inclination and the orbit of Molniya 3-50 in the Methods chapter, the decay in the altitude of the periapse above the surface of the Earth will be assumed to be linear, using the altitude at the beginning and end of the testing year as points off of which to base the trend. Following the tests down to inclination 62.85°, the step size increased for the following three tests to ensure patterns found in the data from the first four tests would continue to much lower inclinations. 62.55°, 62.05° and 61.35° were all tested.

Figure 16 confirms that, in all of the tests, 130° is the closest to the worst-case scenario. Tests done at RAAN 110° and 150° are close to this as well; however, tests run at 90° are found to last approximately a year longer than tests run at the same inclination with RAAN of 130°. Interestingly, orbits with RAAN closer to the worst-case scenario have a shallower slope than those further away, meaning that a decrease in inclination for an orbit with RAAN 130° improves the lifetime of a satellite in orbit less than the same decrease in inclination would improve the lifetime for a satellite in an orbit with RAAN 90°.

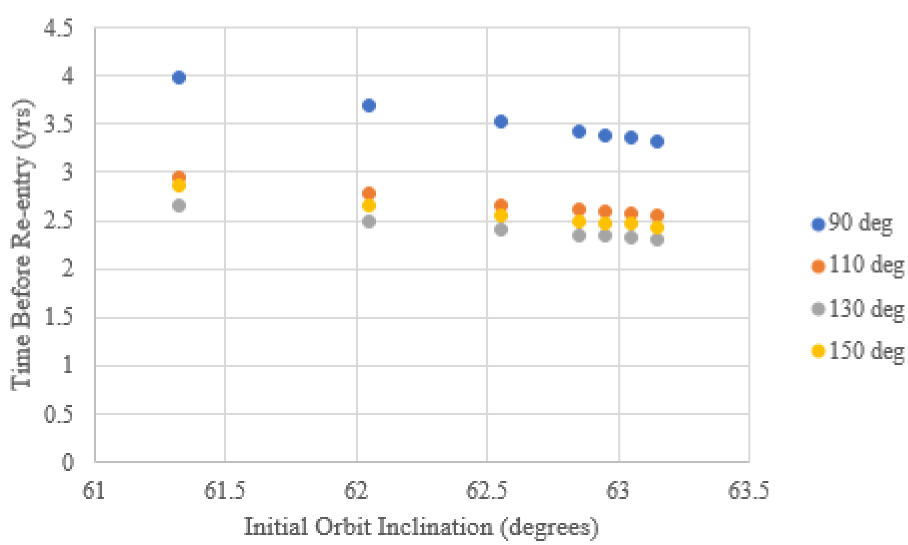

Figure 16. Time to Re-Entry for Inclinations below Critical, Near Worst-Case RAAN

Figure 17 shows that, excluding the data found with RAAN 90° for its extreme difference from the rest of the data, the trend-line (the dotted line in Figure 17) shows that a decrease in 1° inclination will lead to an increase in lifespan of 0.2 years. This information, combined with the data from Figure 10, allows for the selection of an optimal inclination minimizing the change in argument of periapse while maximizing the lifetime (without maintenance).

