THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

DEPARTMENT OF CHEMICAL ENGINEERING

ANALYSIS OF ASYMMETRIC ROCK-PAPER-SCISSORS SOLUTIONS USING CHEMICAL GAME THEORY

ADAM SMOLUK
FALL 2019

A thesis
submitted in partial fulfillment
of the requirements
for a baccalaureate degree
in Chemical Engineering
with honors in Chemical Engineering

Reviewed and approved* by the following:

Darrell Velegol
Distinguished Professor of Chemical Engineering
Honors Adviser and Thesis Supervisor

Themis Matsoukas
Professor of Chemical Engineering
Faculty Reader

* Signatures are on file in the Schreyer Honors College.

ABSTRACT

The objective of this thesis is to compare data from experimental asymmetric rock-paper-scissors (aRPS) games to Nash equilibria (NE) and chemical game theory (CGT) aRPS solutions using perception functions that convert real punishments into pain values used in CGT. aRPS games are a modified form of the traditional rock-paper-scissors game where winning with rock, for example, is more advantageous than winning with scissors or paper. The Nash equilibria and chemical game theory solutions are fully analyzed for both the RPS and aRPS games, and then compared to experimental data for aRPS games where winning with rock has higher payoff than winning with paper or scissors. The NE solution for the same aRPS game with rock as the most valuable play found that paper is played the most often, while the CGT solution found that rock is played the most often. The experimental data resulted in rock as the most probable strategy, which more closely reflects the CGT solution.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

Thank you to my research and thesis adviser, Dr. Darrell Velegol, for supporting me throughout my research and for motivating me to achieve my goals at Penn State. I would also like to thank the Chemical Game Theory lab members, Jacob Scioscia, Jennifer Burke, and Michael Creighton for assisting with this research. A final thanks to my family and close friends that have motivated me every step of the way.

Chapter 1

Introduction

The objective of this thesis is to compare data from experimental asymmetric rock-paper-scissors (aRPS) games to Nash equilibria and chemical game theory (CGT) aRPS solutions using perception functions that convert real punishments into pain values used in CGT.

Rock-Paper-Scissors (RPS) is a classic children's game played into adulthood. The rules of the game are simple and easy to understand. Both players chant "Rock, Paper, Scissors, Shoot" in rhythm together and throw their move on "Shoot." The options are rock, paper, and scissors. The winner is determined based on the moves thrown: rock crushes scissors, scissors cuts paper, and paper covers rock as depicted in **Figure 1.1.**

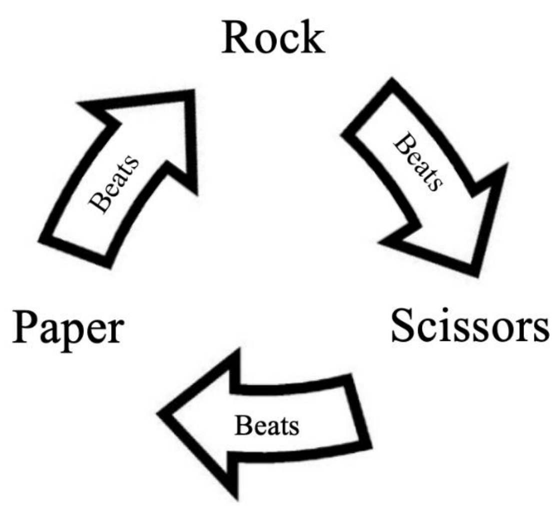

Figure 1.1. Gameplay schematic for an RPS game. The winning play is determined in a cyclical pattern where Rock beats Scissors beats Paper beats Rock and so on.

Asymmetric Rock-Paper-Scissors (aRPS) is a twist on the traditional RPS game where the results of the game are not simply a win or a loss. Winning with rock could be more advantageous than winning with scissors. In this way, the game becomes asymmetric and alters the traditional RPS strategy. The strategy for regular RPS games is well known as many people have played the game many times, but how can the strategy for an aRPS game be found? This thesis will determine these strategies using two different models: traditional game theory (TGT) Nash Equilibria (NE) as well as chemical game theory (CGT). The validity of these models are then compared to experimental game data taken between members in the CGT lab.

1.1 Rock-Paper-Scissors Player Behavior

Humans playing rock-paper-scissors exhibit certain behaviors from game to game which have been studied thoroughly. These studies have collected extensive data on several hundred games worth of human subjects playing games.

Several different behaviors of RPS players have been analyzed. Overall, the chance that a player chooses rock, paper, or scissors is 0.36 ± 0.08, 0.33 ± 0.07 and 0.32 ± 0.06 respectively.[1] Generally, players adopt a win-stay/lose-switch strategy. This strategy indicates that when a player wins they are most likely to throw the same move again, and if they lose, they are more likely to switch from R to P, P to S, or S to R.[1] Players tend to tie about 36.3% of the time, slightly more often than anything else. Ties occur slightly more often because of cases when one player is not sure what move to throw and reflexively mirrors their opponent during the throw.[2]

---

[1] Wang, Z., Xu, B., & Zhou, H.-J. (2014). Social cycling and conditional responses in the Rock-Paper-Scissors game. *Scientific Reports*, *4*(1). The values reported are of the form (mean ± standard deviation). The data was taken from 354 players in 59 separate populations.

[2] Cook, R., Bird, G., Lünser, G., Huck, S., & Heyes, C. (2011). Automatic imitation in a strategic context: players of rock–paper–scissors imitate opponents gestures. *Proceedings of the Royal Society B: Biological Sciences*, *279*(1729), 780–786.

Players tend to update their strategy as they play several games. In the case of playing against a computer which slowly increased its frequency of rock, the human player was able to notice and counter the computer with increasing win rate over time.[3] Even monkeys playing RPS with a reward for winning have been shown to exhibit learning behavior, and can adjust their play to win more often against a computer.[4]

1.2 Rock-Paper-Scissors in Nature

Rock-Paper-Scissors is a cyclical game in which no strategy is the dominant solution because each strategy loses to another. Trends similar to the rock-beats-scissors, scissors-beats-paper, paper-beats-rock scenario tend to appear in nature.

In a nature-like scenario, each strategy in the RPS game is represented by a species of animal or strain of bacteria that has a population competing with other species/strains. Several examples are shown in **Table 1.1**. Side-blotched lizards exhibit cycles of dominant traits among a population between males with different genotypes that represent different behaviors.[5] Bacteria with genetic mutations has also been shown to exhibit an RPS scenario. One study found that three different strains of E. Coli coexisted in a natural setting. Each strain either produced, was sensitive

[3] Stöttinger, E., Filipowicz, A., Danckert, J., & Anderson, B. (2014). The Effects of Prior Learned Strategies on Updating an Opponent’s Strategy in the Rock, Paper, Scissors Game. *Cognitive Science*, *38*(7), 1482–1492.
[4] Lee, D., Mcgreevy, B. P., & Barraclough, D. J. (2005). Learning and decision making in monkeys during a rock–paper–scissors game. *Cognitive Brain Research*, *25*(2), 416–430.
[5] Sinervo, B., Heulin, B., Surget-Groba, Y., Clobert, J., Miles, D. B., Corl, A., … Davis, A. (2007). Models of Density-Dependent Genic Selection and a New Rock-Paper-Scissors Social System. *The American Naturalist*, *170*(5), 663–680.

to, or was resistant to colicin (a bacteriocin).[6] Bacteria has also found to have cyclical dominance in a setting where each of three strains of bacteria both produces a toxin and is resistant to another strain's toxin.[7]

Table 1.1. Examples of Rock-Paper-Scissors relationships that can exist in nature

Population	Rock	Paper	Scissors
Side-blotched lizards[5]	Harem-holding males	Mate-stealing males	Mate-guarding males
Bacteria[6]	Bacteriocin producing strain	Bacteriocin sensitive strain	Bacteriocin resistant strain
Bacteria[7]	Strain with toxin 1, antitoxin 1, and antitoxin 2	Strain with toxin 2, antitoxin 2, and antitoxin 3	Strain with toxin 3, antitoxin 3, and antitoxin 1

[6] Kirkup, B. C., & Riley, M. A. (2004). Antibiotic-mediated antagonism leads to a bacterial game of rock–paper–scissors in vivo. *Nature*, *428*(6981), 412–414.
[7] Liao, M. J., Din, M. O., Tsimring, L., & Hasty, J. (2019). Rock-paper-scissors: Engineered population dynamics increase genetic stability. *Science*, *365*(6457), 1045–1049.

