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SCHREYER HONORS COLLEGE

DEPARTMENT OF AEROSPACE ENGINEERING

OPTIMAL DEFLECTION METHOD FOR A NEAR EARTH ASTEROID USING A LOW-THRUST PROPULSION SYSTEM

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ABSTRACT

Asteroid 4179 Toutatis is a Near Earth Object (NEO) that is also classified as a Potentially Hazardous Object (PHO). On November 5, 2069 it will come within 0.02 AU to Earth making its closest approach since 2024. If anything were to happen to its orbit it is possible that it could impact Earth and cause catastrophic events, for this reason a new method for redirecting its path is looked at. 4179 Toutatis was picked because it's a slow spinning asteroid and because its approach is enough time away to realistically be able to utilize this method of asteroid deflection. A low-thrust propulsion system is assumed to be in place on the asteroid and the effects of the added acceleration in the normal, tangential and/or radial directions are tested. The slight changes in the orbit are observed and the new closest approach point is calculated and compared to the original. The analysis shows that the optimal direction of thrust is in the radial and tangential direction of the orbit and that the optimal duration of thrust is during the entire period of the orbit. It also shows a linear relationship between the thrust applied and the closest approach distance between the asteroid and earth. This is possible due to the large mass of the asteroid and the relatively small thrust applied.

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NOMENCLATURE

I_{sp} – specific impulse

a – semi-major axis

e – eccentricity

i – inclination

Ω - right ascension of ascending node (RAAN)

ω – argument of periapsis

θ – true anomaly

ϖ – longitude of perihelion

L – mean longitude

R_1 – radius vector to the asteroid

R_2 – radius vector to the planet

R_{12} – radius vector from the asteroid to the planet

Chapter 1

Introduction

A Near Earth Object (NEO) is an object that comes within a close proximity to Earth. A Potentially Hazardous Object is a NEO that comes within 0.05 Astronomical Unit (AU) or less to Earth. Asteroid 4179 Toutatis is both of this and will come within 0.02 AU in 2069. This thesis is used to accurately model the path of the asteroid in the years upcoming to 2069 and use a low-thrust propulsion system to change the orbit of the asteroid to make its approach happen at a further distance. The low-thrust system is implemented during the 3 orbits of the asteroid before the 2069 date, each orbit is approximately 4 years, so the calculation begins in 2057.

1.1 Proposed Asteroid Deflection/Redirection Methods

Many methods have been proposed to redirect/deflect and NEO off its course. They can be long duration approaches that take decades to have a significant effect, or short duration schemes that can change the path of an object in minutes.

One of the long duration options is to use the solar radiation pressure to deflect an asteroid. The idea is to use a paintball cloud to paint the surface of the object. The coating of paint increases the albedo of the asteroid and produces a thrust in the radial direction from the Sun [1]. This method is good because it does not require the use of propellant to accelerate the asteroid, but it does require a large amount of time and is only usable for small objects that have a relatively close orbit to the Sun.

Another long duration proposition is to use a gravity tractor. A gravity tractor can be put in place in front of an asteroid or be put into a Keplerian orbit around the asteroid for better results. Placing a spacecraft in a single position requires the use of ion engines to keep it in its desired location and requires a large amount of time to exert the necessary force to manipulate the orbit of an asteroid [2]. Putting a spacecraft in an orbit around the asteroid requires much less propellant and has a larger effect on changing the path of the asteroid as well as the capability of using multiple orbiters for better results [3].

Another way of deflecting an asteroid is using high energy beams as a propulsive method. Laser ablation method uses a high intensity laser light to illuminate a surface of an asteroid and generates a plume of gas that is then ejected like a thruster [4]. A similar method is neutral beam propulsion which uses a plasma thruster to apply force on an asteroid in order to deflect it [5]. Both of these methods are long duration, and require propellant to keep the spacecraft in position; however they are proven to be usable for small type asteroids.

Placing multiple landers on an asteroid around its perimeter in order to thrust it out of its path is an idea that can use both low-thrust and higher thrust propulsion systems [6]. This, with a combination of a tugboat idea which pushes the asteroid out of its current orbit and into a new one, can provide a new way to deflect an asteroid [7]. The combination of these two methods is what is explored further in this thesis.

If an asteroid is to impact Earth within a short period of time a short duration mission has been researched by NASA. A kinetic impact can be used to divert objects away from Earth with and without explosives. These explosives can include a nuclear weapon or non-nuclear impactor [8]. All of these methods have the capability to deflect an asteroid but can lack precision and have a possibility of fracturing as asteroid instead of destroying it.

