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CHEMICAL GAME THEORY: ASYMMETRIC ROCK PAPER SCISSORS DECISION STRATEGY

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ABSTRACT

Traditional game theory has long been used as the standard by which to solve a problem with a quantifiable pain to model and to predict human behavior. The purpose of this thesis is to determine if Chemical Game Theory or CGT is more effective at solving and obtaining the actual human strategy in an asymmetric rock, paper, scissors (aRPS) game than a traditional game theory or TGT model. Asymmetric RPS is a type of rock, paper, scissors game where the pains of winning or losing are not evenly distributed. Additionally, this thesis will compare experimental data with the predictions of both CGT and TGT. TGT indicates that as the pain/payoff associated with rock increases, paper should be played the most while CGT indicates that rock should be played the most. Players seem to follow the general trend that CGT predicts of playing Rock with a higher probability the more the pain/payoff of rock increases, followed by Paper, and finally Scissors.

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Chapter 1

Introduction

Strategic games or games with strategic decisions are often represented by and modeled through traditional game theory (TGT). The objective of this thesis is to show that Chemical Game Theory (CGT) better explains human decision making in an asymmetric rock, paper, scissors (aRPS) game than a TGT model. Additionally, this thesis will compare the trends of the experimental data with the predictions of both CGT and TGT to begin to determine if CGT more accurately models human behavior in an aRPS game than TGT. ARPS is a type of rock, paper, scissors game where the pains of winning or losing are not evenly distributed. For example, winning with rock would be more valuable or have a higher payoff than winning with paper or scissors. Will it be better to play rock more often because it has a higher payoff associated with it or would playing paper to counter this prove to be more effective? How will players choose their strategy and which method will be model this behavior more accurately, CGT or TGT? CGT differs from TGT wildly. While TGT uses merely the pains associated with each outcome to predict what a player ought to do, CGT uses both the pains and principles from Gibbsian thermodynamics as well as chemical engineering topics to predict the actual strategy and decision making of players by modeling their decisions as reactions and their choices as molecules.[1]

Much research has been done in the field of game theory. In their 1944 book, *Theory of Games and Economic Behavior,*[2] John von Neumann and Oskar Morgenstern laid the mathematical groundwork for all future and modern adaptations of game theory. TGT attempts to

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[1] For more information on the topic, read Velegol, Darrell; Suhey, Paul; Connolly, John; Morrissey, Natalie; Cook, Laura. "Chemical Game Theory," *Industrial & Engineering Chemistry Research*, **57**, 13593-13607 (2018).
[2] Von Neumann, J.; Morgenstern, O. Theory of Games and Economic Behavior. Princeton: Princeton University Press, 2007.

determine a player's strategy for a given scenario and therefore functions as a model for how rational people behave.

A very famous game that was studied intensely in the early stages of game theory was the Prisoner's Dilemma game. The game unfolds as two players, A and B, are suspected of robbery. They are questioned by the police separately, not permitted to communicate with one another, and are offered two choices: stay quiet and therefore cooperate with the other player, or tell and betray the other player. In this game, quiet will be represented as Q and tell will be represented as T. Each decision has an associated payoff or pain, in this case it is number of years in prison. However, the payoffs and therefore the outcome are affected by the decisions of both players, not just one player. The possible outcomes to this game are as follows:

1. Both players are quiet (Q/Q): Both players cooperate with one another and only receive 1 year of prison
2. Both players tell (T/T): Both players tell on one another and receive 2 years of prison
3. Player A is quiet and Player B tells: Player A would receive 3 years of prison and Player B would receive no prison time.
4. Player A tells and Player B is quiet: Player A would receive no prison time and Player B would receive 3 years of prison

This game can be represented by Table 1 below:

Table 1: Prisoner's Dilemna Game. The Player A's pains are on the left and Player B's Pains are on the right.

	Player B Quiet	Player B Tell
Player A Quiet	1, 1	3, 0
Player A Tell	0, 3	2, 2

The solution can be determined visually. Consider Player A while holding Player B constant: if Player B is quiet and Player A is quiet, A will get 1 year of prison. If Player B is quiet and A tells, A will have no prison time. So, player A will always tell. Now, if Player A's strategy is held constant to tell, Player B will receive 3 years of prison time if Player B remains quiet, but only 2 years of prison time if Player B tells. Therefore, Player B will always tell, which is a dominant solution and a Nash Equilbrium. The Nash equilibrium was developed by John Nash in his 1951 article, "Non-Cooperative Games." It states that in any non-cooperative game, there exists a stable or stationary equilibrium where no participants can increase their payoff by a change of strategy considering that the others in the game hold their strategies constant. Nash equilibria can be determined either visually if pure Nash Equilibria exist or analytically if there is a mixed-strategy Nash Equilibrium.[3]

For other games, the Nash equilibrium is not as apparent and must be determined mathematically as there is no one dominant strategy but rather a mixed strategy. One such game that TGT and Nash equilibria can be applied to is Rock, Paper, Scissors.

[3] Nash, J. (n.d.). Non-Cooperative Games. Cournot Oligopoly, pp. 82–94.

Chapter 2

The Rock, Paper, Scissors Game

Rock, Paper, Scissors or RPS is a very famous game known worldwide originating in eastern cultures dating back to 200 BC.[4] RPS is a 2-player, zero-sum game where neither player has perfect information or knowledge of what the other will do. Rock beats paper, paper beats scissors, and scissors beats rock. The pain distribution can be seen below. The matrix is written in terms of pain, therefore the pain associated with winning will be represented as negative pain and the pain associated with losing will be represented as positive pain:

Table 2: RPS Game Matrix. Player A has 3 choices: A1 which is rock, A2 which is paper, or A3 which is scissors. Likewise, player B has the same 3 choices. Because this is a zero-sum game, if A wins, B loses the same amount.

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+1**, –1	**–1**, +1
A2: Paper	**–1**, +1	**0**, 0	**+1**, –1
A3: Scissors	**+1**, –1	**–1**, +1	**0**, 0

Because this is a zero-sum game, if A wins, B loses the same amount. As such, the solution for this game can be computed using a mixed strategy Nash equilibrium: each should be played 1/3 of the time or in other words, play in a completely randomized manner. However, is the pain of winning and losing always the same; what if the pains are varied? What if the pain matrix is no longer symmetric but asymmetric?

[4] Moore, M. E., & Sward, J. (2006). Introduction to the Game Industry. Upper Saddle River, NJ: Pearson Prentice Hall: Pearson Prentice Hall.

2.1 The Rock, Paper, Scissors Game in Nature

RPS type interactions exist a great deal in nature and can even be applied across species and to a vast array of ecosystems. Almost always, a cyclical pattern of stability will form for multispecies RPS game. As such, these systems will exist in equilibrium for the traditional one-dimensional RPS game. An example of this cyclical dominance is seen in bacterial colonies. An engineered bacterial colony A will prohibit another bacterial colony B from increasing its population to the point of lysis, inhibiting a colony from either mutating or destroying itself. Additinoally, colony B exerts the same control over colony C, and C exerts the same control over A.[5]

Typically, these systems will exist within 3 states: stable, marginally stable, or unstable. This is a type of law of conservation for the total density of a given system and is applicable to both spatial ecological systems and nonspatial systems.[6] However, this will not always hold true because species do not typically have identical growth rates, rendering the interactions between the species asymmetric as they are fully dependent on these rates. In spatial ecosystems, the most fit or "strongest" species will most likely survive. In nonspatial ecosystems when the system is agitated or fluctuates, a reverse Darwinian rule or "survival of the weakest" can be seen depending on the parameters of the system.[7] Only certain types of asymmetric rock paper scissors games and interactions have been studied with any intensity; these are games that have more possible moves

[5] Liao, M. J. (2019). Rock-paper-scissors: Engineered population dynamics increase genetic stability. Science, 1045–1049.
[6] Ni, X. Y.-X.-C. (2010). Basins of coexistence and extinction in spatially extended ecosystems of cyclically competing species. Chaos: An Interdisciplinary Journal of Nonlinear Science
[7] Venkat, S. & Pleimling, M. (2010). Mobility and asymmetry effects in one-dimensional rock-paper-scissors games. *Physical Review E*, 1-5.

or actions than just the traditional three: rock, paper, scissors, but never games that change the pains or payoffs of the game. Very little work has been done with the aRPS game as defined above.

2.2 The Rock, Paper, Scissors Game Learning Modes

There are several ways players of an RPS game will update their strategy; reinforcement learning is one such method. In reinforcement learning, updates are made using only one piece of information: the outcomes of the games. This type of learning is fairly basic and can been seen even in primates playing RPS. For example, when playing several rounds of RPS, monkeys use their experiences to approximate the optimum strategy. These monkeys play an ocular motor version of the game where they are shown images of rock, paper, or scissors and their line of vision towards one of those three options is recorded as their choice.[8] One extreme form style of this learning is Cournot dynamics where the player makes an update using only the most recent game outcome. The other end of the spectrum is known as fictitious play. Fictitious play is a method to predict what the opponent will do next by considering the entire play history of both players. As such, it is often weighted more heavily towards the most recent plays making it like a blend of both and is the often the strategy many players attempt to employ.

Typically, when playing an RPS game, there is a higher probability of a player throwing the same move in the next game if they had just won with that move in the previous game. Conversely, players who have just lost in a game will likely switch their move and therefore update their strategy for the next game.[9] Furthermore, humans are inclined to tie above all else because

[8] Lee, D. M. (2005). Learning and decision making in monkeys during a rock–paper–scissors game. Cognitive Brain Research, 416–430.
[9] Wang, Z. X.-J. (2014). Social cycling and conditional responses in the Rock-Paper-Scissors game. Scientific Reports.

when not given enough time to process information and determine their next move, humans reflexively mimic their opponent.[10]

Another more complex update method is belief learning. Belief learning utilizes the perception of what another player will do.[11] Both belief learning and reinforcement learning are fairly simple methods to process information at a high rate, predict unfamiliar strategies based on previous data, and make a more informed next decision.[12] In professional RPS matches, some competitors will try to recall how their opponents have performed in similar situations in previous matches.[13] Additionally, both methods have another important similarity: each method differs from the TGT solution of the RPS game.