Chapter 2

Traditional and Chemical Game Theory Methods

2.1 Traditional Game Theory

Nash equilibria (NE) can sometimes show that there is a dominant solution. It states that rational players would only ever pick this solution, but the reality is that human rationality differs from the rationality assumed in Nash equilibria. In addition, humans may not take the time to perfectly analyze something to make a rational decision. Real data shows that the traditional game theory solution is not perfectly accurate.

Traditional game theory (TGT) as referred to in this thesis will describe game theory, a field first formulated by Jon von Neumann and Oskar Morgenstern in 1944.[8] Game theory describes a situation where two or more players have a contested decision in a non-cooperative game. In other words, if two players named A and B are playing a game, player A's outcome is dependent on both player A's choice and player B's choice. For rock-paper-scissors, this means that player A will win if player A's choice beats player B's choice.

The outcome of a game can be represented by the resulting pain of each of the players. For rock-paper-scissors, a positive pain is associated with losing, negative pain (payoff) is associated with winning, and zero pain is associated with a tie. The pain values for both players for each possible outcome are summarized in **Table 2.1**.

[8] Von Neumann, J.; Morgenstern, O. Theory of Games and Economic Behavior. Princeton University Press, 2007.

Table 2.1. Pain matrix for the basic zero sum RPS game. In each cell, the leftmost value represents the pain of player A and the rightmost value represents the pain of player B. Values of +1 and -1 are arbitrarily used to represent losses and wins respectively.

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0, 0	+1, -1	-1, +1
	Paper	-1, +1	0, 0	+1, -1
	Scissors	+1, -1	-1, +1	0, 0

Player A wants to win every game, and player B wants to do the same. However, given the pain matrix in **Table 2.1**, both players cannot win at the same time. As a result of this inability to satisfy both players, each player will try to optimize their strategy in order to win more games. A Nash equilibria is a way to mathematically determine a stable strategy given the pain matrix.[9] In other words, a Nash equilibria stable strategy for both players occurs when player A cannot improve their outcome by changing their strategy assuming that player B also does not change their strategy and vice versa. For detailed explanations on the determination of NE solutions for the RPS games, see appendix A.

At least one Nash equilibrium must exist for a contested game like RPS; Up to infinite Nash equilibria could exist for a single game. The basic RPS game as shown in **Table 2.1** yields a mixed strategy NE of $f_{AR} = f_{AP} = f_{AS} = f_{BR} = f_{BP} = f_{BS} = 1/3$. In other words, the only Nash Equilibrium solution for the basic RPS game is for either player to play rock, paper, or scissors equally and randomly. In reality, humans cannot perfectly randomize their play selection to match

[9] Nash, J. F. (1950). Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences*, *36*(1), 48–49.

the NE solution. Some deviation from the NE solution exists, which can be exploited by strong RPS players to increase a player's wins.

While the TGT NE solution is an important indicator of what the optimum strategy for both players should be, it can fall short in modeling actual contested games game. Nash equilibria solutions assume that both players are perfectly rational and have had time to analyze the game before playing. Humans are not able to make rational decisions that follow Nash's concept of rationality or might not have had time to fully analyze the game before playing. Since the basic RPS game is so simple, the TGT NE solution is close to the literature values for human players. When the RPS game becomes more complicated in the aRPS game, TGT no longer accurately predicts the experimental outcome. The TGT solution for aRPS is covered in depth later in this thesis

2.2 Chemical Game Theory

Chemical Game Theory (CGT) is an alternative method of game theory that uses fictional molecules (dubbed knowlecules) in thermodynamic equilibrium to predict outcomes of strategic contested-decision games such as RPS. An introduction to CGT with extensive examples using the Prisoner's Dilemma game has been published.[10]

CGT uses knowlecules to represent possible decisions and chemical reaction equilibrium from thermodynamics concepts are used to model the outcomes of these decisions. For the RPS game, player A can choose either rock, paper, or scissors, which are modeled as knowlecules

[10] Velegol, D.; Suhey, P.; Connolly, J.; Morrissey, N.; Cook, L. Chemical Game Theory. *Industrial & Engineering Chemistry Research* **2018**, *57* (41), 13593–13607.

labeled a1, a2, and a3 respectively. Player B's choices are identical and are modeled as b1, b2, and b3. In addition to the knowlecules for potential decisions, each player is represented as catalyst A or B. In this way, all of player A's experiences, knowledge, and personality is culminated into the chemical reaction as catalyst A. An example reaction for player A is shown in **Figure 2.1**. Any of the a1, a2, or a3 knowlecules can react with the b1, b2, or b3 knowlecules. The extent of all reactions in CGT is determined using reaction equilibrium concepts based on Gibbs Free Energy. The pain values shown in **Table 2.1** are directly substituted in for $\Delta G/RT$. A positive pain represents an unfavorable outcome much like how positive $\Delta G/RT$ indicates an unfavorable reaction. The trend between pain and $\Delta G/RT$ is the basis for CGT because a higher pain indicates a much more unfavorable outcome and therefore much more unfavorable reaction. In this way, a thermodynamic equilibrium between possible outcomes is determined, resulting in some proportion of final outcomes that yield the CGT solution.

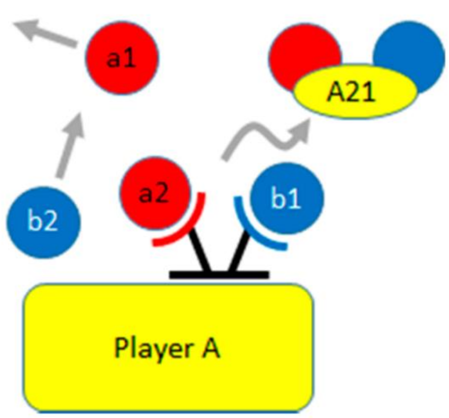

Figure 2.1. Example reaction diagram for player A. In CGT reactions, all decision knowlecules (for RPS they are a1, a2, a3, b1, b2, b3) are present and mixed in solution. Player A is represented by a catalyst that allows a2 to react with b1 to form A21. The reaction notation for is $\mathbf{a2} + \mathbf{b1} + \mathbf{A} \rightarrow \mathbf{A21}$. A21 represents a contribution of player A's thoughts on how often player A will pick

paper (a2) and player B will pick rock (b1). This figure was taken from Figure 4 in "Chemical Game Theory."[10]

The full process flow diagram reactor schematic for the CGT solution is depicted in **Figure 2.2**. An additional parameter present in CGT that is not present in TGT is the ability to incorporate a player's pre-bias into the calculations. Some initial concentration of ai and bj is fed to the A and B reactors, allowing players' pre-biases to be reflected in the game results. Higher initial concentration of a1 compared to a2 or a3 indicates that player A is more likely to select rock than scissors or paper because more a1 will react.

Reactors A and B in **Figure 2.2.** represent the thought process for each player. Nine combinations of a1, a2, or a3 and b1, b2, or b3 can react to form nine different Aij in reactor A or nine different Bij in reactor B. The amount of Aij and Bij produced in each reactor are calculated using reaction equilibrium based on $\Delta G/RT$.