1.2 Low-Thrust Propulsion

The low-thrust propulsion system is assumed to be in place on the asteroid around the entire perimeter and is able to accurately and constantly provide thrust in the necessary direction. To achieve this, it has been proposed that using several low-thrust boosters with variable angles of thrust, it is possible to constantly apply the force in the same direction [6].The thrust is being applied to the normal, tangential and/or radial direction causing the orbit to change and therefore changing the path of the asteroid.

A low-thrust propulsion system such as in this case would use ion thrusters to continually thrust during the entire duration of the mission. Ion thrusters for purposes of space missions have an I_{sp} of 2,000-10,000 seconds [6] and many different kinds of thrusters have been developed and will be developed by mission date. The advantage of using low-thrust system is that it is capable of exerting a force for a long period of time without the need for a large propellant system.

1.3 4179 Toutatis

Asteroid 4179 Toutatis will come within 2.979 X 10^6 km or 7.757 lunar distances and was selected for several reasons. The main reason for its selection is because of the date of its next close approach. Before the year 2069 it is entirely possible to create a method for diverting the asteroid. The usual low-thrust methods take decades which is possible to achieve before its approach. The other reason it was chosen was because it is a slow spinning asteroid. This means that using boosters, it is possible to keep thrusting in the same direction of the orbit. The large mass of 5.05 X 10^{13} kg of the asteroid poses the problem of the thrust being applied having a small effect on the large asteroid

Chapter 2

Orbital Theory

This section goes into the theory behind modeling the orbit and the effects of objects on the orbit of the asteroid. There is a different method for calculating the asteroids orbit and orbit of planets. The planetary positions can be easily calculated using approximations that are valid until the end of the millennia. No such approximation exists for this asteroid; therefore, Gauss' planetary equations are used to determine the change in orbital parameters with time and accurately predict the location of the asteroid in the 12-year cycle that is being looked at.

2.1 Orbital Mechanics

The location of any object in orbit can be determined using its six Classical Orbital Elements: a, e, i, Ω, ω, and θ.

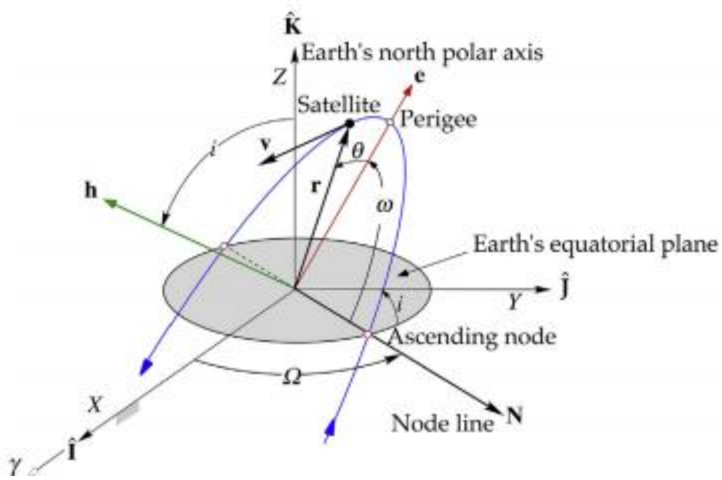

Figure 1: Classical Orbital Elements [9]

To determine the orbit of the asteroid Gauss' planetary equations were used in a MATLAB ode45 function. The starting conditions of the asteroid's location were taken from the JPL Horizons tool [10]. The ode45 function iterated the orbital elements of the asteroid for 12 years, ending on its next close approach to Earth. The gravitational effects of other bodies in the solar system were considered. The biggest gravitational effect came from Jupiter, the location of which was determined during the period of the iteration using the JPL approximate position for major planets tool.

2.2 Gauss' Planetary Equations

Gauss' planetary equations describe the effect of perturbing force (in addition to gravity) to permit the calculation of an object's location in space and some future time [11]. The six perturbations are:

$$\frac{da}{dt} = \frac{2a^2}{h}(esin(\theta)a_r + \frac{p}{r}a_t)$$

$$\frac{de}{dt} = \frac{1}{h}\{psin(\theta)a_r + [(p+r)\cos(\theta) + re]a_t\}$$

$$\frac{di}{dt} = \frac{rcos(\theta + \omega)}{h}a_n$$

$$\frac{d\Omega}{dt} = \frac{rsin(\theta + \omega)}{hsin(i)}a_n$$

$$\frac{d\omega}{dt} = \frac{1}{he}[-pcos(\theta)a_r + (p+r)\sin(\theta)a_t] - \frac{rsin(\theta + \omega)cos(i)}{hsin(i)}a_n$$