Both belief learning and reinforcement learning-based models fit experimental data on games with asymmetric information with far mora accuracy than TGT. Generally, when people play asymmetric information style games, the strategy cannot be approximated accurately by TGT. The result is cyclical movement about the Nash equilibrium without ever approaching it.[14,15] Because studies have found that the Nash Equilibrium is almost never the result of human players in the RPS scenario, this again shows that TGT is not the optimal model for predicting behavior.

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[10] Cook, R. B. (2011). Automatic imitation in a strategic context: players of rock–paper–scissors imitate opponents gestures. Proceedings of the Royal Society B: Biological Sciences, 780-786.
[11] Y.-W. Cheung, D. F. (1997). Individual learning in normal form games: some laboratory results. Games Econ. Behav. 19, 46 – 76.
[12] Stöttinger, E. F. (2014). The Effects of Prior Learned Strategies on Updating an Opponents Strategy in the Rock, Paper, Scissors Game. Cognitive Science, 38(7), 1482–1492.
[13] Mayyasi, A. (2014, September 5). Priceonomics. Retrieved from “Inside the World of Professional Rock Paper Scissors.: https://priceonomics.com/the-world-of-competitive-rock-paper-scissors/
[14] Feltovich, N. (2000). Reinforcement-based vs. belief-based learning models in experimental asymmetric-information games. Econometrica (68), 605-641.
[15] I. Erev, A. R. (1998). Predicting how people play games: reinforcement learning in experimental games with unique, mixed strategy equilibria,. Am. Econ. Rev. 88, 848-881.

2.3 Emotions in the Rock, Paper, Scissors Game: Escalation of Commitment

"When a decision maker discovers that a previously selected course of action is failing, she is faced with a dilemma: Should she pull out her remaining resources and invest in a more promising alternative, or should she stick with her initial decision and hope that persistence will eventually pay off?"[16] Escalation of commitment is the phenomenon that occurs when one is faced with a choice of continuing with their status quo decision or changing. That person decides to commit to their repeated course of action with the hopes that it will pay off, however, it invariably leads to an unfavorable result. Decision makers or players in a game believe they have invested too much in their current way of thinking and acting, that they cannot quit or differentiate by changing their course of action, and so continue to make the same errant decision.[17] This can be applied to RPS as players who begin to lose more with the same play style wish more strongly to win with that move; the pains for losing with that play are intensified. In a similar vein, players who have experienced victory with a certain play would be very disinclined to abandon that strategy. This has significance because a rational player would not let themselves fall into an escalation of commitment trap. They would remove the desire to win with one play instead of another and choose their plays in an unbiased manner.

There are many ways this form of thinking can be explained. Self-justification or feeling personally responsible for a failure of action that caused one pain exacerbates the threat of another errant choice and makes that person or player more prone to justify the original decision to themselves. In this way, the player attempts to prove to themselves that they will succeed if they only intensify their current course of action. Another justification for this behavior is confirmation

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[16] Kelly, T. &. (2013). Escalation of Commitment. Encyclopedia of Management.
[17] Staw, B. M. (1981). The Escalation of Commitment to a Course of Action. The Academy of Management Review, 6(4), 577.

bias; accepting biased information as a source to validate their decisions. After a decision is made, the player will weigh evidence that aligns with their decision more heavily than evidence that does not and actively search for the former, intensifying the issue. Additionally, there is loss aversion. A player that is experiencing failure with their current course of action is faced with the dilemma that changing their ways will cause them to lose both the potential rewards associated with the current course of action and the previous resources they have committed to that course of action. Lastly, it should be noted that even individuals who use a calculated strategy when solving problems are just as likely as those who are non-calculating to fall into this way of thinking.[18] This brings up two important points. Firstly, this phenomenon can be directly applied to a game of RPS. As a player keeps playing the same move and losing more, their frustration rises and they are stuck in a dilemma; do they continue to play the same move with the thought "surely they will change moves and then I will finally win with the move I have been playing" or do they risk changing their course? This is not how a purely rational player would think. The second point is that even calculated rational players can and will act irrationally, falling into the escalation of commitment trap and biasing their choices of what to play next. However, TGT assumes all players are perfectly rational which is clearly not the case. Therefore, if the game were to be played, it follows that TGT would not be able to predict accurately what the players will do.

2.4 Emotions in the Rock, Paper, Scissors Game: Sunk Costs

A major factor that influences escalation of commitment is sunk costs. Sunk costs are previous investments or courses of action that have already been completed, failing to pay off. The

[18] Conlon, E. J. (1980). The moderating effects of strategy, visibility, and involvement on allocation behavior: An extension of staws escalation paradigm. Organizational Behavior and Human Performance, 26(2), 172-192.

more resources that have been spent, the more a player is likely to escalate commitment.[19] Players are reluctant to accept those erroneous decisions as lost and factor them into their next decision; "[They are] too focused on the costs of abandoning the current approach as compared with the costs of missing other possible opportunities, and too unwilling to acknowledge that the original investments were a mistake. They thus invest additional resources in the hope that an eventual payoff will erase their losses and vindicate their actions."[20]

Within the lens of RPS, sunk costs can account for fair number of the decisions of players, especially if players are not given time to think about their decision. It is important to note that when given time to reflect, the tendency to make a sunk cost error is considerably reduced.[21] This can be directly applied to an RPS scenario. As multiple games are played in rapid succession and the player is not given time to think while they are becoming frustrated from losing with the same play, the player can easily fall into the fallacy of a sunk cost mentality. Again, a rational player would be able to think through the sunk cost trap. However, this occurs when players become flustered and aggravated during games, which can very easily happen. As their anger or frustration rises, they can bias their moves and act in an irrational way; a way that TGT is not able to predict accurately.

2.5 The Asymmetric Rock, Paper Scissors Game

RPS in its traditional form is a universally known and used game. It has been studied in great depth and has applied to many different natural systems as seen above. However, RPS with

[19] Arkes, H. R. (1985). The psychology of sunk cost. Organizational Behavior and Human Decision Processes 35(1), 124–140.
[20] Molden, D. C. (2010). Promoting De-Escalation of Commitment. Psychological Science, 8-12.
[21] Hafenbrack, A. C. (2013). Debiasing the Mind Through Meditation. Psychological Science, 25(2), 369–376.

asymmetric payoffs or pains has never been studied. For example, what if the rules of the game are changed so that the pain associated with winning by using rock were –3 instead of –1. The pain matrix for the asymmetric game is shown below.

Table 3: aRPS Pain Matrix. Player A has 3 choices: A1 which is rock, A2 which is paper, or A3 which is scissors. Likewise, player B has the same 3 choices.

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+1**, –1	**–3**, +3
A2: Paper	**–1** +1	**0**, 0	**+1**, –1
A3: Scissors	**+3**, –3	**–1**, +1	**0**, 0

The game shown is still zero-sum, yet it no longer has equal payoffs/pains across all the possible choices; winning with rock is clearly the best and yields the highest payoff. However, if players will use rock very often, would it not be better to play more paper to counter this? What is the optimal strategy a player can use? The solution for this game has become far more complicated and has deviated far from the simple solution of playing each 1/3 of the time.

Chapter 3

A New Model: Chemical Game Theory

The solution for this game will be derived using 2 different methods: through TGT and through chemical game theory or CGT. Within the method of CGT, each player is described by a reactor and the reactions that occur within this reactor are the decisions made by the player in their mind.

Because CGT utilizes the principles of Gibbsian thermodynamics, each reaction has an associated Gibbs free energy; the lower this energy, the more favorable the reaction. The pains associated with an action will correspond to the Gibbs free energy for that reaction or decision. Furthermore, there are required initial concentrations for the reactions to begin in the forward direction. These initial concentrations refer to a player's preconceived notions about an action or their bias towards an action. They are represented in CGT as "knowlecules" and the knowlecules of player A (a_i) can react with knowlecules of another player (b_j) taking into account player A's knowledge, history, personality (A), to form a decision player A can make (A_{ij}). (See Figure 1)

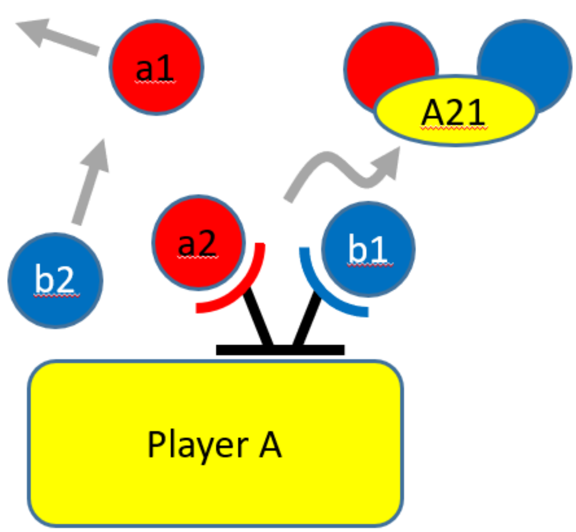

Figure 1: Molecule A Reacts with Knowlecules a_2 and b_1 to form Possible Decision A21.[1]

For example, if initial concentration a_1 is larger than a_2, then Player A has more of a bias towards option 1 than option 2. This is the same for any number of n players and can be modified to include any number of decisions. (See Figure 2).