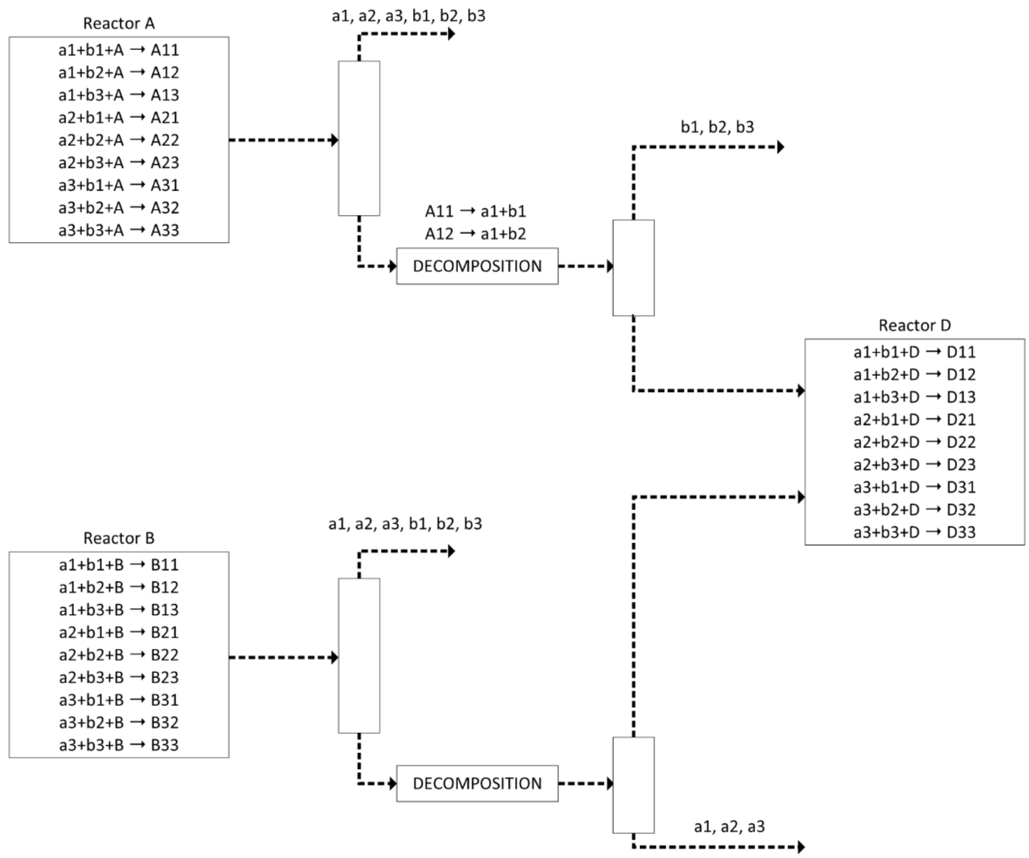

Figure 2.2. Reactor schematic for the CGT solution of the RPS game. Nine separate reactions occur in the A and B reactor. The resulting Aij's and Bij's are decomposed into ai and bj before being fed into the final decision reactor. In the D reactor, ai and bj reacts to form nine different Dij's which are the final percentages of each outcome for the RPS game. This figure was taken from Jacob Scioscia's honors thesis.[11]

The Aij products from Reactor A are isolated and decomposed back into ai and bj. The resulting ai is isolated and fed into the D reactor to represent player A's final choices. The same decomposition is done for player B to produce bj. All ai's and bj's then react within the D reactor to form the final Dij's which are normalized to produce the percent chance of each of the nine outcomes occurring. The reactions in the D reactor are given a $\Delta G/RT = -1$ to encourage a final outcome to be made.

[11] Jacob Scioscia. Chemical Game Theory: Asymmetric Rock Paper Scissors Decision Strategy. SHC thesis. **2019**

In previous work with CGT, the resulting Aij and Bij products from reactors A and B were fed directly into the D reactor to form the final Dij products using the reaction form $\mathrm{Aij + Bij + D \rightarrow Dij}$.[12] This method produces a fundamental problem. For an RPS game, **Eq. 2.1** and **2.2** describe a way to calculate the fractional outcome of A choosing rock and B choosing rock

(2.1) $\mathrm{fa1 = D11 + D12 + D13}$

(2.2) $\mathrm{fb1 = D11 + D21 + D31}$

where fa1 and fb1 are the fractions of time that players A and B respectively choose rock, D11, D12, and D13 are the outcomes where A chooses rock, and D11, D21, and D31 are the outcomes where B chooses rock. Considering **Eq. 2.1** and **2.2**, it should hold that **Eq. 2.3** is true as well to check the validity of **Eq. 2.1** and **2.2**.

(2.3) $\mathrm{D11 = fa1 * fb1}$

where D11 is the percentage chance that both players choose rock. Unfortunately, if decomposition of Aij and Bij is neglected, **Eq. 2.1**, **2.2**, and **2.3** do not agree. For this reason, decomposition was used to calculate the final Dij outcomes.

In calculating the equilibrium for all reactions in reactors A, B, and D, the solution is a system of nonlinear equations. To make the calculations easier, a linear approximation with a small amount of error was used instead.

If both players are unbiased, meaning the initial concentrations fed to both the A and B reactor are Ca1 = Ca2 = Ca3 = 1/3 and Cb1 = Cb2 = Cb3 = 1/3, the CGT solution is identical to the TGT solution. All nine outcomes happen 1/9 of the time and both players select rock, paper,

[12] Previous work with CGT refers to theses written by Natalie Morrissey **2018**, Laura Cook **2018**, Frank Gentile **2019**, and Sneha Srinivasan **2018**.

and scissors 1/3 of the time. This outcome is not true of real RPS gameplay as discussed in section 1.1 of this thesis, indicating that players have an initial pre-bias.

Varying the initial pre-bias for the RPS game yields the results in **Figure 2.3**. A higher initial pre-bias of rock results in selecting rock more often. When increasing or decreasing pre-biases in **Figure 2.3**, both players are assumed to know the pre-biases for both themselves and their opponent. Even though the pre-biases for scissors and paper are equal, the actual frequency with which scissors or paper are played is not equal. Scissors is played more often than paper when rock has a pre-bias below 1/3 and scissors is played less often than paper when rock has a pre-bias above 1/3. This trend occurs because scissors is more likely to win or tie when rock has a low pre-bias while paper is more likely to win or tie when rock has a higher pre-bias. Eventually the pre-bias for rock becomes too great for both players to the point where paper and scissors are barely played at all. The scenario where rock has a pre-bias close to 0 or close to 1 are unlikely for an RPS game between human players.

Sample calculations from excel for a CGT solution can be found in appendix B.

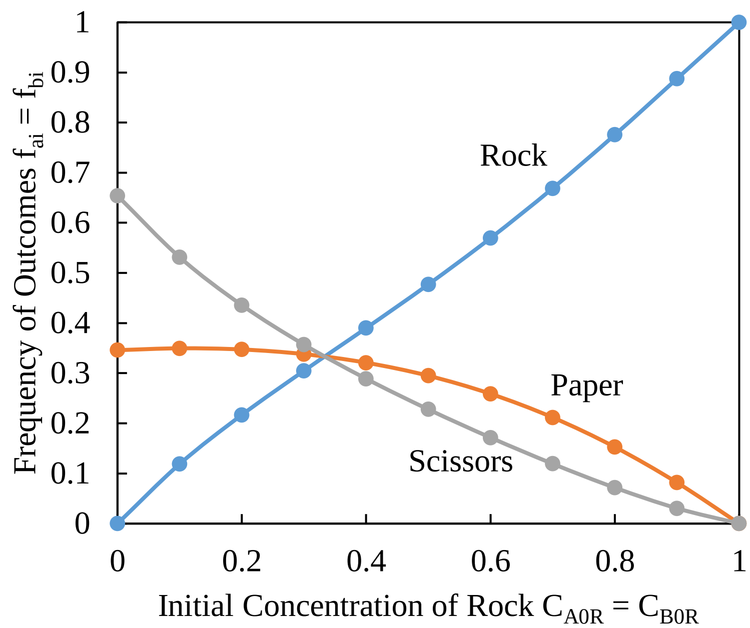

Figure 2.3. The CGT solution for basic RPS game while varying the pre-bias. The initial concentrations of all species (ai's and bj's) in the A reactor are equal to their counterpart in the B reactor. The initial concentrations were all varied, keeping the initial concentrations of paper and scissors equal (i.e. $C_{A0R} = 0.2$, $C_{A0P} = C_{A0S} = 0.4$), so that the sum of the initial concentrations always equals 1.