$$\frac{d\theta}{dt} = \frac{h}{r^2} + \frac{1}{eh}[pcos(\theta)a_r - (p+r)\sin(\theta)a_t]$$

where

$$p = a(1 - e^2)$$

$$r = \frac{p}{1 + ecos(\theta)}$$

$$h = \sqrt{\mu p}$$

These six equations of the orbital elements accurately determine the location of an object in orbit, letting us know the R_1 vector for the asteroid. To accurately calculate the location, it is necessary to input the acceleration due to gravity from planets and the thrust provided by the propulsion system.

2.3 Multiplanetary Trajectory

Knowing the location of the planets in the solar system with respect to the asteroid at any given time allows us to determine the gravitational force that it exerts on the asteroid. The JPL Keplerian Elements for Approximate Positions of the Major Planets paper allows us to accurately calculate the location of each planet in the solar system at any given time [12].

The planetary parameters (a, e, i, Ω, ϖ, and L) for each planet are given with equations such as

$$a = a_0 + \dot{a}T$$

where

$$T = (T_{eph} - 2451545.0)/36525$$

and T_{eph} is the starting date looked at, a_0, $\dot{a}$ are values given. The argument of periapsis and mean anomaly are calculated using

$$\omega = \varpi - \Omega$$

$$M = L - \varpi - \mathrm{b}T^2 + ccos(fT) + ssin(fT)$$

where the values of b c s & f are given for outer planets and are 0 for inner planets. From there an iterative method is used to calculate eccentric anomaly with a low tolerance of 10^{-6} using these equations

$$|\Delta E| = 10^{-6}$$

$$E_0 = M$$

$$F = (E_0 - esin(E_0) - M)$$

$$F' = (1 - ecos(E_0))$$

$$E_{new} = E_0 - \frac{F}{F'}$$

$$E_o = E_{new}$$

$$\Delta E = |E_{new} - E_0|$$

True anomaly is calculated for later use using

$$\theta = 2\tan(\sqrt{\frac{1+e}{1-e}})\tan\left(\frac{E}{2}\right)$$

Then the planets heliocentric coordinates in the r' plane is calculated using

$$x' = a(\cos(E) - e$$

$$y' = a\sqrt{1-e^2}$$

$$z' = 0$$

and then converted to ecliptic plane using

$$x_{ecl} = (\cos(\omega)\cos(\Omega) - \sin(\omega)\sin(\Omega)\cos(I))x' + (-\sin(\omega)\cos(\Omega) - \cos(\omega)\sin(\Omega)\cos(I))y'$$

$$x_{ecl} = (\cos(\omega)\sin(\Omega) + \sin(\omega)\cos(\Omega)\cos(I))x' + (-\sin(\omega)\sin(\Omega) + \cos(\omega)\cos(\Omega)\cos(I))y'$$

$$z_{ecl} = (\sin(\omega)\sin(I))x' + (\cos(\omega)\cos(\mathrm{I}))y'$$

To convert into heliocentric coordinate system several direct cosine matrixes were used:

$$C_3(\Omega) = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3(\omega) = \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_1(\mathrm{I}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\mathrm{i}) & \sin(\mathrm{i}) \\ 0 & -\sin(\mathrm{i}) & \cos(\mathrm{i}) \end{bmatrix}$$

$$C^{EP} = C_3(\omega)C_1(i)C_e(\Omega)$$

$$r = \frac{p}{1 + ecos(\theta)}$$

$$\vec{R} = \begin{bmatrix} rcos(\theta) \\ rsin(\theta) \\ 0 \end{bmatrix}$$

$$\vec{R}_2 = [C^{EP}]^T\vec{R}$$

This location lets us know the R_2 vector for the planet.

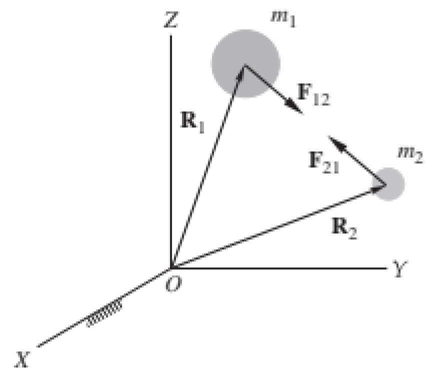

Figure 2: 2-body Problem with Relative Location and Gravitational Attraction [9]

After knowing the location of both objects, we can determine their relative location through and direction that the gravitational force will act upon using:

$$\vec{R}_{12} = \vec{R}_2 - \vec{R}_1$$

Then using the gravitational force equation,

$$\vec{F} = G\frac{m_1 m_2}{R_{12}^2} * \frac{\vec{R}_{12}}{R_{12}}$$

the force magnitude and direction exerted on the asteroid can be calculated and input to the Gauss' planetary equations.

