THE PENNSYLVANIA STATE UNIVERSITY
SCHREYER HONORS COLLEGE

DEPARTMENT OF CHEMICAL ENGINEERING

CHEMICAL GAME THEORY: STRATEGY IN REPEATED GAMES

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SPRING 2020

A thesis
submitted in partial fulfillment
of the requirements
for a baccalaureate degree
in Chemical Engineering
with honors in Chemical Engineering

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ABSTRACT

The purpose of this thesis is to provide a new method for solving repetitive game theory problems. A game theory problem consists of players with two or more possibilities, and the outcome is dependent on the choices of all of the players' decisions. A repetitive game theory problem includes playing many iterations of the same game, incorporating aspects of learning for both players. Solutions to contested decisions or strategic games are typically represented by game theory, which will be called Classical Game Theory in this thesis. For repetitive games, tit-for-tat strategy and grim trigger solutions are most popularly used in Classical Game Theory. This thesis will use a new approach called Chemical Game Theory to analyze how players learn throughout repetitive games and update their strategy from game to game. The shortcomings of either Classical Game Theory or Chemical Game Theory in repetitive games will be discussed.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Velegol for his support and encouragement during my time in the CGT research lab. Dr. Velegol has not only supported my work in research but has given me career and life advice that I will continue to use and never forget. I would also like to thank the members of the CGT lab including Michael Creighton, Adam Smoluk, Jacob Scioscia, Ellie Alberti, Kadambari Prabakar, and Timothy Schmitt. It is very motivating and exciting to be a part of such an accomplished team. To my parents and sister, thank you for always sharing your positive attitudes and for your continued support throughout my four years at Penn State.

Chapter 1

Introduction

Chemical Game Theory provides an alternative approach to representing and solving both one-shot game theory problems, as well as repetitive games. This method produces unique results from Classical Game Theory. A main contribution of Chemical Game Theory is that it accounts for what player's *actually* do, whereas Classical Game Theory only considers what rational players *should* do. Chemical Game Theory can do this by incorporating chemical engineering fundamentals such as decision reactions, thermodynamic principles, and perception functions. The key question that will be answered in this thesis is:

Does the process control learning model of Chemical Game Theory explain experimental data better than Classical Game Theory learning models?

To answer this question, the probability that a player will choose a decision throughout a repetitive game, while updating his strategy from round to round will be analyzed. Existing learning algorithms and biology comparisons will also be examined to show significance of feedback control, which is a main component in the Chemical Game Theory model. [1] Additionally, the methods for calculating both the Classical and Chemical Game Theory solutions in repetitive games will be shown. The results will then be compared directly to experimental data, testing both Classical Game Theory and Chemical Game Theory solutions in repetitive games. [2] The answer to this key question can be summarized in Table 15.

[1] Velegol, Darrell. Physics of Community Course Notes for Fall 2019.

[2] Velegol, D.; Suhey, P.; Connolly, J.; Morrissey, N.; Cook, L. Chemical Game Theory. Industrial & Engineering Chemistry Research 2018, 57 (41), 13593–13607.

Chapter 2

Classical Game Theory

2.1 Classical Game Theory in One-Shot Prisoner's Dilemma Game

Classical Game Theory has been studied to find solutions to strategic games in which two or more players make decisions that directly affect each other. John von Neumann and Oskar Morgenstern describe solutions to these games in their work, *Theory of Games and Economic Behavior*. [3] Much of modern game theory is based on the mathematical models within this work. These models have a very critical assumption, which is that each player acts rationally. Acting rationally indicates that all players act in their own self-interest and are making decisions that will offer them the greatest utility. It is also assumed that players know their self-interest and are able to compute it.

A common game that is solved within Classical Game Theory is called the Prisoner's Dilemma.[4] This game is played between two players, Player A and Player B. In this game, these players have started to commit a robbery together but are caught by the police before completing the act. They are taken to the police station, having no contact with each other. In separate rooms, the district attorney gives them each the decision to "tell "or "be quiet". By "telling", the player will be revealing that the other individual was planning on robbing the bank. By "being quiet", the player will not reveal any information about the other player's intentions. Based on the players' decisions, there are four possible outcomes and payoffs. These are described below:

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[3] Von Neumann, J.; Morgenstern, O. Theory of Games and Economic Behavior. Princeton: Princeton University Press, 2007.
[4] Gintis, H. "Game Theory Evolving." Princeton University Press, 2000.