One of the largest differences between TGT and CGT is that CGT does not stop after the players of the game have made their decisions – there are several more steps. Firstly, any unused initial concentrations or knowlecules, a_i and b_i, are separated and discarded; only the player decision knowlecules, A_{ij} and B_{ij}, continue in the process. These player decision knowlecules then go through a decomposition reaction back to their initial a_i and b_j pre-conceived notions of their own strategies. This is done in order to determine the exact pre-bias associated with each decision reaction. Next these pre-biases go through another separation unit. In this unit, Player A's pre-biases, a_i, are kept and sent on to the last reactor while Player A's conception of Player B's pre-biases, b_j, are discarded. Conversely, Player B's pre-biases, b_j, are kept and sent on to the last reactor while Player B's conception of Player A's pre-biases, a_i, are discarded. Finally, there is the last reactor, a decider reactor (Reactor D), that these player decisions A_{ij} and B_{ij} are fed into. Within Reactor D, these player decisions are turned into the final decision outcomes (D_{ij}).

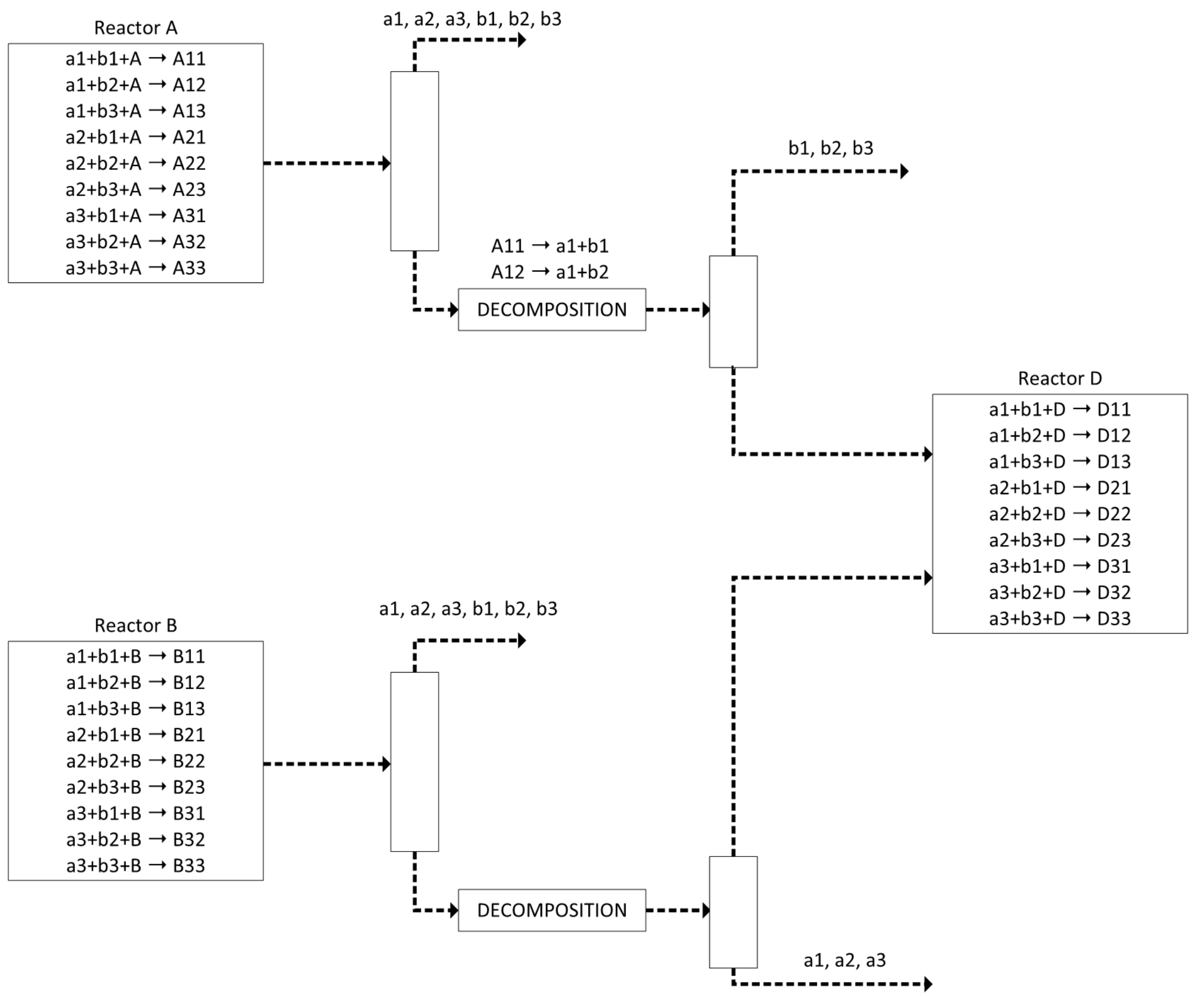

Figure 2: Process Flow Diagram of the Decision Reaction System:
A block flow diagram for any style RPS game. It is initially assumed that Players A and B have the same knowledge, so the game is only asymmetric in its payoffs, not in its information. However, this system can be used for asymmetric information games. There are separation units to remove the A and B decision reactants and decomposition units in order to remove the starting knowlecule prebiases into the decision reactor D.

CGT is able to overcome some of the largest shortcomings of TGT by including the ability to incorporate both entropic effects and pre-bias. Pre-bias effects how the game is played and can very drastically shift the equilibrium solution in CGT. This is very important because certain players of an aRPS game might be inclined to play more or less of a certain throw depending on what their strategy is. The figure below shows how changing the pre-bias for a game of RPS can affect the strategy.

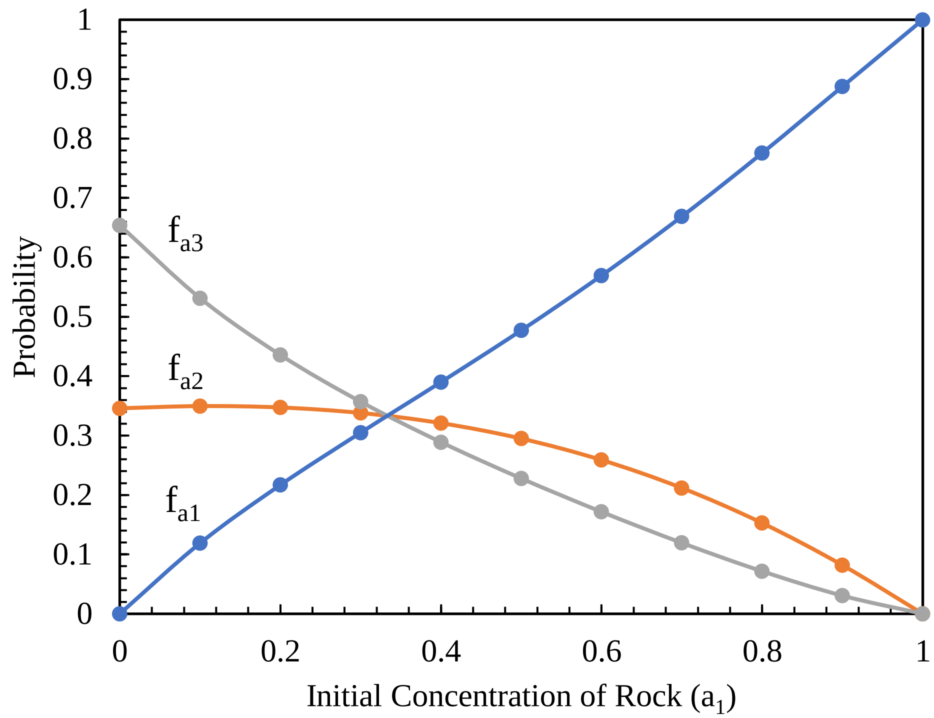

Figure 3: The probabilities of playing Rock (f_{a1}), Paper (f_{a2}), and Scissors (f_{a3}) during a normal game of RPS. The pre-bias of Rock (a_1) is increased by 0.1 each game with the pre-biases of Paper (a_2) and Scissors (a_3) being both equivalent and summing with Rock (a_1) to a pre-bias of 1 each game. At equal pre-biases, the probabilities of each play are also equal.

From the figure it is shown that at an equal pre-bias of Rock, Paper, and Scissors (a1 = a2 = a3 = $^1/_3$), the probabilities of each play are equal ($f_{a1} = f_{a2} = f_{a3} = {}^1/_3$); this is what TGT claims to always be the case. TGT claims that rational players have no preference for any choice before the game, however this is not how all people act. TGT has no mechanism to account players with preconceived notions of how they will play the game. Furthermore, entropic effects in chemistry stipulated that a reaction can never go to total completion. This can easily be applied to an aRPS game: each play has a finite probability that is not 100% which is due to the entropic effect in the choices of the player.

Chapter 4

The Theoretical Game Theory Solutions

4.1 The Chemical Game Theory Solution

In order to help visualize all the different reactions happening simultaneously, a SpICEY table will be used. This stands for Species, Initial, Change, Equilibrium, and Y which stands for mole fraction. The Species column will contain what types of knowlecules are present in the reactor, the Initial column will have the initial concentrations or pre-biases of the players. The Change column will indicate how much the initial concentration will change. The Equilibrium column will show what the equilibrium concentration of that species is after the reaction has occurred, and lastly, the mole fraction will show what the concentration of each reactant and product is at the end of the reaction.