Chapter 3

Analysis

This section discusses the results of the testing done. The effects of each planet and its gravity are discussed and the optimal planetary system it displayed. The distance change from different thrusts are calculated at the last point and the closest approach during the 12 years of thrusting.

3.1 Planetary Effects

Each planet has a different effect on the path of the asteroid. The biggest effect comes from Jupiter since it's the largest planet that gets close to the asteroid. The locations for the inner planets are calculated using the estimation equations and their effects are minimal to the asteroids' path. The effects of the outer planets are calculated to a lesser degree of accuracy, but still accurate enough to predict their location. Looking at Table 1, the gravitational effect of Jupiter is seen to be the largest on the orbit of the asteroid.

Table 1: Gravitation Effect of Planets on the Orbit of the Asteroid

Planet	Closest Approach	Closest Approach Difference	Exact Approach Expected	Exact Approach Difference	Closest to Exact Approach Difference
None	0.019150274	0	0.019917767	0	0
Mercury	0.019157846	0.0000075715	0.024192607	0.0042748403	-0.000759921468
Venus	0.019252607	0.0001023333	0.024379850	0.0044620834	-0.000665159738
Earth	0.019512188	0.0003619140	0.024881227	0.0049634600	-0.000405578992
Mars	0.019132944	-0.0000173300	0.024142436	0.0042246691	-0.000784823031
Jupiter	0.02144894	**0.0022986664**	0.028026021	0.0081082540	0.001531173370
Saturn	0.020174054	0.0010237801	0.025707982	0.0057902154	0.000256287102
Uranus	0.019852646	0.0007023718	0.025518332	0.0056005649	-0.000065121167
Neptune	0.019186491	0.0000362172	0.024255667	0.0043379000	-0.000731275793
All	**0.024795664**	**0.0056453900**	**0.031484127**	**0.0115663596**	**0.004877897053**

When combining the effects off all the planets the closest approach gets further from the expected value. This means that the estimation of the planetary locations is not accurate enough. Only the planets which have a noticeable effect on the asteroid were used which is why Earth, Mars, Jupiter, Saturn and Uranus were the planets incorporated into the calculation.

3.2 Optimal Low-Thrust Direction

Thrust can be applied in radial, normal, tangential direction or any combination of the three. Figure 4 shows the effect of 5, 10, 15 and 20 Newtons of thrust in all the possible directions of the asteroid's obit.