1. Both players can remain quiet and not give any information away to the district attorney, receiving one year of prison time each.
2. Both players can tell on each other, betraying each other, and each player will receive two years of prison time.
3. Player A can tell on Player B while Player B remains quiet. This will cause Player A to receive zero years in prison and Player B will receive three years in prison.
4. Player B can tell on Player A while Player A remains quiet. This will cause Player B to receive zero years in prison and Player A will receive three years in prison.

These scenarios can be described in normal form by a payoff matrix. A payoff matrix has three main components, which include players, payoffs, and strategies. In the Prisoner's Dilemma game, the players are Player A and Player B, the strategies are to stay quiet or tell, and the payoffs are number of years in prison. The payoff matrix for Prisoner's Dilemma is shown in Table 1.

Table 1: Payoff matrix for a Prisoner's Dilemma game

	Player B Quiet	Player B Tell
Player A Quiet	1,1	3,0
Player A Tell	0,3	2,2

The Nash Equilibrium solution to this one-shot game, a game that is only played once, is for both players to choose to tell on the other player. This solution is considered a Nash equilibrium, which is a stable state in which no player can gain more utility by changing his or her strategy if the strategy of the other player remains unchanged.[5] To come to this solution, the

[5] Nash, J. (n.d.). Non-Cooperative Games. Cournot Oligopoly, pp. 82–94.

players are considered individually. For example, it is assumed that Player B's choice is held constant at tell. If Player A stays quiet, Player A will receive three years of prison time. However, if Player A chooses to tell then he will only receive two years of prison time. In this scenario, Player A will self-interestingly always choose to tell. Instead, Player B's choice can be held constant at quiet. If Player A choses to quiet, he will receive one year of prison. If Player A chooses to tell, he will receive zero years in prison. In this scenario, Player A will also always choose to tell. This logic is concurrent for Player B, which results in the same strategy of both players telling.

2.2 Tit-for-Tat and Grim Trigger Solutions in Iterated Prisoner's Dilemma Game

The Nash Equilibrium model is used particularly for one-shot games in Classical Game Theory; however, different models are used for repetitive games. These include the tit-for-tat and grim trigger strategies, which are the currently the most widely accepted solutions to reoccurring games. A repetitive game indicates that players will interact repeatedly with the same payoffs and available choices. A distinguishing feature of the repetitive games is that the players have the opportunity to learn about both the other player and the game itself. This enables the player to update their strategy from game to game. An iterated Prisoner's Dilemma, a repetitive game, can be solved using both tit-for-tat and grim trigger.

The tit-for-tat strategy was introduced by Anatoni Rapoport and is noted for its simplicity in direct competition.[6] A player using tit-for-tat strategy will tend to initially cooperate. For example, in the Iterated Prisoner's Dilemma game, the players will initially choose not to tell on

[6] Nowak, M., Sigmund, K. (1993). A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game. *Nature*, 364, 56–58.

each other. The players will then duplicate their opponent's previous action. If the opponent previously chose not to tell, then the player will also choose not to tell. If the opponent previously chose to tell, then the player will also choose to tell. This tit-for-tat solution is shown in Table 2 and Table 3, where “T” indicates tell and “Q” indicates to remain quiet.

Table 2: Tit-for-tat solutions to an Iterated Prisoner’s Dilemma game when both player’s start with the same play. Vertical axis indicates the player’s identity and the horizontal axis indicates the trial number

	1	2	3	4	5
Player A	Q	Q	Q	Q	Q
Player B	Q	Q	Q	Q	Q

Table 3: Tit-for-tat solutions to an Iterated Prisoner’s Dilemma game when both player’s start with different play. Vertical axis indicates the player’s identity and the horizontal axis indicates the trial number

Trial Number	1	2	3	4	5
Player A	Q	T	Q	T	Q
Player B	T	Q	T	Q	T

This strategy heavily relies on the input of the opponent. A visual representation of the tit-for-tat strategy is shown in Figure 1.