Table 4: SpICEY Table For Reactor A

Species	Initial	Change	Equilibrium	y (Mole Fraction)
a1	0.33	$-(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$	$0.33 - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$	$[0.33 - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)] / \sum$
a2	0.33	$-(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$	$0.33 - (\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$	$[0.33 - (\varepsilon_4 + \varepsilon_5 + \varepsilon_6)] / \sum$
a3	0.33	$-(\varepsilon_7 + \varepsilon_8 + \varepsilon_9)$	$0.33 - (\varepsilon_7 + \varepsilon_8 + \varepsilon_9)$	$[0.33 - (\varepsilon_7 + \varepsilon_8 + \varepsilon_9)] / \sum$
b1	0.33	$-(\varepsilon_1 + \varepsilon_4 + \varepsilon_7)$	$0.33 - (\varepsilon_1 + \varepsilon_4 + \varepsilon_7)$	$[0.33 - (\varepsilon_1 + \varepsilon_4 + \varepsilon_7)] / \sum$
b2	0.33	$-(\varepsilon_2 + \varepsilon_5 + \varepsilon_8)$	$0.33 - (\varepsilon_2 + \varepsilon_5 + \varepsilon_8)$	$[0.33 - (\varepsilon_2 + \varepsilon_5 + \varepsilon_8)] / \sum$
b3	0.33	$-(\varepsilon_3 + \varepsilon_6 + \varepsilon_9)$	$0.33 - (\varepsilon_3 + \varepsilon_6 + \varepsilon_9)$	$[0.33 - (\varepsilon_3 + \varepsilon_6 + \varepsilon_9)] / \sum$
A11	0	$+\varepsilon_1$	ε_1	$\varepsilon_1 / \sum$
A12	0	$+\varepsilon_2$	ε_2	$\varepsilon_2 / \sum$
A13	0	$+\varepsilon_3$	ε_3	$\varepsilon_3 / \sum$
A21	0	$+\varepsilon_4$	ε_4	$\varepsilon_4 / \sum$
A22	0	$+\varepsilon_5$	ε_5	$\varepsilon_5 / \sum$
A23	0	$+\varepsilon_6$	ε_6	$\varepsilon_6 / \sum$
A31	0	$+\varepsilon_7$	ε_7	$\varepsilon_7 / \sum$
A32	0	$+\varepsilon_8$	ε_8	$\varepsilon_8 / \sum$
A33	0	$+\varepsilon_9$	ε_9	$\varepsilon_9 / \sum$

total	$\sum_0 = 2.00$	$-(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \varepsilon_7 + \varepsilon_8 + \varepsilon_9)$	$\sum = 2.00 - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \varepsilon_7 + \varepsilon_8 + \varepsilon_9)$	1.00

Table 5: SpICEY Table For Reactor B

Species	Initial	Change	Equilibrium	y (Mole Fraction)
a1	0.33	$-(\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12})$	$0.33 - (\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12})$	$[0.33 - (\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12})] / \sum$
a2	0.33	$-(\varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15})$	$0.33 - (\varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15})$	$[0.33 - (\varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15})] / \sum$
a3	0.33	$-(\varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18})$	$0.33 - (\varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18})$	$[0.33 - (\varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18})] / \sum$
b1	0.33	$-(\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16})$	$0.33 - (\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16})$	$[0.33 - (\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16})] / \sum$
b2	0.33	$-(\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17})$	$0.33 - (\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17})$	$[0.33 - (\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17})] / \sum$
b3	0.33	$-(\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18})$	$0.33 - (\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18})$	$[0.33 - (\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18})] / \sum$
B11	0	$+\varepsilon_{10}$	ε_{10}	$\varepsilon_{10} / \sum$
B12	0	$+\varepsilon_{11}$	ε_{11}	$\varepsilon_{11} / \sum$
B13	0	$+\varepsilon_{12}$	ε_{12}	$\varepsilon_{12} / \sum$
B21	0	$+\varepsilon_{13}$	ε_{13}	$\varepsilon_{13} / \sum$
B22	0	$+\varepsilon_{14}$	ε_{14}	$\varepsilon_{14} / \sum$
B23	0	$+\varepsilon_{15}$	ε_{15}	$\varepsilon_{15} / \sum$
B31	0	$+\varepsilon_{16}$	ε_{16}	$\varepsilon_{16} / \sum$
B32	0	$+\varepsilon_{17}$	ε_{17}	$\varepsilon_{17} / \sum$
B33	0	$+\varepsilon_{18}$	ε_{18}	$\varepsilon_{18} / \sum$
total	$\sum_0 = 2.00$	$-(\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15} + \varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18})$	$\sum = 2.00 - (\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15} + \varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18})$	1.00

Table 6: SpICEY Table for Reactor D

species	initial	change	end	y mole fraction
a1	$\varepsilon_1 + \varepsilon_2 + \varepsilon_3$	$-\varepsilon_{19} - \varepsilon_{20} - \varepsilon_{21}$	$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - \varepsilon_{19} - \varepsilon_{20} - \varepsilon_{21}$	$(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - \varepsilon_{19} - \varepsilon_{20} - \varepsilon_{21}) / \sum$
a2	$\varepsilon_4 + \varepsilon_5 + \varepsilon_6$	$-\varepsilon_{22} - \varepsilon_{23} - \varepsilon_{24}$	$\varepsilon_4 + \varepsilon_5 + \varepsilon_6 - \varepsilon_{22} - \varepsilon_{23} - \varepsilon_{24}$	$(\varepsilon_4 + \varepsilon_5 + \varepsilon_6 - \varepsilon_{22} - \varepsilon_{23} - \varepsilon_{24}) / \sum$
a3	$\varepsilon_7 + \varepsilon_8 + \varepsilon_9$	$-\varepsilon_{25} - \varepsilon_{26} - \varepsilon_{27}$	$\varepsilon_7 + \varepsilon_8 + \varepsilon_9 - \varepsilon_{25} - \varepsilon_{26} - \varepsilon_{27}$	$(\varepsilon_7 + \varepsilon_8 + \varepsilon_9 - \varepsilon_{25} - \varepsilon_{26} - \varepsilon_{27}) / \sum$
b1	$\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16}$	$-\varepsilon_{19} - \varepsilon_{22} - \varepsilon_{25}$	$\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16} - \varepsilon_{19} - \varepsilon_{22} - \varepsilon_{25}$	$(\varepsilon_{10} + \varepsilon_{13} + \varepsilon_{16} - \varepsilon_{19} - \varepsilon_{22} - \varepsilon_{25}) / \sum$
b2	$\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17}$	$-\varepsilon_{20} - \varepsilon_{23} - \varepsilon_{26}$	$\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17} - \varepsilon_{20} - \varepsilon_{23} - \varepsilon_{26}$	$(\varepsilon_{11} + \varepsilon_{14} + \varepsilon_{17} - \varepsilon_{20} - \varepsilon_{23} - \varepsilon_{26}) / \sum$
b3	$\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18}$	$-\varepsilon_{21} - \varepsilon_{24} - \varepsilon_{27}$	$\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18} - \varepsilon_{21} - \varepsilon_{24} - \varepsilon_{27}$	$(\varepsilon_{12} + \varepsilon_{15} + \varepsilon_{18} - \varepsilon_{21} - \varepsilon_{24} - \varepsilon_{27}) / \sum$
D11	0	$+\varepsilon_{19}$	ε_{19}	$\varepsilon_{19} / \sum$
D12	0	$+\varepsilon_{20}$	ε_{20}	$\varepsilon_{20} / \sum$
D13	0	$+\varepsilon_{21}$	ε_{21}	$\varepsilon_{21} / \sum$
D21	0	$+\varepsilon_{22}$	ε_{22}	$\varepsilon_{22} / \sum$
D22	0	$+\varepsilon_{23}$	ε_{23}	$\varepsilon_{23} / \sum$

D23 D31 D32 D33	0 0 0 0	$+\varepsilon_{24}$ $+\varepsilon_{25}$ $+\varepsilon_{26}$ $+\varepsilon_{27}$	ε_{24} ε_{25} ε_{26} ε_{27}	$\varepsilon_{24} / \sum$ $\varepsilon_{25} / \sum$ $\varepsilon_{26} / \sum$ $\varepsilon_{27} / \sum$
total	$\sum_0$	Δ	$\sum = \sum_0 + \Delta$	1.00

The Gibbs free energy or ΔG of a reaction measures the favorability and extent of a reaction by determining the equilibrium constant associated with that reaction. The relationship between these two is described by Equation 1

EQ 1: $$\Delta G° = -RT\ln(K)$$

where R is the ideal gas constant, T is the temperature of the system, and K is the equilibrium constant.

The equilibrium constant, K, of a reaction is defined by Equation 2

EQ 2: $$K = \prod_i a_i^{\nu_i}$$

where a_i is the activity coefficient of a particular species or knowlecule and v_i is the stoichiometric coefficient of a particular species or knowlecule.

The activity coefficient a_i can be defined further by Equation 3

EQ 3: $$a_i = y_i \Phi_i \left(\frac{P}{P_0}\right)$$

where y_i is the mole fraction of a particular species or knowlecule, Φ_i is the fugacity coefficient of a particular species or knowlecule, P is the pressure of the system, and P_0 is the reference pressure. If the system, which is being considered as an ideal system, is operating at standard pressure where P is 1 bar, the equation will reduce to:

$$a_i = y_i$$

Therefore, the equilibrium constant equation can be rewritten as:

$$K = \prod_i y_i^{\nu_i}$$

As such, Equation 4 can be now written as follows:

EQ 4: $$\frac{\Delta G^\circ}{-RT} = ln\left(\prod_i y_i^{\nu_i}\right)$$

Where the non-dimensionalized -ΔG/RT stands for the pain associated with a win or loss. This linear approximation enables the game to be solved quickly with minimal error. The SpICEY tables above enable the equilibrium mole fractions to be determined for all species in any reactor and the above equations illustrate the relationship between those equilibrium mole fractions and the Gibbs free energy of the reactions.