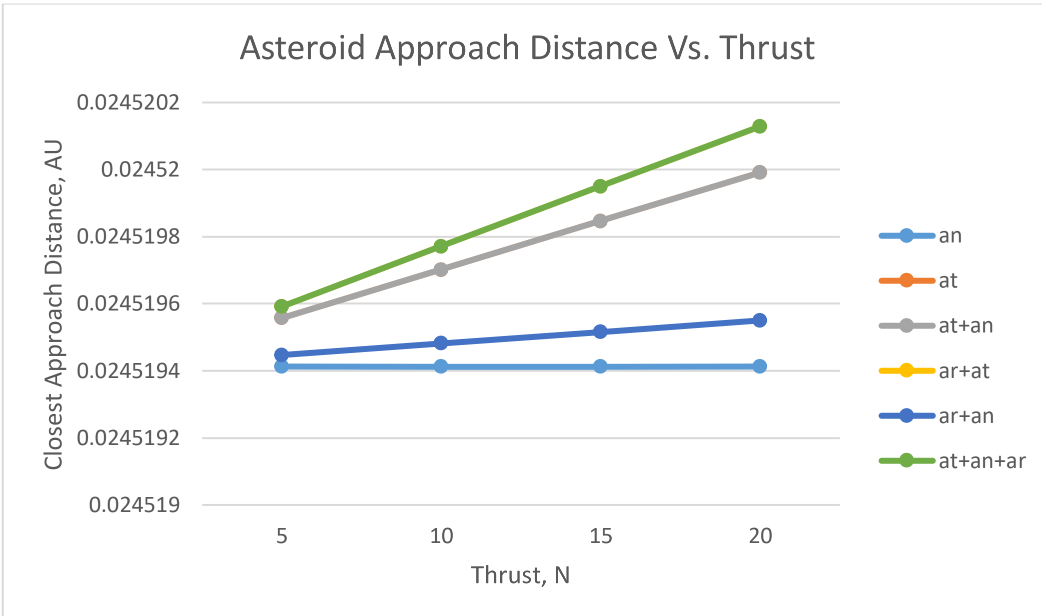

Figure 3: Asteroid Approach Distance Vs. Thrust

From Figure 3 it is clear that applying thrust in the normal direction has no effect on the closest approach distance and is considered a waste of propellant. For later calculations, thrust was applied in radial and tangential direction because those have the largest effect on the asteroid.

3.3 Optimal Low-Thrust Duration

Different values of thrust were calculated to see how it affects the orbit. Table 2 shows the change in distance with thrust ranging from 0 to 50 Newtons in the indicated directions.

Table 2: Thrust vs Change in Distance

Thrust Radial, N	Thrust Tangential, N	Thrust Normal, N	Distance, AU	Delta Distance, AU
0	0	0	0.024519413	0
5	5	0	0.024519592	1.78979E-07
10	10	0	0.024519771	3.57959E-07
15	15	0	0.02451995	5.36943E-07
20	20	0	0.024520129	7.15927E-07
25	25	0	0.024520308	8.94913E-07
30	30	0	0.024520487	1.0739E-06
35	35	0	0.024520666	1.25289E-06
40	40	0	0.024520845	1.43188E-06
45	45	0	0.024521024	1.61087E-06
50	50	0	0.024521203	1.78987E-06

The relationship between thrust and distance shows a linear expression between the two. The most likely reason for a linear change instead of exponential is that the thrust size when compared to the size of the asteroid, provides very small acceleration and therefore a very small change in orbit, and it is likely that if the asteroid were smaller or thrust was much larger, the change in orbit would be exponential.

Figure 4 shows the distance between the asteroid and earth and the change with thrusting at different times during the orbit. The idea is that thrusting at the apoapsis and/or at the periapsis would change the semi-major axis enough to create a larger difference upon the approach while requiring less propellant.

Once again because the thrust is small when considering the large mass of the asteroid, the optimal time to thrust is throughout the entire duration of the orbit.