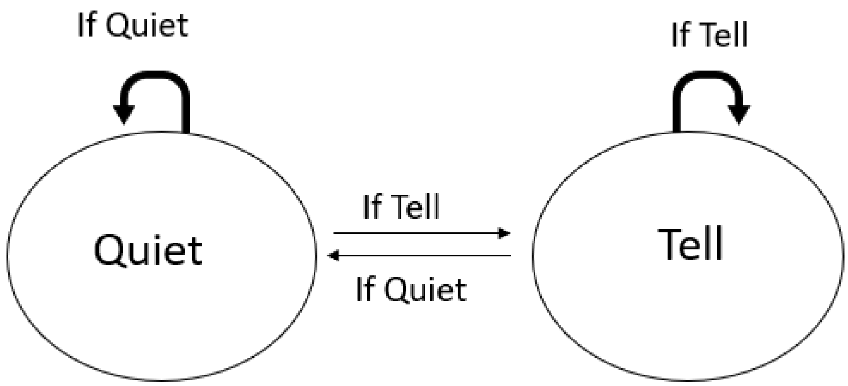

Figure 1: Visual representation of tit-for-tat strategy in an Iterated Prisoner's Dilemma game

Another popular strategy for iterated games is called grim trigger. [7] A player using grim trigger strategy will always cooperate at first. When his opponent defects, the player will then defect for the rest of the game. This indicates that one defect by the opponent triggers a permanent defection. Table 4 shows the solution to an Iterated Prisoner's Dilemma game using the grim trigger strategy.

Table 4: Grim trigger solutions to an Iterated Prisoner's Dilemma game when both players initially cooperate. Vertical axis indicates the player's identity and the horizontal axis indicates the trial number

	1	2	3	4	5
Player A	Q	Q	T	T	T
Player B	Q	T	T	T	T

[7] Chincarini, Ludwig B., Experimental Evidence of Trigger Strategies in Repeated Games (2003). *Rydex Working Paper*.

Chapter 3

Chemical Game Theory

3.1 Chemical Game Theory Addresses Shortcomings of Classical Game Theory

Chemical Game Theory utilizes a distinctive model to solve game theory problems, including Prisoner's Dilemma. Chemical game theory, or CGT, uses different methods than Classical Game Theory and arrives at different solutions. This model was created to address the apparent shortcomings of Classical Game Theory. A major shortcoming of Classical Game Theory is the assumption of rational players. [8] Many experimental players do not act rationally or act only in self-interest. There are many reasons why players may not choose the Nash equilibrium. Some these reasons could include an existing relationship between players, previous experiences with playing the game, or altruistic personalities. Due to this, CGT models each player as a reactor and the reactions are called "decision reactions". The entropy of each "decision reaction" causes the CGT solution to yield different results than Classical Game Theory. A reaction in Gibbsian thermodynamics includes an associated Gibbs free energy and an initial concentration. In CGT, the Gibbs free energy is correlated to the "pain" of the decision. A more negative Gibbs free energy of a decision reaction, or less "pain", will cause the reaction to move in a more favorable direction. The "initial concentration" is correlated to the pre-bias associated with a decision. In CGT, these possibilities and players are represented by "knowlecules", or metaphorical molecules. Figure 2 is a visual representation of these knowlecules in Player A's mind.

[8] Kaplow, L., Shavell, S. *Fairness Versus Welfare*. Harvard University Press. 2006.

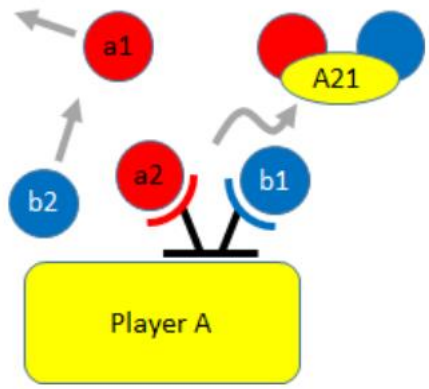

Figure 2: A21 is formed when knowlecules a2 and b1 react. A21 is an intermediate decision of Player A

