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LET’S MAKE IT A TRUE DAILY DOUBLE: WAGERING TO WIN ON JEOPARDY!

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ABSTRACT

The quiz show *Jeopardy!* is comprised of three rounds and 61 total questions. Hidden within the game board are three Daily Doubles, questions where players select a portion of their current game score to wager on their answer. Wagering behavior of top players indicates that Daily Doubles play a significant role in determining the outcome of *Jeopardy!* games, but there exists significant variation among players on how to approach these events. This thesis addresses the subject of how to wager on Daily Doubles in a way that best positions a player to win on *Jeopardy!*. First, this paper confirms that Daily Doubles play a critical role in *Jeopardy!* game results. Then, it develops a model that establishes a statistically optimal betting strategy for players at different points throughout gameplay by using simulation of game results. Finally, it generalizes the model's results so that they can be used by players to boost their chances of winning on the show. This approach shifts the framework of *Jeopardy!* wagering from simple reliance on gut instinct to a method statistically designed to maximize the probability of winning.

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Chapter 1 : What is Jeopardy?

Introduction

Entering question 16 of Double Jeopardy in the finals of 2014's Battle of the Decades, Roger Craig had staked a slight lead over both Ken Jennings and Brad Rutter, *Jeopardy!*'s all-time respective leaders in games and money won. Having just correctly provided the answer of "stenosis," he selected the $800 clue under the category Medical Terms, but the question did not appear on the board. Rather, the words "Daily Double" flashed on screen, and Roger faced a difficult decision. Should he choose to protect his lead, wagering a small amount that he could easily afford to lose? Should he bet big, seizing the opportunity to build an even greater lead over two extremely talented opponents? As the co-holder of the all-time highest Daily Double bet and a programmer who had extensively used data mining to develop his game strategy, Roger barely even paused (NPR, 2011). "Everything," he announced with a shrug, eliciting supportive cheers from the studio audience (*Jeopardy!* Episode #6840, 2014). Unfortunately for Roger, the correct answer of "edema" eluded him, sending him careening into a distant third place, where he would finish the tournament. His wager proved to be a divisive choice to *Jeopardy!* pundits across the nation, as observers either praised or derided the audacity of such a high wager in his position. The event likely pushed future *Jeopardy!* contestants to shy away from large bets, and certainly caused many to carefully consider their Daily Double strategies. Did Roger make the right move? Is there an optimal way to approach Daily Double betting?

This thesis approaches the subject of Daily Double wagering on *Jeopardy!* using a combination of statistics, modeling, and game theory. Chapter 1 briefly highlights the game's history and cultural significance before explaining the rules of gameplay to facilitate understanding of the basis of the model. Chapter 2 describes the process of obtaining and cleaning the *Jeopardy!* game data later used in modeling and analysis. Chapter 3 demonstrates the importance of Daily Doubles in winning a game of *Jeopardy!*. In order to look at Daily Doubles, it is essential to understand the position a person must hold to maximize the chances of winning a game upon reaching Final Jeopardy. Therefore, the paper works backwards, first developing a model for Final Jeopardy wagering in Chapter 4 in order to set ending objectives and target intervals for Daily Double betting. Chapter 5 then presents the Daily Double betting model, describing its inputs, components, and methodology. Finally, Chapter 6 summarizes the results of the Daily Double wagering model into an easily remembered set of guidelines that players can use when competing on the show.

History

Amid the quiz show craze of the 1950s, Merv Griffin and his wife Julann devised a show premise with a twist: the game's host would ask "answers" and the contestants would provide the "questions" (Lidz, 1989). This brainstorm begat the show *Jeopardy!*, whose current iteration with host Alex Trebek has been continuously aired since 1984 and has received 34 Daytime Emmy awards and a Peabody Award for "encouraging, celebrating and rewarding knowledge" (*Jeopardy!* Awards, 2019). Beyond its daily episodes, *Jeopardy!* has grown to regularly host specialty competitions for teens, college students, and teachers (among others), as well as periodic tournaments wherein prior winners face off to crown larger-scale *Jeopardy!* royalty (*Jeopardy!* Contestant Zone, 2019). As the viewership of the show has expanded, so has its prospective player pool, as *Jeopardy!* has grown past Los Angeles-only auditions and now filters applicants through a combination of an online test and in-person auditions at sites across the nation. In 2018, about 80,000 people took the online test, 3,000 attended in-person tryouts, and approximately 450 appeared on the show (Hinds, 2018). Established as a staple in households across the nation, and claiming an avid viewership, there is little dispute to *Jeopardy!*'s claim of being "America's Favorite Quiz Show."

Rules

On each episode of *Jeopardy!*, three contestants compete in a three-round game. Money is earned by answering questions correctly, and at the end of each game, the player with the highest accumulated earnings wins and returns to play the next day against two new opponents. The first two rounds, respectively called the Jeopardy and Double Jeopardy rounds, are each comprised of six categories of five questions apiece, as shown in Figure 1. In the Jeopardy round the clues are valued at $200, $400, $600, $800, and $1,000; in the Double Jeopardy round these amounts are doubled. Higher clue values generally correspond to more difficult clues. For each clue, the player who last answered a question correctly selects a category and clue value. A clue is revealed and is read by the host. As soon as the host finishes reading the question, all the players may use their signaling devices to ring in. Buzzing too early results in the signaling device to be momentarily 'locked out', so players must be precise in listening for the end of the question (Harris, 2006, p. 35). The first player to buzz in may provide a response to the clue, and the response must be phrased in the form of a question in a tribute to the show's original concept of contestants providing the "questions" instead of the "answers". If the player's response is correct, they gain the value of the clue to their total as well as control of the board in order to select the next clue; if it is incorrect, they lose the value of the clue and the other contestants have the opportunity to ring in and answer correctly. No penalty is given for not buzzing on a question, and if time expires before all of a round's clues can be read the rest are left unrevealed.

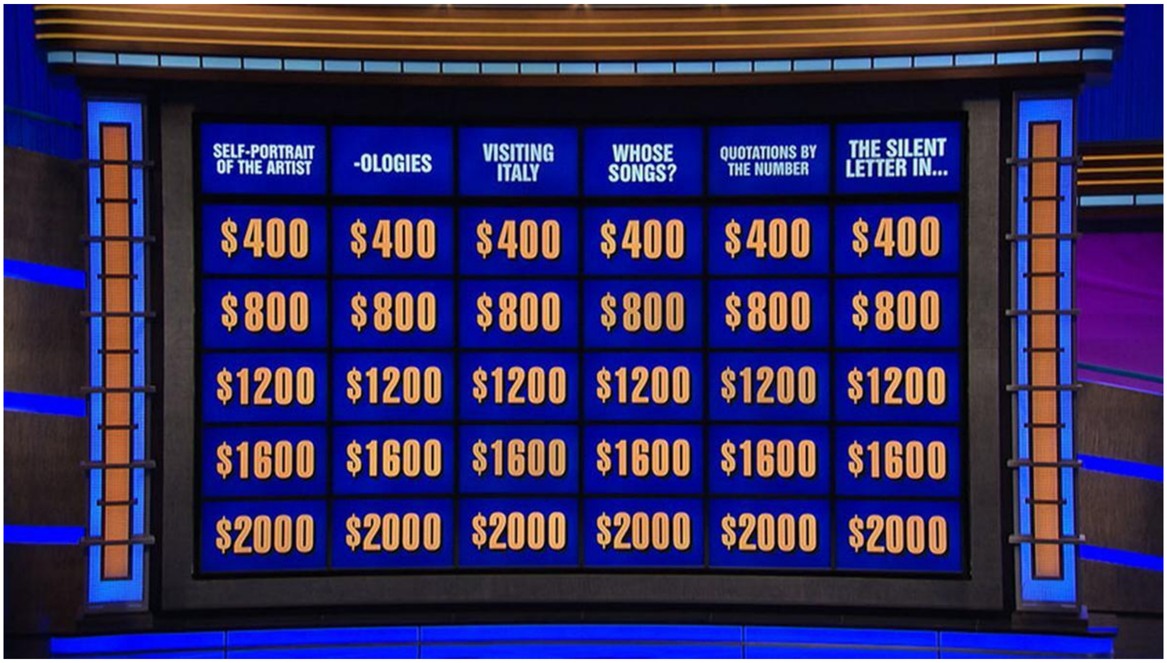

Figure 1. Sample Double Jeopardy Game Board (J!Buzz, 2015)

Hidden throughout the game board are three special clues known as Daily Doubles, with one positioned in the Jeopardy round and the other two in the Double Jeopardy round. When a player selects a clue that is revealed to be a Daily Double, they must quickly select a wager of any portion of their current money holdings on their response, giving the player the effective potential to double their money, lose it all, or fall anywhere in between on that clue. Contestants must bet at least five dollars, and if a player has less than $1,000 in the Jeopardy Round or

$2,000 in the Double Jeopardy round, they may still wager up to $1,000 and $2,000, respectively. Players also have a few more seconds to answer Daily Doubles as compared to ordinary questions, and no other player will have the opportunity to answer a Daily Double besides the player who originally uncovered it.

Final Jeopardy follows Double Jeopardy as the third and last round of the game. Having seen up to 60 clues already, all the contestants are shown only a category for a final question and then have as much time as they like to write down a wager of between zero and all of their holdings on their answer to the question. If any contestant enters the round with zero dollars or less, they are automatically eliminated before the Final Jeopardy question and leave the stage. Once all participants have chosen a wager, the clue is read and the players have 30 seconds to write their answers. At the end of this time, each player's answer and wager is revealed, starting with the player who entered Final Jeopardy with the least money. Whoever ends Final Jeopardy with the most money wins the game, keeps their money, and returns to play the next day against two new opponents, while the second-place contestant goes home with $2,000 and the third-place contestant with $1,000. In the rare case that Final Jeopardy ends in a tie for first place, the tied players enter into tiebreaker procedures until one winner is determined, and if all players finish Final Jeopardy with zero dollars, no one wins or returns for the next show (J!Buzz, 2016).

Chapter 2 : Data Preparation

In order to begin constructing a model to determine optimal Daily Double betting strategy, it is necessary to have thorough data on *Jeopardy!* gameplay. Fortunately, *Jeopardy!* enthusiasts are blessed with an abundance of publicly available data in the form of the J! Archive. The J! Archive is a database of all previous *Jeopardy!* clues, games, players, and trends, and is completely fan-maintained for the enjoyment of others (J! Archive, 2018). To extract the data from J! Archive webpages to Excel for analysis, I used a Google Chrome web scraper extension (Version 0.3.8; webscraper.io, 2019). After clearing several hundred games with missing data points there were still well over 6,000 games left to use for modeling. Next, an investigation into historic trends revealed *Jeopardy!* doubled its clue values in 2001, making it necessary to compensate by doubling all clue values and scores prior to episode #3966 (Zak, 2012).

Analysis by Game Type

As *Jeopardy!* holds a variety of themed tournaments in addition to its ordinary games, it seemed prudent to investigate whether they all operate on the same difficulty level and could be used in the model. Separating the types of games showed that there have been sixteen different types of *Jeopardy!* games and tournaments throughout its 35-year run. In order to evaluate the comparative difficulty for each, I built confidence intervals for the average rates of correct answers in Final Jeopardy, which I believe to be the most straightforward and controllable source of game difficulty. Eight of the sixteen types of competitions have seen 76 or more games, while the others have all had 26 or less games. Since the players all answer the same Final Jeopardy clue in each game, I counted games and not responses when considering sample sizes in order to create large enough sample sizes to ignore dependencies within individual games. Furthermore, I only counted players who had made a non-zero wager before hearing the Final Jeopardy question, since players whose ending scores are completely unaffected by the accuracy of their answer have little motivation to deduce the correct answer. In order to construct confidence intervals, I used normal approximations to the binomial situation of answering Final Jeopardy correctly or incorrectly. Therefore, it made sense to discard any of the game types with less than 30 games played, since the Central Limit Theorem advises to only use the normal approximation for situations with 30 or more independent observations. Using Equation (2.1) with $z_{99\%} = 2.575$, I calculated the 99% confidence intervals for the proportion of correct Final Jeopardy answers for each type of game. Figure 2 graphs these intervals. Each bar represents the proportion of

$$CI_{99\%} = \bar{x} \pm z_{99\%} * \sqrt{\frac{\bar{x} * (1 - \bar{x})}{n}}$$

(Equation 2.1)

correct Final Jeopardy answers for that type of game, and the bars are both ordered by and display the number of observations of game type. The black line at 48.6% shows the rate of Final Jeopardy correct answers for regular games and serves as a baseline, while the red dashed lines demonstrate 99% confidence intervals for Final Jeopardy answer rates. Green bars failed to achieve inclusion in the model. The eight rightmost bars failed for the aforementioned small sample sizes, which led to enormous confidence intervals, while the 99% confidence interval for the Teen bar did not include the baseline rate of Final Jeopardy correct answers. Therefore, the

Teen Tournament is suspected to have a different question difficulty than other game types and not be a good approximation for ordinary games, and Teen games were removed from the data.

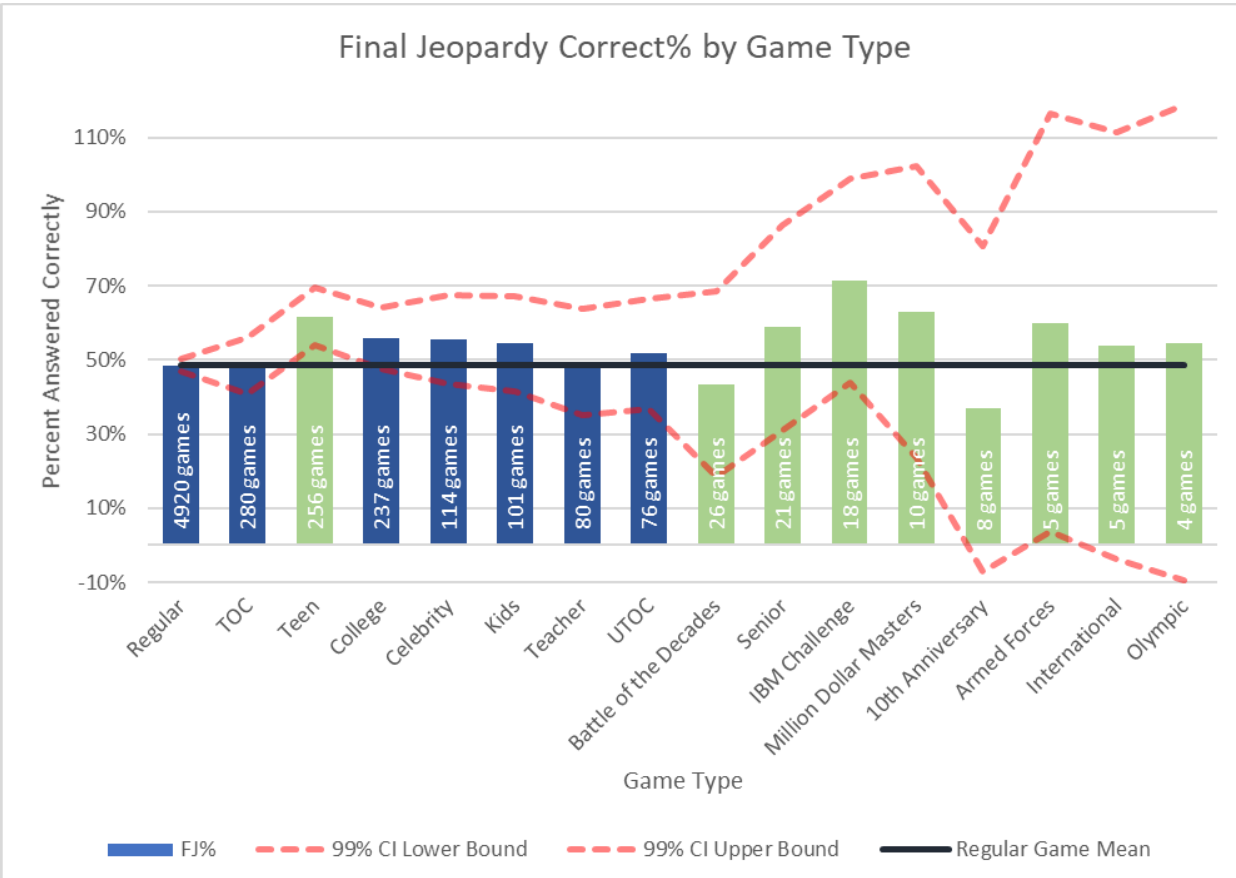

Figure 2: Final Jeopardy Difficulty by Game Type

For each game type, if there were less than 30 games or if the 99% confidence interval for the correct answer rate did not include the regular Final Jeopardy correct answer rate, the game type was removed from the data. Green bars failed to achieve inclusion in the model, while blue bars were included.

Analysis by Season

Having pared nine game types from the model, I turned my attention to variations in question difficulty by season. From personal observation, I theorized that the *Jeopardy!* production team may periodically adjust question difficulty in response to viewership trends or as a reputational tool, so I wanted to see whether any season's questions appeared to be different enough as to have an undue impact on overall expected rates of question accuracy. Similarly to the game types approach, I constructed 99% confidence intervals for the Final Jeopardy correct answer rate for each season and looked for seasons where the confidence interval didn't include the Final Jeopardy correct answer rate across all seasons. As shown in Figure 3, Seasons 14 and 27 both had abnormally high rates of correct answers in Final Jeopardy, indicating that those seasons had been intentionally made easier and would thus be inappropriate to include in a model for a standard game.