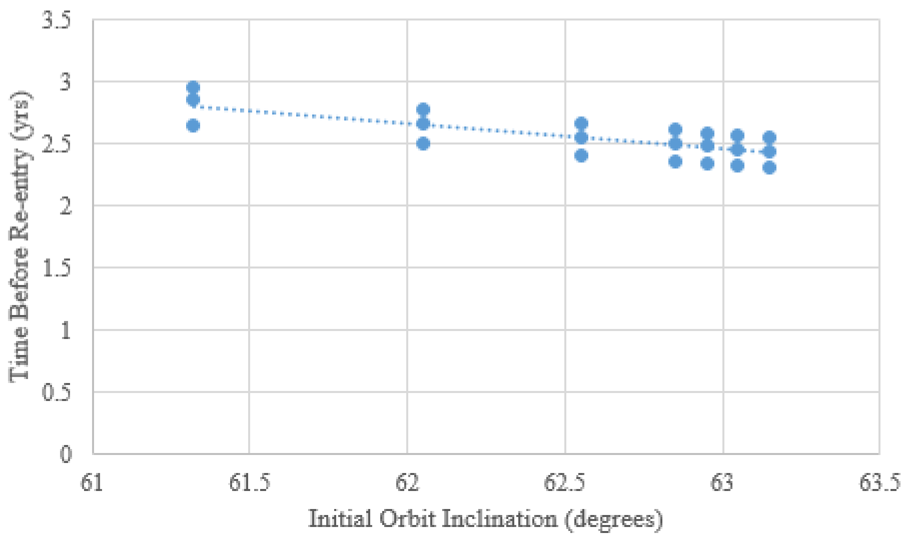

Figure 17. Time to Re-Entry Trend near Worst-Case RAAN

It was previously determined that satellite Molniya 3-50, with no maintenance done to its orbit (62.11° inclination) as of February 14, 2019, would last for approximately 3.09 years, and its argument of periapse changes by about 4° over the course of a year. This is a rather large argument of periapse; maintenance focusing on keeping the satellite above the top of the atmosphere is simpler than maintenance focusing in fixing the argument of periapse, so some of the durability of this orbit can be sacrificed with the goal of decreasing the annual change in argument of periapse.

For this reason, 62.5° is determined to be the optimal inclination for a highly elliptical orbit. Without maintenance, a satellite in an orbit with this inclination will see its argument of periapse change approximately 3° every year, down from 4° from the Molniya 3-50 orbit.

However, a satellite in such an orbit would only last for 2.56 years. For the purposes of balancing efficiency between maximizing time in orbit and minimizing the rate of change of the argument of periapse, this orbit is selected as the ideal.

Now that an ideal orbit inclination has been selected, the most propellant efficient method for de-orbiting a satellite from such an orbit must be found. While such a disposal would not occur immediately after orbit maintenance is performed, this test for the ideal disposal method will be performed using the ideal orbital elements, as it is the only orbital state it is known for certain the satellite will be found in at any point. This orbit is described by the orbital elements in Table 2 (all except for the inclination are kept from the Molniya 3-50 orbit) which apply in finding the best method of disposal.

Table 2. Orbital Elements of Ideal Highly Elliptical Orbit

Inclination (i)	62.5°
Semi-major Axis (a)	26554 km
Eccentricity (e)	0.7194429
Radius of Periapse (r_p)	8740.5 km
Argument of Periapse (ω)	271.1684°

The first possible method for disposal is a pure inclination burn performed at the ascending node to move the satellite into an orbit with the critical inclination. The Δv required for such a maneuver is described in equation (4).

$$\Delta v = 2v sin(\frac{\Delta i}{2}) \tag{4}$$

To solve for this Δv, the velocity of the satellite at the ascending node is necessary; this can be found using equations (5), (6), (7) and (8). The standard gravitational parameter for the Earth, μ, is 3.986x10^5 km^3/s^2.

$$\theta_{AN} = 360° - \omega \tag{5}$$

$$p = a(1 - e^2) \tag{6}$$

$$r = \frac{p}{1 + e cos\theta} \tag{7}$$

$$v = \sqrt{2(-\frac{\mu}{2a} + \frac{\mu}{r})} \tag{8}$$

The velocity of the satellite at the ascending node is found to be 8.73 km/s. The inclination needs to be changed by 0.9349°, so the Δv required for this maneuver is found to be 0.1424 km/s.

The next potential method for disposal, which is utilized for most satellites being forced to re-enter the atmosphere, is an apoapse burn intended to force the satellite down into the atmosphere. Such a maneuver slows the satellite down at apoapse, reducing the semi-major axis and lowering the periapse into the atmosphere.

The radius of the periapse is found using equation (9). The desired new semi-major axis is then also found using equation (9), replacing the radius of periapse with an assumed radius of the top of the atmosphere (6471 km).

$$a = \frac{r_p + r_a}{2} \tag{9}$$

At this point, the velocity of the satellite before the maneuver and the necessary velocity of the satellite after the maneuver are both found with equation (8). The Δv is the difference between these velocities. For this maneuver, the required Δv is found to be 0.0921 km/s.

The last option for a maneuver causing the satellite to re-enter the atmosphere is an inclination and RAAN change to force the orbit to the critical inclination, this time at the apoapse. This maneuver is displayed in Figure 18.