Figure 2 shows one of four possible decision reactions that occur in Player A's mind. [9] The knowelecules are represented as a1, a2, b1, and b2 in Player A's brain. In a Prisoner's Dilemma game, a2 is Player A's pre-bias towards telling and b1 is Player A's perception of Player B's pre-bias towards staying quiet. These two knowlecules combine with "A", which is a knowlecule that holds all of Player A's personality and history. This process is aided with a catalyst to form the product of A21. This process occurs in parallel within Player B's mind, however, the product is B21. Instead, the knowlecule of "B" holds all of Player B's personality and history. The four products for Player A are: A11, A12, A21, B22. The four products for Player B are: B11, B12, B21, and B22. This process is further detailed in Figure 3.[10]

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[9] This figure was adopted from Velegol, D.; Suhey, P.; Connolly, J.; Morrissey, N.; Cook, L. Chemical Game Theory. Industrial & Engineering Chemistry Research 2018, 57 (41), 13593–13607.
[10] This figure was adopted from Jacob Scioscia, *Chemical Game Theory: Asymmetric Rock Paper Scissors Design Strategy*, Fall 2019 SHC thesis

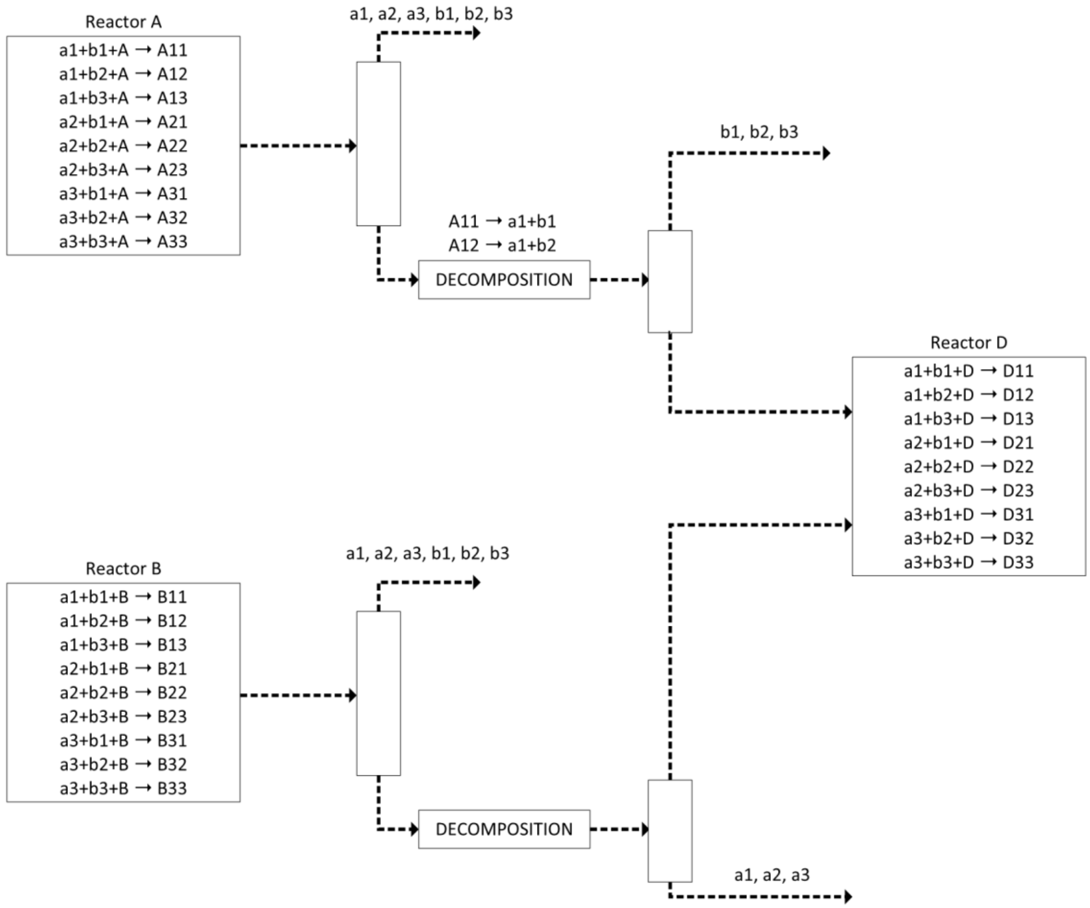

Figure 3: The block flow diagram of the decision reaction system.