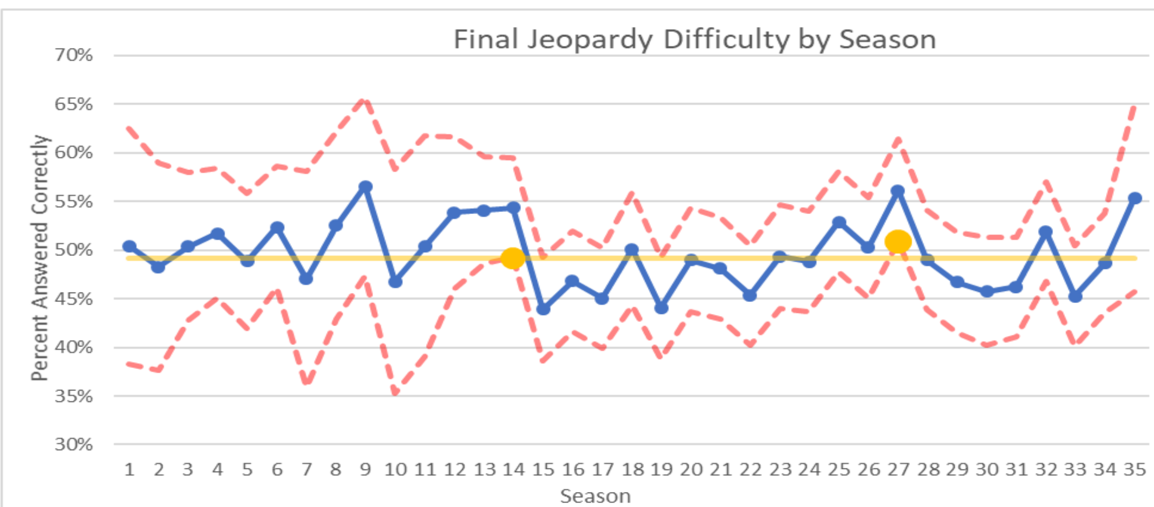

Figure 3: Final Jeopardy Difficulty by Season

For each season, if the 99% confidence interval for the rate of correct answers in Final Jeopardy did not include the overall Final Jeopardy correct answer rate, it was marked with a yellow dot and removed from the data. This occurred in seasons 14 and 27.

Chapter 3 : Importance of Daily Doubles

Observed Behavior of Top Players

In an ordinary game of *Jeopardy!*, contestants will usually begin the Jeopardy and Double Jeopardy rounds with a low-dollar clue at the top of the gameboard and gradually shift down the gameboard to higher-value clues as the round progresses. Such a strategy helps players gain familiarity with patterns within the category, eliminate potential answers that may have confused them on higher-value clues, and make mental associations and connections with facts that may help them on the more difficult clues further down the board.

A viewer watching *Jeopardy!* for the first time between February 20th and March 5th, 2019, however, would never have surmised this tendency. Those two weeks comprised *Jeopardy!*'s All-Star Games, and welcomed back 18 of the most outstanding and memorable players from past seasons to participate in the show's first-ever team tournament. With *Jeopardy*'s best and brightest going head-to-head, it was apparent that the players recognized the difficulty of their task. Rather than waiting for their opponents to slip up and beat themselves, all the players had decided to approach the tournament with the mindset of playing to win. In the All-Star Games, each match consisted of two games of *Jeopardy!*. Each of a team's three players played one of the three rounds in the first game, and a different round in the second game. Most teams quickly slotted their strongest players in the Double Jeopardy round of one game and Single Jeopardy of the other. Even though Final Jeopardy is the ultimate round of the game and all a team's money can be won or lost in the round, top players often consider Daily Doubles to essentially be embedded Final Jeopardy rounds where the other players can't gain any money.

With two such Daily Doubles in the Double Jeopardy round, each team unquestionably considered Double Jeopardy the most high-leverage round in the tournament.

When the top players began a Double Jeopardy round, they allowed themselves no time to adjust to the categories. Rather, they adopted variations on the "Forrest Bounce". Pioneered by Season 3 Tournament of Champions winner Chuck Forrest, the technique involves selecting clues "not in simple vertical lines but by hopscotching back and forth across the game board, continually changing categories" (Harris, 2006, p. 73). Ricocheting around the board in this manner has the double effect of disorienting opponents and allowing one to hunt for Daily Doubles, which are generally concentrated in the middle and bottom rows, as shown in Figure 4. Bottom-row Daily Doubles are significantly more common in the Jeopardy round, while third-row Daily Doubles occur with significantly greater frequency in the Double Jeopardy round. The second and sixth columns are more likely than others to feature pop culture or wordplay questions, which may be why they are relatively less frequent Daily Double locations (Tesauro, 2013, p. 213).

In the Double Jeopardy round of the second game of the All Star Games' wildcard match, Daily Double hunting was on full display. The contestants had undoubtedly studied location frequency charts, as the first-column, fourth-row clue was the first off the board, and by question seven, five of the six categories had been sampled. Upon finding the first Daily Double, Alex Jacob, who had previously won the 2006 United States Poker Championship, happily went all-in to double his $7,400 holdings, only to dramatically lose it all on the next Daily Double nine clues later (DeCwikiel-Kane, 2019; *Jeopardy!* Episode #7945, 2019). Not until both Daily Doubles had been found were any first- or second-row clues even attempted. *Jeopardy!*'s best players clearly believe Daily Doubles to be the most pivotal questions in the game, and this chapter includes statistical approaches that support this position.

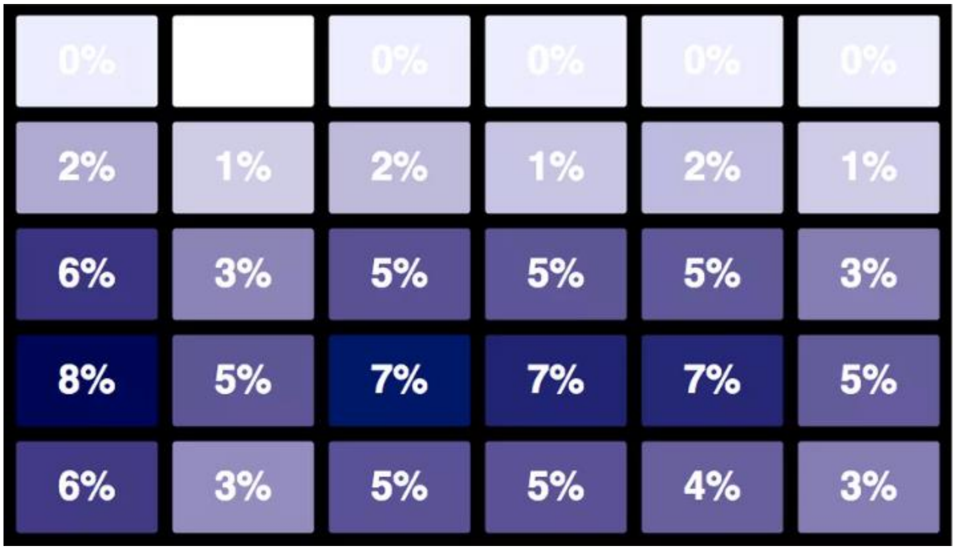

Figure 4 : Daily Double Location Frequency (Yau, 2015)

Daily Doubles are positioned most frequently in the fourth row of the game board, but almost never in the first or second. The second and sixth columns also have a relative paucity of Daily Doubles.

Actual Value

The most straightforward method of valuating the worth of Daily Doubles is through analyzing the Actual Value that players historically wager on the questions. As seen in Table 1, the pure values that players wager on Daily Doubles make them frequently the most valuable clues among the clues of the Jeopardy and Double Jeopardy rounds. Displaying results from the second half of the Jeopardy round separately from the statistics for the entire round shows the difference in wagering once players have accumulated some earnings by question 15. The second column shows the maximum face value of the round’s clues. The third column demonstrates the average wager made in the round in question, with Daily Doubles in the Jeopardy and Double

Jeopardy rounds and the Final Jeopardy bet in the last row. The fourth column describes the round's average wager as a percentage of the player's pre-wager holdings. The fifth column shows the proportion of Daily Double wagers that are greater or equal to that round's maximum clue face value, while the sixth column only shows the proportion of wagers that are greater to that round's maximum clue face value.

Round	Max Clue Actual Value	Average Wager Actual Value	As % of Holdings	%DD >= Max Clue Value	%DD > Max Clue Value
Jeopardy	1,000	1,451	54%	85%	50%
Jeopardy (second half)	1,000	1,655	44%	87%	62%
Double Jeopardy	2,000	2,800	31%	75%	50%
Final Jeopardy	11,254	6,360	57%	n/a	n/a

Table 1 : Daily Double Actual Value Statistics

Average wagers increase throughout the game but are highest, both in terms of average wager and percent of holdings, in Final Jeopardy. During the first two rounds, the second half of the Jeopardy round has the highest wagers in relation to the board's clue values.

In the Jeopardy and Double Jeopardy rounds, Daily Doubles are generally the most valuable clues by Actual Value. In each round, the average wager is greater than the maximum clue face value. Moreover, Daily Doubles are at least tied as the clues with the greatest change in money in 75-85% of rounds, and stand alone as the highest-value clues in over 50% of rounds. Furthermore, there is a Daily Double with a wager greater than $2,000 – the maximum face value of any normal clue – in 73% of Jeopardy games.

However, Daily Doubles are not the most valuable questions of the game by Actual Value. That title belongs to Final Jeopardy questions, whose average wagers and wagers as a percent of pre-question holdings surpass those of Daily Doubles. A whopping 99% of games include at least one Final Jeopardy wager greater than $2,000, a much greater proportion than games with Daily Double wagers clearing the same threshold.

If Actual Value were the only method of evaluating Daily Double worth, it would appear that they are merely the most valuable clues in the first two rounds of Jeopardy and not in the entire game. However, wagering tendencies and considerations show that Actual Value may not be the most effective way of valuing Daily Doubles.

Opportunity Value

Evaluating Daily Doubles on what players retrospectively wager does not return the best reflection of their true value, as players often fail to utilize the full value of the opportunity to grow their earnings. Rather, they should be considered in terms of their potential effect on the game if employed to their maximum. Based on their wagers, this was how the players in the *Jeopardy!* All-Star Games regarded Daily Doubles, and their backgrounds certainly afford them recognition as experts on *Jeopardy!* gameplay.

When calculating the Opportunity Value of a Daily Double, its clue's face value must be ignored. Incorporating a clue's initial dollar value into a wager provides no benefit and is fundamentally illogical, as the clue is an opportunity to put whatever money on the table best benefits a player's game position. Despite this, over 50% of wagers are placed within $500 of the clue's face value (Jetter, 2016, p. 11). In reality, it would be logical to wager less money on Daily Doubles with higher initial values, as clues further down on the game board are generally more difficult. The only other clue-related factor that should be included when setting a Daily Double bet is personal confidence in the category. In this paper, I assume that players have thoroughly prepared for their *Jeopardy!* appearances and are equally confident in all categories. Even if a difficulty factor was included, it would be near-impossible for players to mentally quantify each question's confidence and immediately input it into a model when prompted for a Daily Double wager. In the event of a Daily Double in one of a player's best or worst categories, it is fully logical to bet an extremely high amount or next to nothing in response.

Upon landing on a Daily Double, a player has their entire holdings available to wager on the clue. Not only can a player increase their personal score, but by selecting a Daily Double, they also prevent any opponents from landing on the clue and doubling their own scores. Thus, the Opportunity Value of a Daily Double is the sum of the score of the player who selects the clue and the score of their leading opponent, as this represents the potential point swing had another player landed on the clue.

Explained formulaically, let X be the score of the player in consideration and Y be the score of their leading opponent. Maximum score differences in each player's favor occur when they wager all their holdings and answer correctly.

X finds Daily Double: Max score difference = $2*X-Y$

Y finds Daily Double: Max score difference = $X-2*Y$

Opportunity Value = |Max diff. after X finds DD – Max diff. after Y finds DD|

$$= |2*X-Y - (X-2*Y)| = |2*X - X - Y + 2*Y| = |X + Y| = \underline{X + Y}$$

(Equation 3.1)

The Opportunity Value of Final Jeopardy clues, on the other hand, is limited to only a player's own holdings, as their opponents also have the chance to play the round and earn points. When devising metrics for measuring the value of Daily Doubles, it is essential to consider not only the change in a player's score but also the lost potential point change for other players, as shown in Table 2.

		Daily Doubles Correct			
	Win%	**0**	**1**	**2**	**3**
Daily Doubles Found	**0**	18%	-	-	-
	1	20%	40%	-	-
	2	27%	44%	62%	-
	3	28%	46%	73%	85%

Table 2 : Game Win Percentage based on Number of Daily Doubles Found and Correct

Historically, as the number of Daily Doubles a player answers correctly increases, so do the rates at which players win. Uncovering Daily Doubles and answering them incorrectly also increases the rates at which players win, both since opponents cannot play the clues and because better players find more Daily Doubles.

It should not be surprising that answering more Daily Doubles correctly correlates to marked increases in winning percentage of *Jeopardy!* games. However, it is counterintuitive that landing on a Daily Double and answering incorrectly actually correlates to a higher winning percentage than not landing on the question at all. Granted, players who find Daily Doubles have likely been answering other questions correctly, which inherently relates to an increased probability of winning. Even with this fact acknowledged, there is still a need to include a factor for the lost prospect of opponent score increases when valuating Daily Doubles, which is accomplished in Opportunity Value by adding in the greatest possible opponent wager at the time a clue is revealed.

Defining Daily Doubles in terms of their Opportunity Value gives them much more clout as compared to other clues, as shown in Table 3 (whose columns are analogous to the columns in Table 1.)

Round	Max Clue Opportunity Value	Average Wager Opportunity Value	As % of Holdings	%DD >= Max Clue Value	%DD > Max Clue Value
Jeopardy	2,000	5,529	204%	87%	84%
Jeopardy (second half)	2,000	7,698	205%	99%	99%
Double Jeopardy	4,000	19,017	207%	99%	99%
Final Jeopardy	11,254	11,254	100%	n/a	n/a

Table 3 : Daily Double Opportunity Value Statistics

Though regular clues have relatively small Opportunity Values in the first two rounds, Daily Double Opportunity Values in Double Jeopardy show them as the most valuable clues in the game, both in terms of average worth, value as percent of holdings, and the percent that are greater than the maximum regular clue value.

Not only does the Opportunity Value of Daily Doubles surpass the value of other regular clues, but it is also noticeably higher than the Opportunity Value of Final Jeopardy clues. Though amounts are lower towards the beginning of the game since overall scores are lower, by Double Jeopardy, the average Opportunity Value is over $19,000. Throughout the game, Opportunity Values average stand at over 200% of a player's pre-wager totals. No other clue can even approach this total, as Final Jeopardy can only increase a player's holdings by 100%, and even the most valuable regular clues cannot grow holdings by more than 200% unless a player has less than $1,000 to begin with. In 83% of games, the greatest Daily Double Opportunity Value is larger than the largest pre-Final Jeopardy score, meaning that a Daily Double was the most consequential clue in the game. This valuation fully establishes Daily Doubles as extremely important events in a game of *Jeopardy!* and demonstrates why appropriate wagering strategy is essential.

Chapter 4 : Final Jeopardy Strategy and Wagering

In order to devise a Daily Double wagering model, it is essential to know how the position from which a player enters Final Jeopardy affects their chances of winning. For example, if having 50% more money than one's closest competitor at the end of Double Jeopardy leads to a significantly higher winning percentage for the game than does having 49% more money, it may be advantageous to adopt a slightly riskier Daily Double approach in order to drastically increase one's chances of winning before the Final Jeopardy clue is even revealed. This chapter thus develops a model to quantify each player's chances of winning a game of Jeopardy based off of the scores of all three players entering Final Jeopardy.

Final Jeopardy Strategy Research

In contrast to Daily Double strategy, there is a significant amount of quantitative-based literature evaluating optimal Final Jeopardy approaches. In the paper "Last Round Betting", Ferguson and Meloidakis (1997) applied game theory principles to a two-person zero-sum game to develop a Final Jeopardy wagering strategy. This was expanded upon by Abramson, Collina, and Gasarch (2017), who used a maximin approach and defined payoffs in terms of expected dollar winnings in the paper "Maximizing Winnings on Final Jeopardy!" Though their papers are significant in recognizing Final Jeopardy as a game that can be quantitatively approached, both fall short in making an accurate model that can be used in gameplay.

Modeling a three-person Final Jeopardy game with a two-player model leads to significant inaccuracies. Granted, the player in the lead may often only bet with the player in second place in consideration. However, the player in second should closely monitor the player in third and compute a wager that accounts for both overtaking the player in the lead and staying abreast of the player in third, depending on the outcome of the question.

Some of the strategic recommendations made by Abramson et. al. are unfavorable in practice. For example, they recommend that when the leading player feels they have more than a 50% probability of answering Final Jeopardy correctly, they should always wager all their money, as this maximizes the expected value of their holdings at the end of the game. In practice, this is unwise, as there are many cases when the leading player has more than double the money of the player in second. In these cases, the player in the lead can guarantee a win by wagering a small amount, while if they bet everything, they can lose whenever they don't answer correctly. To be fair, Abramson et. al. acknowledge that they are considering payoffs in terms of money won instead of winning the game, but if they were to account that future wins and potential Tournament of Champions winnings for longer winning streaks can make each win worth between about $19,000 and $51,000, they would likely find that it is financially beneficial to play for the win (Saunders, 2017).

Another helpful resource for Final Jeopardy wagering is the Wagering Calculator on the J! Archive website (2018). A significant improvement over more theoretical papers described before, the Wagering Calculator takes scores for three players entering Final Jeopardy and gives wagering suggestions for each of them, considering what approach gives them the best chance to win given logical wagers by the other players. It also lists the rules and break points that it uses to determine betting strategy. This is an extremely helpful tool for prospective Jeopardy players to practice making logical bets, and the rules it provides are well-reasoned and mathematically sound. However, like the papers mentioned above, the wagers suggested are calculated using a normative rather than a positive approach; they are rooted in how contestants should bet rather than how contestants actually do bet. Aside from players in first place betting to cover those in second place doubling their scores, contestants only wager according to the principles prescribed by J! Archive Wagering Calculator strategy approximately 57% of the time, so presuming that other players are going to wager logically is a significant and dangerous assumption.

There was one model that did address the others' shortcomings. In February 2011, *Jeopardy!* legends Ken Jennings and Brad Rutter participated in a widely-publicized match against WATSON, an IBM computer that had been programmed using artificial intelligence to master the natural language skills needed to compete on the show. When creating WATSON, IBM programmers had to not only prepare their computer to deliver correct answers, but also to wager as to maximize its chances of winning (Markoff, 2011). In preparing WATSON to bet, researchers first devised a contestant model with which to model human opponent wagering. To do so, they found average Final Jeopardy accuracy percentages based off historical data for different contestant skill levels, calculated the correlation of in-game contestant accuracy, and analyzed historic contestant wagering trends (Tesauro, 2013). Using this information, they created a human Final Jeopardy model. To create a wagering model for WATSON itself, they programmed the computer to compute a "Best-Response" strategy to the human model by calculating an accuracy confidence interval given the category title, deriving probabilities for each right/wrong answer combination for the three players, running Monte Carlo simulations of human wagers, and solving for the bet that gives WATSON the greatest likelihood of winning. This approach is extremely comprehensive, relying on both normative and positive methods to deduce a Final Jeopardy bet that optimizes the chances of winning in a three-player game. It was therefore an exemplary framework for the human Final Jeopardy model presented in this thesis.