Chapter 3

Asymmetric Rock-Paper-Scissors

Asymmetric Rock-Paper-Scissors (aRPS) is an alternative form of the RPS game which modifies the pain values in **Table 2.1** such that the pain for some outcomes is higher than others. The simplest form of an aRPS game is summarized in **Table 3.1** which shows how the pains differ from the normal form of the RPS game. Since the pains have changed, both the TGT and CGT solutions will be different.

Table 3.1. Pain matrix example for an aRPS game. The only difference between this aRPS pain matrix and the basic RPS pain matrix is pains for the outcomes with rock and scissors. A player that wins with rock has a pain of -2 while winning with paper or scissors only gives a pain of -1. In addition, losing with scissors is a pain of +2 while losing with rock or paper only gives a pain of +1.

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0, 0	+1, -1	-2, +2
	Paper	-1, +1	0, 0	+1, -1
	Scissors	+2, -2	-1, +1	0, 0

3.1 aRPS in Traditional Game Theory

The aRPS solution for TGT is solved in the same way as the basic RPS game. No pure strategy TGT NE solutions exist upon analysis of **Table 3.1**, just like the basic RPS game. The mixed strategy NE can be found using the pain values in **Table 3.1** along with the equations in

Appendix A, resulting in $f_{AP} = 0.5$, $f_{AR} = f_{AS} = 0.25$, $f_{BP} = 0.5$, and $f_{BR} = f_{BS} = 0.25$. Paper is played twice as often as rock or scissors according to the Nash Equilibrium solution.

Consider a similar game of aRPS where the pains are represented in **Table 3.2.**

Table 3.2. General pain matrix for aRPS with X-1-1 pain distribution. Winning with rock has a pain of -X while winning with paper or scissors only gives a pain of -1. In addition, losing with scissors is a pain of +X while losing with rock or paper only gives a pain of +1.

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0, 0	+1, -1	-X, +X
	Paper	-1, +1	0, 0	+1, -1
	Scissors	+X, -X	-1, +1	0, 0

Replacing X in **Table 3.2** with a pain value other than 1 results in the aRPS game. When X = 3, the resulting aRPS game has a NE solution of $f_{AP} = 0.6$, $f_{AR} = f_{AS} = 0.2$, $f_{BP} = 0.6$, and $f_{BR} = f_{BS} = 0.2$. Here, paper is played the most just like the previous aRPS game, but paper is now played three times as much as rock or scissors. **Figure 3.1** shows an increasing frequency of paper with an increase in X. As the pain distribution becomes more disparate with larger X, the Nash Equilibrium maintains stability through an increase in the frequency of paper. Since winning with rock has a large negative pain, paper is played much more often in the NE to quell the advantage of playing rock.

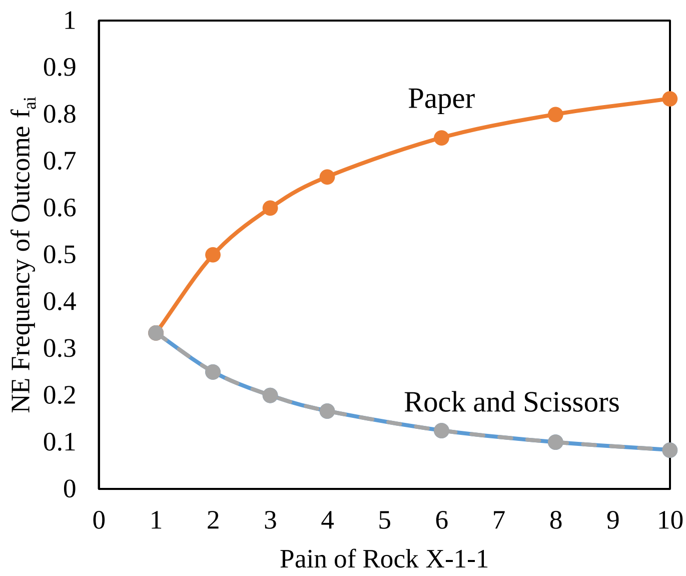

Figure 3.1. NE solution of the aRPS game X-1-1 where –X is the pain for winning with rock and +X is the pain for winning with scissors. As X increases, paper is played X times as often as rock or scissors.

3.2 aRPS in Chemical Game Theory

Chemical Game Theory results are calculated the same way as regular RPS games. The only difference between the RPS solution and the aRPS solution is the pains used for ΔG. The difference in ΔG results in a different chemical equilibrium and therefore different predicted outcomes for the game.

The CGT solution for the aRPS game can be compared to the basic RPS game depending on the pre-bias.

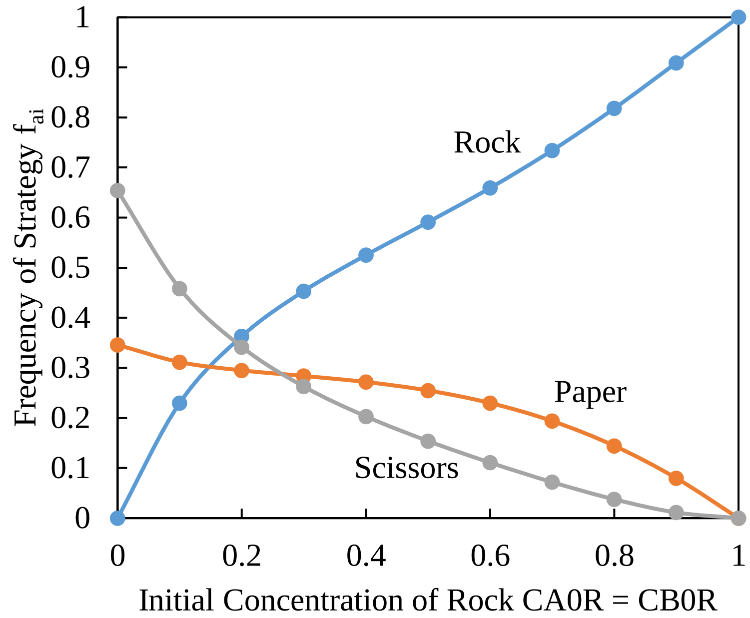

Figure 3.2. The CGT solution for aRPS game where X = 6 while varying the pre-bias. The initial concentrations of all species (ai's and bj's) in the A reactor are equal to their counterpart in the B reactor. The initial concentrations were all varied, keeping the initial concentrations of paper and scissors equal (i.e. C_{A0R} = 0.2, C_{A0P} = C_{A0S} = 0.4), so that the sum of the initial concentrations always equals 1.

Comparing **Figure 3.2** to **Figure 2.3** reveals that the difference in pain between the aRPS game and RPS game does not affect the general trend for resulting frequency of outcomes. Instead, the curves become much sharper. The frequency of rock increases more quickly at low initial concentrations of rock, and scissors decreases more quickly at low initial concentrations of rock. The frequency with which each strategy is played is dependent upon both the pains and the initial concentrations of each strategy.

The aRPS CGT solution for unbiased players as the pains change can be seen in **Figure 3.3**.