Figure 3 shows that after reactor A and B form products, a decomposition step occurs. This causes the products to return into a1, a2, b1, and b2 knowlecules. These products then enter a third reactor that is the “decision reactor” or “Reactor D” to form the final products of D11, D12, D21, and D22. For the Prisoner Dilemma’s game, this reactor belongs to the final decider, the attorney general. The purpose of this reactor is to incorporate the influence of a third party. However, if there is no third party, the Gibbs free energies of each reaction are set to large negative quantities, which will ensure that the reactions will virtually go to full conversion. By doing this, “Reactor D” essentially has no influence on the final outcomes and the products can be calculated based on the concentrations of the reactants.

3.2 Perception Functions and Pain Values

In CGT, the pain values or Gibbs free energies are determined using a perception function. This perception function takes the form of a Weber-Fechner function and requires surveying from either the players or a representative sample population. [11] The purpose of using a perception function is to find a dimensional, scaled value of the pain or Gibbs free energy of a reaction decision for a player. Each player will not experience the same level of pain for each decision due to personality differences, which may include level of competitiveness. Equation 1 takes the form of the Weber-Fechner function and Equation 2 is in slope-intercept form,

$$p = p_0 ln\frac{s}{s_0} \qquad (1)$$

$$p = p_0 lns - p_0 lns_0 \qquad (2)$$

where p is pain and s is the variable that is causing this pain. Since Equation 2 is in slope-intercept form, y=mx+b, the following relationships apply.

$$x = lns$$

$$b = -p_0 lns_0$$

$$m = p_0$$

$$y = p$$

When collecting data from the players for p and s, it is possible to fit a line to find m and b. This can be done using linear regression analysis. When these values are determined, the perception function for pain is established, giving a more precise representation of the players. A perception

[11] Nutter, F.W., Esker, P.D. The Role of Psychophysics in Phytopathology: The Weber–Fechner Law Revisited. *Eur J Plant Pathol* **114**, 199–213 (2006).

function has similarities to a chemical potential in thermodynamics. This process is described in Appendix A.

3.3 Chemical Game Theory Solutions to One-Shot Prisoner's Dilemma Game

D11 in a Prisoner's Dilemma game indicates that Player A and Player B both stay quiet. D12 indicates that Player A stayed quiet while Player B told. D21 indicates that Player A told while Player B stayed quiet. D22 indicates that both Player A and Player B tell on each other. The final concentrations, D11, D12, D21, and D22 are normalized to represent the final probabilities of each decision. These probabilities are represented by y_{D11}, y_{D12}, y_{D21}, and y_{D22}. For a game with two players that have two choices, this system can be solved with Excel. The chemical game theory solution for the Prisoner's Dilemma game with pains of 0-1-2-3 is $y_{D11} = 0.523$, $y_{D12} = y_{D21} = 0.183$, and $y_{D22} = 0.111$.[12] These results indicate that the decision of both players staying quiet happens 52.3% of the time, the decision of one player staying quiet and the other telling happens 18.3% of the time, and the decision of both players telling on each other happens 11.1% of the time. This is different from Classical Game Theory that has one solution of both players telling, 100% of the time. These CGT results for a one-shot game show a more accurate correlation to experimental data than Classical Game Theory results. [13]

[12] Natalie Morrissey, Chemical Game Theory: Entropy in Strategic Decision-Making, Spring 2018 SHC Thesis
[13] Nash, J. Non-cooperative Games. *Annals of Mathematics* **1951**, 54 (2), 286-295.

3.4 Chemical Game Theory Solutions to Iterated Prisoner's Dilemma Game

Chemical Game Theory's solution to an iterative game uses the same methods, however, it incorporates an updating function. As a player learns from game to game, their initial concentrations or pre-biases towards a decision change. This change in concentrations for a decision shows that players learn about their opponent, themselves, and the game itself throughout trials. Chemical Game Theory utilizes process control to represent iterative games. This is shown by the process flow diagram in Figure 4.