Final Jeopardy Individual Model

What is a player's probability of answering Final Jeopardy correctly? This is the central question at the foundation of any Final Jeopardy model and the first question addressed in this model. Other models assume that each player has the overall historic probability of 49% of answering correctly, or round up to 50%. The WATSON model assigns 50% for average contestants, 60% for past winners, and 66% for elite players (Tesauro, 2013, p. 214). I would like to take a more accurate approach than simply assigning all players the same probability, but when a player is facing two opponents, the player does not know if either should be considered an average, good, or elite player based off total number of games won, as there is no hindsight of knowing how many games those opponents will end up winning. Therefore it is necessary to make use of the only available predictive information to set a probability. By the end of a game, players have had 60 questions to demonstrate their skill as a player, so their results from the Jeopardy and Double Jeopardy rounds can be used to gauge the probability that they answer Final Jeopardy correctly.

Named after Season 12 two-game champion Karl Coryat, the Coryat score is:

> A player's score if all wagering is disregarded. In the Coryat score, there is no penalty for forced incorrect responses on Daily Doubles, but correct responses on Daily Doubles earn only the natural values of the clues, and any gain or loss from the Final Jeopardy! Round is ignored. (J! Archive, 2018)

Coryat scores can be used to estimate a player's skill, as they reward correct answers while removing the undue effects that betting might inflict on the player's score. Both a cursory look and common sense indicate that a higher Coryat score correlates with a better probability of answering Final Jeopardy correctly, as seen in Figure 5.

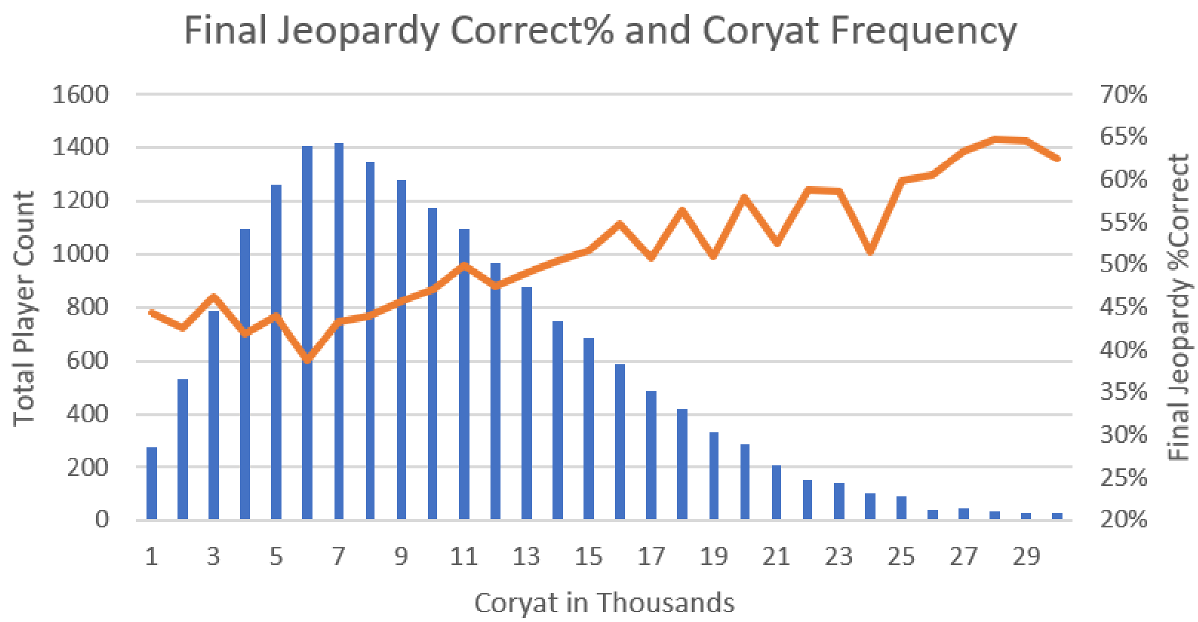

Figure 5: Final Jeopardy Accuracy by Coryat and Coryat Frequency

Coryat scores follow a distribution that is skewed left, with the mode at approximately 7,000. On the whole, players with higher Coryats answer Final Jeopardy questions at a higher rate.

By regressing players' Coryat scores with their Final Jeopardy results, a model can be made that predicts Final Jeopardy accuracy from player performance in the first two rounds. There are two possible approaches: either a logistic regression that relates a player's Coryat score to the binary result of Final Jeopardy (getting the question right or wrong), or a regression linking a grouping of all players with Coryats in a certain range to the overall Final Jeopardy accuracy of that group. The other factor to consider was the combined Coryat of a player's opponents; if a player happened to be matched with exceptionally skilled opponents, it could artificially deflate their Coryat below the expected level based on their true ability. Therefore, I tested a number of regression models with combinations of these components, shown in Table 4.

Model	R-Squared	Player Coryat P-Value	Opponent Coryat P-Value
Linear Groups - Player - Logged	0.8257	1.60E-12	-
Linear Groups - Player	0.8168	3.31E-12	-
Linear Groups - Player & Opp	0.0987	0.0206	0.0055
Poly No Groups - Player	0.0091	2.13E-28	-
Poly Groups - Player	0.0091	1.34E-14	-
Linear No Groups - Player	0.0090	5.66E-31	-
Linear No Groups - Player & Opp	0.0090	3.86E-29	0.6731
Logistic - Player	0.0066	2.29E-30	-
Logistic - Player & Opp	0.0066	2.06E-28	0.7369

Table 4: Tested Regression Models

Linear = Linear Regression; Poly = Polynomial Regression; Logistic = Logistic Regression
Groups = Players with Coryats in the same range of one thousand grouped together
Player = Player's Coryat is only factor;
Player & Opp = Both player and combined opponent Coryat used as factors
The linear logged groups and linear groups models have the highest R-Squared in predicting Final Jeopardy correct answer rates from Coryat scores. The player's Coryat score is significant in predicting Final Jeopardy correct answer rates in nearly every model, but the opponents' combined Coryat score is not.

The p-value of a player's Coryat score is extremely significant in all the models at the α=0.05 significance level, indicating that there is undoubtedly a positive relationship between Coryat score and Final Jeopardy accuracy. The p-value of the combined opponents' Coryat, on the other hand, is not significant at the α=0.05 level in two of the three models that included the opponents' Coryat as a factor. The exception is the Linear Groups – Player & Opponent model that inexplicably determined Final Jeopardy accuracy to be negatively related to a player's Coryat score, and should thus be ignored. On the whole, Opponent Coryat can be discarded as a factor. The data in groups naturally has a better R-Squared than the ungrouped data, as the

Coryat scores predict a binary Final Jeopardy outcome for each player in the ungrouped data, while the Coryat scores predict the overall Final Jeopardy accuracy percentage for a collection of players in the grouped data. The R-Squared and p-values are difficult to evaluate, though, so it is rational to also take a visual approach to comparing the models, shown in Figure 6. The actual game data is shown in blue, and is compared to each of the models.

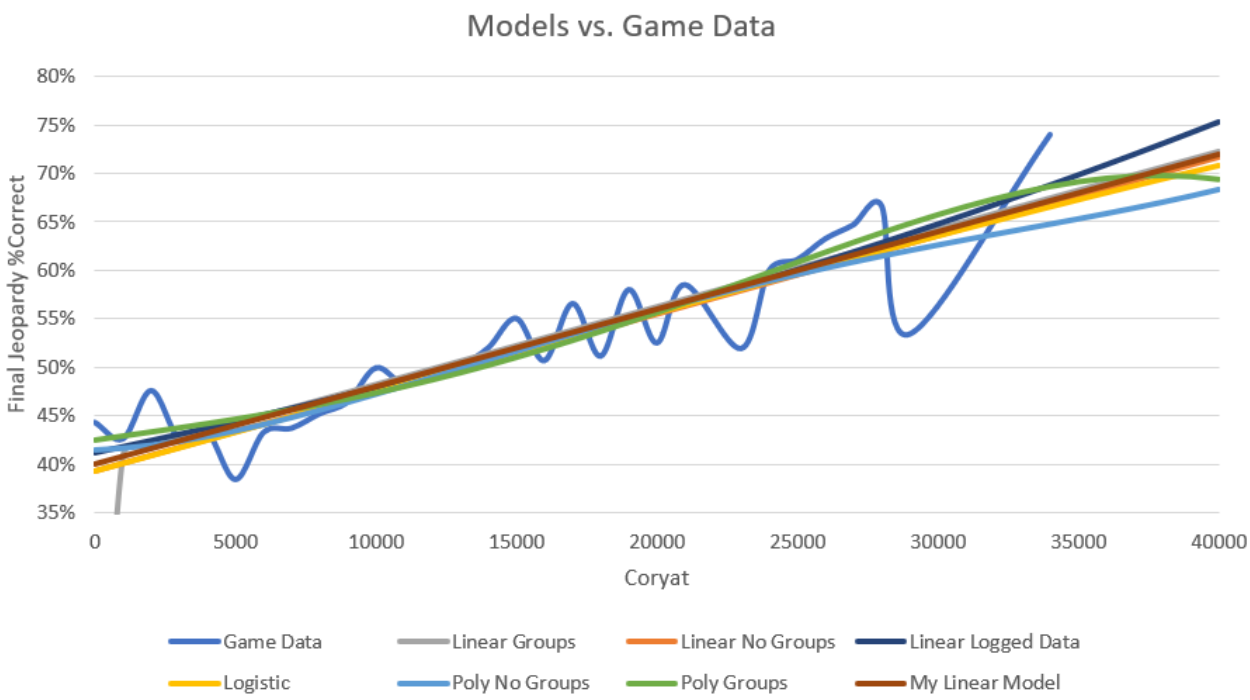

Figure 6: Regression Model Results vs. Game Data

All the models tested gave almost exactly the same predictions of Final Jeopardy correct answer rate between the Coryats of 4,000 and 25,000.

Despite the differences in R-Squared and factor p-values in the different models, all seven displayed are virtually indistinguishable between the Coryats of 4,000 and 25,000. Considering that 88% of players fall in this range, it makes little sense to pursue needlessly complex models when the simple ones are just as effective for nearly all contestants. Former *Jeopardy!* champion Melanie Spratford recounts her surprise that,

> When it came time to [wager] on the show, I would be standing, under the lights, with competitors on either side, contestant coordinators flitting around, a makeup artist hovering nearby to touch up my shiny bits, and a stealthy woman with a clipboard standing behind me waiting to take my official wager. And I'd be doing my calculations on a ½ sheet of paper with a Sharpie marker. I freaked out. (Kenkel, 2012)

Even J! Archive founding archivist Robert Knecht Schmidt found it difficult to correctly do simple algebra during his appearance on the show, and took so long trying to formulate his bet that the program assistants urgently prompted him to write a wager until he gave up and wrote zero (Schmidt, 2010). Therefore, it is advantageous to use the most simplistic models whenever possible to aid contestants under extreme pressure. The linear model using groups is thus a logical choice. Before using it as a model, it is needed to check whether it satisfies normality. Plotting the model's residuals against its fitted values produces the graph in Figure 7.

There does not appear to be any worrisome relationship between the predicted and residual values for the linear grouped model, as there is expected to be slight upwards and downwards trends due to the predicted values projecting binary values. On the whole, the plot appears sufficiently random. Proceeding forward, the linear model can be approximated as:

$$\text{Probability of Answering FJ Correctly} = 40\% + 0.8\% * (\text{Coryat} / 1000)$$

(Equation 4.1)

Key points on the linear model are shown in Figure 8.

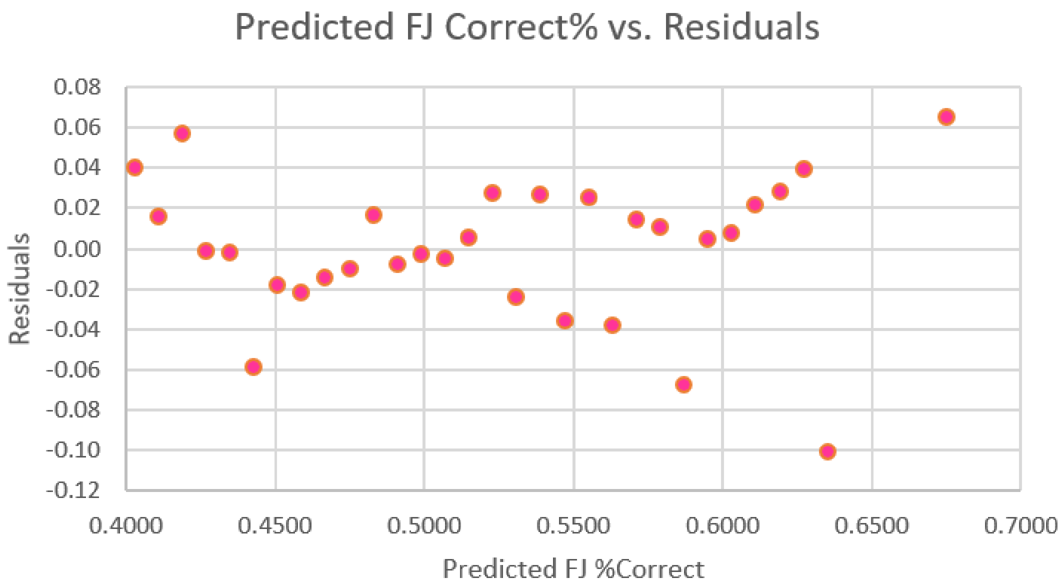

Figure 7: Predicted FJ Accuracy vs. Residuals for Linear Grouped Regression Model

As expected, there is a slight upward and downward trend between the predicted and residual values for predicting the linear grouped model for Final Jeopardy correct answer rate from Coryat score, since a binary value is being predicted. On the whole, the relationship is sufficiently random.

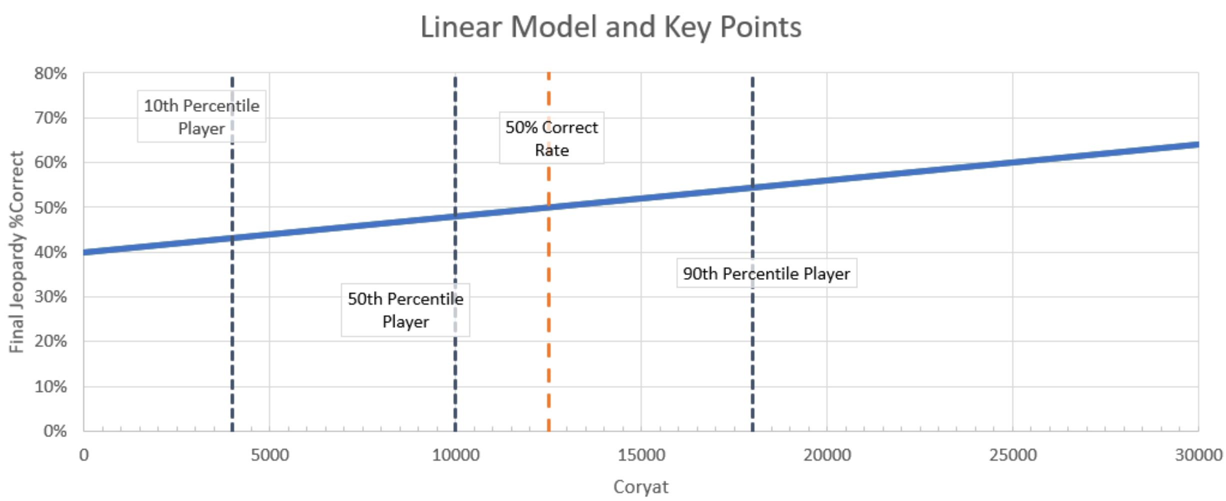

Figure 8: Key Points on Individual Final Jeopardy Model

In the model for predicting the probability of a correct Final Jeopardy answer from a Coryat score, the middle 80% of Coryats will have Final Jeopardy probabilities between 43% and 55%. A Coryat of 12,500 relates to a 50% probability of answering Final Jeopardy correctly.

Final Jeopardy Wagering Model

Correlations and Contingencies

A model for individual performance on Final Jeopardy is helpful, but it is important to remember that Final Jeopardy is a game for three players, not one. Performances for the three players in a single game cannot be considered independent, as all three players are answering the same questions and competing against each other to buzz in. (Final Jeopardy is played with one or two players in about 6% of games.) When programming WATSON, the IBM researchers found a correlation coefficient of about $\rho=0.3$ between contestants in Final Jeopardy; other estimates place the measure closer to $\rho=0.2$ (Tesauro, 2013, p. 214; Williams, 2015). I was concerned that a correlation value would be inaccurate if applied evenly across all Final Jeopardy outcomes. Therefore, I began by finding the historic probability of each Final Jeopardy outcome by score position entering the round. These results are displayed in Table 5. (If a player did not wager anything on Final Jeopardy, they are not considered as submitting an answer, as they had no motivation to seriously attempt the question.) The letter "R" denotes right answers, while "W" denotes wrong answers. The leftmost letter represents the player in the lead entering Final Jeopardy, so that a sequence of "WRW" signifies that the player in the lead entering Final Jeopardy answered the question wrong, the player in second place answered right, and the player in last answered wrong.

Clearly, correlation exists between contestant accuracy, as Triple Stumpers – questions where no player reaches the correct answer – are the most frequent results, followed by questions where all three players deduce the correct answer. As expected, though, the correlation could not be applied evenly for all situations, as between the games where only one player answered correctly, the player in the lead entering Final Jeopardy was most likely to answer correctly, followed by the player in second.

Then, I calculated the probability of each player answering correctly based off the answers of the player ahead of them; this figure could be easily integrated with the results from the individual Final Jeopardy model in later steps. These dependencies are displayed in Table 6. In general, it appears that the player in second has about a 60% chance of achieving the same result as the player in first entering Final Jeopardy, while players in third have between a 65% and 70% chance of achieving the same result as the first two players if the first two players both answer the same, and a 57% probability of answering the Final Jeopardy question incorrectly if exactly one of the other two players also answers incorrectly.