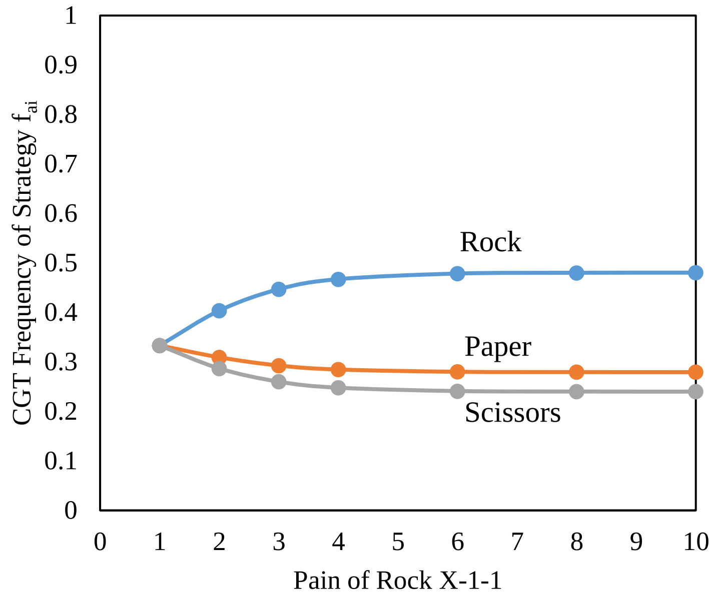

Figure 3.3. CGT solution of the aRPS game X-1-1 where –X is the pain for winning with rock and +X is the pain for winning with scissors. As X increases, rock is played the most often and scissors is played least often. The fraction in which each strategy is played reaches a plateau at higher values of X. Pre-bias for this figure was set to $C_{A0R} = C_{A0P} = C_{A0S} = 1/3$.

As the pain distribution becomes more disparate with larger X, the CGT solution finds that rock is played the most frequently, and is played more often at higher X. In addition, paper is played more often than scissors, but less than rock. In the context of CGT, the pain of losing with paper is less than that for scissors, so paper will be played slightly more often than paper to handle that. Also, since the CGT model solves the equilibrium of all 9 reactions simultaneously, the model can account for the fact that rock has the largest negative pain for winning and counteract that by playing paper more often.

Comparing **Figure 3.1.** and **Figure 3.3.** reveals a fundamental difference between the TGT and CGT solutions for the aRPS game is the most probable strategy. In TGT, the NE solutions finds a the most frequent strategy of paper while CGT results in rock. Since the two models both provide different solutions, which model more accurately predicts the actual aRPS strategy when the game is played between two human players?

Chapter 4

Data and Comparison to Model

The CGT research team has collected data playing games of RPS and aRPS amongst ourselves. Given the small sample size used when collecting this data, the games were used as a basis for general trends and habits of play.

4.1 Collection of data

Traditional RPS games were played in a rapid-fire sequence of 100 games between two players. Each set of 100 games was completed between two different people. Overall, out of the 600 games that were played, rock, paper, and scissors were played 0.308, 0.342, and 0.350 of the time respectively. These experimental games are within the error of the literature values: 0.36 ± 0.08, 0.33 ± 0.07 and 0.32 ± 0.06.

The aRPS setup was achieved using pushups as a consequence for losing. Using pushups, the number of pushups for each strategy can be set to achieve asymmetry. In addition, pushups are a tangible consequence that can be used to generate pain values. When playing games with pushups, matches were played in 20 rounds, and most of these matches were played between the same two lab members. Every time a round is lost, pushups are added to the player's total, and every time a round is won, pushups are subtracted from the player's total. In this way, players are competing with an actual goal to make their opponent do pushups while minimizing the number of pushups done themselves. Three different pushup schemes in were used in these games to create the asymmetry: 4-4-4, 6-4-4, 8-4-4, (R-P-S) such that the number corresponding to R, P, or S, correlates to the number of pushups the opponent has added to their total and the number of pushups subtracted from the player's total. Since winning with rock adds more pushups to the opponent's total than paper or scissors in two of the schemes, the game follows the aRPS set up.

Upon playing five matches of the 4-4-4 pushup scheme between the same two players, player A was biased towards scissors and against paper as summarized in **Table 4.1.**, while player B had a relatively light bias towards rock.

Table 4.1. Fractions of each strategy for player A and B at all three pushup schemes. At higher levels of asymmetry, rock is played more often while scissors is played less often.

	Player A			Player B		
	Rock	Paper	Scissors	Rock	Paper	Scissors
4-4-4	0.340	0.170	0.490	0.420	0.310	0.270
6-4-4	0.388	0.350	0.263	0.413	0.350	0.238
8-4-4	0.475	0.388	0.138	0.513	0.313	0.175

Four matches were played for both the 6-4-4 and 8-4-4 pushup schemes. **Table 4.1.** shows that for the aRPS schemes, both player A and B played rock more often and scissors less often when compared to the 4-4-4 RPS scheme. The polarization of strategy is greater at the higher level of asymmetry 8-4-4 than 6-4-4.

4.2 Conversion of pushups to pain using a perception function

In the context of the model so far, arbitrary pain values have been selected when modeling the results of the aRPS game with TGT or CGT. How is it possible to quantify the difference between a pain of +1 and +2? Based on the Weber-Fechner law, humans tend to perceive stimuli on a logarithmic scale.[13] For example, the difference between spending $5 and $10 on an item is a lot larger than the difference between spending $105 and $110. In this way, a logarithmic function can be constructed to represent the average stimulus for pushups, which allows the conversion of number of pushups to pain and ΔG. The Weber-Fechner law takes the form

$$(4.1) \quad p = c \ln \frac{pu}{pu_0}$$

where p is the pain, pu is the number of push-ups to be completed, and c and pu_0 are parameters that must be fit using data.

In order to fit the parameters in **Eq. 4.1**, 5 members of the CGT lab were asked how many pushups they would consider to be insignificant (pain = 0), easy (pain = 1), medium (pain = 2), hard (pain = 3), or impossible (pain = 4) to complete. After running a linear regression on the data, the perception function takes the form

$$(4.2) \quad p = 1.15 \ln \frac{pu}{1.97}$$

where the parameters in **Eq. 4.1** c and pu_0 are equal to 1.15 ± 0.09 and 1.97 ± 0.45 respectively.[14]

The perception function with parameters in **Eq. 4.2** was used to convert pushups into pain values to be used in the CGT and TGT solutions. **Table 4.2.** shows the resulting calculations from the conversion of pushups to pain.

---

[13] Fechner, G. T. *Elements of Pyschopypsics*; 1860.
[14] The parameters c and pu_0 are calculated with standard errors of 0.09 and 0.45 respectively.

Table 4.2. Pain values calculated based on number of pushups using **Eq. 4.2**. 8 pushups has a pain value slightly smaller than twice the pain value from 4 pushups. This difference in value comes as a result from the perception function being logarithmic.

Pushups	Pain
4	0.82
6	1.28
8	1.61

4.3 TGT and CGT models using the calculated pains from perception function

The pain values calculated from the perception function are used in the pain matrix to calculate the TGT and CGT solutions. **Table 4.3.** shows an example pain matrix for the 6-4-4 game.

Table 4.3. Pain matrix for the 6-4-4 aRPS pushup scheme using pain values calculated from the perception function. This pain matrix is still in the same general format as the aRPS game shown in **Table 3.2.**

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0, 0	+0.82, -0.82	-1.28, +1.28
	Paper	-0.82, +0.82	0, 0	+0.82, -0.82
	Scissors	+1.28, -1.28	-0.82, +0.82	0, 0

The pain matrices for all three pushup schemes are used to calculate the TGT NE and CGT solutions in **Table 4.4**.

Table 4.4. Fractions of each strategy for the TGT solution, CGT solution, and experimental data for both players at each of the three pushup schemes. Both players have the same fractional strategy for the TGT and CGT solutions. The players were assumed to be unbiased for the calculation of the CGT solution. The experimental data combines the strategy of both player A and B into one overall fraction.

	TGT NE Solution			CGT Solution			Experimental Data		
	R	P	S	R	P	S	R	P	S
4-4-4	0.333	0.333	0.333	0.333	0.333	0.333	0.380	0.240	0.380
6-4-4	0.281	0.438	0.281	0.354	0.329	0.316	0.400	0.350	0.250
8-4-4	0.252	0.495	0.252	0.367	0.327	0.307	0.494	0.350	0.156

The calculations in **Table 4.4** have the same general trends as the previous NE and CGT calculations made in Chapters 2 and 3. For the basic RPS game, the pain values do not have to be -1, 0, and +1 to give a NE and CGT solution of 1/3 for all three strategies. In this case, for 4 pushups the pains are all -0.82, 0, and +0.82 and still give equal 1/3 fraction for all three strategies.