4.2 The Traditional Game Theory Solution

The TGT solution was derived two player 3 decision RPS game as follows. The expected pains of Player A and Player B were determined by multiplying the probability Player A would play a certain move, for example rock, by both the probability Player B would play a certain move, again rock, and the pain associated with the moves that both players made. The equation for the expected pain of Player A is shown below in Equation 5

EQ 5: $$E_A = f_{A_R} * f_{B_R} * P_{A_{RR}} + f_{A_R} * f_{B_P} * P_{A_{RP}} + f_{A_R} * \left(1 - f_{B_R} - f_{B_P}\right) * P_{A_{RS}} + f_{A_P} * f_{B_R} * P_{A_{PR}} + f_{A_P} * f_{B_P} * P_{A_{PP}} + f_{A_P} * \left(1 - f_{B_R} - f_{B_P}\right) * P_{A_{PS}} + f_{A_S} * f_{B_R} * P_{A_{SR}} + f_{A_S} * f_{B_P} * P_{A_{SP}} + f_{A_S} * \left(1 - f_{B_R} - f_{B_P}\right) * P_{A_{SS}}$$

Next, the partial derivative of these expected pains for both Player A and Player B were taken with respect to the probability of 2 of the moves, rock and paper, and solved for those probabilities. The equation for the fraction that Player A will play rock is shown below in Equation 6. The equation for the fraction that Player B will play rock is very similar but unlike

EQ 6: $$f_{A_R} = -\left(\frac{\left(\left(P_{B_{SP}} - P_{B_{SS}}\right) * \left(P_{B_{PR}} - P_{B_{PS}} - P_{B_{SR}} + P_{B_{SS}}\right) - \left(P_{B_{SR}} - P_{B_{SS}}\right) * \left(P_{B_{PP}} - P_{B_{PS}} - P_{B_{SP}} + P_{B_{SS}}\right)\right)}{\left(\left(P_{B_{PR}} - P_{B_{PS}} - P_{B_{SR}} + P_{B_{SS}}\right) * \left(P_{B_{RP}} - P_{B_{RS}} - P_{B_{SP}} + P_{B_{SS}}\right) - \left(P_{B_{RR}} - P_{B_{RS}} - P_{B_{SR}} + P_{B_{SS}}\right) * \left(P_{B_{PP}} - P_{B_{PS}} - P_{B_{SP}} + P_{B_{SS}}\right)\right)}\right)$$

The equation for the fraction that Player B will play rock is very similar but unlike Equation 6, it only depends on the pains associated with Player A; see Equation 7 below.

EQ 7: $$f_{B_R} = -\left(\frac{\left(\left(P_{A_{PS}}-P_{B_{SS}}\right)*\left(P_{A_{RP}}-P_{A_{RS}}-P_{A_{SP}}+P_{B_{SS}}\right)-\left(P_{A_{RS}}-P_{A_{SS}}\right)*\left(P_{A_{PP}}-P_{A_{PS}}-P_{A_{SP}}+P_{A_{SS}}\right)\right)}{\left(\left(P_{A_{PR}}-P_{A_{PS}}-P_{A_{SR}}+P_{A_{SS}}\right)*\left(P_{A_{RP}}-P_{A_{RS}}-P_{A_{SP}}+P_{A_{SS}}\right)-\left(P_{A_{RR}}-P_{A_{RS}}-P_{A_{SR}}+P_{A_{SS}}\right)*\left(P_{A_{PP}}-P_{A_{PS}}-P_{A_{SP}}+P_{A_{SS}}\right)\right)}\right)$$

Only 2 probabilities were solved for because the probability of the third move would be able to be determined by taking the difference between 1 and the probabilities of the two moves that were just calculated. The equation for the fraction that Player A will play rock can be seen in Equation 7 below:

EQ 7: $$f_{A_S} = 1 - f_{A_R} - f_{A_P}$$

The solutions gathered from these equations were verified via comparison against several online TGT solvers.[22]

[22] Several online solvers were used. They can be found at: https://www.math.ucla.edu/~tom/gamesolve.html, https://www.zweigmedia.com/RealWorld/gametheory/games.html, http://gte.csc.liv.ac.uk/gte/builder/

Chapter 5

Testing

5.1 Computational Game Testing

The means by which the CGT method will be pitted against TGT method in order to determine which is more effective at solving for the optimal strategy involved random number generation and 1000s of aRPS games. Both CGT and TGT will solve a game of aRPS using the pains associated with each of the outcomes and determine the probability of playing R, P, and S. Once these probabilities have been calculated, numbers between 0 and 1 will be randomly generated. These random numbers will fall within the range of one of the play probabilities, either R, P, or S. The corresponding play probability will be the play of that method for that specific game of aRPS.

For example, in the case of a 3-1-1 pain distributed game of aRPS (See Table 3) TGT gives the optimal probabilities of RPS are as follows: f_{a1} = 0.2 (R), f_{a2} = 0.6 (P), and f_{a3} = 0.2 (S). So, if the randomly generated number falls between 0 to 0.2, the play is rock by TGT, from 0.2 to 0.8, the play is paper by TGT, and lastly from 0.8 to 1.0, the play is scissors by TGT. For an unbiased game of CGT, the optimal probabilities are: f_{a1} = 0.447 (R), f_{a2} = 0.293 (P), and f_{a3} = 0.260 (S). Similarly, if the randomly generated number falls between 0 to 0.447, the play is rock by CGT, from 0.447 to 0.74, the play is paper by CGT, and lastly from 0.74 to 1.0 the play is scissors by CGT.

5.2 Computational Game Results

Several different asymmetric RPS games were solved using this method assuming unbiased players. Each game was played 150,000 times and the averages were computed in groups of 300. The first 5 games that were played had the pains randomly generated. The first game that was tested was a 5-1-3 pain distributed game of RPS. The pain and probability matrices along with the statistical analysis information are shown in table format in Appendix A.

GAME 1: After 150,000 games at a pain distribution of 5-1-3, CGT had a much higher average of points at 270.80 with an error of 1.13 and a standard deviation of 5.67 while TGT had an average of 265.32 points with an error of 0.96 and a standard deviation of 4.81. Additionally, CGT had a 54%-win rate while TGT had only 46% as shown in Table 15.

GAME 2: The next game that was tested was an aRPS game with a pain distribution of 2-0.4-1.6. After 150,000 games, CGT had a very slightly lower point average at 100.63 with a standard error of 0.50 and a standard deviation of 2.48 while TGT had a higher point average at 100.79 points with a standard error of 0.35 and a standard deviation of 1.73. Additionally, CGT had a 49%-win rate while TGT had 51% as shown in Table 18.

GAME 3: The third game that was tested was an aRPS game with a pain distribution of 1-0.2-0.6. After 150,000 games, CGT had a higher point average of 53.73 with a standard error of 0.32 and a standard deviation of 1.00 while TGT had a higher point average of 53.19 points with a standard error of 0.39 and a standard deviation of 1.25. Additionally, CGT had a 54%-win rate while TGT had only 46% as shown in Table 21.

GAME 4: The fourth game that was tested was an aRPS game with a pain distribution of 9-2-1. After 150,000 games, CGT had a far higher point average of 265.60 with a standard error of 1.57 and a standard deviation of 7.84 while TGT had a higher point average of 258.99 points with a standard error of 1.18 and a standard deviation of 5.92. Additionally, CGT had a 58%-win rate while TGT had only 42% as shown in Table 24.

GAME 5: The fifth and final game that was tested was an aRPS game with a pain distribution of 1.1-1.3-0.8. After 150,000 games, CGT had a slightly higher point average of 104.51 with a standard error of 0.26 and a standard deviation of 1.17 while TGT had a slightly lower point average of 104.21 with a standard error of 0.40 and a standard deviation of 1.77. Additionally, CGT had a 49%-win rate while TGT had a 51% as shown in Table 27.

The first 3 games that were played had the pains reduced each round. Between the first and second game there was an equal pain reduction of 60% and between the second and third game there was a pain reduction of 50%. The data indicated that in certain instances CGT was the superior method after 150,000 games having a higher point average at 270.80 with a standard error of 1.13 compared to the TGT average of 265.32 with a standard error of 0.96. Similarly, the win rate for CGT is 54% while for TGT it is only 46%. However, far more games would need to be played in order to confirm this.

Furthermore, as the pains become closer together it is even less clear which, if either, method is better. In some games tested like the 2-0.4-1.2 game, TGT has both a higher point average and win rate although CGT is only slightly lower and when the standard errors are taken into consideration, it cannot be determined which is a more successful method without playing more games. This is confirmed by the results of game 3 and game 5, both of which have pains that are very similar to one another and have results that do not clearly indicate a winner; this makes logical sense. As the pains approach one another, it turns into a symmetric game of RPS no matter what numerical value is assigned to the pains.

Conversely, as the pains diverge from each other, like in the case of game 1 or game 4 where there is a large difference between the pains associated with losing to rock compared to losing with paper or scissors one of the methods will be better at solving these asymmetric games. The results of game 4 support this theory as the point average for CGT is 265.60 with a standard error of only 1.57 while the TGT point average is 258.99 with a standard error of 1.18. Nevertheless, additional games would need to be played in order to definitively state this.

However, the same claims cannot be made with win rate. CGT aims to ensure the most points are scored while TGT tries to limit the amount of high pain inflicted on a player. This can be seen in the probability matrices for each game. As the pains are similar, such as in game 5, CGT predicts playing more paper with $f_{a2} = 0.365$ (see Table 26) because this yields the highest point value at 1.3. Conversely, TGT predicts playing more scissors with $f_{a3} = 0.406$ even though this is the lowest point value at 0.8. This is further supported in game 4, where CGT predicts playing the most rock with $f_{a1} = 0.435$ because of its incredibly high point value of 9 while TGT predicts that almost every play should be paper with $f_{a2} = 0.75$ despite the payoff of winning with paper being only 2! As such, TGT seems more likely to have a higher win rate than CGT because it will try to limit the amount of pain inflicted by countering the high payoff plays that CGT favors. This can clearly be seen in the calculation for the probability of a TGT play; the probability of any player choosing a move using the TGT strategy is based solely on the pains that the other player experiences.

Several more games were played slowly increasing the pain of rock from a 2-1-1 game of RPS to a 10-1-1 game of RPS using the same randomization method. The probabilities of playing R, P, or S varied widely as the pain/payoff associated with rock increased as shown in Figure 4.