FJ Answers	Proportion of All Games
WWW	16%
RRR	15%
RWW	10%
RRW	9%
WRW	9%
RWR	8%
WWR	7%
WRR	7%
WW	5%
RR	5%
RW	4%
WR	3%
R	1%
W	1%

Answer Contingency	Probability
P(R|R)	57%
P(W|R)	43%
P(R|W)	39%
P(W|W)	61%
P(R|RR)	63%
P(W|RR)	37%
P(R|WR)	43%
P(W|WR)	57%
P(R|RW)	43%
P(W|RW)	57%
P(R|WW)	31%
P(W|WW)	69%

Table 5 : FJ Answer Contingent Probabilities

Table 6 : Proportion of FJ Answer Arrangements (W = Wrong, R = Right)

The most frequent Final Jeopardy outcomes are all of the players answering incorrectly and all answering correctly. The trailing players experience the same outcome as the leading players about 60% of the time.

Theoretical Wagering

One of the elements most appealing about WATSON's model is its reliance on actual human wagering trends rather than assuming rational wagers by opponents. To incorporate this into my model, I first needed to know how often players actually make the most logical wager in Final Jeopardy. To do so, I recreated the rules found in the logic-based J! Archive wagering calculator (2018). A player's betting strategy should be based on the ratios between the different players' scores, not the absolute values of the scores. The J! Archive analysts have identified five break points at which player's wagering strategy should change. These points - one-half, two-thirds, three-quarters, four-fifths, and one – exist both between the scores of the second- and first-place players and between the scores of the third- and second-place players. This leads to one hundred different scoring scenarios, as there are five break points plus five ranges between break points, and two different player ratios. Added to these hundred are three unique scoring scenarios with independent wagering approaches: the Faith Love scenario, in which the 1st-place player's score is equal to twice the difference between the scores of the 2nd- and 3rd-place players; the Evenly Spaced scenario, in which 2nd-place player's score is equidistant from that of the 1st-place and 3rd-place players' scores; and the First equals Second plus Third scenario, which is self-explanatory (J! Archive, 2018). Some scenarios have multiple acknowledged strategies, though one is typically more accepted than others. I compiled the recommended strategies for each of the three players in each of these 103 situations, shown in Appendix A.

Once the correct bets for each situation could be ascertained, the natural next step was to investigate how often contestants abide by the rules of rational wagering. One would infer that *Jeopardy!* players, having accumulated massive amounts of trivia knowledge both throughout their lives and in particular preparation for the show, would have also prepared wagering strategy, especially for the closed-game situation of Final Jeopardy. However, this conclusion is far from true, as less than 50% of contestants tend to wager in accord with the logically prescribed approaches, and this falls to close to 40% when situations when lock games – situations where the player in the lead has over double the score of their nearest opponent – are disregarded, as players in the lead usually know enough not to blow a guaranteed winning situation. Table 7 displays the proportions of theoretically accurate wagers made by each contestant, as well as whether the players tended to overbet or underbet. Semicorrect wagers follow less-accepted but still acknowledged theoretical approaches, and only exist in limited situations. All players appear to systemically overbet, with non-leading players wagering too high about as often as they wager according to the theoretically correct methods.

	Correct Wager	Semicorrect Wager	Low Wager	High Wager
Player 1	62%	0%	14%	24%
Player 1 excluding lock games	50%	1%	18%	31%
Player 2	37%	4%	18%	42%
Player 3	44%	3%	11%	41%

Table 7 : Proportion of Wagering Accuracy in Final Jeopardy

Aside from the leading player in lock games, players systematically wager more than is logically prescribed in Final Jeopardy. Only about 40% of trailing players wager logically.
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At this point, it would be possible to quantify probabilities of winning the game when using theoretically accurate, low, and high wagers from previous game data. However, despite there being 7,890 historic games of *Jeopardy!* at the time of the data pull, 5,298 of which were available on the database and survived data cleaning, there were not a significant number of games for each of the 103 Final Jeopardy scenarios. Particularly, many of the situations with exact ratios between players, such as the first player having exactly 50% more money than the player in second and the player in second having 25% more money than they player in third, had only occurred once or twice in *Jeopardy!* history. Therefore, it was necessary to simulate more games.

Final Jeopardy Simulator

The goal of the simulator was to determine when the theoretically correct wagering approach actually gives the best probability of a player to win, considering players' tendency not to bet as theoretically expected, as well as the value of that probability. Each show in the historic data is used as a starting point, inputting each player's earnings and Coryat score at the start of Final Jeopardy. For each player, 5,000 Final Jeopardy outcomes are simulated from a theoretically correct wagering approach, 5,000 from an underbetting approach, 5,000 from an overbetting approach, and 5,000 from a semicorrect wagering approach, if one exists. To simulate a game, the player in question is randomly assigned a bet in the prescribed range (either correct, semicorrect, low, or high). Next, the simulator randomly decided the range – correct, semicorrect, low, or high – into which each of the opponents' wagers fell, based on historic wagering patterns between players with similar score ratios. After the range was determined, a random point in that range was selected. To arrive at the probability of each player answering Final Jeopardy correctly, the Final Jeopardy Individual Model (Equation 4.1) was used to calculate the probability of the player in the lead answering correctly, and the answer dependencies broken down in Table 6 were used to create a distribution of three-person right/wrong outcomes. Finally, an outcome was randomly chosen from the answer distribution, and given each player's wager, each player's final score was calculated and a winner determined.

(Winners were randomly assigned from ties according to Coryat ratio.) Then, the probabilities of each player winning if they wagered either theoretically correct, semicorrect, low, or high could be calculated, and the simulator compiled these results for each historic game situation inputted into the simulator. From there, the best approaches in each situation and the probabilities of winning given said approach were determined, and set aside for use in the Daily Double model.

Final Jeopardy Wagering Takeaways

The results of the simulation model surprised me. As identified earlier, players tend to systemically overbet, so I expected the model to dictate aggressive wagering strategies to keep up with opponents. Rather, it frequently suggested wagering lower than the theoretically suggested amount, particularly for the second- and third-place players entering Final Jeopardy. This is highly reflective of the correlation between player responses in Final Jeopardy, as there is a much higher probability that both a leading and trailing player will answer a question incorrectly than a leading player missing the question and the trailing player answering correctly. First-place players followed a more expected pattern, as it is almost always recommended for them to wager to cover the second player doubling their score.

As mentioned in Chapter 3, an important factor that the model does not consider is differences in player confidence and ability between categories. Rather, this should largely be up to the discretion of individual players. If a contestant hears a Final Jeopardy category that is in their wheelhouse, it would be absolutely reasonable for them to make a higher wager than generally recommended, while if their weakest category appears, betting nothing is not necessarily irrational. However, I believe that if a player is a serious contender on *Jeopardy!*, they should have thoroughly prepared before their appearance, so that they have no glaringly weak areas that would require addressing in the model. Furthermore, Final Jeopardy clues usually include some sort of twist that can confuse even the most knowledgeable participants. Former 74-game winner Ken Jennings describes,

> The goal of this kind of question is to ask something *nobody* knows the answer to, but to include just the right clues so that, with a little bit of common sense, deduction, or lateral thinking, the listener can have a sudden ‘Aha!’ flash of insight and get the answer. (2006, p. 116)

Therefore, players should use caution and avoid being overconfident even in their strongest categories, for if the correct “flash of insight” eludes them, they could easily miss a question built not to test pure knowledge but ability to make the appropriate mental connections. The question style of Final Jeopardy thus supports a generally even difficulty assumption across categories.

Though the model provided recommended wagering strategy for each of the 103 Final Jeopardy situations, there were some general player-by-player patterns, summarized below. Full numeric results are displayed in Appendix B.

Leading Player:

- If you have a lock game, you have a guaranteed win; don't bet so that the second-place player can catch you.
- If the second player has 2/3 or less of your score, always bet to cover them doubling and force them to answer correctly. This approach is known as Boyd's Rule (J! Archive, 2018).
- If the second player has between 2/3 and exactly 4/5 of your score, consider not wagering to cover the second player doubling if the third player has less than 2/3 of the score of the second player; the second player will often only wager a small amount to cover the third player. This strategy verifies Shore's Conjecture (J! Archive). If the third player has more than that, wager to cover the second player doubling. Wagering to win – covering the second player doubling – is always a defensible strategy, though.
- If the second player has more than 4/5 of your score, bet to cover them doubling. If you are tied, wager everything. (Though if you both have more than twice the score of the third player, betting nothing is also valid.)
- In special situations, bet according to theoretically prescribed rules.

<u>Second Player</u>:

- If you have 1/2 or less of the first player's score, you're probably not going to win, so your wager is probably irrelevant. Either bet to maintain second, or if you can tie for first by wagering everything, it's worth a shot.
- If you have between 1/2 and 2/3 of the first player's score, wager to at least take the lead and consider wagering everything. Either way, you have to get the question right and the first player has to get it wrong for you to win.
- If you have exactly 2/3 of the first player's score, wager everything.
- If you have between 2/3 and 3/4 of the first player's score, your best move is to wager so that you win if the first player bets to cover you doubling but answers incorrectly.
- If you have exactly 3/4 of the first player's score, any wager gives approximately the same chance of winning. Wagering to both pass a zero bet by the first player and the third player doubling is strategically defensible, but all results are more or less equal in practice.
- If you have between 3/4 and 4/5 of the first player's score, a small wager that aims to win when all three players answer incorrectly may actually be best your move.
- If you have exactly 4/5 of the first player's score, wagering only a small amount gives you a slightly better chance of winning than does betting to cover the third player doubling, but not so much so that betting to cover the third player and to pass a zero bet by the first player isn't defensible.

- If you have more than 4/5 of the first player's score but aren't tied, a small bet is often effective when the third player has 3/4 or more of your score. If they don't, wagering to cover them doubling gives about the same probability.
- If you are tied with the first player, you have about the same chance of winning if you go all in, wager nothing, or bet to cover the third player doubling.
- In special situations, either follow prescribed wagering rules or consider going higher, as the player in the lead may wager small and give you the chance to win.

Third Player:

- If the first player has a lock game or the second player has exactly 1/2 of the score of the first, make sure not to wager everything and hope that they miraculously mess up.
- If the second player has 2/3 or less of the score of the first player, you shouldn't be able to win, but if you wager close to everything, get Final Jeopardy correct, and the others mess up, you have a chance at winning.
- If the second player has exactly 2/3 of the score of the first player, wager to at least pass a zero bet by the second player, and more if you can pass a zero bet by the first player.
- If the second player has more than 2/3 of the score of the first player, wager a small enough amount to win regardless of your answer if both other players miss. If you can pass a zero bet by the second player or first player while doing so, make that bet, but your best chance to win is by staying mostly where you are.
- In special situations, wager according to the prescribed theoretical rules.

Chapter 5 : Daily Double Wagering Model

In Chapter 3, it was confirmed that Daily Doubles play an extraordinarily important role in games of *Jeopardy!*. Particularly when observing top players and analyzing the Opportunity Value of the clues, Daily Doubles are the centerpieces of gameplay and the most crucial questions in nearly any *Jeopardy!* game. Somewhat surprisingly, even answering Daily Doubles incorrectly correlates with a higher expected game winning percentage, as opponents are prevented from answering them correctly.

However, it is not enough to simply recognize the importance of Daily Doubles; a winning wagering strategy must be developed to best capitalize on such game-changing opportunities. Quantifying an optimal Final Jeopardy wagering strategy laid a good foundation for a Daily Double wagering model, as Daily Doubles can be more accurately examined with an end-goal for the clues in mind. To construct the Daily Double model, it was logical to begin in an analogous fashion, working backwards from the last Daily Double and adding layers of prediction and uncertainly while moving earlier into the game.

Difficulty by Gameboard Location

The probability of correctly answering played a central role in WATSON's Daily Double model. Using confidence rates from the category title and previous questions in the category, the computer could calculate precisely how certain it was that it was going to answer the clue correctly (Tesauro, 2013). However, human players do not have the luxury of immediate and exact question confidence probabilities, especially as all players have different strengths and knowledge bases that cannot be easily quantified. Drawing on historic answer rates by gameboard location can help incorporate gameboard location difficulty adjustments into the human Daily Double model. Though it is generally observable and very natural that clues that are worth more money on the *Jeopardy!* gameboard are more difficult, there are long-standing rumors that Daily Double difficulty is unrelated to face value of the clue. Investigating the veracity of these rumors by comparing answer rate probabilities can help produce a more accurate approach towards optimizing Daily Double wagers.

Defining what questions players do and do not know in *Jeopardy!* is a challenging task. Most contestants on *Jeopardy!* later report that the game is more of a buzzer race than a domination of wits; acclaimed player Bob Harris focuses at least as much on timing the buzzer than actually answering questions in his book Prisoner of Trebekistan (2006). Therefore, it would be unfair to count players as not knowing the answers to questions when they are beaten to the buzzer; at the same time, it is impossible to know when one player answers a clue that the others truly don't know. Accounting for this, I defined knowing a question in the most straightforward way possible – answering it correctly – while considering not knowing a question as either buzzing in and answering incorrectly or getting caught on a Triple Stumper, a clue where no one rings in with the correct answer, since this is evidence that the contestant in fact does not know the answer. From this definition it is possible to construct question accuracy rates that can be used to predict Daily Double accuracy. Figure 9 displays four charts of findings: in the top row, answer rates maps for normal questions in the Jeopardy and Double Jeopardy rounds, and in the bottom row, answer rates maps for Daily Doubles in the same two rounds.

J Correct %	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Total
Row 1	82%	81%	81%	80%	80%	80%	81%
Row 2	76%	76%	77%	76%	77%	78%	76%
Row 3	71%	70%	71%	72%	72%	73%	72%
Row 4	64%	65%	66%	66%	67%	68%	66%
Row 5	52%	54%	53%	54%	53%	57%	54%
Total	68%	69%	69%	69%	69%	71%	69%

DJ Correct %	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Total
Row 1	81%	83%	80%	82%	82%	84%	82%
Row 2	71%	74%	73%	73%	74%	75%	73%
Row 3	63%	66%	64%	64%	64%	68%	65%
Row 4	54%	56%	54%	56%	55%	59%	56%
Row 5	42%	44%	41%	44%	42%	46%	43%
Total	61%	63%	61%	62%	62%	65%	63%

DD J Correct %	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Total
Row 1	100%	0%	0%	0%	0%	100%	67%
Row 2	72%	76%	73%	77%	63%	74%	72%
Row 3	63%	75%	73%	73%	73%	67%	70%
Row 4	66%	63%	69%	65%	67%	67%	66%
Row 5	58%	60%	61%	64%	62%	64%	61%
Total	63%	66%	68%	67%	66%	67%	66%

DD DJ Correct %	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Total
Row 1	50%	100%	40%	0%	0%	100%	67%
Row 2	67%	69%	68%	73%	66%	72%	69%
Row 3	67%	67%	62%	61%	63%	65%	64%
Row 4	66%	68%	61%	60%	59%	61%	63%
Row 5	59%	64%	62%	59%	59%	56%	59%
Total	65%	67%	62%	61%	61%	62%	63%

Figure 9: Question Difficulty by Round, Question Type, and Board Location

The top two charts show that regular questions are slightly harder in the Double Jeopardy round than in the Jeopardy round, especially for higher-value clues. For higher-value clues, Daily Doubles are converted at a higher rate than regular clues but see similar rates of correct answers otherwise. Question difficulty does not vary substantially by column.

More expensive questions are ostensibly more difficult, though admittedly less so for Daily Doubles. Such expansive question records make each of these proportions statistically significant, even though the heat maps demonstrate that the differences aren't always tremendous. Interestingly enough, the Daily Doubles are actually converted at a higher rate than many of the questions, particularly more difficult question in Double Jeopardy. I propose that this is due to the fact that better players who answer more questions retain control of the board for longer and find more Daily Doubles. Since better players play more Daily Doubles, they artificially enhance the Daily Double answer rate. Therefore, I split players into good, average, and poor players when determining their answer accuracy for the model, setting cutoffs as the 33rd and 67th percentiles for historic Coryat scores at each question to determine a player's individual placement. These individual answer rates by location, found in Appendix C, were then used as correct answer probabilities for both regular questions and Daily Doubles.

Other Daily Double Models

The Daily Double model in this thesis was heavily inspired by the Daily Double model made for WATSON. As described by Tesauro et. al. (2013, p. 221), the researchers began with a "description of a current game state… and [output] an estimate of the probability that WATSON will win from the current game state." They calculated the equity of different wagers according to the formula:

$$E(bet) = p_{DD} * V(S_W + bet, \ldots) + (1 - p_{DD}) * V(S_W - bet, \ldots)$$

(Equation 5.1)

From this, they obtained "an optimal risk-neutral bet by evaluating E(bet) for every legal bet, and selecting the bet with highest equity." This was the general framework around which I constructed the Daily Double model in this thesis.

Model of Third Daily Double

Just as it is best to begin modeling *Jeopardy!* wagering strategy by starting with a Final Jeopardy model, so is it best to start looking at Daily Doubles from the third, final one. The final Daily Double presents a situation with an adequate amount of game data, a decently near vision of the end of the game, and no future Daily Doubles to introduce high uncertainty. Similar to the Final Jeopardy model, the Daily Double model relies heavily not on players' actual scores but more on the ratios between player scores. This is particularly true for the Third Daily Double model, which is very geared towards setting the player up for end-game scenarios.

The Daily Double model operates by inputting information from past games of *Jeopardy!* and testing each potential wager for the bettor, from the minimum of five dollars to the maximum of their entire holdings, in increments of five dollars. For each potential wager, it calculates the ratio between player scores that could occur based on the player's response. Then, it references a table of score trends to see the historic distribution of Coryat score ratios at the end of the game given the potential outcomes of the current question as a starting point. Therefore, if the model looks at a player who, when finding a Daily Double on the 57^{th} question, has a score of \$10,000 against an opponent's score of \$5,000, it would consider a how a \$2,000 wager would affect the prospects of a game by looking at the probability distribution of ending score ratios over the final three questions when beginning with the possible score ratios of five-eighths or five-twelfths (lower score in the numerator). This distribution is multiplied by the distribution of Final Jeopardy win probabilities for a set of score ratios, found from the model described in Chapter 4, to give a win probability for a scoring position as soon as Daily Doubles are gone. The win probability when getting the Daily Double correct is multiplied by the player's probability of answering the question right given its location and their personal prowess, then added to the product of the win probability when answering incorrectly and the probability of answering incorrectly. This returns a win share metric that can be compared across potential wagers to find one that optimizes the probability of winning. This process generally assumes that Coryat scores are accumulated fairly evenly throughout the game, so that a score ratio at one point will continue unless acted upon by the outside influence of a Daily Double.