The modified pain values to represent pushups are important for the asymmetric game. The Nash Equilibrium solution follows the same general trend as before where paper is the most frequent strategy. However, the actual fraction for each strategy depends on the ratio of pains. The ratio is about 1.5:1 for the 6-4-4 scheme and about 2:1 for the 8-4-4 scheme. The solutions are not exactly 1.5:1 or 2:1 because the perception function is logarithmic.

The CGT solution follows the same general trend as before just like the NE solution. Rock is the most frequent strategy just like the previous calculations with rock becoming more probable as the game becomes more asymmetric.

4.4 Comparison of data to model

The calculated solutions and experimental results in **Table 4.4.** are compared.

For the basic RPS game, the TGT and CGT solutions are identical and somewhat close to the experimental results in the 4-4-4 pushup scheme. The notable differences are for player A because player A has a very low fraction of paper played and very high fraction of scissors played compared to the TGT and CGT solutions. The difference between the experimental data and the NE solution occurs because the players are not perfectly rational and are unable to reach the stable NE solution as a result. The CGT solution is different from the experimental data because of the pre-bias. The calculations in **Table 4.4.** were made assuming the players are unbiased, but that is not the case given the experimental data.

The CGT solution models the trends of the real data more closely than the TGT solution. The most probable outcome resulting from the experimental games of aRPS was rock, and the least probable outcome was scissors. Likewise, the CGT solution using unbiased players results in rock as the most probable outcome and scissors as the least probable outcome, which mirrors the data from the experimental games. The TGT solution using NE does not reflect the results from the experimental games since the TGT solution finds that paper is the most probable outcome with rock and scissors tied for least probable outcome. The fractions of each outcome for the CGT solution do not perfectly match the experimental data due in part to the small number of games played between lab members and the assumption that the players are unbiased. However, the trends between the fractions of each outcome for the CGT and the aRPS experimental data match.

The initial pre-bias can be backwards calculated from the results. The experimental results from **Table 4.1.** for each pushup scheme were used in the CGT solution as the final fraction of each strategy to get the values in **Table 4.5.**

Table 4.5. Initial pre-biases backwards calculated using the CGT solution in reverse and the experimental results in **Table 4.1.** These values are the initial concentrations that enter the A and B reactor in **Figure 2.2.** which means these values are not the fraction with which each strategy is played.

	Player A			Player B		
	Rock	Paper	Scissors	Rock	Paper	Scissors
4-4-4	0.281	0.118	0.602	0.411	0.319	0.271
6-4-4	0.400	0.332	0.267	0.434	0.336	0.230
8-4-4	0.531	0.331	0.138	0.647	0.211	0.142

Table 4.5. shows the initial pre-biases that would be input to the CGT solution for the strategy that the CGT solution calculates to equal the experimental results. In the cases where a player had a very high or very low fraction of one of the strategies, the player's pre-bias is even more extreme than the result. For example, the fraction which player A chose paper in the RPS 4-4-4 pushup scheme was 0.170 which resulted in an initial pre-bias of 0.118. The result of calculating the pre-bias for each player shows that the basic trend calculated by the unbiased CGT solution becomes exacerbated due to player's biases. The basic trend calculated in the CGT solution between unbiased players becomes exacerbated in the experimental data because the players are highly biased towards rock and against scissors. The players think about the game in such a way that the general trend of their pre-bias (rock more often than paper more often than scissors) matches the unbiased CGT solution.

Chapter 5

Conclusion and Future Work

This thesis provides two different models, TGT Nash Equilibria and CGT, to find the solution to an asymmetric RPS game. Upon comparing the solutions for each of these two models to experimental data taken from aRPS games between CGT lab members in **Table 4.4.**, the CGT solution was found to more closely model the trends of the experimental data compared to the TGT Nash Equilibria solution. This conclusion is drawn assuming both players are unbiased when calculating the CGT solution. The players were found to have a bias, which follows the same trend as the CGT solution where rock is the most probable outcome and scissors is the least probable outcome.

5.1 Future Work

In the future, the aRPS game and CGT solution could be analyzed in greater detail. More aRPS games should be played to get more accurate experimental data and better comparisons to the model. In addition, data should be collected from aRPS games with several matchups between different players instead of allowing the same two people to compete. Perhaps the initial pre-biases for these players could be determined before playing the game rather than doing a reverse calculation in the CGT model. If the initial pre-biases can be calculated, the CGT model would become extremely useful in predicting future outcomes as opposed to modeling prior data.

The CGT lab has hypothesized that all the information in the pain matrices cannot be processed all at once. In other words, there is some limit to the information that players can process when formulating their strategy and most probable outcomes. When looking at the pain matrix, do players only really focus on the fact that winning with rock is the best outcome and would that influence the results of the CGT model?

In addition, players tend to learn and improve their strategy over time. The CGT solution is modeled using equilibrium reactions, so the solution implies that many games have been played at a given set of pre-biases. If the equilibrium, or initial pre-bias, of the players is changing constantly, then the model may become inaccurate. A proposed solution to this would be to use a PID or PI controller to constantly modify the initial concentrations/pre-biases entering the A and B reactors in **Figure 2.2.**

Appendix A

Solving for Nash Equilibria

At least one Nash equilibrium must exist for a contested game like RPS; Up to infinite Nash equilibria could exist for a single game. The Nash equilibrium occurs in a situation when player A is not able to reduce their pain by changing their play assuming that player B does not change their play. In addition to that criteria, player B must also be unable to reduce their pain by changing their play again with the assumption that player A does not change their play. The Nash equilibria for some games can be found through inspection of the pain matrix. In **Table 1.2**, If we assume player B chooses rock and player A chooses rock, the pain of each player is 0, and player A will be able to reduce their pain to -1 by switching to paper. This analysis means the rock-rock outcome is not a Nash equilibrium. Next if player A chooses paper and player B chooses rock, player A has a pain of -1 and player B has a pain of +1. In this case, player A will not be able to reduce their pain any lower because -1 is already the lowest. Now looking at player B, if we assume player A stays at paper, player B can switch to scissors to reduce their pain. Since player B can switch to reduce their pain, the paper-rock outcome is not a Nash equilibrium either. The result of no Nash equilibrium will be found for all nine squares in **Table 1.2** upon repeating this exercise, which means that no pure strategy Nash equilibrium exists. This means that the optimal strategy for either player A or B is not to choose the same play 100% of the time.

Instead, RPS has a mixed strategy Nash Equilibrium, which is when some normalized fraction of each option satisfies the Nash Equilibrium requirements. The solution to finding a mixed strategy Nash equilibrium for a 2 player, 3 decision game is found in **Equations A.1, A.2, A.3, A.4, A.5, A.6**.

(A.1) $f_{BR} =$

$$\frac{-(-P_{A22} + P_{A23} + P_{A32} - P_{A33})(P_{A13} - P_{A33}) - (-P_{A12} + P_{A13} + P_{A32} - P_{A33})}{(-P_{A12} + P_{A23} + P_{A31} - P_{A33})(-P_{A12} + P_{A13} + P_{A32} - P_{A33}) - (-P_{A11} + P_{A13} + P_{A31} - P_{A33})(-P_{A22} + P_{A23} + P_{A32} - P_{A33})}$$

(A.2) $f_{BP} =$

$$\frac{P_{A13}(P_{A21} - P_{A31}) + P_{A23}P_{A31} - P_{A21}P_{A33} + P_{A11}(-P_{A23} + P_{A33})}{P_{A22}(-P_{A31} + P_{A33}) + P_{A23}(P_{A31} - P_{A32}) + P_{A21}(P_{A32} - P_{A33}) + P_{A13}(P_{A21} - P_{A22} - P_{A31} + P_{A32}) + P_{A12}(-P_{A21} + P_{A23} + P_{A31} - P_{A33}) + P_{A11}(P_{A22} - P_{A23} - P_{A32} + P_{A33})}$$