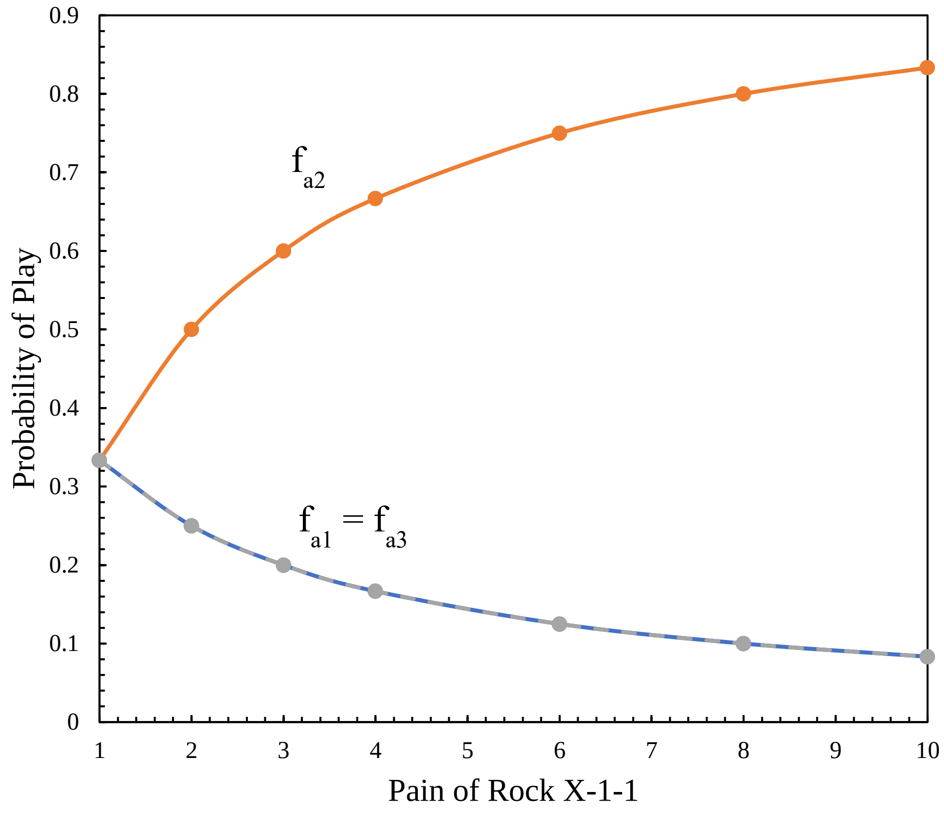

Figure 4: The TGT Probability of Playing Rock (f_{a1})**, Paper** (f_{a2})**, and Scissors** (f_{a3}) **varying the pain associated with Rock only**. The probabilities of playing Rock (f_{a1}) and Scissors (f_{a3}) are equal in each variation. The probability of Paper (f_{a2}) is the highest probability while Rock and Scissors are both equal and far lower.

The probability of playing Paper increases dramatically as the pains begin to diverge from a classic game of RPS with the biggest increase occurring between the 1-1-1 and 1-1-2 pain distributed games of RPS. As the pain of rock increases, the probability of playing paper (f_{a2}) reaches over 83% while the probability of playing Rock (f_{a1}) or Scissors (f_{a3}) are only approximately 8.3% each; the TGT strategy is to play Paper more than 4 out of every 5 games! This demonstrates that the TGT solution is a risk averse one and that the focus of TGT is a type of damage mitigation and counter to the high pain/payoff of Rock. Alternatively, CGT offers a far more neutral solution to the increasing Rock pain/payoff in the aRPS games.

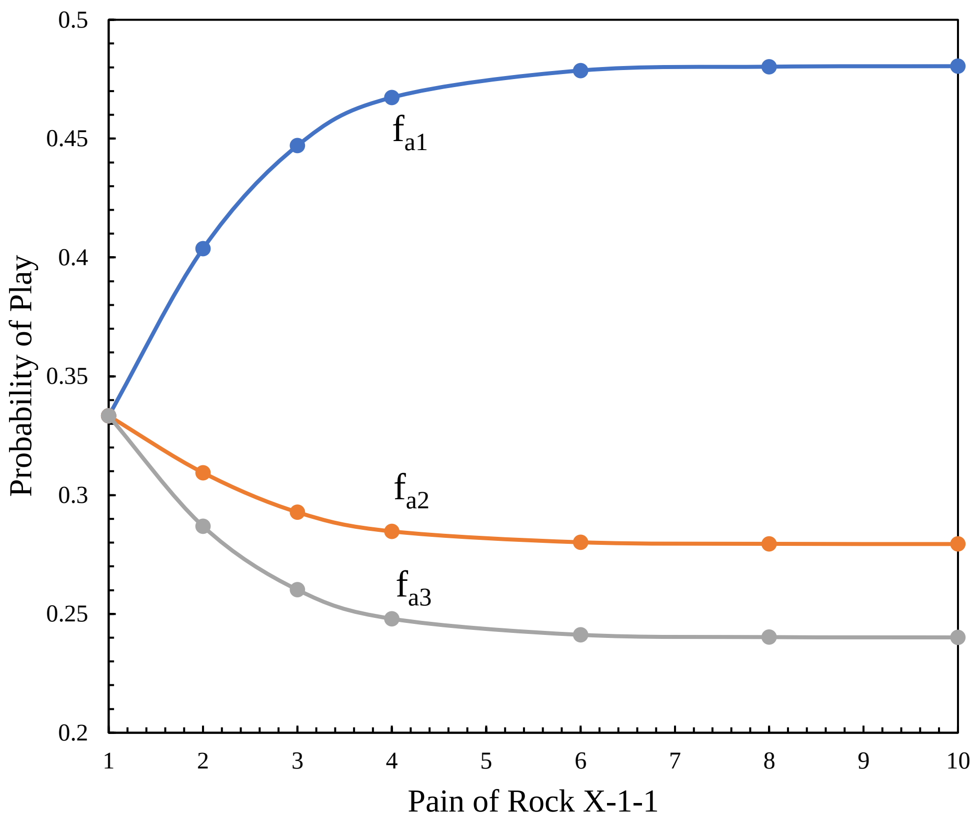

Figure 5: The CGT Probability of Playing Rock (f_{a1})**, Paper** (f_{a2})**, and Scissors** (f_{a3}) **varying the pain associated with Rock only.** For these solutions, the prebiases associated with each decision (Rock, Paper, or Scissors) were kept at $^{1}/_{3}$ each. Rock (f_{a1}) is the highest, followed by Paper (f_{a2}), and lastly Scissors (f_{a3}).

Similar to the TGT solution, the probabilities diverge rapidly from each other as the pain/payoff associated with Rock begins to increase with the biggest difference occurring between the 1-1-1 and 1-1-2 pain distributed games of RPS. However, unlike the TGT solution, the CGT probabilities quickly plateau around the 6-1-1 pain distributed game of aRPS at approximately a 47% play of rock. CGT only plays Paper (f_{a2}) a maximum of 33.3% at the traditional RPS pain distributed game of 1-1-1. As the pain/payoff of Rock increases, CGT begins to favor Rock (f_{a1}) and the probability of a Rock play reaches a maximum of 48% at a 10-1-1 game, with a Paper play of 28% and a Scissors play of 22%, differing drastically from the TGT solution; CGT tries to maximize the largest payoff available.

The expected pains E_A and E_B when Player A utilizes CGT and Player B uses TGT at each X-1-1 game of RPS varying X from 2 to 10 as done above equate to 0, indicating that the two methods ought to have similar point averages after multiple rounds of the game are played.

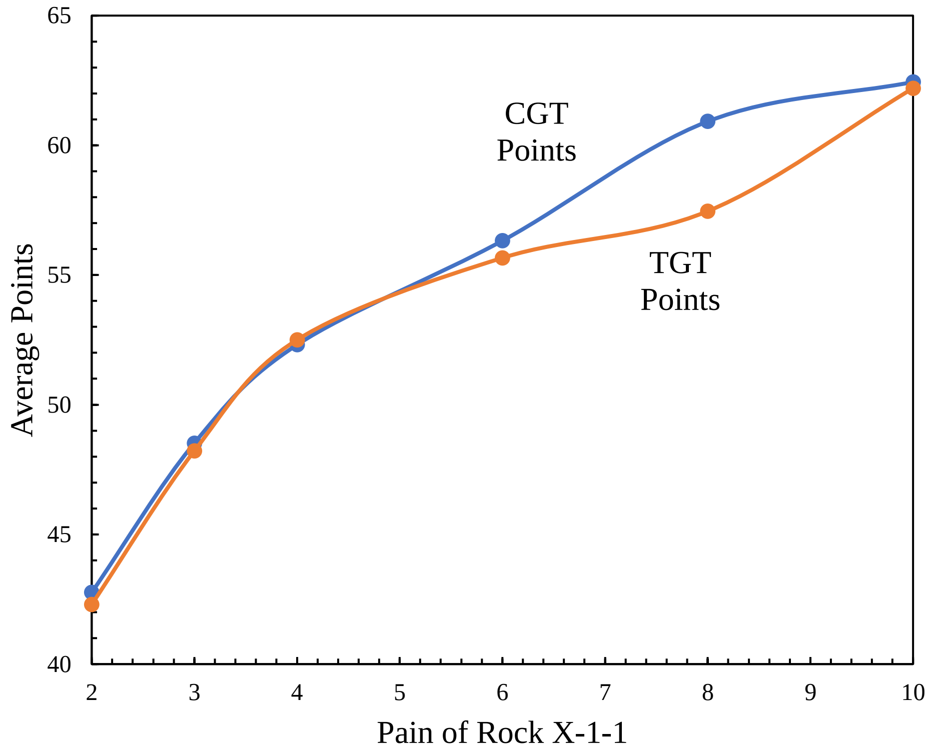

Figure 6: Average Points of both CGT and TGT solution methods. For the CGT solutions, the prebiases associated with each decision (Rock, Paper, or Scissors) were kept at $^1/_3$ each. For each X-1-1 data point, 10,000 games were played. The methods used to calculate and solve these games are outlined in Section 4.1.