Model of First Two Daily Doubles

Unsurprisingly, the models for the first and second Daily Doubles followed very closely to that of the third, with only the additional complexity of future Daily Doubles added. To account for these, random simulations were used, following historic expected wagers for other players and the rules established by the prior models for the user. Players were randomly assigned to select Daily Doubles in proportion to their Coryat score in relation to the other players, as Coryat scores are a measure of in-game question prowess and board control as compared to opponents. Since the location of these Daily Doubles were unknown, players were assigned the general Double Jeopardy answer rate of 60% on Daily Doubles. For the model of the second Daily Double, the last Daily Double was calculated according to the end-game strategies deduced from its model. For the model of the first Daily Double, two possibilities for the middle Daily Double were calculated based on the potential outcomes of the first Daily Double, and only one possibility was again calculated for the last Daily Double.

Chapter 6 : Application of Daily Double Model

Though the Daily Double model in this thesis may have been similar in organization to the Daily Double model used by WATSON, it had one crucial difference. My Daily Double model was built to be used by humans who have to make split-second wagering decisions using only mental math, not computers that can run thousands of calculations in the span of a millisecond. To make the findings of the model usable in gameplay, the model results must be carefully analyzed to find overarching patterns and trends. From there, a simple set of easily remembered and quickly calculated rules can be derived to make the Daily Double model applicable to humans playing *Jeopardy!*.

Third Daily Double Guidelines

It would be highly optimistic to expect the Daily Double model to return a complete set of hard-and-fast rules for wagering. After all, if incredibly clear patterns existed, they likely would have become widely evident at some point in the show's over 35-year run. However, the model did present some definitive findings and patterns that can be adopted as a recommended wagering strategy. Graphs of suggested Daily Double wagers for the players in each of the ordinal score positions are shown in Figures 10 through 12, plotted with the proposed wager as a percent of current holdings on the y-axis and the ratio between player scores on the x-axis.

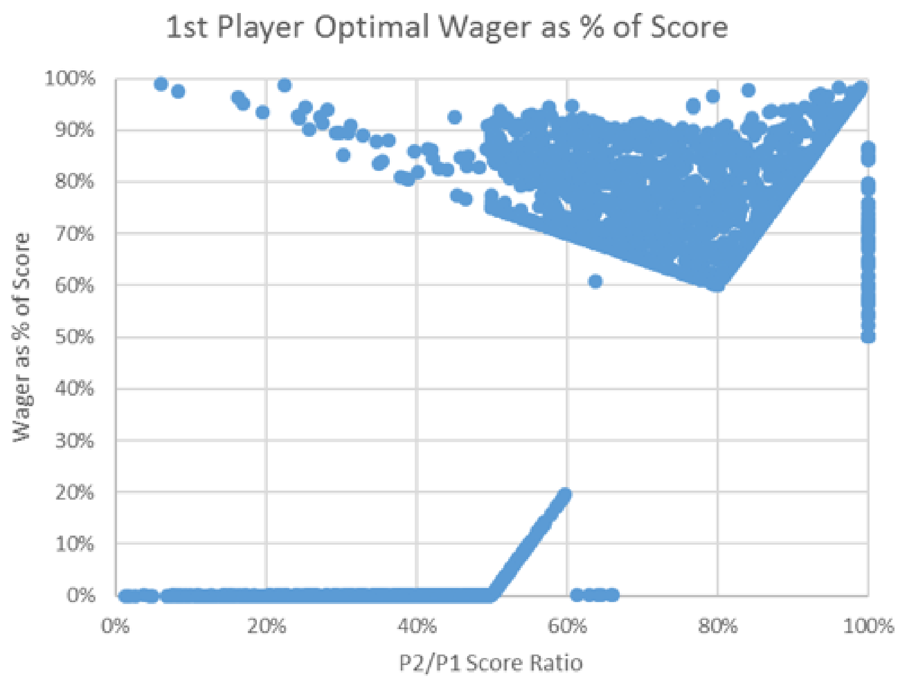

Figure 10 : Leading Player Optimal Wager against Score Ratio for Last Daily Double

For P2/P1 score ratios of less than 50%, the model recommends the leading player should wager nothing on the last Daily Double. For larger P2/P1 ratios, the model suggests the leading player wager aggressively to be in a position with at least twice the second player's score, but with no less than half the second player's score.

The model clearly respects the goal of ending Double Jeopardy with more than twice the money of one's leading opponent. If the second-place player has less than 50% of the score of the leading player, the model almost always suggests that the leader wager next to nothing, especially close to the end of the game. When the model suggests wagering minor amounts, it means as little as possible – the model completely rejects any anchoring tendencies of wagering similar amounts to the face values of questions, and usually prefers to bet either a significant amount or the smallest amount allowed. Upon the Player 2 to Player 1 score ratio increasing past 0.5, though, strategy changes, as Player 1 must now bet in order to still cover Player 2 potentially doubling their score. The model recommends making the smallest possible bet to cover Player 2 doubling until the score ratio P2/P1 hits 0.6, at which point a more aggressive strategy is adopted. Between the ratio values of P2/P1 = 0.5 and 0.8 (and weakly even for lesser values), the model suggests wagering so that if a player were to miss the question, they would have exactly half the score of their nearest opponent. However, larger bets are also valid, as the model clearly recognizes the value of locked games (when one player has clinched victory), and seeks to maximize the chances of locking out opponents while not getting personally locked out. The model hits a vertex at P2/P1 = 0.8, recommending a 60% wager so as to have exactly twice the score of the trailing player if answering correctly and exactly half the trailing player's score if not. Beyond this, it advocates for wagering everything on tied games and a linear increase in wagers in between. Wagering strategy for leading players on the last Daily Double is could thus be called the "Rule of Two": if you have more than twice your opponent's score, keep it that way. If you don't have more than twice your opponent's score, wager to at least that plateau, but beware falling beneath half the score of your opponent.

Trends are also apparent for the second player on the last Daily Double. In general, the optimal wagers are nearly everything when the second player has between 30% and 50% of the score of the first player, as the second player is trying not to get locked out of contention. As they obtain more money in relation to the first player, though, their wagers decrease. If players have the luxury to avoid being shut out of an opportunity to win during Final Jeopardy after missing a Daily Double, the model suggests they should restrain themselves. This seems counterintuitive, as many would expect players in second to seize the opportunity of a Daily Double to take the lead.

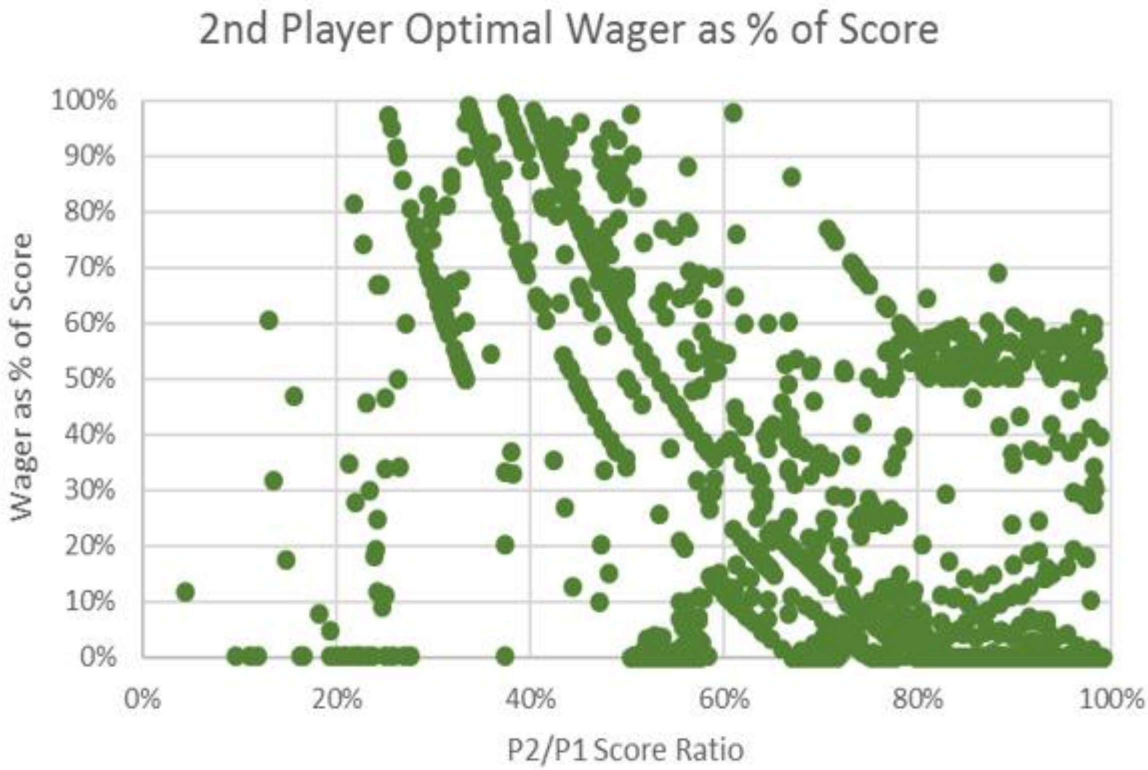

Figure 11 : Second Player Optimal Wager against Score Ratio for Last Daily Double

The model suggests the player in second wagering large amounts on the last Daily Double when the P2/P1 score ratio is less than 50% and gradually decreasing to about 0 for ratios greater than 80%. However, wagers of 50%-60% for high P2/P1 score ratios are also fair game. In general, the model recommends to wager so that the resulting score falls along Final Jeopardy break points.

When the scores between the second and first place contenders are very close, two alternate strategies emerge. The first is a conservative one. By wagering very little – ten percent or less of one's score – a player can win if the leader messes up Final Jeopardy, regardless of whether the trailing player answers either the Daily Double or Final Jeopardy correctly. Another cluster has the player in second wagering between fifty and sixty percent of their holdings to pass the current holdings of the player in first but avoid falling out of contention in case of a miss.

Though the most significant behavior has been noted, it is also interesting that among the trends of smaller wagers for higher P2/P1 score ratios are a number of more correlated lines.

These mark the different break points between Final Jeopardy wagering strategies. To optimize winning strategies, players may try to wager in a way that approaches these points in the ratios. For example, the leftmost line shows strategies that should get the second player to around 50% of the first player's points. From left to right, the other lines represent approaches towards two-thirds, three-quarters, four-fifths, one (faintly), and even six-fifths of the first player's ratio.

From these takeaways, it seems like it is most pertinent for the second player to remember to try to get to as close to or in the lead if possible on the last Daily Double, but to hedge to avoid being subject to a runaway in the case of an incorrect answer. Particularly efficient players will aim for Final Jeopardy wagering break points.

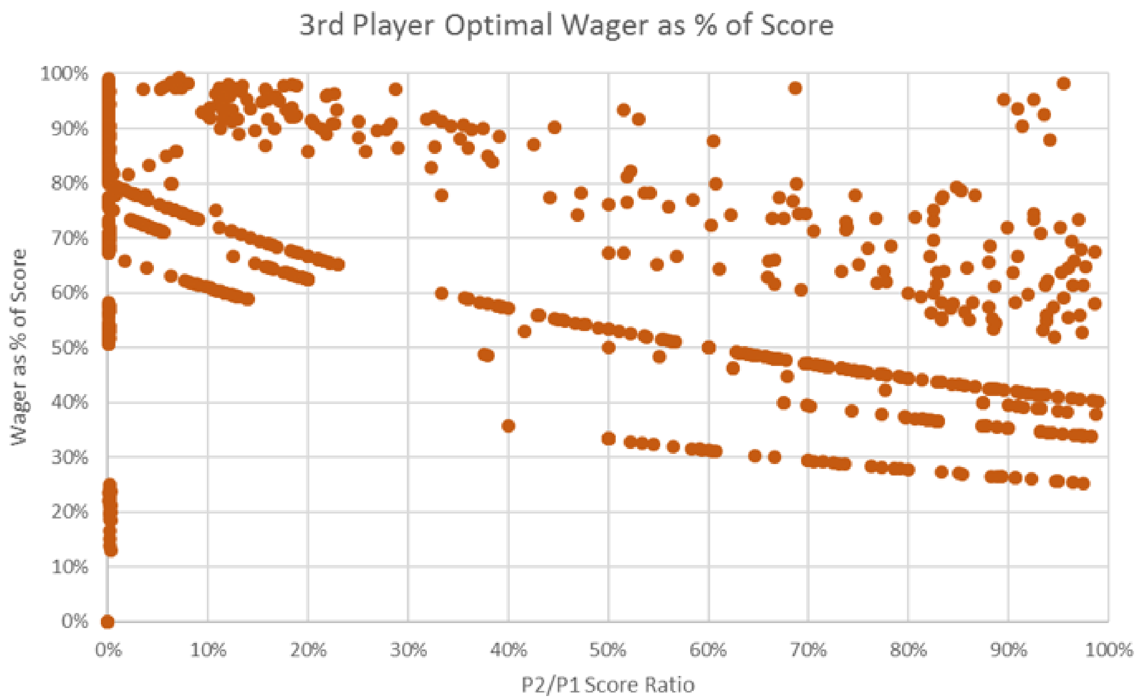

Figure 12 : Last Player Optimal Wager against Score Ratio for Last Daily Double

The model recommends the player in third to wager large amounts on the final Daily Double if there is a small P3/P2 score ratio, and to gradually wager less as the ratio increases, with wagers of about 40% of score when P3/P2 is greater than 90%. Wagering to fall along Final Jeopardy break points is suggested.

The third player's wagering strategy for the third Daily Double is similar to that of the second player. It is recommended to go nearly all in for very low scores, but to begin to temper that as the score relative to that of the second player increases. Unlike the middle player, the lowest player is virtually always suggested to wager at least thirty percent of their holdings. Player 3 also has noticeable bands of wagering strategies at break points, though these typically represent the lower wagers and are separate from a large band of less-correlated wagers that never fall below 65% of holdings. The biggest takeaway for the third player should be that, with the opportunity of a Daily Double in the final few questions, to be prepared to wager a significant amount – though not necessarily all – of their holdings to get back in contention.

Though other factors exist for quantifying Daily Doubles, none displayed anywhere near the amount of correlation and relationships with wagering strategy as could be gleaned from the coupling of player score ratios and wager percentage as a percent of score. Expected probability of answering correctly and sequential question number, among others, were investigated but showed no useful results.

Second Daily Double Guidelines

Continuing to proceed backwards through gameplay, the model's findings for the second Daily Double – the first one in the Double Jeopardy round – were very similar to the findings of the last Daily Double. The leading two players had virtually indistinguishable patterns from the third Daily Double's iteration of the model, so it is unnecessary to repeat all the recommendations for them.

The model did, however, advocate a slightly less aggressive strategy for the last-place player, as shown in Figure 13 below.

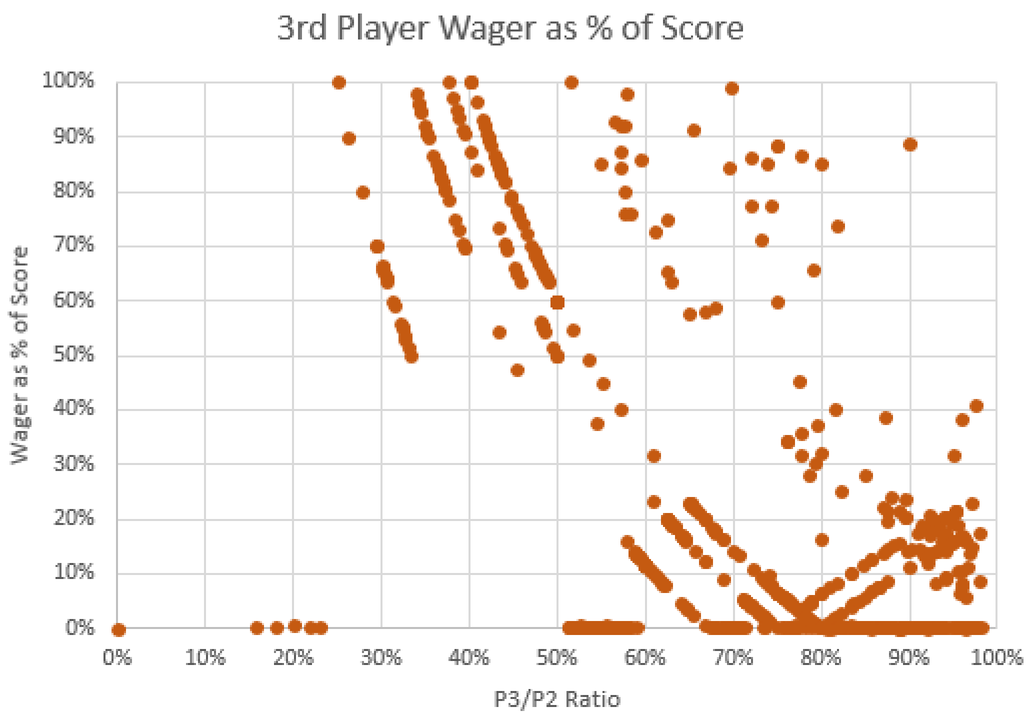

Figure 13 : Last Player Optimal Wager Against Score Ratio for Second Daily Double

For players in a distant third, the model recommends large wagers on the second Daily Double. However, if the third player's score is close to that of the second, the model recommends a relatively small wager between 0% and 20% of holdings in order to take second place but not fall below 50% of the second player's score.

Just as for the final Daily Double, the model advocates for wagering between 60% and 100% of holdings if the third player has less than half the money of the second. (To note, for all the P3/P2 ratios less than 20%, the third player has less than \$2,000 and is thus recommended to wager more than 100% of their holdings, since anyone may wager up to \$2,000 on any Double Jeopardy Daily Double.) The recommended wager, however, falls to the minimum of 5 by the time the player reaches a P3/P2 ratio of 70% - 80%, before rising slightly between 80% - 100%. Like the second player on the last Daily Double, the model does not want the game to fall completely out of reach with a missed question at this point, and recommends instead playing conservatively and allowing the other players to err.

First Daily Double Guidelines

The first-round Daily Double recommendations are notedly different than the guidelines for Daily Doubles in Double Jeopardy. Little correlation is apparent between player score ratios and recommended wagers, in harsh contrast to the highly visible patterns in the second round. It does seem, however, that when so little money has been accumulated, it is advantageous to build a quick lead. Besides giving players a psychological advantage, which isn't quantified in the model but is certainly present, early points can be exponentiated once later Daily Doubles are found. Therefore, the model overwhelmingly recommends "True Daily Doubles", or Daily Doubles where the maximum amount is wagered, for the first fifteen clues. This is shown in Figure 14.