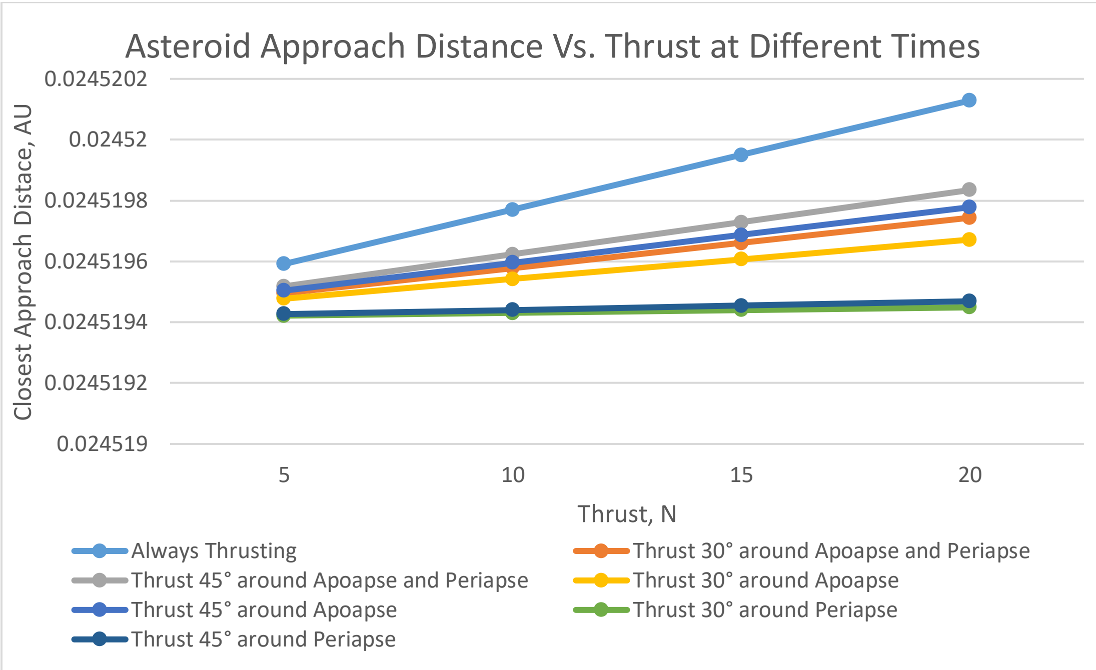

Figure 4: Thrust vs Distance

Chapter 4

Conclusion

Due to the size of the asteroid the small thrust provided by the low-trust system provides a minimal effect on changing the orbit of the asteroid. The largest effect of 1.789 X 10^{-6} AU or 268 km distance change was achieved when thrust was applied during the entire duration of the 12-year period. Thrusting in the normal direction of the orbit has a negligible effect and would just be a waste of propellant. A linear relationship was also seen between different magnitudes of thrust and the change of the asteroids closest approach distance, again this is likely due to the small overall effect that the thrust has on the asteroid due to its size. The small additional distance produced by this method could be increased by using more thrusters.

For future work a better model of the orbits should be calculated without using the approximate location of major planets. This would result in a more accurate location of the asteroid in years to come. A different, smaller asteroid could be used to see the larger effect of the low-thrust propulsion to see if this method is plausible for large distance deflection. The work in this thesis considered only fixed-direction thrust. Future work would examine the use of optimal control theory to determine if varying the thrust direction over time would result in a greater miss distance.

Appendix A

4179 Toutatis Always Thrusting

```
%% Igor Kobzarenko

%% 4179 Toutatis
clc
clear

global mu o

% Canonical Units
TU = 365.25*24*60*60;                      % Year on Earth (from seconds)
LU = 149597870700;                         % AU (from meters)
MU = 1.99e30;                              % Mass of Sun (from kg)

% Orbit Parameters
ra = 4.122170177225009;                    %LU - Aphelion
rp = 0.9398164707421598;                   %LU - Perihelion

% Asteroid 12 years before
a = 2.555261910258848;                     %LU - Semi-major axis
e = 0.6183221859580477;                    %N/A - eccentricity
M = 57.50671552509408/180*pi;              %rad - Mean anomaly
i = 0.4508988417606777/180*pi;             %rad - Inclination
omega = 124.2914119403076/180*pi;          %rad - Lomgitude of ascending node
w = 276.9308668428299/180*pi;              %rad - Argument of perihelion
theta2057 = 85.06800105112556/180*pi;
mu = 4*pi^2;                               %km^3/sec2 - Heliocentric gravitational
constant
T = 2*pi*sqrt(a^3/mu);                     %TU - Period
albedo = 0.1346;

% Asteroid Parameters
diameter = 5.4*1000/LU;                    %LU - Diameter
mass = 5.05e13;                            %Kg - Mass
density = 2.1/1000/MU*LU^3;                %MU/LU^3 - Density
RotP = 176*60*60/TU;                       %TU - Rotational period
RA2069 = [7.061794502932175E-01, 6.683978371798898E-01, -7.583538216614760E-03];
% Radius of asteroid on November 6 2069

Thrust = 0/mass/LU*TU^2;                   %LU/TU^2/MU - Thrust (Acceleration)

%% Gauss Function

% Initial Conditions
a2069 = 2.555261910258848;
e2069 = 0.6183221859580477;
theta2069 = 85.06800105112556/180*pi;
i2069 = 0.4508988417606777/180*pi;
omega2069 = 124.2914119403076/180*pi;
w2069 = 276.9308668428299/180*pi;

gAcellJA = [0 0 0];
```

```
AacelM = [0 0 0];
R = zeros(1201,3);
RE = zeros(1201,3);
k=1;
for o = 0:0.01:12

       % 3 Orbits with Thrust
       ar = -4.0672079e-13+AacelM(1)+gAcellJA(1)+Thrust;
       at = -6.3865388e-15+AacelM(2)+gAcellJA(2)+Thrust;
       an = AacelM(3)+gAcellJA(3);

       x0 = [a2069,e2069,i2069,omega2069,w2069,theta2069, ar, at, an];
       tspan = o:0.01:o+0.01;
       options = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
       [~,x] = ode45('Gauss_Function', tspan, x0, options);

       % new ones for 12 years before
       a2069 = x(end,1);
       e2069 = x(end,2);
       i2069 = x(end,3);
       omega2069 = x(end,4);
       w2069 = x(end,5);
       theta2069 = x(end,6);
       p2069 = a2069*(1-e2069^2);

       %% Ateroid on November 6, 2069
       C3 = [cos(omega2069)  sin(omega2069) 0;
            -sin(omega2069)  cos(omega2069) 0;
             0               0              1];

       C3w = [cos(w2069)  sin(w2069) 0;
             -sin(w2069)  cos(w2069) 0;
             0            0          1];

       C1 = [1    0           0;
             0    cos(i2069)  sin(i2069);
             0   -sin(i2069)  cos(i2069)];

       CEP = C3w*C1*C3;

       r2=p2069/(1+e2069*cos(theta2069));

       RAP2069= [r2*cos(theta2069)
                 r2*sin(theta2069)
                 0];
       CEPinv= transpose(CEP);

       RAH2069New= CEPinv*RAP2069;                 % Radius vector of Asteroid in
       Heliocentic coordinates
       R(k,:) = RAH2069New;
       k = k + 1;

       %% Jupiters position
       [gAcellJA] = AaccelJ(RAH2069New);

       %% Mercury position
       [AacelM] = AaccelM(RAH2069New);

       %% Earth position
```

```
       PE = Earth_Position();
       RE(k-1,:) = PE;
end
fprintf("Change in orbital elements while Thrust is aplied for 3 rotations\n");
fprintf([repmat('%4.14g\t', 1, size(x, 2)) '\n'], x')
fprintf("\n");

%% Earth on November 6, 2069
REactual = [.7213172063462266, .6789612208268408,-.0001015123422083197];

%% Distance between the Earth and Asteroid
Distance0 = sqrt((REactual(1)-RA2069(1))^2+(REactual(2)-RA2069(2))^2+(REactual(3)-
RA2069(3))^2);                    % Distance without any change
Distancelast = sqrt((REactual(1)-RAH2069New(1))^2+(REactual(2)-
RAH2069New(2))^2+(REactual(3)-RAH2069New(3))^2);     % Distance at te last moment
Distance = sqrt((RE(:,1)-R(:,1)).^2+(RE(:,2)-R(:,2)).^2+(RE(:,3)-R(:,3)).^2);
% Closest distance
MinD = min(Distance);
fprintf("Distance between Earth and Asteroid would be %4.14f AU\n", Distance0);
fprintf("Distance between Earth and Asteroid at last moment would be %4.14f AU\n",
Distancelast);
fprintf("Minimum distance between Earth and Asteroid is now %4.14f AU\n", MinD);
```