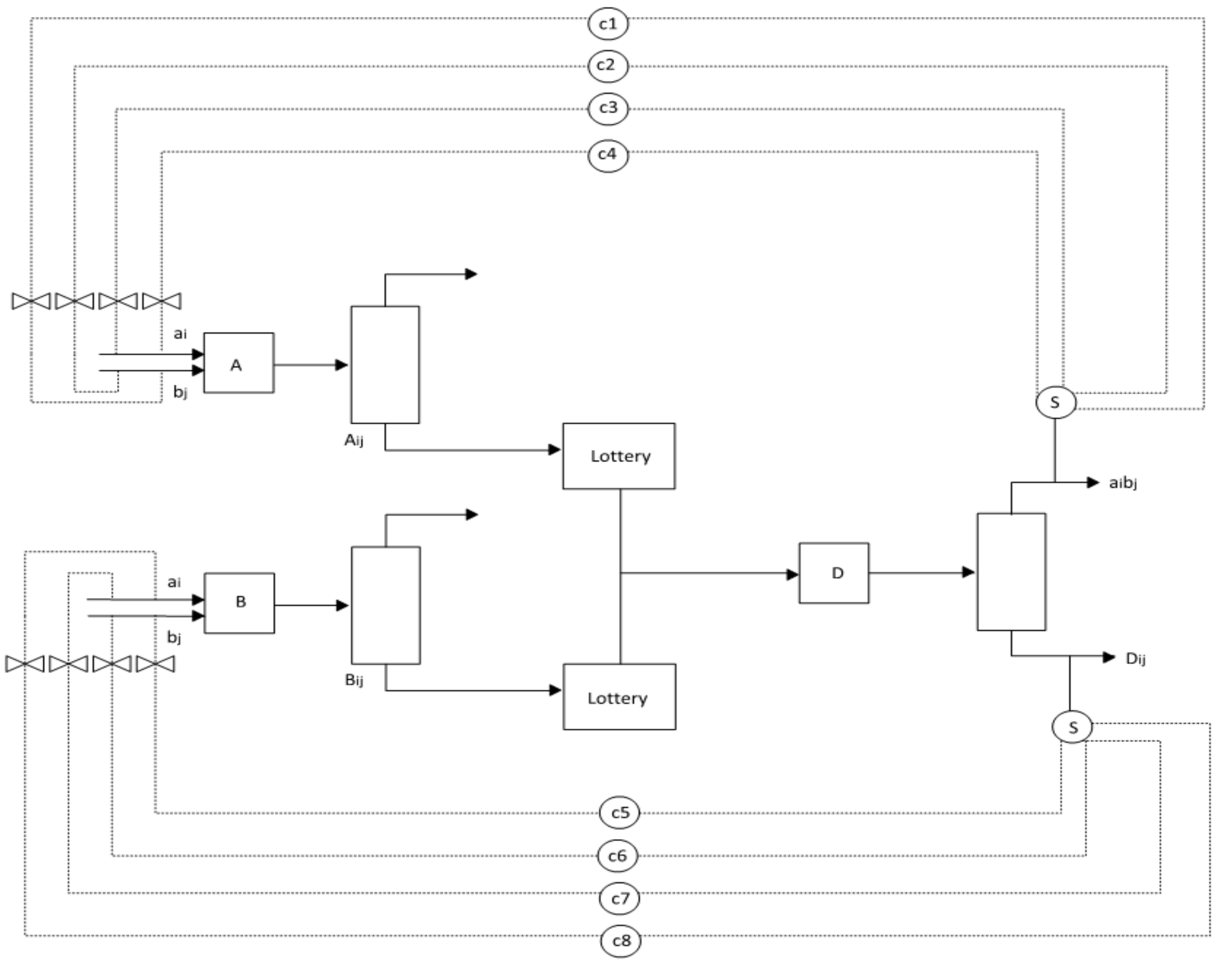

Figure 4: Chemical Game Theory Process Flow Diagram for Iterative Games and Updating Strategies

In Figure 4, Player A and Player B both have initial pre-biases and therefore both players have initial concentrations that enter their respective reactors. These knowlecules react to form A12, A21, A11, and A22 in reactor A and B12, B21, B11, and B22 in reactor B. It is implied that any unreacted reactants are removed from the product stream. These products then undergo a decomposition step in which they decompose back into a1, a2, b1, and b2 knowlecules. After this process, the knowlecules are fed into reactor D. The normalized concentrations of D12, D21, D11, and D22 are the final probabilities that each decision will occur. The final step of the system is a decomposition of D11, D12, D11, and D22 into a1, a2, b1, and b2. The concentrations of these knowlecules are then measured using a sensor. The sensor measures the error, which is the difference between the set point and the control variable. Depending on the magnitude of the error, the controllers signal a need for adjustment in the final control element. The final control elements are the valves that are positioned on the feed streams going into reactors A and B. When signaled, the valve will either increase or decrease its openness percentage. This is based on whether or not the concentration of a specific type of knowlecule needs to increase or decrease from the previous game. The use of process control in CGT allows the model to represent how humans learn from game to game. It is important to note that the set point for each player may differ based on personal interests and stakes in the game. This process control system is an example of a negative feedback loop, which utilizes the output of the system to reduce fluctuations and error. [14]