For the rest of the Jeopardy round, it was very difficult to discern any patterns within the model. The model continued to generally advocate for significant wagers, as shown in Figure 15, but it had tempered its propensity to wager everything. Likely, it recognized that while players can risk complete setbacks early on in the game, the risk is worth the potential reward of an early lead. However, by the middle of the Jeopardy round, players have begun to accumulate substantial amounts of holdings, and completely erasing these would be difficult to overcome. Therefore, it is fitting to take a measured approach, wagering a considerable amount that will significantly help if answered correctly but not be completely decimating if missed. There was no substantial difference between approaches for players in different positions throughout the

Jeopardy round, showing that any contestant still has a fair chance at winning so early in the game, and by capitalizing on Daily Doubles, anyone can similarly improve their chances of winning.

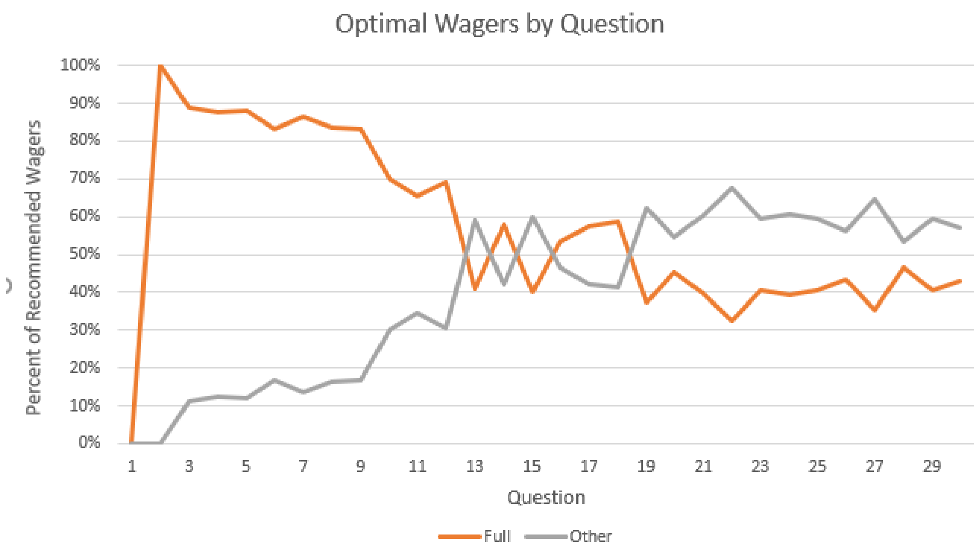

Figure 14 : Recommended True Daily Doubles by Question

The model overwhelmingly recommends wagering all a player's holdings over the first 10-15 questions of the Jeopardy round, but generally does not support it for the remainder of the round.

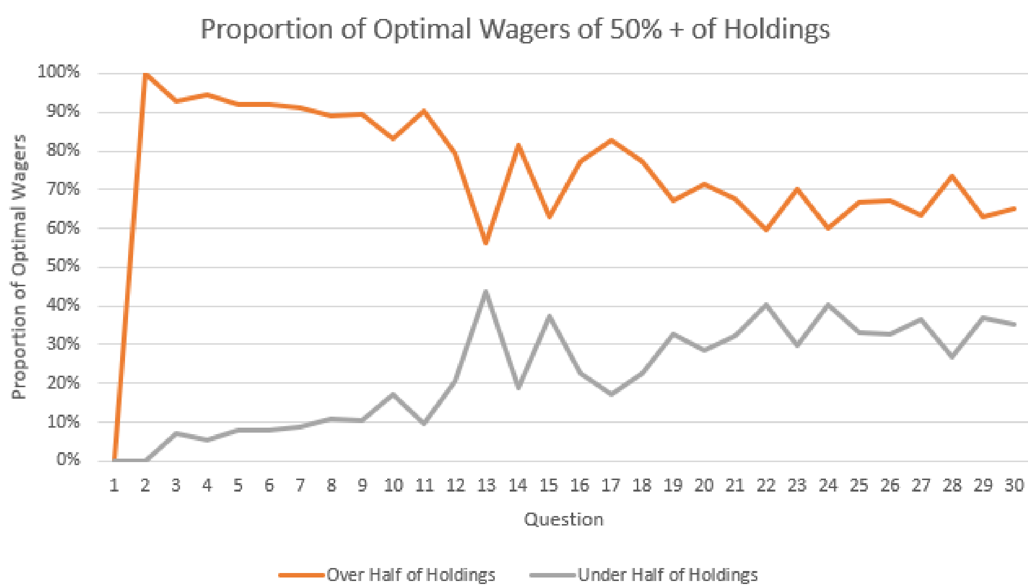

Figure 15 : Proportion of Optimal Wagers Suggesting Wagering Over Half of Holdings

The model suggests wagering at least 50% of a player's holdings on most Daily Doubles in the Jeopardy round.

Reaction to Guidelines

One of the primary conclusions of WATSON's Daily Double analysis was that "humans systematically err on the conservative side in [Daily Double] wagering" (Tesauro, 2013, p. 237). WATSON's model, on the other hand, advocated for much larger Daily Double wagers, to the point that the IBM researchers included a penalty term for wager volatility and set a limit on the "downside risk" of a wager (p. 221). I was therefore concerned that my model's suggested wagers would be much too large, as I did not include any risk adjustment factors. Though my model's proposed wagers were more aggressive than those ordinarily made by contestants, the recommendations were on the whole much more moderate than I originally expected. Particularly for the second- and third-place players, the model seemed more concerned with not allowing them to fall to less than half the holdings of the leading player than with taking the lead. Apparently the ~0% chance of winning when in such a position at the end of Double Jeopardy overrides the elevated winning probability when holding a lead.

On a larger-scale level, this approach is sensible and backed by historical data. However, on a game-by-game basis, I believe that it would be difficult to follow this advice and wager conservatively, knowing the method to risk losing when a player knows the answer and is simply not aggressive enough. Most of the time being conservative might work, but since each player only has one opportunity to compete on *Jeopardy!*, they are not concerned with "most of the time", but rather with "this time". Therefore, a player must recognize that many of the recommendations in the model are implemented using historic difficulty rates and answer probabilities, without specific knowledge of each player's category specialties and risk tolerance. If a player believes they have significantly higher than average knowledge in a category or feels inordinately confident in a clue, the guidelines can certainly be overridden. Even the models had a number of data points that strayed from the general patterns and recommended much higher or lower wagers; these situations likewise arise in real gameplay from time to time. However, for the average player in the average situation, the trends and guidelines derived from the Daily Double model proposed in this thesis should maximize the probability of victory. In the long run, the best chances of winning will be found by following these rules.

Chapter 7 : Conclusion

After establishing the importance of Daily Doubles to a game of *Jeopardy!* and constructing wagering models for both Final Jeopardy and Daily Doubles, some guidelines can be established to help players wager more optimally on *Jeopardy!*. These are summarized in Table 8.

Round	Player	Guideline
Jeopardy	All	- Wager everything in the first half of the round; wager over fifty percent of holdings but not everything in the second half of the round
Double Jeopardy	Leader	- Follow the "Rule of Two": wager to have twice or half of the second player's score
Double Jeopardy	Second	- Take the lead if possible, but make sure not to have less than half of leader's score if wrong
Double Jeopardy	Third	- The further the score gap, the greater wager needed. For close scores, don't fall out of contention if wrong.
Final Jeopardy	Leader	- Don't let anyone else catch you
Final Jeopardy	Second	- Try to take the lead, but be in a position to win if the leader misses
Final Jeopardy	Third	- Be in a position to win if the other players miss

Table 8 : Summarized Wagering Strategies

Just as Daily Doubles are chronically undervalued on *Jeopardy!*, so too is wagering strategy undervalued when preparing for an appearance on the show. It is essential for all contestants to do their due diligence when preparing for the show to ensure that they have a coherent wagering methodology. This thesis's approach of analyzing a combination of historic game trends, logical positions, and simulated results generates a complete and effective approach to *Jeopardy!* wagering. The theories and guidelines proposed in this thesis provide contestants with a wagering strategy that will maximize their chances of winning on *Jeopardy!*.

Appendix A : Logical Final Jeopardy Wagering Strategy

This appendix lays out the prescribed logical Final Jeopardy wagering strategies for each of the first-, second-, and third-place players entering Final Jeopardy based on each player's relative score position. P1 denotes Player 1 or Player 1's score entering Final Jeopardy, and so on. The first two columns of bets give the minimum and maximum of the normally accepted range of optimal wagers, while the second two note other acknowledged but less accepted wagering strategies.

Player 1

P2/P1	P3/P2	P1 Min 1	P1 Max 1	P1 Min 2	P1 Max 2
<0.5	<0.5	0	P1-P2*2-1		
<0.5	0.5	0	P1-P2*2-1		
<0.5	0.5 - 0.67	0	P1-P2*2-1		
<0.5	0.67	0	P1-P2*2-1		
<0.5	0.67 - 0.75	0	P1-P2*2-1		
<0.5	0.75	0	P1-P2*2-1		
<0.5	0.75 - 0.8	0	P1-P2*2-1		
<0.5	0.8	0	P1-P2*2-1		
<0.5	0.8 - 1	0	P1-P2*2-1		
<0.5	1	0	P1-P2*2-1		
0.5	<0.5	1	P1-P3*2-1		
0.5	0.5	1	P1-P3*2		
0.5	0.5 - 0.67	1	P1-P3*2-1		
0.5	0.67	1	P1-P3*2-1		
0.5	0.67 - 0.75	1	P1-P3*2-1		
0.5	0.75	1	P1-P3*2-1		
0.5	0.75 - 0.8	1	P1-P3*2-1		
0.5	0.8	1	P1-P3*2-1		
0.5	0.8 - 1	1	P1-P3*2-1		
0.5	1	1	P1-P3*2		
0.5 - 0.67	<0.5	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.5	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.5 - 0.67	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.67	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.67 - 0.75	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.75	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.75 - 0.8	P2*2-P1+1	P1-P2-1		
0.5 - 0.67	0.8	P2*2-P1+1	P1-2*P3-1		
0.5 - 0.67	0.8 - 1	P2*2-P1+1	P1-P3-1		
0.5 - 0.67	1	P2*2-P1+1	MAX(2*P3-P1-1, P2*2-P1+1)		

0.67	<0.5	P1-P2	P1-P2		
0.67	0.5	P1-P2	P1-P2		
0.67	0.5 - 0.67	P1-P2	P1-P2		
0.67	0.67	P1-P2	P1-P2		
0.67	0.67 - 0.75	P1-P2	P1-P2		
0.67	0.75	P1-P2	P1-P2		
0.67	0.75 - 0.8	P1-P2	P1-P2		
0.67	0.8	P1-P2	P1-P2		
0.67	0.8 - 1	P1-P2	P1-P2		
0.67	1	P1-P2	P1-P2		
0.67 - 0.75	<0.5	2*P2-P1+1	IF(P1-P3*2-1<2*P2-P1+1, 2*P2-P1+1, P1-P3*2-1)	IF(2*(P2-P3)-P1+1>=0, 2*(P2-P3)-P1+1, "", IF(P3=0, "", 0)	IF(P3=0, "", IF(2*(P2-P3)-P1+1>=0, 2*(P2-P3)-P1+1, ""))
0.67 - 0.75	0.5	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.5 - 0.67	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.67	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.67 - 0.75	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.75	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.75 - 0.8	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.8	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	0.8 - 1	2*P2-P1+1	2*P2-P1+1		
0.67 - 0.75	1	2*P2-P1+1	2*P2-P1+1		

P2/P1	P3/P2	P1 Min 1	P1 Max 1	P1 Min 2	P1 Max 2
0.75	<0.5	2*P1-2*P2	2*P1-2*P2		
0.75	0.5	2*P1-2*P2	2*P1-2*P2		
0.75	0.5 - 0.67	2*P1-2*P2	2*P1-2*P2		
0.75	0.67	2*P1-2*P2	2*P1-2*P2		
0.75	0.67 - 0.75	2*P1-2*P2	2*P1-2*P2		
0.75	0.75	2*P1-2*P2	2*P1-2*P2		
0.75	0.75 - 0.8	2*P1-2*P2	2*P1-2*P2		
0.75	0.8	2*P1-2*P2	2*P1-2*P2		
0.75	0.8 - 1	2*P1-2*P2	2*P1-2*P2		
0.75	1	2*P1-2*P2	2*P1-2*P2		
0.75 - 0.8	<0.5	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.5	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.5 - 0.67	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.67	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.67 - 0.75	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.75	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.75 - 0.8	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.8	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	0.8 - 1	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.75 - 0.8	1	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.8	<0.5	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.5	2*P2-P1	2*P2-P1	2*P2-P1	2*P2-P1
0.8	0.5 - 0.67	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.67	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.67 - 0.75	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.75	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.75 - 0.8	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.8	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	0.8 - 1	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8	1	2*P2-P1	2*P2-P1	2*P2-P1+1	2*P2-P1+5
0.8 - 1	<0.5	2*P2-P1+1	IF(P1-2*P3-1<2*P2-P1+1, 2*P2-P1+1, P1-2*P3-1)		
0.8 - 1	0.5	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.5 - 0.67	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.67	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.67 - 0.75	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.75	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.75 - 0.8	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.8	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	0.8 - 1	2*P2-P1+1	2*P2-P1+1		
0.8 - 1	1	2*P2-P1+1	2*P2-P1+1		
1	<0.5	P1	P1	0	0
1	0.5	P1	P1	0	0
1	0.5 - 0.67	P1	P1	0	0
1	0.67	P1	P1	0	0
1	0.67 - 0.75	P1	P1	0	0
1	0.75	P1	P1	0	0
1	0.75 - 0.8	P1	P1	0	0
1	0.8	P1	P1	0	0
1	0.8 - 1	P1	P1	0	0
1	1	P1	P1	0	0
Faith Love Scenario		0	0		
Evenly Spaced		P3	P3		
First equals second plus third		2*P2-P1	2*P2-P1		

Player 2

P2/P1	P3/P2	P2 Min 1	P2 Max 1	P2 Min 2	P2 Max 2
<0.5	<0.5	0	IF(AND(P2=0, P3=0), 0, P2-P3*2-1)		
<0.5	0.5	0	0		
<0.5	0.5 - 0.67	P3*2-P2+1	P3*2-P2+1		
<0.5	0.67	P2/3	P2/3		
<0.5	0.67 - 0.75	P3*2-P2+1	P3*2-P2+1		
<0.5	0.75	P3*2-P2+1	P3*2-P2+1		
<0.5	0.75 - 0.8	P3*2-P2+1	P3*2-P2+1		
<0.5	0.8	P3*2-P2+1	P3*2-P2+1		
<0.5	0.8 - 1	P3*2-P2+1	P3*2-P2+1		
<0.5	1	P2	P2	0	0
0.5	<0.5	P2	P2		
0.5	0.5	P2	P2		
0.5	0.5 - 0.67	P2	P2		
0.5	0.67	P2	P2		
0.5	0.67 - 0.75	P2	P2		
0.5	0.75	P2	P2		
0.5	0.75 - 0.8	P2	P2		
0.5	0.8	P2	P2		
0.5	0.8 - 1	P2	P2		
0.5	1	P2	P2	0	0
0.5 - 0.67	<0.5	P1-P2+1	IF(2*P1-3*P2>P1-P2+1, 2*P1-3*P2, P1-P2+1)		
0.5 - 0.67	0.5	P1-P2+1	P1-P2+1		
0.5 - 0.67	0.5 - 0.67	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.67	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.67 - 0.75	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.75	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.75 - 0.8	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.8	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	0.8 - 1	MAX(P3*2-P2+1, P1-P2+1)	P2-1		
0.5 - 0.67	1	P2	P2		
0.67	<0.5	P2	P2		
0.67	0.5	P2	P2		
0.67	0.5 - 0.67	P2	P2		
0.67	0.67	P2	P2		
0.67	0.67 - 0.75	P2	P2		
0.67	0.75	P2	P2		
0.67	0.75 - 0.8	P2	P2		
0.67	0.8	P2	P2		
0.67	0.8 - 1	P2	P2		
0.67	1	P2/2	P2/2	0	0
0.67 - 0.75	<0.5	0	3*P2-2*P1	min(3*P2-2*P1+2, P2-P3-1)	P1-P2+1
0.67 - 0.75	0.5	0	0	P2	P2
0.67 - 0.75	0.5 - 0.67	2*P3-P2+1	3*P2-2*P1		
0.67 - 0.75	0.67	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	0.67 - 0.75	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	0.75	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	0.75 - 0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	0.8 - 1	2*P3-P2+1	P2	0	3*P2-2*P1
0.67 - 0.75	1	P2	P2	0	0