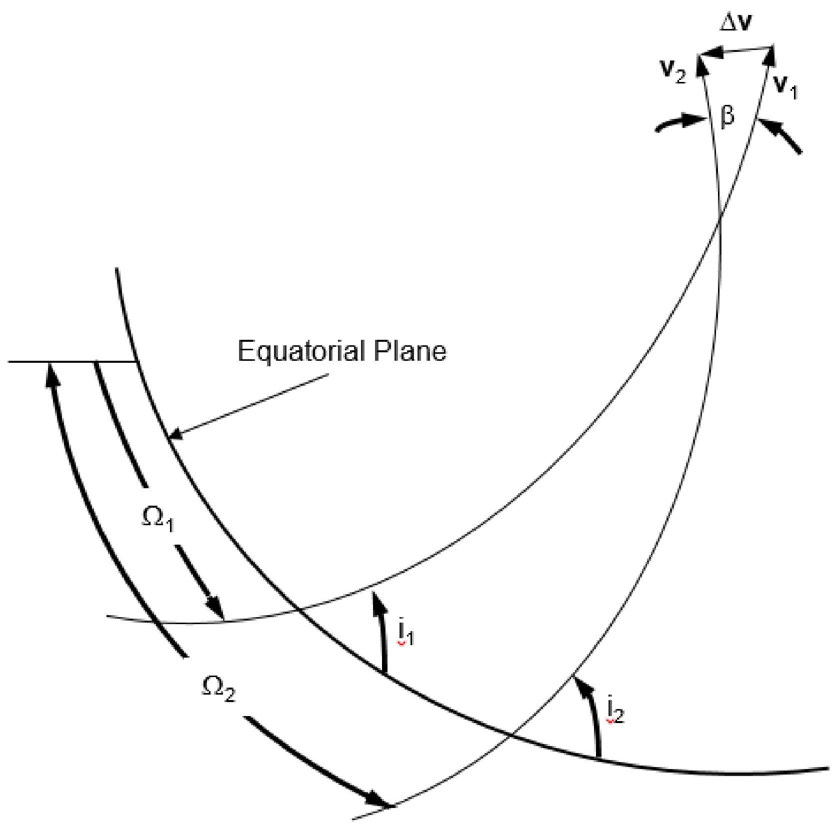

Figure 18. Maneuver to Change Inclination and RAAN

To calculate the necessary Δv, first, the angle by which the orbit must be affected, α, is calculated using equations (10) and (11).

$$u_{initial} = \omega + \theta \tag{10}$$

$$\cos(\pi - i_{final}) = -\cos(\alpha)\cos(i_{initial}) + \sin(\alpha)\sin(i_{initial})\cos(u_{initial}) \tag{11}$$

This angle α is used in equation (12) to solve for the change in RAAN that has to correspond with the desired change in inclination.

$$\cos(u_{initial}) = \frac{\sin(i_{final})\cos(\Delta\Omega) - \cos(\alpha)\sin(i_{initial})}{\sin(\alpha)\cos(i_{initial})} \tag{12}$$

Finally, with the initial and final inclinations and the change in RAAN, the angle β can be found with equation (13), which allows for the solving of the necessary Δv for this maneuver through equation (14), a modified version of equation (4).

$$\cos(\beta) = \cos(i_{initial})\cos(i_{final}) + \sin(i_{initial})\sin(i_{final})\cos(\Delta\Omega) \tag{13}$$

$$\Delta v = 2v sin(\frac{\beta}{2}) \tag{14}$$

The necessary Δv for this maneuver is found to be 2.7629 km/s.

The burn at apoapse is found to be the most propellant-efficient method for forcing an atmosphere from the found ideal orbit to re-enter the atmosphere.

Chapter 4

CONCLUSIONS

This thesis set to search for the optimal inclination. This all centers around the effects of the geopotential forces, specifically J_2 and J_3, the two most prominent. At the critical inclination 63.9349°, J_2 effects are completely negated, setting the rate of change of the argument of periapse to zero. This, however, allows J_3 to dominate; this very quickly sends satellites back into the atmosphere. Thus, the optimal inclination for a highly elliptical orbit would be one that minimizes the effect of J_2, thereby reducing the rate of change of the argument of periapse, while preventing the effects of J_3 from dominating and quickly forcing a satellite in this orbit to re-enter the atmosphere.