(A.3) $f_{BS} = 1 - f_{BR} - f_{BP}$

(A.4) $f_{AR} =$

$$\frac{-(-P_{B22} + P_{B23} + P_{B32} - P_{B33})(P_{B31} - P_{A33}) - (-P_{B21} + P_{B31} + P_{B23} - P_{B33})}{(-P_{B21} + P_{B23} + P_{B31} - P_{B33})(-P_{B12} + P_{B13} + P_{B32} - P_{B33}) - (-P_{B11} + P_{B13} + P_{B31} - P_{B33})(-P_{B22} + P_{B23} + P_{B32} - P_{B33})}$$

(A.5) $f_{AP} =$

$$\frac{P_{B13}(P_{B32} - P_{B31}) + P_{B12}(P_{B31} - P_{B33}) + P_{B11}(-P_{B32} + P_{B33})}{P_{B22}(-P_{B31} + P_{B33}) + P_{B23}(P_{B31} - P_{B32}) + P_{B21}(P_{B32} - P_{B33}) + P_{B13}(P_{B21} - P_{B22} - P_{B31} + P_{B32}) + P_{B12}(-P_{B21} + P_{B23} + P_{B31} - P_{B33}) + P_{B11}(P_{B22} - P_{B23} - P_{B32} + P_{B33})}$$

(A.6) $f_{AS} = 1 - f_{AR} - f_{AP}$

where f_{BR}, f_{BP}, and f_{BS}, are the fractions of rock, paper, and scissors respectively, played by B in the NE solution, and P_{Aij} is the pain for player A in the ith row and jth column of **Table 1.2**. In addition, f_{AR}, f_{AP}, and f_{AS}, are the fractions of rock, paper, and scissors respectively, played by A in the NE solution, and P_{Bij} is the pain for player B. The full derivation for these six equations can be found below. Something worth noting about **Equations A.1** through **A.6** is the NE strategy for player B depends on the pains of player A and vice versa. Using all 6 of these equations with the pain values given by the basic RPS game as shown in **Table 1.2** yields a mixed strategy NE of $f_{AR} = f_{AP} = f_{AS} = f_{BR} = f_{BP} = f_{BS} = 1/3$.

$$f_{AS} = 1 - f_{AR} - f_{AP}$$

$$f_{BS} = 1 - f_{BR} - f_{BP}$$

$$E_A = f_{AR}f_{BR}p_{A11} + f_{AR}f_{BP}p_{A12} + f_{AR}(1 - f_{BR} - f_{BP})p_{A13} + f_{AP}f_{BR}p_{A21} + f_{AP}f_{BP}p_{A22} + f_{AP}(1 - f_{BR} - f_{BP})p_{A23} + (1 - f_{AR} - f_{AP})f_{BR}p_{A31} + (1 - f_{AR} - f_{AP})f_{BP}p_{A32} + (1 - f_{AR} - f_{AP})(1 - f_{BR} - f_{BP})p_{A33}$$

$$\frac{\partial E_A}{\partial f_{AR}} = 0 = f_{BR}p_{A11} + f_{BP}p_{A12} + p_{A13} - f_{BR}p_{A13} - f_{BP}p_{A13} - f_{BR}p_{A31} - f_{BP}p_{A32} - p_{A33} + f_{BR}p_{A33} + f_{BP}p_{A33}$$

$$\frac{\partial E_A}{\partial f_{AP}} = 0 = f_{BR}p_{A21} + f_{BP}p_{A22} + p_{A23} - f_{BR}p_{A23} - f_{BP}p_{A23} - f_{BR}p_{A31} - f_{BP}p_{A32} - p_{A33} + f_{BR}p_{A33} + f_{BP}p_{A33}$$

$$E_B = f_{AR}f_{BR}p_{B11} + f_{AR}f_{BP}p_{B12} + f_{AR}(1 - f_{BR} - f_{BP})p_{B13} + f_{AP}f_{BR}p_{B21} + f_{AP}f_{BP}p_{B22} + f_{AP}(1 - f_{BR} - f_{BP})p_{B23} + (1 - f_{AR} - f_{AP})f_{BR}p_{B31} + (1 - f_{AR} - f_{AP})f_{BP}p_{B32} + (1 - f_{AR} - f_{AP})(1 - f_{BR} - f_{BP})p_{B33}$$

$$\frac{\partial E_B}{\partial f_{BR}} = 0 = f_{AR}p_{A11} + f_{AP}p_{B21} + p_{B31} - f_{AR}p_{B31} - f_{AP}p_{B31} - f_{AR}p_{B13} - f_{AP}p_{B23} - p_{B33} + f_{AR}p_{B33} + f_{AP}p_{B33}$$

$$\frac{\partial E_B}{\partial f_{BP}} = 0 = f_{AR}p_{B12} + f_{AP}p_{B22} + p_{B32} - f_{AR}p_{B32} - f_{AP}p_{B32} - f_{AR}p_{B13} - f_{AP}p_{B23} - p_{B33} + f_{AR}p_{B33} + f_{AP}p_{B33}$$

Solve a system of equations for f_{AR}, f_{AP}, f_{BR}, and f_{BP} using the above four partial differential equations set to 0 to yield equations A.1 through A.6

Appendix B

Sample CGT calculations from excel

The following sample calculations show the results of the CGT solution using the three-reactor diagram in **Figure 2.2.** In these sample calculations, the basic RPS game given by **Table 2.1.** will be used, the pre-bias for player A will be $C_{0AR} = C_{0AP} = C_{0AS} = 1/3$, and the pre-bias for player B will be $C_{0BR} = 0.45$, $C_{0AP} = 0.25$, and $C_{0AS} = 0.3$.

The following tables will show the species (name) of each knowlecule in the reactor, the initial concentration, the change in concentration, the equilibrium concentration, and the mole fraction at equilibrium.

Reactor A				
SP	Initial	Change	End	Y frxn
a1	0.5000	-0.1152	0.3848	0.1786
a2	0.5000	-0.1276	0.3724	0.1728
a3	0.5000	-0.1023	0.3977	0.1845
b1	0.4500	-0.1529	0.2971	0.1379
b2	0.2500	-0.0878	0.1622	0.0753
b3	0.3000	-0.1044	0.1956	0.0908
A11	1E-10	0.0387	0.0387	0.0180
A12	1E-10	0.0077	0.0077	0.0036
A21	1E-10	0.0991	0.0991	0.0460
A22	1E-10	0.0197	0.0197	0.0091
A13	1E-10	0.0688	0.0688	0.0319
A31	1E-10	0.0151	0.0151	0.0070
A32	1E-10	0.0604	0.0604	0.0280
A33	1E-10	0.0268	0.0268	0.0125
A23	1E-10	0.0088	0.0088	0.0041
inert	1E-10	0.0000	0.0000	0.0000
Sum	2.5000	-0.3450	2.1550	1.0000

Reactor B					
SP	Initial	Change	End	Y frxn	y check
a1	0.5000	-0.1083	0.3917	0.1818	0
a2	0.5000	-0.1076	0.3924	0.1821	0
a3	0.5000	-0.1291	0.3709	0.1721	0
b1	0.4500	-0.1530	0.2970	0.1378	0
b2	0.2500	-0.0877	0.1623	0.0753	0
b3	0.3000	-0.1044	0.1956	0.0908	0
B11	1E-10	0.0399	0.0399	0.0185	0
B12	1E-10	0.0588	0.0588	0.0273	0
B21	1E-10	0.0147	0.0147	0.0068	0
B22	1E-10	0.0217	0.0217	0.0101	0
B13	1E-10	0.0096	0.0096	0.0045	0
B31	1E-10	0.0983	0.0983	0.0456	0
B32	1E-10	0.0072	0.0072	0.0033	0
B33	1E-10	0.0236	0.0236	0.0110	0
B23	1E-10	0.0712	0.0712	0.0330	0
inert	1E-10	0.0000	0.0000	0.0000	0
Sum	2.5000	-0.3451	2.1549	1.0000	0

Reactor D					
SP	Initial	Change	End	Y frxn	y check
a1	0.1152	-0.0421	0.0731	0.1296	0
a2	0.1276	-0.0466	0.0809	0.1435	0
a3	0.1023	-0.0374	0.0649	0.1151	0
b1	0.1530	-0.0559	0.0970	0.1721	0
b2	0.0877	-0.0321	0.0556	0.0986	0
b3	0.1044	-0.0382	0.0663	0.1175	0
D11	1E-10	0.0187	0.0187	0.0331	0
D12	1E-10	0.0107	0.0107	0.0190	0
D21	1E-10	0.0207	0.0207	0.0366	0
D22	1E-10	0.0118	0.0118	0.0210	0
D13	1E-10	0.0127	0.0127	0.0226	0
D31	1E-10	0.0166	0.0166	0.0294	0
D32	1E-10	0.0095	0.0095	0.0169	0
D33	1E-10	0.0113	0.0113	0.0201	0
D23	1E-10	0.0141	0.0141	0.0250	0
inert	1E-10	0.0000	0.0000	0.0000	0

Sum	0.6901	-0.1261	0.5640	1.0000	0

After normalizing the mole fractions of the Dij's coming out of the D reactor, the following table shows the actual fraction of each outcome that results from this RPS game.