P2/P1	P3/P2	P2 Min 1	P2 Max 1	P2 Min 2	P2 Max 2
0.75	<0.5	P2	P2	P1-P2	P1-P2
0.75	0.5	0	0	P1-P2	P1-P2
0.75	0.5 - 0.67	P1-P2	P1-P2	P3*2-P2+1	P3*2-P2+1
0.75	0.67	P1-P2	P1-P2	P2	P2
0.75	0.67 - 0.75	P3*2-P2+1	P2	0	P1-P2
0.75	0.75	P3*2-P2+1	P2	0	P1-P2
0.75	0.75 - 0.8	P3*2-P2+1	P2	0	P1-P2
0.75	0.8	P3*2-P2+1	P2	0	P1-P2
0.75	0.8 - 1	P3*2-P2+1	P2	0	P1-P2
0.75	1	P2	P2	0	0
0.75 - 0.8	<0.5	P1-P2+1	3*P2-2*P1		
0.75 - 0.8	0.5	P1-P2+1	P2-P3-1		
0.75 - 0.8	0.5 - 0.67	MAX(P1-P2+1, 2*P3-P2+1)	MAX(P1-P2+1, 2*P3-P2+1)		
0.75 - 0.8	0.67	2*P3-P2+1	P2-P3+1		
0.75 - 0.8	0.67 - 0.75	2*P3-P2+1	3*P2-2*P1	0	3*P2-2*P1
0.75 - 0.8	0.75	2*P3-P2+1	P2	0	3*P2-2*P1
0.75 - 0.8	0.75 - 0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.75 - 0.8	0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.75 - 0.8	0.8 - 1	2*P3-P2+1	P2	0	3*P2-2*P1
0.75 - 0.8	1	P2	P2	0	0
0.8	<0.5	IF(P1-P2>P3, 2*P1-2*P2, P2-2*P3-1)	IF(P1-P2>P3, 2*P1-2*P2, P2-2*P3-1)		
0.8	0.5	P2	P2		
0.8	0.5 - 0.67	P3*2-P2+1	P2-P3-1		
0.8	0.67	P3*2-P2+1	P2/2		
0.8	0.67 - 0.75	P3*2-P2+1	P2/2		
0.8	0.75	P2/2	P2/2	P2	P2
0.8	0.75 - 0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.8	0.8	2*P3-P2+1	P2	0	3*P2-2*P1
0.8	0.8 - 1	2*P3-P2+1	P2	0	3*P2-2*P1
0.8	1	P2	P2	0	0
0.8 - 1	<0.5	2*P1-2*P2+1	Min(3*P2-2*P1, P2-2*P3-1)		
0.8 - 1	0.5	0	0	P2-P3-1	P2-P3-1
0.8 - 1	0.5 - 0.67	2*P3-P2+1	P2-P3-1		
0.8 - 1	0.67	P2-P3	P2-P3		
0.8 - 1	0.67 - 0.75	2*P3-P2+1	3*P2-2*P1	0	P2-P3-1
0.8 - 1	0.75	2*P2-2*P3	2*P2-2*P3	2*P2-2*P3+1	P2
0.8 - 1	0.75 - 0.8	2*P3-P2+1	IF(3*P2-2*P1<2*P3-P2+1, 2*P3-P2+1, 3*P2-2*P1)	0	P2-P3-1
0.8 - 1	0.8	2*P3-P2+1	3*P2-2*P1	0	P2-P3-1
0.8 - 1	0.8 - 1	2*P3-P2+1	P2	0	3*P2-2*P1
0.8 - 1	1	P2	P2	0	0
1	<0.5	P2	P2	0	0
1	0.5	P2	P2	0	0
1	0.5 - 0.67	P2	P2	0	0
1	0.67	P2	P2	0	0
1	0.67 - 0.75	P2	P2	0	0
1	0.75	P2	P2	0	0
1	0.75 - 0.8	P2	P2	0	0
1	0.8	P2	P2	0	0
1	0.8 - 1	P2	P2	0	0
1	1	P2	P2	0	0
Faith Love Scenario		P1-P2	P1-P2		
Evenly Spaced		P2-P3	P2-P3	P2	P2
First equals second plus third		P2	P2		

Player 3

P2/P1	P3/P2	P3 Min 1	P3 Max 1	P3 Min 2	P3 Max 2
<0.5	<0.5	0	IF(P3=0, 0, P3-1)		
<0.5	0.5	0	P3-1		
<0.5	0.5 - 0.67	2*P2-3*P3	P3-2		
<0.5	0.67	P3	P3		
<0.5	0.67 - 0.75	0	3*P3-2*P2		
<0.5	0.75	0	3*P3-2*P2		
<0.5	0.75 - 0.8	P2-P3+1	3*P3-2*P2		
<0.5	0.8	P2-P3+1	3*P3-2*P2		
<0.5	0.8 - 1	P2-P3+1	3*P3-2*P2		
<0.5	1	P3	P3	0	0
0.5	<0.5	0	P3-1		
0.5	0.5	0	P3-1		
0.5	0.5 - 0.67	P2-P3+1	3*P3-2*P2		
0.5	0.67	P2-P3+1	3*P3-2*P2		
0.5	0.67 - 0.75	P2-P3+1	3*P3-2*P2		
0.5	0.75	P2-P3+1	3*P3-2*P2		
0.5	0.75 - 0.8	P2-P3+1	3*P3-2*P2		
0.5	0.8	P2-P3+1	3*P3-2*P2		
0.5	0.8 - 1	P2-P3+1	3*P3-2*P2		
0.5	1	P3	P3	0	0
0.5 - 0.67	<0.5	0	IF(P3=0, 0, max(P3-2*P2+P1, 0))		
0.5 - 0.67	0.5	0	max(P3-2*P2+P1, 0)		
0.5 - 0.67	0.5 - 0.67	max(P3-2*P2+P1, 0)	max(P3-2*P2+P1, 0)		
0.5 - 0.67	0.67	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	0.67 - 0.75	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	0.75	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	0.75 - 0.8	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	0.8	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	0.8 - 1	max(P3-2*P2+P1, 0)	P3-1		
0.5 - 0.67	1	P3	P3		
0.67	<0.5	IF(P3=0, 0, P2/2-P3+1)	IF(P3=0, 0, P3-1)		
0.67	0.5	P3	P3		
0.67	0.5 - 0.67	P2-P3+1	P3-1		
0.67	0.67	P2-P3+1	P3-1		
0.67	0.67 - 0.75	P2-P3+1	P3-1		
0.67	0.75	P2-P3+1	P3-1		
0.67	0.75 - 0.8	P2-P3+1	P3-1		
0.67	0.8	P2-P3+1	P3-1		
0.67	0.8 - 1	P2-P3+1	P3-1		
0.67	1	P3/2	P3/2	0	0
0.67 - 0.75	<0.5	If(P3*2>2*P1-2*P2-1, 2*P1-2*P2-P3, 0)	P3		
0.67 - 0.75	0.5	P3	P3		
0.67 - 0.75	0.5 - 0.67	P2-P3+1	P3		
0.67 - 0.75	0.67	P2-P3+1	P3		
0.67 - 0.75	0.67 - 0.75	P2-P3+1	P3		
0.67 - 0.75	0.75	P1-P3+1	P3	P2-P3+1	P3
0.67 - 0.75	0.75 - 0.8	P1-P3+1	P3	P2-P3+1	P3
0.67 - 0.75	0.8	P1-P3+1	P3	P2-P3+1	P3
0.67 - 0.75	0.8 - 1	P2-P3+1	IF(P3-2*P1+2*P2-1<0, P1-P3+1, P3-2*P1+2*P2)		
0.67 - 0.75	1	P3	P3	0	0

P2/P1	P3/P2	P3 Min 1	P3 Max 1	P3 Min 2	P3 Max 2
0.75	<0.5	IF(2*P2-P1-P3+1>P3, P3, 2*P2-P1-P3+1)	IF(2*P2-P1-P3+1>P3, P3, P3-1)	0	
0.75	0.5	2*P2-P1-P3+1	P3-1	P3	P3
0.75	0.5 - 0.67	2*P2-P1-P3+1	P3-1	P2-P3+1	P3
0.75	0.67	P3	P3		
0.75	0.67 - 0.75	P2-P3+1	P3-1		
0.75	0.75	P2-P3+1	P3-1		
0.75	0.75 - 0.8	P2-P3+1	P3-1		
0.75	0.8	P2-P3+1	P3-1		
0.75	0.8 - 1	P2-P3+1	P3-1		
0.75	1	P3	P3	0	0
0.75 - 0.8	<0.5	0.9*P3	P3		
0.75 - 0.8	0.5	0.9*P3	P3		
0.75 - 0.8	0.5 - 0.67	0.9*P3	P3		
0.75 - 0.8	0.67	0	P3-2*P1+2*P2	P2-P3+1	P3
0.75 - 0.8	0.67 - 0.75	0	MAX(P2-P3+1, P3-2*P1+2*P2)	P2-P3+1	P3
0.75 - 0.8	0.75	0	MAX(P2-P3+1, P3-2*P1+2*P2)	P2-P3+1	P3
0.75 - 0.8	0.75 - 0.8	0	P3-2*P1+2*P2	P2-P3+1	P3
0.75 - 0.8	0.8	0	IF(P3-2*P1+2*P2<P2-P3+1, P2-P3+1, P3-2*P1+2*P2)	P2-P3+1	P3
0.75 - 0.8	0.8 - 1	0	P3-2*P1+2*P2	P2-P3+1	P3
0.75 - 0.8	1	P3	P3	0	0
0.8	<0.5	IF(P3=0, 0, P3-2)	P3		
0.8	0.5	0	0		
0.8	0.5 - 0.67	0	P3-2*P1+2*P2-1		
0.8	0.67	P3-2*P1+2*P2	P3-2*P1+2*P2	P1-P3+1	P3
0.8	0.67 - 0.75	P3-2*P1+2*P2	P3-2*P1+2*P2	P1-P3+1	P3
0.8	0.75	P3	P3		
0.8	0.75 - 0.8	P2-P3+1	P3-2*P1+2*P2		
0.8	0.8	P2-P3+1	P3-2*P1+2*P2		
0.8	0.8 - 1	P2-P3+1	P3-2*P1+2*P2		
0.8	1	P3	P3	0	0
0.8 - 1	<0.5	0	MAX(0, P3-2*P1+2*P2-1)		
0.8 - 1	0.5	P3	P3	P3-2*P1+2*P2	P3-2*P1+2*P2
0.8 - 1	0.5 - 0.67	0	P3-2*P1+2*P2-1	P2-P3+1	P2-P3+1
0.8 - 1	0.67	P1-P3+1	P3	0	0
0.8 - 1	0.67 - 0.75	P2-P3+1	P3	0	3*P3-2*P2
0.8 - 1	0.75	P2-P3	P2-P3	P3	P3
0.8 - 1	0.75 - 0.8	P2-P3+1	3*P3-2*P2		
0.8 - 1	0.8	P2-P3+1	3*P3-2*P2		
0.8 - 1	0.8 - 1	P2-P3+1	P3-2*P1+2*P2		
0.8 - 1	1	P3	P3	0	0
1	<0.5	0	P3-1		
1	0.5	0	0	P3	P3
1	0.5 - 0.67	P1-P3+1	P3-1		
1	0.67	P1-P3+1	P3-1		
1	0.67 - 0.75	P1-P3+1	P3-1		
1	0.75	P1-P3+1	P3-1		
1	0.75 - 0.8	P1-P3+1	P3-1		
1	0.8	P1-P3+1	P3-1		
1	0.8 - 1	P1-P3+1	P3-1		
1	1	P3	P3	0	0
Faith Love Scenario		P3	P3		
Evenly Spaced		P3	P3		
First equals second plus third		P3	P3		

Appendix B : Final Jeopardy Simulation Model Results

If any game type is missing from the model, there were no games with that situation on record and that game type was not worth simulating. "Semi" denotes an acknowledged but less accepted wagering strategy. The first column shows the winning percentage for the leading player when they wager rationally for the situation described by the P2/P1 and P3/P2 ratios; the optimal response for those ratios and its associated win probability are shown in the last two columns.

Player 1

P1 Correct W%	P1 Low W%	P1 High W%	P1 Semi W%	P2/P1	P3/P2	P1 Best Strategy	P1 Win Prob
100%	100%	77%	0%	<0.5	<0.5	Correct	100%
100%	100%	75%	0%	<0.5	0.5	Correct	100%
100%	100%	75%	0%	<0.5	0.5 - 0.67	Correct	100%
100%	100%	74%	0%	<0.5	0.67	Correct	100%
100%	100%	74%	0%	<0.5	0.67 - 0.75	Correct	100%
100%	100%	75%	0%	<0.5	0.75	Correct	100%
100%	100%	74%	0%	<0.5	0.75 - 0.8	Correct	100%
100%	100%	74%	0%	<0.5	0.8	Correct	100%
100%	100%	71%	0%	<0.5	0.8 - 1	Correct	100%
100%	100%	70%	0%	<0.5	1	Correct	100%
89%	89%	61%	0%	0.5	<0.5	Low	89%
87%	87%	65%	0%	0.5	0.5 - 0.67	Correct	87%
82%	82%	64%	0%	0.5	0.67	Low	82%
87%	86%	64%	0%	0.5	0.67 - 0.75	Correct	87%
87%	88%	68%	0%	0.5	0.75 - 0.8	Low	88%
94%	94%	71%	0%	0.5	0.8	Low	94%
87%	87%	67%	0%	0.5	0.8 - 1	Correct	87%
82%	79%	68%	0%	0.5 - 0.67	<0.5	Correct	82%
82%	79%	66%	0%	0.5 - 0.67	0.5	Correct	82%
81%	68%	65%	0%	0.5 - 0.67	0.5 - 0.67	Correct	81%
77%	67%	66%	0%	0.5 - 0.67	0.67	Correct	77%
76%	67%	65%	0%	0.5 - 0.67	0.67 - 0.75	Correct	76%
76%	65%	65%	0%	0.5 - 0.67	0.75	Correct	76%
74%	65%	64%	0%	0.5 - 0.67	0.75 - 0.8	Correct	74%
77%	68%	68%	0%	0.5 - 0.67	0.8	Correct	77%
73%	60%	63%	0%	0.5 - 0.67	0.8 - 1	Correct	73%
73%	41%	68%	0%	0.5 - 0.67	1	Correct	73%
74%	63%	70%	0%	0.67	<0.5	Correct	74%
65%	61%	62%	0%	0.67	0.5 - 0.67	Correct	65%
65%	60%	66%	0%	0.67	0.67	High	66%
65%	59%	64%	0%	0.67	0.67 - 0.75	Correct	65%
55%	56%	56%	0%	0.67	0.75	High	56%
67%	55%	61%	0%	0.67	0.8	Correct	67%
68%	55%	61%	0%	0.67	0.8 - 1	Correct	68%
61%	81%	53%	0%	0.67 - 0.75	<0.5	Low	81%
59%	76%	55%	0%	0.67 - 0.75	0.5	Low	76%
64%	76%	55%	0%	0.67 - 0.75	0.5 - 0.67	Low	76%
66%	61%	58%	0%	0.67 - 0.75	0.67	Correct	66%
68%	59%	61%	0%	0.67 - 0.75	0.67 - 0.75	Correct	68%
69%	53%	63%	0%	0.67 - 0.75	0.75 - 0.8	Correct	69%
67%	51%	61%	0%	0.67 - 0.75	0.8	Correct	67%
60%	54%	53%	0%	0.67 - 0.75	0.8 - 1	Correct	60%
68%	37%	65%	0%	0.67 - 0.75	1	Correct	68%

P1 Correct W%	P1 Low W%	P1 High W%	P1 Semi W%	P2/P1	P3/P2	P1 Best Strategy	P1 Win Prob
67%	60%	65%	0%	0.75	<0.5	Correct	67%
57%	74%	50%	0%	0.75	0.5 - 0.67	Low	74%
63%	63%	58%	0%	0.75	0.67 - 0.75	Low	63%
62%	58%	56%	0%	0.75	0.75 - 0.8	Correct	62%
63%	51%	59%	0%	0.75	0.8 - 1	Correct	63%
59%	68%	54%	0%	0.75 - 0.8	<0.5	Low	68%
64%	65%	56%	0%	0.75 - 0.8	0.5	Low	65%
57%	64%	54%	0%	0.75 - 0.8	0.5 - 0.67	Low	64%
56%	66%	50%	0%	0.75 - 0.8	0.67 - 0.75	Low	66%
70%	58%	69%	0%	0.75 - 0.8	0.75	Correct	70%
52%	54%	51%	0%	0.75 - 0.8	0.75 - 0.8	Low	54%
56%	54%	56%	0%	0.75 - 0.8	0.8	Correct	56%
50%	47%	50%	0%	0.75 - 0.8	0.8 - 1	Correct	50%
66%	35%	64%	0%	0.75 - 0.8	1	Correct	66%
56%	68%	50%	52%	0.8	<0.5	Low	68%
58%	57%	52%	57%	0.8	0.5	Correct	58%
46%	64%	45%	45%	0.8	0.5 - 0.67	Low	64%
52%	64%	51%	51%	0.8	0.67 - 0.75	Low	64%
50%	50%	50%	51%	0.8	0.8 - 1	Semicorrect	51%
54%	57%	51%	0%	0.8 - 1	<0.5	Low	57%
52%	55%	51%	0%	0.8 - 1	0.5	Low	55%
52%	54%	51%	0%	0.8 - 1	0.5 - 0.67	Low	54%
51%	48%	50%	0%	0.8 - 1	0.67	Correct	51%
57%	48%	53%	0%	0.8 - 1	0.67 - 0.75	Correct	57%
54%	51%	53%	0%	0.8 - 1	0.75	Correct	54%
51%	45%	50%	0%	0.8 - 1	0.75 - 0.8	Correct	51%
49%	41%	48%	0%	0.8 - 1	0.8	Correct	49%
51%	38%	51%	0%	0.8 - 1	0.8 - 1	Correct	51%
66%	33%	65%	0%	0.8 - 1	1	Correct	66%
43%	47%	43%	52%	1	<0.5	Semicorrect	52%
45%	42%	45%	36%	1	0.5 - 0.67	Correct	45%
51%	42%	52%	33%	1	0.67	Correct	52%
40%	39%	40%	35%	1	0.67 - 0.75	Correct	40%
54%	40%	54%	34%	1	0.75 - 0.8	Correct	54%
48%	34%	48%	35%	1	0.8 - 1	Correct	48%
67%	67%	51%	0%	Faith Love		Low	67%
49%	47%	44%	0%	Evenly Spaced		Correct	49%
55%	50%	52%	0%	First equals second plus third		Correct	55%