Through tests in STK of highly elliptical orbits like that of Molniya 3-50 with varying inclinations and RAANs, 62.5° was found to be the optimal inclination for these orbits. Tests at RAANs evenly distributed around the globe showed that such an inclination would produce an annual change in argument of periapse of only 3° per year. Through these tests, it was also found that a RAAN of 120° was near the worst-case scenario - the RAAN that, given the position of the Sun and the Moon at the start time of these tests, leads to the satellite re-entering the atmosphere the fastest. Tests around RAAN 120° led to the conclusion that a highly elliptical orbit with inclination 62.5° would last approximately 2.56 years without maintenance prior to re-entry.

In reality, satellites are constantly seeing their orbits maintained through small maneuvers. A satellite’s lifetime is found to be over when the amount of propellant remaining is just enough to force it to re-enter the atmosphere. Any more maintenance maneuvers at this point would prevent a satellite from being capable of bringing itself back down to the top of the atmosphere and would render it space debris stranded in orbit until it re-entered naturally. Thus, the less propellant necessary to cause re-entry, the longer a satellite can remain in use.

Three options for forcing re-entry were tested - a pure inclination change at the ascending node, an apoapse burn to lower the periapse and an apoapse burn that changes the inclination and the RAAN simultaneously. The necessary Δv for each type of maneuver was found; the values are displayed in Table 3.

Table 3. Change in Velocity Required for each Maneuver Type

Pure Inclination Change	Lowering Periapse	Inclination and RAAN Change
0.1424 km/s	0.0921 km/s	2.7629 km/s

The data in Table 3 shows that the burn at apoapse to lower the periapse is the most propellant-efficient. This makes logical sense; the other methods can be expected to require a large Δv, as the satellite is moving very fast at the ascending node and the inclination and RAAN change is a complex maneuver. This method is also the most time efficient: it causes the satellite to re-enter after just one-half orbit (the next time it reaches the periapse) while forcing the orbit to critical inclination takes slightly longer to force the satellite to re-enter. It can therefore be said that a burn at apoapse to lower the periapse of the orbit is the best method for removing a satellite from a highly elliptical orbit.

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ACADEMIC VITA

Education

The Pennsylvania State University *May 2019*
Bachelor of Science in Aerospace Engineering, Minor in Physics
The Schreyer Honors College

Study Abroad

National University of Singapore – Mechanical Engineering Product Design Programme *May 2016*

- Prototyped form-fitting dust-bin to account for non-uniform draining system throughout Singapore
- Consulted with cleaning staff on NUS campus to understand their needs and integrate them into our design

Engineering Experience

Arconic Technology Center, Aerospace and Defense, New Kensington, PA *May-Aug 2018*
Aerospace Intern

- Established standard expectations for Fatigue Crack Growth of 2524 Aluminum across a range of humidity levels
- Added expected curves to NASGRO to be used by Arconic in marketing and selling 2524 Al based products
- Utilized in-house software to model crack growth in 2524 Al plates, varying size and humidity, to confirm findings

Lord Corporation, Manufacturing, Dayton, OH *May-July 2017*
Student Associate

- Observed machining processes and created detailed instruction sets to cut down on training time for new machinists
- Organized a catalog of 50,000 different tools used in the machine shop
- Modeled a re-usable sleeve to cover necessary surfaces during tilt-rotor painting process

Digital Sign I.D., Product Development, Richboro, PA *June-Aug 2016*
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- Comparatively analyze of DSID photobooths against similar products to create competitive pricing model
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Research

Station Keeping of Molniya Orbits to Balance Third Body Effects of the Sun and Moon*2018-Present*
Honors Thesis coordinated with Dr. David Spencer (~10 hrs/wk)

- Utilizing STK to search for most efficient methods to counteract Third Body Effects from Sun and Moon
- Strategizing to minimize propellant cost for End of Life disposal of satellites in Molniya Orbit

Leadership/Activities

American Institute of Aeronautics and Astronautics, University Park, PA *2015-Present*
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2017/18 Special Events Chair, 2018/19 Community Chair (~25 hrs/wk)

- 2017/18 - Planned fundraising events throughout the year to help raise over $165,000 to fight pediatric cancer
- 2018/19 - Coordinating recruitment and member retention efforts to continue raising funds

Movin' On – Penn State's Annual Music Festival, University Park, PA *2016-18*
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Skills

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Dean's List