AaccelJ

```
function AaccelJ = AaccelJ(RAH2069New)

global o

% Givens
T = (2472672.5-2451545.00+365.25*o)/36525;
MassS = 1.9885e30;
MassJ = 1.89813e27;
mu = 4*pi^2*MassJ/MassS;

% Original 6
aJ = 5.20248019-0.00002864*T;
eJ = 0.04853590+0.00018026*T;
iJ = (1.29861416-0.00322699*T)/180*pi;
LJ = (34.33479152+3034.90371757*T)/180*pi;
wbarJ = (14.27495244+0.18199196*T)/180*pi;
OmegaJ = (100.29282654+0.13024619*T)/180*pi;

% Calculated
wJ = wbarJ-OmegaJ;
MJ = LJ-wbarJ;

tol = 10^-6;
Eold=MJ;
Enew=0;
errors=1;
while errors > tol
   F = (Eold-eJ*sin(Eold)-MJ);
   Fprime = (1-eJ*cos(Eold));
   Enew= Eold-F/Fprime;
   errors = abs(Enew-Eold);
```

```
   Eold=Enew;
end
EJ=Enew;

xH = aJ*(cos(EJ)-eJ);
yH = aJ*sqrt(1-eJ^2)*sin(EJ);
zH = 0;

% ECL Coords
xecl = (cos(wJ)*cos(OmegaJ)-sin(wJ)*sin(OmegaJ)*cos(iJ))*xH+(-sin(wJ)*cos(OmegaJ)-
cos(wJ)*sin(OmegaJ)*cos(iJ))*yH;
yecl = (cos(wJ)*sin(OmegaJ)+sin(wJ)*cos(OmegaJ)*cos(iJ))*xH+(-
sin(wJ)*sin(OmegaJ)+cos(wJ)*cos(OmegaJ)*cos(iJ))*yH;
zecl = (sin(wJ)*sin(iJ))*xH+(cos(wJ)*sin(iJ))*yH;
RHJ = [xecl;
       yecl;
       zecl];

thetaJ = 2*atan(sqrt((1+eJ)/(1-eJ))*tan(EJ/2))*180/pi+360;
pJ = aJ*(1-eJ^2);

DistJA = sqrt((RHJ(1)-RAH2069New(1))^2+(RHJ(2)-RAH2069New(2))^2+(RHJ(3)-
RAH2069New(3))^2);
RJA = RAH2069New-RHJ;
gAcellJ = -mu/DistJA^2*(RJA/DistJA);     % Acceleration from Jupiter on Asteroid in
Heliocentric coordinates

% Heliocentric to Asteroids
C3JA = [cos(OmegaJ)  sin(OmegaJ)  0;
       -sin(OmegaJ)  cos(OmegaJ)  0;
       0             0            1];

C3wJA = [cos(wJ+thetaJ)  sin(wJ+thetaJ)  0;
        -sin(wJ+thetaJ)  cos(wJ+thetaJ)  0;
         0               0               1];

C1JA = [1    0        0;
        0    cos(iJ)  sin(iJ);
        0   -sin(iJ)  cos(iJ)];

CHPJA = C3wJA*C1JA*C3JA;
CHPinvJA= transpose(CHPJA);

AaccelJ= CHPinvJA*gAcellJ;              % Acceleration from Jupiter on Asteroid in
asteroids coordinates
end
```