Player 2

P2 Correct W%	P2 Low W%	P2 High W%	P2 Semi W%	P2/P1	P3/P2	P2 Best Strategy	P2 Win Prob
1%	1%	1%	0%	<0.5	<0.5	Low	1%
1%	1%	1%	0%	<0.5	0.5	Low	1%
1%	1%	1%	0%	<0.5	0.5 - 0.67	Correct	1%
1%	1%	1%	0%	<0.5	0.67	High	1%
1%	1%	1%	0%	<0.5	0.67 - 0.75	Low	1%
1%	1%	1%	0%	<0.5	0.75	Low	1%
1%	1%	1%	0%	<0.5	0.75 - 0.8	Low	1%
1%	1%	1%	0%	<0.5	0.8	High	1%
1%	1%	1%	0%	<0.5	0.8 - 1	High	1%
1%	1%	1%	1%	<0.5	1	Correct	1%
4%	8%	4%	0%	0.5	<0.5	Low	8%
6%	7%	5%	0%	0.5	0.5 - 0.67	Low	7%
6%	8%	7%	0%	0.5	0.67	Low	8%
6%	7%	6%	0%	0.5	0.67 - 0.75	Low	7%
6%	6%	6%	0%	0.5	0.75 - 0.8	Correct	6%
5%	5%	5%	0%	0.5	0.8	Low	5%
5%	5%	5%	0%	0.5	0.8 - 1	Correct	5%
21%	16%	22%	0%	0.5 - 0.67	<0.5	High	22%
19%	14%	21%	0%	0.5 - 0.67	0.5	High	21%
21%	16%	22%	0%	0.5 - 0.67	0.5 - 0.67	High	22%
21%	14%	22%	0%	0.5 - 0.67	0.67	High	22%
21%	13%	22%	0%	0.5 - 0.67	0.67 - 0.75	High	22%
21%	13%	22%	0%	0.5 - 0.67	0.75	High	22%
22%	13%	23%	0%	0.5 - 0.67	0.75 - 0.8	High	23%
23%	6%	32%	0%	0.5 - 0.67	0.8	High	32%
21%	13%	22%	0%	0.5 - 0.67	0.8 - 1	High	22%
18%	9%	18%	0%	0.5 - 0.67	1	Correct	18%
31%	22%	31%	0%	0.67	<0.5	Correct	31%
30%	24%	30%	0%	0.67	0.5 - 0.67	Correct	30%
32%	21%	31%	0%	0.67	0.67	Correct	32%
32%	21%	32%	0%	0.67	0.67 - 0.75	Correct	32%
39%	23%	39%	0%	0.67	0.75	Correct	39%
27%	20%	27%	0%	0.67	0.8	Correct	27%
30%	17%	30%	0%	0.67	0.8 - 1	Correct	30%
44%	45%	28%	34%	0.67 - 0.75	<0.5	Low	45%
40%	41%	27%	23%	0.67 - 0.75	0.5	Low	41%
31%	31%	24%	0%	0.67 - 0.75	0.5 - 0.67	Correct	31%
25%	29%	25%	35%	0.67 - 0.75	0.67	Semicorrect	35%
24%	25%	24%	31%	0.67 - 0.75	0.67 - 0.75	Semicorrect	31%
24%	21%	24%	28%	0.67 - 0.75	0.75 - 0.8	Semicorrect	28%
24%	23%	23%	30%	0.67 - 0.75	0.8	Semicorrect	30%
24%	23%	24%	32%	0.67 - 0.75	0.8 - 1	Semicorrect	32%
19%	17%	19%	29%	0.67 - 0.75	1	Semicorrect	29%

P2 Correct W%	P2 Low W%	P2 High W%	P2 Semi W%	P2/P1	P3/P2	P2 Best Strategy	P2 Win Prob
32%	32%	32%	34%	0.75	<0.5	Semicorrect	34%
32%	39%	24%	41%	0.75	0.5 - 0.67	Semicorrect	41%
23%	29%	30%	31%	0.75	0.67 - 0.75	Semicorrect	31%
24%	27%	32%	32%	0.75	0.75 - 0.8	High	32%
23%	22%	31%	27%	0.75	0.8 - 1	High	31%
43%	45%	27%	0%	0.75 - 0.8	<0.5	Low	45%
33%	39%	24%	0%	0.75 - 0.8	0.5	Low	39%
38%	34%	28%	0%	0.75 - 0.8	0.5 - 0.67	Correct	38%
27%	40%	23%	40%	0.75 - 0.8	0.67 - 0.75	Semicorrect	40%
15%	24%	17%	25%	0.75 - 0.8	0.75	Semicorrect	25%
22%	32%	24%	35%	0.75 - 0.8	0.75 - 0.8	Semicorrect	35%
20%	28%	21%	32%	0.75 - 0.8	0.8	Semicorrect	32%
24%	26%	24%	30%	0.75 - 0.8	0.8 - 1	Semicorrect	30%
19%	22%	18%	31%	0.75 - 0.8	1	Semicorrect	31%
42%	48%	28%	0%	0.8	<0.5	Low	48%
28%	34%	29%	0%	0.8	0.5	Low	34%
48%	49%	27%	0%	0.8	0.5 - 0.67	Low	49%
37%	37%	23%	0%	0.8	0.67 - 0.75	Low	37%
24%	28%	33%	32%	0.8	0.8 - 1	High	33%
48%	49%	35%	0%	0.8 - 1	<0.5	Low	49%
42%	41%	37%	40%	0.8 - 1	0.5	Correct	42%
42%	42%	31%	0%	0.8 - 1	0.5 - 0.67	Low	42%
39%	35%	36%	0%	0.8 - 1	0.67	Correct	39%
37%	34%	25%	33%	0.8 - 1	0.67 - 0.75	Correct	37%
32%	35%	24%	24%	0.8 - 1	0.75	Low	35%
27%	37%	24%	37%	0.8 - 1	0.75 - 0.8	Semicorrect	37%
28%	37%	24%	36%	0.8 - 1	0.8	Low	37%
24%	29%	24%	30%	0.8 - 1	0.8 - 1	Semicorrect	30%
19%	26%	19%	31%	0.8 - 1	1	Semicorrect	31%
37%	43%	37%	50%	1	<0.5	Semicorrect	50%
33%	39%	33%	34%	1	0.5 - 0.67	Low	39%
32%	33%	32%	30%	1	0.67	Low	33%
39%	38%	39%	35%	1	0.67 - 0.75	Correct	39%
29%	29%	29%	30%	1	0.75 - 0.8	Semicorrect	30%
32%	28%	32%	33%	1	0.8 - 1	Semicorrect	33%
14%	6%	25%	0%	Faith Love		High	25%
15%	14%	15%	19%	Evenly Spaced		Semicorrect	19%
19%	13%	19%	0%	First equals second plus third		Correct	19%

Player 3

P3 Correct W%	P3 Low W%	P3 High W%	P3 Semi W%	P2/P1	P3/P2	P3 Best Strategy	P3 Win Prob
0%	0%	0%	0%	<0.5	<0.5	Low	0%
0%	0%	0%	0%	<0.5	0.5	High	0%
0%	0%	0%	0%	<0.5	0.5 - 0.67	Correct	0%
0%	0%	0%	0%	<0.5	0.67	Correct	0%
0%	0%	0%	0%	<0.5	0.67 - 0.75	Low	0%
0%	0%	0%	0%	<0.5	0.75	Low	0%
0%	0%	0%	0%	<0.5	0.75 - 0.8	Low	0%
0%	0%	0%	0%	<0.5	0.8	Low	0%
0%	1%	0%	0%	<0.5	0.8 - 1	Low	1%
0%	1%	0%	1%	<0.5	1	Semicorrect	1%
2%	2%	2%	0%	0.5	<0.5	Low	2%
3%	3%	3%	0%	0.5	0.5 - 0.67	Correct	3%
4%	4%	3%	0%	0.5	0.67	Low	4%
4%	4%	4%	0%	0.5	0.67 - 0.75	Low	4%
3%	4%	4%	0%	0.5	0.75 - 0.8	Low	4%
3%	3%	3%	0%	0.5	0.8	Low	3%
3%	4%	3%	0%	0.5	0.8 - 1	Low	4%
0%	0%	0%	0%	0.5 - 0.67	<0.5	Low	0%
1%	1%	1%	0%	0.5 - 0.67	0.5	Low	1%
2%	2%	3%	0%	0.5 - 0.67	0.5 - 0.67	High	3%
5%	2%	5%	0%	0.5 - 0.67	0.67	Correct	5%
7%	2%	7%	0%	0.5 - 0.67	0.67 - 0.75	High	7%
8%	3%	9%	0%	0.5 - 0.67	0.75	High	9%
9%	3%	9%	0%	0.5 - 0.67	0.75 - 0.8	High	9%
2%	1%	2%	0%	0.5 - 0.67	0.8	High	2%
11%	7%	11%	0%	0.5 - 0.67	0.8 - 1	Correct	11%
13%	8%	13%	0%	0.5 - 0.67	1	Correct	13%
1%	1%	1%	0%	0.67	<0.5	Low	1%
10%	4%	10%	0%	0.67	0.5 - 0.67	High	10%
9%	4%	9%	0%	0.67	0.67	Correct	9%
10%	5%	10%	0%	0.67	0.67 - 0.75	Correct	10%
12%	7%	12%	0%	0.67	0.75	High	12%
12%	6%	12%	0%	0.67	0.8	Correct	12%
12%	6%	12%	0%	0.67	0.8 - 1	Correct	12%
1%	0%	1%	0%	0.67 - 0.75	<0.5	High	1%
4%	2%	4%	0%	0.67 - 0.75	0.5	Correct	4%
9%	4%	9%	0%	0.67 - 0.75	0.5 - 0.67	Correct	9%
12%	7%	11%	0%	0.67 - 0.75	0.67	Correct	12%
11%	8%	11%	0%	0.67 - 0.75	0.67 - 0.75	Correct	11%
12%	11%	11%	11%	0.67 - 0.75	0.75 - 0.8	Correct	12%
12%	13%	12%	13%	0.67 - 0.75	0.8	Low	13%
18%	19%	13%	0%	0.67 - 0.75	0.8 - 1	Low	19%
13%	16%	13%	25%	0.67 - 0.75	1	Semicorrect	25%

P3 Correct W%	P3 Low W%	P3 High W%	P3 Semi W%	P2/P1	P3/P2	P3 Best Strategy	P3 Win Prob
3%	1%	3%	2%	0.75	<0.5	Correct	3%
7%	1%	8%	10%	0.75	0.5 - 0.67	Semicorrect	10%
11%	9%	11%	0%	0.75	0.67 - 0.75	High	11%
12%	15%	13%	0%	0.75	0.75 - 0.8	Low	15%
15%	23%	15%	0%	0.75	0.8 - 1	Low	23%
4%	2%	4%	0%	0.75 - 0.8	<0.5	Correct	4%
9%	7%	8%	0%	0.75 - 0.8	0.5	Correct	9%
10%	8%	10%	0%	0.75 - 0.8	0.5 - 0.67	High	10%
15%	22%	12%	11%	0.75 - 0.8	0.67 - 0.75	Low	22%
14%	15%	9%	8%	0.75 - 0.8	0.75	Low	15%
23%	24%	15%	13%	0.75 - 0.8	0.75 - 0.8	Low	24%
22%	23%	15%	12%	0.75 - 0.8	0.8	Low	23%
26%	27%	18%	16%	0.75 - 0.8	0.8 - 1	Low	27%
13%	21%	13%	28%	0.75 - 0.8	1	Semicorrect	28%
0%	0%	0%	0%	0.8	<0.5	High	0%
7%	6%	9%	0%	0.8	0.5	High	9%
4%	3%	7%	0%	0.8	0.5 - 0.67	High	7%
9%	17%	11%	13%	0.8	0.67 - 0.75	Low	17%
25%	26%	19%	0%	0.8	0.8 - 1	Low	26%
4%	4%	3%	0%	0.8 - 1	<0.5	Low	4%
4%	5%	4%	5%	0.8 - 1	0.5	Semicorrect	5%
12%	12%	11%	10%	0.8 - 1	0.5 - 0.67	Low	12%
12%	14%	12%	13%	0.8 - 1	0.67	Low	14%
15%	21%	15%	22%	0.8 - 1	0.67 - 0.75	Semicorrect	22%
17%	24%	13%	11%	0.8 - 1	0.75	Low	24%
24%	25%	19%	0%	0.8 - 1	0.75 - 0.8	Low	25%
26%	26%	22%	0%	0.8 - 1	0.8	Low	26%
26%	28%	22%	0%	0.8 - 1	0.8 - 1	Low	28%
13%	24%	13%	28%	0.8 - 1	1	Semicorrect	28%
17%	18%	17%	0%	1	<0.5	Low	18%
20%	22%	20%	0%	1	0.5 - 0.67	Low	22%
19%	20%	20%	0%	1	0.67	High	20%
23%	24%	23%	0%	1	0.67 - 0.75	Low	24%
20%	21%	20%	0%	1	0.75 - 0.8	Low	21%
25%	26%	24%	0%	1	0.8 - 1	Low	26%
0%	0%	0%	0%	Faith Love		Correct	0%
11%	8%	11%	0%	Evenly Spaced		Correct	11%
2%	2%	2%	0%	First equals second plus third		Low	2%

Appendix C: Question Difficulty by Location

This appendix presents the probabilities of players of different abilities (delineated as "Good", "Average", and "Poor"; determined by Coryat score at time of question) answering *Jeopardy!* questions at different locations on the game board. These are broken into proportions in both the Jeopardy and Double Jeopardy rounds.

J Round							
Good Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	91%	90%	90%	89%	90%	91%	90%
Row 2	89%	89%	89%	86%	88%	90%	88%
Row 3	88%	86%	86%	87%	86%	88%	87%
Row 4	84%	83%	83%	84%	83%	85%	84%
Row 5	76%	76%	76%	75%	73%	77%	76%
Total	85%	85%	85%	84%	84%	86%	85%
J Round							
Average Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	85%	83%	85%	82%	82%	82%	83%
Row 2	79%	77%	78%	79%	80%	81%	79%
Row 3	72%	70%	73%	73%	74%	74%	73%
Row 4	63%	65%	66%	64%	68%	68%	65%
Row 5	48%	51%	49%	51%	50%	57%	51%
Total	69%	69%	70%	69%	70%	72%	70%
J Round							
Poor Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	61%	61%	60%	62%	61%	61%	61%
Row 2	48%	51%	53%	55%	55%	56%	53%
Row 3	38%	43%	44%	45%	49%	47%	44%
Row 4	30%	32%	33%	34%	36%	38%	34%
Row 5	17%	20%	20%	21%	24%	24%	21%
Total	37%	41%	41%	43%	44%	45%	42%

DJ Round							
Good Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	86%	87%	86%	87%	87%	88%	87%
Row 2	80%	82%	81%	82%	82%	84%	82%
Row 3	76%	79%	77%	77%	77%	80%	77%
Row 4	71%	73%	71%	72%	71%	74%	72%
Row 5	62%	62%	60%	63%	60%	65%	62%
Total	74%	76%	74%	75%	74%	77%	75%

DJ Round							
Average Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	82%	84%	81%	83%	84%	85%	83%
Row 2	71%	74%	74%	72%	74%	75%	73%
Row 3	63%	66%	63%	64%	65%	68%	65%
Row 4	54%	52%	53%	54%	53%	59%	54%
Row 5	38%	41%	39%	41%	39%	43%	40%
Total	61%	63%	61%	62%	62%	65%	62%

DJ Round							
Poor Player	**Column 1**	**Column 2**	**Column 3**	**Column 4**	**Column 5**	**Column 6**	**Total**
Row 1	73%	76%	72%	74%	72%	77%	74%
Row 2	58%	61%	59%	59%	61%	64%	60%
Row 3	42%	46%	43%	43%	45%	49%	45%
Row 4	30%	33%	32%	33%	31%	36%	33%
Row 5	17%	20%	18%	21%	19%	23%	68%
Total	43%	46%	43%	45%	44%	48%	59%

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Academic Vita

Paul R. Birch

EDUCATION

The Pennsylvania State University | Schreyer Honors College — **University Park, PA**
Bachelor of Science | Risk Management – Actuarial Science Option | Smeal College of Business — *Class of 2019*
Minors: Statistics, History

Indiana Area Senior High School — **Indiana, PA**
Valedictorian: Ranked first out of 190 students — *Graduated Jun 2015*

ACTUARIAL EXAMS

Exams P/1, FM/2, IFM/3F, SRM – *Passed* — *Mar 2019*
Exam LTAM – *Sitting* — *Oct 2019*
VEE: Economics; Corporate Finance; Applied Statistical Methods – *Credit Earned* — *Dec 2017*

WORK EXPERIENCE

Aetna | Actuarial Intern – International Americas — **Blue Bell, PA**
– Created trend model for use in pricing and forecasting functions on $500 million business block — *May 2018 – Aug 2018*
– Normalized for 4 leading trend factors and investigated historic patterns to bolster future trend determination
– Collaborated on interdisciplinary proposal integrating CVS merger to improve member experience for 400,000+ members

Liberty Mutual Insurance | Actuarial Intern – Life Reserving — **Boston, MA**
– Converted model for over 23,000 policies from unsupported database system to Excel — *Jun 2017 – Aug 2017*
– Upgraded run-time of model by 25% and enhanced user interface while exactly matching nearly $10 million in reserves
– Designed flowcharts and glossary of commutation functions and formulas to enhance and expedite life reserving training

Kuzneski Insurance Group | Benefits Intern — **Indiana, PA**
– Researched and assisted presentation of optimal benefits packages purchased by 5 companies — *May 2016 – Aug 2016*
– Contributed to overarching branding initiative and established website posting consistency through engaging weekly blogs

LEADERSHIP

Penn State Actuarial Science Club | President — **University Park, PA**
– Led 12-person executive board tasked with providing opportunities for actuarial students — *August 2015 – Jan 2019*
– Oversaw events including career fair, speaker events, newsletters, and club meals to support student development
– Initiated a club points system, Lunch with Leaders program, and actuarial case competition to increase club involvement
– Organized 2-day trip for 18 students to Washington, DC including actuarial firm site visits and alumni networking dinner

Society of Actuaries | Actuary of the Future - Section Council Intern — **University Park, PA**
– Conducted regular interviews with experienced actuaries for distribution as informative podcasts — *Dec 2017 – Dec 2018*

Presidential Leadership Academy | Member — **University Park, PA**
– Travel on biannual trips across country to gain a broader perspective on social and political issues — *Apr 2016 – Present*
– Coauthored policy paper proposing initiative using mobile technology to facilitate campus mental health support
– Advise University President Barron on PSU budget, mental health offerings, and execution of novel leadership courses

Penn State Quiz Bowl | President — **University Park, PA**
– Coordinate transportation, funding, lodging, and overall logistics for 17 players to travel to 7 annual tournaments in locations including Chicago, Rochester, University of Maryland, and Philadelphia — *Aug 2015 – Present*
– Finished 2nd overall in individual scoring at 2016 Penn State Invitational, demonstrating breadth of academic knowledge

Penn State Music Service Club | Member — **University Park, PA**
– Volunteer as pianist on over 10 annual trips sharing therapeutic properties of music at nursing homes — *Oct 2015 – Present*

Sapphire Leadership Academic Program | Distinguished Member — **University Park, PA**
– Translate skills learned from program developing leadership through professional events, speakers, business etiquette, and community service into leadership positions in Quiz Bowl and the Actuarial Science Club — *Aug 2015 – Present*

TECHNICAL SKILLS & INTERESTS

– **Technical Skills:** VBA, SQL, SAS, R, Java, Microsoft Excel/Access/Word/PowerPoint
– **Interests:** Piano (jazz, classical, small group), Trivia competitions, Ping-pong, Bicycling, Fantasy sports, US history