Figure 6 helps to confirm the above statement that the expected pains are the same throughout all the games played with the slight exception of the 8-1-1 aRPS game which could just be considered an outlier.

5.3 Experimental Game Testing

Next, experimental data was collected using lab members as test subjects to gain a preliminary and rough idea of the general accuracy of the unbiased CGT solution with respect to aRPS games. The same two subjects participated in all games. A big challenge for experimental testing was determining what physical phenomenon could be substituted for the pains associated with winning or losing the aRPS game. For this experiment, the group decided the pains would be manifested in the form of pushups; this was something that all subjects could participate in. However, pushups are certainly not applicable to all players of an aRPS game. Further research would need to be done in order to determine what and how the pains of the game relate to real world phenomenon that players can experience.

Three different sets of games were played ranging from 60 games to 100 games. The games had pain distributions of 4-4-4, 6-4-4, and 8-4-4. Because it is unknown how the pain of doing a pushup is related to the theoretical pain associated with winning or losing, the pushups associated with each play were divided by the play with the lowest payoff or least number of pushups to determine what the pains were for the sake of calculation. Therefore, the pain distributions for the games became 1-1-1, 1.5-1-1, and 2-1-1.

5.4 Experimental Game Results

GAME 1: After 100 games at a pain distribution of 1-1-1 R-P-S, both players deviated fairly heavily from both the CGT and TGT solutions as both these solutions computationally yielded probabilities of $^{1}/_{3}$ for each play. It is important to note that Scissors and Rock for Player A and B respectively are considerably higher than Paper despite there not being any higher pain associated with those plays.

Table 7: Probabilities of TGT, CGT, Player A, and Player B for the 1-1-1 RPS game.

	CGT	TGT	Player A	Player B
F_1 (Rock)	0.33	0.33	0.34	0.42
F_2 (Paper)	0.33	0.33	0.17	0.31
F_3 (Scissors)	0.33	0.33	0.49	0.27

Table 8: Percent Differences of Player A and Player B between CGT and TGT for the 1-1-1 RPS game.

	CGT % Diff A	TGT % Diff A	CGT % Diff B	TGT % Diff B
F_1 (Rock)	3%	3%	21%	21%
F_2 (Paper)	94%	94%	6%	6%
F_3 (Scissors)	33%	33%	22%	22%

One potential reason for both Rock and Scissors being far higher than the expected $^1/_3$ for a normal game of RPS is the fact that the hand when playing the game is usually in the form of a Rock play as both players countdown to play the game. Because games happen in rapid succession and the players are not given time to think, they might simply default to this play. Another possible reason is because when beginning the game, players typically shout the possible plays in the following order, "Rock! Paper! Scissors!" before casting their plays. This initial yelling of Rock or final yelling of Scissors could also be another factor in why both subjects favored that play so strongly. Further tests and analysis would need to be conducted to determine if these influence a player's decision.

GAME 2: After 60 games at a pain distribution of 1.5-1-1 R-P-S, both players had highly similar strategies, favoring Rock, then Paper, then Scissors. The results are shown below in Table 9

Table 9: Probabilities of TGT, CGT, Player A, and Player B for the 1.5-1-1 aRPS game.

	CGT	TGT	Player A	Player B
F_1 (Rock)	0.37	0.29	0.42	0.4
F_2 (Paper)	0.32	0.43	0.33	0.35
F_3 (Scissors)	0.31	0.29	0.25	0.25

Table 10: Percent Differences of Player A and Player B between CGT and TGT for the 1.5-1-1 aRPS game.

	CGT % Diff A	TGT % Diff A	CGT % Diff B	TGT % Diff B
F_1 (Rock)	11%	31%	7%	29%
F_2 (Paper)	4%	29%	8%	22%
F_3 (Scissors)	23%	14%	23%	14%

As seen in table XX, both Player A and Player B had incredibly similar strategies, differing from each other by no more than 2%. Both had the largest percent differences comparing their actual plays to what TGT predicts they should play with respect to Rock. Furthermore, these percent differences were for both players were far smaller for the CGT solution than the TGT solution in all aspects except for the probabilities of playing Scissors which for both players were approximately 23%. This could be attributed to the fact that only 60 games were played. If more games were played, the percent differences for CGT may very well change.

GAME 3: After 80 games at a pain distribution of 2-1-1 R-P-S, both players had highly similar strategies favoring Rock, then Paper, then Scissors. The results are shown below in Table 11.

Table 11: Probabilities of TGT, CGT, Player A, and Player B for the 2-1-1 aRPS game.

	CGT	TGT	Player A	Player B
F_1 (Rock)	0.40	0.25	0.48	0.51
F_2 (Paper)	0.31	0.50	0.39	0.31
F_3 (Scissors)	0.29	0.25	0.14	0.18

Table 12: Percent Differences of Player A and Player B between CGT and TGT for the 1.5-1-1 aRPS game.

	CGT % Diff A	TGT % Diff A	CGT % Diff B	TGT % Diff B
F_1 (Rock)	15%	47%	21%	51%
F_2 (Paper)	20%	29%	1%	60%
F_3 (Scissors)	109%	82%	64%	43%

Both Player A and Player B had fairly similar strategies, both heavily favoring rock and playing it nearly or over half the time. Similarly, both played scissors roughly the same amount, differing by only 4%. Both Player A and B have their largest percent differences between the probability of playing Scissors and what CGT predicts the probability of Scissors will be. This is in part due to the low percentage that scissors is both played and predicted. Furthermore, the same high percent differences can be seen for the TGT prediction again due to the low probability of a

Scissors play. The percent differences for the other two plays differ by no more than 22% with respect to the CGT prediction, while the percent differences between TGT predicts for both players ranges from 29% to 60%.

While there are some large percent differences between CGT and both players, they follow the general trend that CGT predicts of playing Rock with a higher probability the more the pain/payoff of rock increases, followed by Paper, and finally Scissors. It is clear then that the players do not follow the trend that TGT predicts of increasing the play of Paper the most and decreasing Scissors and Rock equally. This is not clearly shown by the percent difference calculations because both players increased their plays of Rock heavily and Paper slightly while decreasing the play of Scissors drastically through the course of all the games. This trend is exactly as CGT predicts and it happens that the prediction by TGT for scissors in the final game was closer to what was experimentally played than what CGT predicted. Since the sample size was so small for the experimental data, more games would need to be played in order to confirm these trends.

Chapter 6

Conclusion

Despite CGT winning the majority of the first 5 aRPS games played, it cannot be definitively said that one method is superior to another based on the number of theoretical computational solutions that were played. It is possible that after many more games, one method may appear to be better, however the expected pain calculation for both Player A using CGT and Player B using TGT for each of the X-1-1 games of aRPS yield that each player should be neutral despite the large differences in the probabilities of playing Rock, Paper, or Scissors for each method. As a result, it is inconclusive which method yields the optimal strategy using a theoretical computational solution when CGT experiences no pre-bias towards a particular play.

That being said, CGT, unlike TGT, has the ability to introduce pre-bias into the decisions which would sway the probabilities of what a player would choose to do. Future work would include playing more aRPS games with CGT and TGT but including some prebias in the CGT method which may indicate that CGT is a superior and more versatile method simply because it has the ability to favor which play has a higher or lower pain/payoff associated with it.

For the experimental data, as the pain/payoff associated with Rock increased, the players followed the same general trend that CGT predicted by playing the most Rock, followed by Paper and lastly Scissors. Despite some of the large percent differences between the CGT prediction and what was played, the trend was starkly different than the TGT prediction (playing Paper more followed by an equal playing of Rock and Scissors the higher the pain/payoff of Rock increased). There are a few important caveats to these findings, the first of which is the simplification made to the pain matrix. It was assumed that the pain matrix of pushups could be normalized to 1 in order to convert the pushups to pains that are able to be used within the framework of CGT. This may not be an accurate assumption and a method of converting between the two would need to developed for this game in particular. Future work would include determining both a physical phenomenon that all players of the game can participate in and how this relates to the pains/payoffs of any game. Secondly, only 240 games in total were played; these conclusions about the general trends of the data and how they compare to the CGT and TGT solutions are drawn from a very small sample of data; many more games would need to be played to begin to make these statements with any firm validity. Lastly, the same two players were used for every game. There could be an issue with these players learning the strategies and play styles of one another. In order to determine if CGT applies to all players, the sample size of players would need to be drastically increased.

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Appendix A

Game Pain Matrices, Probabilities, and Statistics

GAME 1:

Table 13: Pain Matrix for a 5-1-3 game of aRPS

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+1**, –1	**–5**, +5
A2: Paper	**–1** +1	**0**, 0	**+3**, –3
A3: Scissors	**+5**, –5	**–3**, +3	**0**, 0

Table 14: The probabilities of Rock, Paper, and Scissors computed by both CGT and TGT for the 5-1-3 game of aRPS

Probability	CGT Solution	TGT Solution
f_{a1}	0.422	0.333
f_{a2}	0.223	0.556
f_{a3}	0.335	0.111

Table 15: Results of 5-1-3 game of aRPS. 150,000 games were played.