Gauss Function

```
function xdot=Gauss_Function(~,x)
global mu
xdot = zeros(9,1);

p = x(1)*(1-x(2)^2);
h = sqrt(mu*p);
r = p/(1+x(2)*cos(x(6)));

xdot(1) = 2*x(1)^2/h*(x(2)*sin(x(6))*x(7)+p/r*x(8));
xdot(2) = 1/h*(p*sin(x(6))*x(7)+((p+r)*cos(x(6))+r*x(2))*x(8));
xdot(3) = (r*cos(x(6)+x(5))*x(9))/h;
xdot(4) = (r*sin(x(6)+x(5))*x(9))/(h*sin(x(3)));
xdot(5) = 1/(h*x(2))*(-p*cos(x(6))*x(7)+(p+r)*sin(x(6))*x(8))-
(r*sin(x(6)+x(5))*cos(x(3))*x(9))/(h*sin(x(3)));
xdot(6) = h/r^2+1/(x(2)*h)*(p*cos(x(6))*x(7)-(p+r)*sin(x(6))*x(8));
end
```

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ACADEMIC VITA

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EDUCATION:

Bachelor of Aerospace Engineering (Graduation: May 2019)
The Pennsylvania State University, University Park, PA
Schreyer Honors College

SKILLS:

MATLAB	Teamwork	Russian
C++	Data Analysis	Ukrainian
STK	Time Management	Spanish

RESEARCH:

The Pennsylvania State University, State College, PA August 2018 – May 2019
Capstone, Sample return mission to the 16 Psyche asteroid

- Leaded several subsystems for a sample return mission to the 16 Psyche asteroid
- Conducted trade studies for launch vehicles and worked closely with other subsystems
- Helped in creating STK model for the mission to the asteroid and back

The Pennsylvania State University, State College, PA January 2018 – May 2018
Aerodynamics

- Did CFD computation of an airfoil at different angles of attack and Mach numbers
- Analyzed and visualized the two-dimensional freestream flow over an airfoil
- Discussed the change in density, lift, drag, pressure, and moment coefficients as well as Mach number over the airfoil
- Analyzed the possibility of boundary layer separation at high Mach numbers with high angles of attack

The Pennsylvania State University, Abington, PA August 2017 – May 2018
ACURA Research

- Learned and understand the basics of radio astronomy and high frequency spinning pulsars
- Was able to collect and analyze data from 20m telescope in Green Bank Observatory
- Collaborated closely with a faculty research mentor, and other participants in the research

EXPERIENCE:

McDonalds, Richboro, PA August 2015 – August 2017
Floor Supervisor (Sept. 2016 – Aug. 2017)

- Lead a team of 10-15 people during high-volume, fast-paced hours of operations
- Resolved customer complaints quickly and appropriately
- Accurately counted cash deposits and ensured all the money was in order
- Supported other areas as needed (i.e. answering telephones, completing transactions)

Line Cook (Aug. 2015 – Sept. 2016)

- Verified that prepared food met all quality and quantity standards
- Met customers’ expectations with fast and well-prepared food
- Knew all jobs in and had the ability to multitask on multiple parts of food assembly process

LEADERSHIP:

LionTech Rocket Labs, *member*

- Worked in a group on building the structure of that year's rocket to launch at the NASA Student Launch Projects

Impact, *member*

- Volunteered and helped the Penn State Thon and those impacted by childhood cancer