3.4 Thermodynamic Analogies in Chemical Game Theory

Analogies can be established between thermodynamics and Chemical Game Theory. Each thermodynamic property can be related to a factor within a game such as Prisoner's Dilemma. These definitions are shown in Table 5.

[14] J.F.MacGregor ; T.Kourti.(1995), Statistical process control of multivariate processes. *Control Engineering Practice*. *3,* 403-414.

Table 5: Chemical Game Theory definitions in terms of thermodynamic properties

Thermodynamics Term	Definition in CGT
Gibbs Free Energy	Pain associated with choosing a particular decision
Enthalpy	The player's utility
Entropy	The randomness of a decision based on external circumstances or the "fairness" in a game
Temperature	Excitement level or the inverse of choosiness
Extent of Reaction	The percentage of how often a player will choose a particular decision
Concentrations	Initial and final (pre-bias)
Chemical Potential	Perception function

Chapter 4

Definitions of Learning

4.1 Existing Definitions of Learning and Biology Comparisons

There are many existing definitions of learning in the areas of cognitive psychology, behavioral ecology, and machine learning. One specific definition of learning in psychology is that "learning refers to the process by which an animal (human or non-human) interacts with its environment and becomes changed by this experience so that its subsequent behavior is modified."[15] This suggests that the learning process is a change in behavior because of previous experiences, which is essentially the framework that CGT uses to define the learning.[16]

Additionally, the immune system is one of the most prevalent representations of learning in the human body. The human immune system specifically uses a negative feedback loop to maintain bodily functions. There are two main divisions of the immune system; one division is the adaptive immune system and the other is the innate immune system. The adaptive immune system will be specifically described in this paper because it illustrates how the body is learning based on its environment. The adaptive immune system both flags foreign invaders that are not initially recognized and eradicates them from the body. It will then produce antibodies that will be able to recognize the invader in a subsequent attack, resulting in a more efficient immune response. This response for identifying foreign invaders in the body is a result of trial and error of the linkages between B cell receptor sites and the invaders. After multiple iterations, the B cell adapts its receptor sites and effectively tags the invader for destruction. This phenomenon is an example of a negative feedback loop, reducing error in trials to reach the desired state. These antibodies are analogous to the "memory" of humans within the Chemical Game Theory model. It is hypothesized that the way in which players update their decision-making in strategic games is analogous to the learning of the human adaptive immune system.

Using process control in regard to biological phenomena is a relatively new study. There are therefore few experiments that have been performed. Despite this, Proportional-Integral control, or PI control, has been used as the most common control mode to represent biological

[15] Sih, A., Ferrari, M. C., & Harris, D. J. (2011). Evolution and behavioural responses to human-induced rapid environmental change. *Evolutionary applications*, 4(2), 367–387.
[16] Kadambari Prabakar, current CGT lab member

processes. This is because PI control incorporates delay times for these non-instantaneous processes. One experiment was performed to test the effectiveness of PI control within the context of protein expression.[17] In this experiment, the concentration of glucose and galactose were controlled to either promote or terminate protein expression until the desired amount of protein was produced. It was found that PI control was successful for a total of 24 hours in producing the desired quantity of proteins given a significant delay time and disruptions. Figure 5 represents how PI control operates within this experiment.