	PTS CGT	PTS TGT	Win Rate CGT	Win Rate TGT
Mean	270.80	265.32	0.54	0.46
Standard Error	1.13	0.96	0.02	0.02
Median	270.70	265.45	0.55	0.45
Mode	270.95	262.25	0.55	0.45
Standard Deviation	5.67	4.81	0.10	0.10
90% Confidence Level	1.94	1.65	0.03	0.03

GAME 2:

Table 16: Pain Matrix for a 2-0.4-1.2 game of aRPS

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+0.4**, –0.4	**–2**, +2
A2: Paper	**–0.4**, +0.4	**0**, 0	**+1**, –1
A3: Scissors	**+2**, –2	**–1**, +1	**0**, 0

Table 17: The probabilities of Rock, Paper, and Scissors computed by both CGT and TGT for the 2-0.4-1.2 game of aRPS

Probability	CGT Solution	TGT Solution

f_{a1}	0.431	0.333
f_{a2}	0.255	0.556
f_{a3}	0.314	0.111

Table 18: Results of a 2-0.4-1.2 game of aRPS. 150,000 games were played.

	PTS CGT	PTS TGT	Win CGT	Win TGT
Mean	100.63	100.79	0.49	0.51
Standard Error	0.50	0.35	0.02	0.02
Median	100.28	101.12	0.45	0.55
Mode	106.84	103.46	0.45	0.55
Standard Deviation	2.48	1.73	0.11	0.11
90% Confidence Level	0.85	0.59	0.04	0.04

GAME 3:

Table 19: Pain Matrix for a 1-0.2-0.6 game of aRPS

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+0.2**, –0.2	**–1**, +1
A2: Paper	**–0.2**, +0.2	**0**, 0	**+0.6**, –0.6
A3: Scissors	**+1**, –1	**–0.6**, +0.6	**0**, 0

Table 20: The probabilities of Rock, Paper, and Scissors computed by both CGT and TGT for the 1-0.2-0.6 game of aRPS

Probability	CGT Solution	TGT Solution
f_{a1}	0.391	0.333
f_{a2}	0.294	0.556
f_{a3}	0.314	0.111

Table 21: Results of a 1-0.2-0.6 game of aRPS. 150,000 games were played.

	PTS CGT	PTS TGT	WIN RATE CGT	WIN RATE TGT
Mean	53.73	53.19	0.54	0.46
Standard Error	0.32	0.39	0.02	0.02
Median	53.75	52.99	0.53	0.48
Mode	55.22	55.54	0.60	0.55
Standard Deviation	1.00	1.25	0.07	0.07
90% Confidence Level	0.58	0.72	0.04	0.04

GAME 4:

Table 22: Pain Matrix for a 9-2-1 game of aRPS

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+2**, –2	**–9**, +9
A2: Paper	**–2**, 2	**0**, 0	**+1**, –1
A3: Scissors	**+9**, –9	**–1**, +1	**0**, 0

Table 23: The probabilities of Rock, Paper, and Scissors computed by both CGT and TGT for the 9-2-1 game of aRPS

Probability	CGT Solution	TGT Solution
f_{a1}	0.435	0.083
f_{a2}	0.341	0.750
f_{a3}	0.224	0.167

Table 24: Results of a 9-2-1 game of aRPS. 150,000 games were played.

	PTS CGT	PTS TGT	Win CGT	Win TGT
Mean	265.60	258.99	0.58	0.42
Standard Error	1.57	1.18	0.02	0.02
Median	265.10	260.20	0.60	0.40
Mode	274.00	259.20	0.60	0.40
Standard Deviation	7.84	5.92	0.09	0.09
90% Confidence Level	2.68	2.03	0.03	0.03

GAME 5:

Table 25: Pain Matrix for a 1.1-1.3-0.8 game of aRPS

	B1: Rock	B2: Paper	B3: Scissors
A1: Rock	**0**, 0	**+1.3**, –1.3	**–1.1**, +1.1
A2: Paper	**–1.3**, +1.3	**0**, 0	**+0.8**, –0.8
A3: Scissors	**+1.1**, –1.1	**–0.8**, +0.8	**0**, 0

Table 26: The probabilities of Rock, Paper, and Scissors computed by both CGT and TGT for the 1.1-1.3-0.8 game of aRPS

Probability	CGT Solution	TGT Solution

f_{a1}	0.330	0.250
f_{a2}	0.365	0.344
f_{a3}	0.305	0.406

Table 27: Results of a 1.1-1.3-0.8 game of aRPS. 150,000 games were played.

	CGT PTS	TGT PTS	CGT Win Rate	TGT Win Rate
Mean	104.51	104.21	0.49	0.51
Standard Error	0.26	0.40	0.02	0.02
Median	104.44	104.07	0.50	0.50
Mode	106.97	107.35	0.55	0.60
Standard Deviation	1.17	1.77	0.10	0.10
90% Confidence Level	0.45	0.68	0.04	0.04

Appendix B

2-Player RPS TGT Mathematica Solution

Ea = fa1*fb1*pa11+fa1*fb2*pa12+fa1(1-fb2-fb1)*pa13+fa2*fb1*pa21+fa2*fb2*pa22+fa2(1-fb2-fb1)*pa23+(1-fa2-fa1)*fb1*pa31+(1-fa2-fa1)fb2*pa32+(1-fa2-fa1)(1-fb2-fb1)*pa33;

Eb = fa1*fb1*pb11+fa1*fb2*pb12+fa1(1-fb2-fb1)*pb13+fa2*fb1*pb21+fa2*fb2*pb22+fa2(1-fb2-fb1)*pb23+(1-fa2-fa1)*fb1*pb31+(1-fa2-fa1)fb2*pb32+(1-fa2-fa1)(1-fb2-fb1)*pb33;

Solve System of Equations:
$d_{fa1}Ea = 0$
$d_{fa2}Ea = 0$
$d_{fb1}Eb = 0$
$d_{fb2}Eb = 0$

fa1 = 1.*(-1)*(((pb32-pb33) (pb21-pb23-pb31+pb33)-(pb31-pb33) (pb22-pb23 pb32+pb33))/((pb21-pb23-pb31+pb33) (pb12-pb13-pb32+pb33)-(pb11-pb13-pb31+pb33) (pb22-pb23-pb32+pb33)))

fa2 = 1.*(-1)*((-pb12 pb31+pb13 pb31+pb11 pb32-pb13 pb32-pb11 pb33+pb12 pb33)/(-pb12 pb21+pb13 pb21+pb11 pb22-pb13 pb22-pb11 pb23+pb12 pb23+pb12 pb31-pb13 pb31-pb22 pb31+pb23 pb31-pb11 pb32+pb13 pb32+pb21 pb32-pb23 pb32+pb11 pb33-pb12 pb33-pb21 pb33+pb22 pb33))

fa3 = 1-fa1-fa2

fb1 = 1.*(-1)*(((pa23-pa33) (pa12-pa13-pa32+pa33)-(pa13-pa33) (pa22-pa23-pa32+pa33))/((pa21-pa23-pa31+pa33) (pa12-pa13-pa32+pa33)-(pa11-pa13-pa31+pa33) (pa22-pa23-pa32+pa33)))

fb2 = 1.*(-1)*((-pa13 pa21+pa11 pa23+pa13 pa31-pa23 pa31-pa11 pa33+pa21 pa33)/(-pa12 pa21+pa13 pa21+pa11 pa22-pa13 pa22-pa11 pa23+pa12 pa23+pa12 pa31-pa13 pa31-pa22 pa31+pa23 pa31-pa11 pa32+pa13 pa32+pa21 pa32-pa23 pa32+pa11 pa33-pa12 pa33-pa21 pa33+pa22 pa33))

fb3 = 1-fb1-fb2

Academic Vita

The Pennsylvania State University Schreyer Honors College

Bachelor of Science in Chemical Engineering **Graduation Date**: **December 2019**

Skills:

HTRI, Flowmaster, Aspen Hysys, Wolfram Mathematica, Public Speaking, Leadership, Teaching

Work Experience:

Production Engineer: *Dow Chemical Company – Midland, MI* *May 2019 to August 2019*

- Stretched capacity on a highly leveraged product by 5% while implementing plans to stretch another 10% saving the company $300,000 during the summer and $600,000 after implementation of all solutions
- Sole project manager for valve replacement project; charged with research, speccing, and implementation
- Served as the man-rep for a column bottoms replacement project

Process Engineer: *Dow Chemical Company – Plaquemine, LA* *May 2018 to August 2018*

- Performed technical engineering support for various plants calculations with an emphasis on heat exchanger design and fluid mechanics
- Saved the company thousands in engineering costs by contributing to a comprehensive model of their entire hydrocarbons cooling water system based on underground isometric schematics and P&IDs

Production Engineer: *Dow Chemical Company – Bristol, PA* *September 2017 to December 2017*

- Drafted and updated technical procedures. Developed and updated P&ID
- Uncovered a major discrepancy between logistics and production, which enabled the company to focus production on other products, saving the company over $100,000 in production costs
- Aided in a major capital reactor expansion project

Business Analyst: *Grant Street Group – Pittsburgh, PA* *May 2017 to August 2017*

- Served as the liaison between stakeholders and software developers in a project to design an online widget that monitored the performance of the company's sites.
- Performed iterative testing of the site internally to gauge effectiveness and ease of use
- Directly responsible for coming up with and meeting project deliverables

Extra-Curricular Activities:

Lion Ambassadors: *Spring 2018 to Present*

- Selected member of the Penn State Alumni Association Student Corps
- Organized the student section for sporting events, scheduled and executed campus wide projects, tour guide
- Strategic Planning Committee

Crotona Program: *Summer 2016*

- Volunteered at a program that takes place over the summer in which several counselors teach underprivileged kids in the South Bronx of New York City

Appalachia Mission Trip: *Summer 2016*

- Built homes for the impoverished in West Virginia with in teams of high school and college men and women

EON Mentorship Program: *Fall 2016 to Fall 2017*

- Mentor of freshman engineering students which helps to ease the transition the students face in changing from high school to college

Crossfit Club *Spring 2015 to Present*

Awards:

- Lee and Mary Eagleton Award for Excellence in Design
- Penn State College of Engineering – EDSGN 100 GE Transportation Most Innovative Design Award by People's Choice