Outcome	Fraction
D11	0.1479
D12	0.0848
D21	0.1010
D22	0.1639
D13	0.0939
D31	0.1119
D32	0.1314
D33	0.0754
D23	0.0897

Using **Eq. 2.1** and **2.2**, the fraction in which each player plays their respective strategies are in the following table:

R	fa1	0.3337813	R	fb1	0.443244
P	fa2	0.3696953	P	fb2	0.254113
S	fa3	0.2965234	S	fb3	0.302643

BIBLIOGRAPHY

1. Wang, Z., Xu, B., & Zhou, H.-J. (2014). Social cycling and conditional responses in the Rock-Paper-Scissors game. *Scientific Reports*, *4*(1).
2. Cook, R., Bird, G., Lünser, G., Huck, S., & Heyes, C. (2011). Automatic imitation in a strategic context: players of rock–paper–scissors imitate opponents gestures. *Proceedings of the Royal Society B: Biological Sciences*, *279*(1729), 780–786.
3. Stöttinger, E., Filipowicz, A., Danckert, J., & Anderson, B. (2014). The Effects of Prior Learned Strategies on Updating an Opponent's Strategy in the Rock, Paper, Scissors Game. *Cognitive Science*, *38*(7), 1482–1492.
4. Lee, D., Mcgreevy, B. P., & Barraclough, D. J. (2005). Learning and decision making in monkeys during a rock–paper–scissors game. *Cognitive Brain Research*, *25*(2), 416–430.
5. Sinervo, B., Heulin, B., Surget-Groba, Y., Clobert, J., Miles, D. B., Corl, A., … Davis, A. (2007). Models of Density-Dependent Genic Selection and a New Rock-Paper-Scissors Social System. *The American Naturalist*, *170*(5), 663–680.
6. Kirkup, B. C., & Riley, M. A. (2004). Antibiotic-mediated antagonism leads to a bacterial game of rock–paper–scissors in vivo. *Nature*, *428*(6981), 412–414.
7. Liao, M. J., Din, M. O., Tsimring, L., & Hasty, J. (2019). Rock-paper-scissors: Engineered population dynamics increase genetic stability. *Science*, *365*(6457), 1045–1049.
8. Von Neumann, J.; Morgenstern, O. Theory of Games and Economic Behavior. Princeton University Press, 1944.
9. Nash, J. F. (1950). Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences*, *36*(1), 48–49.
10. Velegol, D.; Suhey, P.; Connolly, J.; Morrissey, N.; Cook, L. (2018). Chemical Game Theory. *Industrial & Engineering Chemistry Research*, *57* (41), 13593–13607.
11. Jacob Scioscia. (2019). Chemical Game Theory: Asymmetric Rock Paper Scissors Decision Strategy. SHC thesis.
12. Natalie Morrissey. (2018). Chemical Game Theory: Entropy in Strategic Decision-Making. SHC thesis.
13. Laura Cook. (2018). Chemical Game Theory and Cooperation in Experimental Prisoner's Dilemma Games. SHC thesis.
14. Frank Gentile. (2019). n-Player Tragedy of The Commons and Chemical Game Theory. SHC thesis
15. Sneha Srinivisan. (2019). Representing The Agenda-Setting Process with Chemical Game Theory. SHC thesis.
16. Fechner, G. T. *Elements of Pyschopysics*; 1860.

ACADEMIC VITA

Adam Smoluk
adam.smoluk@gmail.com

EDUCATION

The Pennsylvania State University | The Schreyer Honors College **University Park, PA**
Bachelor of Science in Chemical Engineering *December 2019*

Relevant Coursework: Intro to C++, Water and Wastewater Treatment, Intro to Materials Science

RELEVANT EXPERIENCES

Penn State Chemical Engineering Department **University Park, PA**
Instructional Aide for Thermodynamics II and Material Balances *Aug 2018-present*

- Reduce highly technical concepts into simple terms for students to easily understand
- Achieve a high level of understanding of course material to transfer that knowledge to students
- Demonstrate patience and eagerness to teach so students find motivation to develop understanding of course material

Capstone Project – Final Grade A **University Park, PA**
Chemical Plant Design *Jan 2019-May 2019*

- Collaborated as a team of 6 to write a 248-page technical document detailing all decisions and specifications chosen during design, results from Aspen+ simulations, and a thorough economic evaluation including capital spending and depreciation
- Simulated 2 design solutions as a team in Aspen+ with full economic evaluations in Excel to find the most profitable solution
- Calculated cost estimations for 74 unique pieces of equipment including pumps, heat exchangers, distillation columns, etc

University Park Allocation Committee **University Park, PA**
Allocating Team Sub-Chair, Student Contact Team Sub-Chair *Aug 2016-present*

- Schedule and simultaneously coordinate 3 separate meeting rooms to review a total of 6-12 student orgs' budgets each week
- Facilitate discussion alongside thorough review of budget details to ensure allocation according to handbook
- Propose and implement innovative changes and improvements to UPAC and the UPAC rulebook to better allocate $4 million

Penn State Assmann Laboratory **University Park, PA**
Lab Assistant *Sep 2016-May 2018*

- Prepared buffers, solutions, glassware, and rice plants to assist in DNA research on drought resistance of rice plants
- Practiced consistent lab safety procedures for handling potentially dangerous chemicals and/or hazardous situations

Physical Chemistry Lab Special Research Project **University Park, PA**
Group Member *Mar 2018-May 2018*

- Collected measurements to determine viscosity of propylene glycol at glass-transition temperatures
- Completed regression analysis on collected data for final paper, in-class presentation, and poster presentation

LEADERSHIP EXPERIENCES

Wegmans Food Markets **Warrington, PA**
Produce Customer Service, Cashier, Knowledge Based Seller *Apr 2014-present*

- Communicate with team leaders and manager on department needs in order to reduce shrink and maximize sales
- Trained and mentored 3 new employees on rotation, shrink, educating customers, and other important daily tasks

Schreyer Honors Orientation (SHO) Time **University Park, PA**
Mentor Aug 2017

- Aided 13 first-year students in the transition to college by answering questions about school-life during the 3-day orientation
- Encouraged new students to learn about Penn State and have fun at games and activities to prepare them for the semester

ADDITIONAL EXPERIENCES

- THON OPPerations Committee; 2-year member; University Park, PA; Oct 2017-Feb 2018
- Springfield Special Interest THON Org; 3-year member, 1-year Logistics Captain; University Park, PA; Aug 2016-Present
- Chemical Game Theory (CGT) Research; Undergrad Researcher; University Park, PA; Feb 2018-present

SKILLS

- **Software:** Excel, Java, C++, Mathematica, MatLab, Aspen+, SolidWorks, Chemdraw
- **Machines and Techniques:** IR, ^{1}H NMR, ^{13}C NMR, GC-MS, UV-VIS, Fluorescence Spectroscopy, Laser Photolysis, Distillation, Column Chromatography