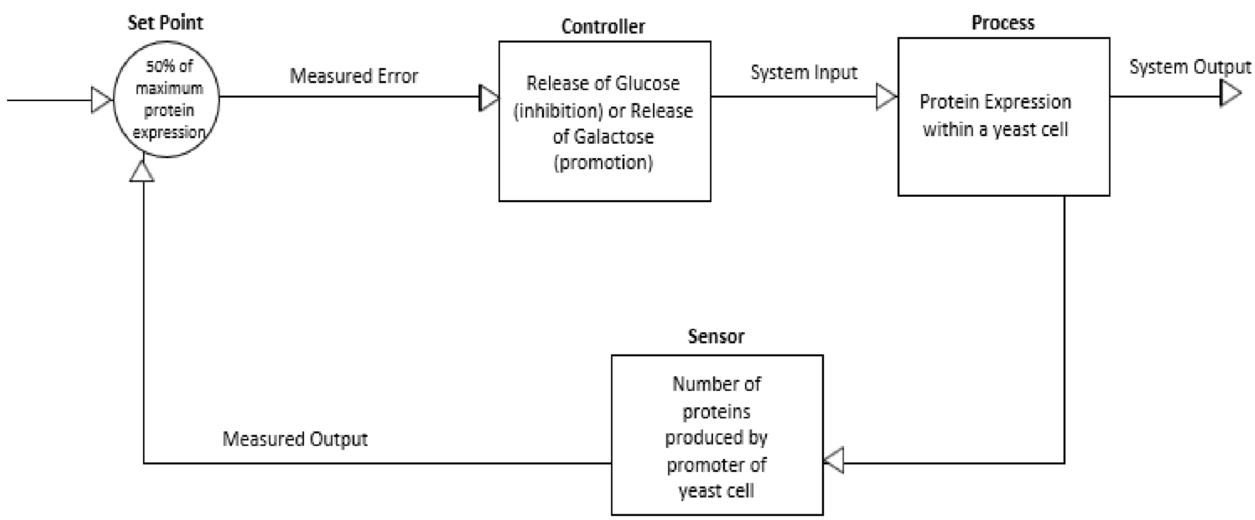

Figure 5: The process-integral control loop utilized for glucose and galactose expression

This use of PI control in a biological phenomenon will be further explored for the Chemical Game Theory model. It will be determined if PI control of initial concentrations is the most accurate type of control to represent the continuous process of learning.

[17] Menolascina, F., Fiore, G., Orabona, E., De Stefano, L., Ferry, M., Hasty, J., di Bernardo, M., & di Bernardo, D. (2014). In-vivo real-time control of protein expression from endogenous and synthetic gene networks. *PLoS computational biology*, 10(5).

4.1 Chemical Game Theory's Definition of Learning

The Chemical Game Theory model utilizes these biology comparisons to formulate a definition of learning. The definition of learning in this model is characterized by the following parameters. *First, changes in decision-making are based on knowledge drawn from previous experience.* This previous experience is defined as an individual's direct participation in an event or accumulated knowledge through observations. *Second, learning is indicated by several continuous changes in decision-making rather than a single permanent one*.[18] As knowledge is accumulated through previous experience, decision-making strategies adapt as a response to this new experience. *Third, individuals have a finite capacity for processing information.* This is described as bounded rationality, which accounts for an individual's limited cognitive capacity, with regards to both limitations of knowledge and computational capacity. [19] *Fourth, memory in the context of this model is the utilization of previous experience to influence current behavior.* After initial learning, humans utilize their memories to apply the knowledge, which can be considered relearning. *Finally, learning will be measured based on short-term timescale.*[20]

Chapter 5

Testing

5.1 Experiment Prompt

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[18] Ellie Alberti, current CGT lab member
[19] Simon, H. A.; "Models of Bounded Rationality." MIT Press, 1997
[20] Kadambari Prabakar, current CGT lab member

To test the accuracy of Chemical Game Theory's learning model in comparison to Classical Game Theory's learning models, an IRB (International Review Board) was approved to test 125 Penn State Students' ability to learn in a game that is similar to Iterated Prisoner's Dilemma. Portions of the IRB can be seen in Appendix C. A "lemonade stand" game was used as the game that the students would have to learn how to play. This game was run as a trial between a group of 3 students. The following prompt was given:

> You and your neighbor are both selling lemonade today and you set up your stands directly across the street from each other. Your neighborhood has a fixed number of customers and it has been determined that the quality of both of your lemonade is equal. You both start by selling your lemonade for the same price. As a seller, you have the option to either lower the price of your lemonade or keep it the same.

With this prompt, the following scenarios are possible:

1. Both players keep the same price and each get roughly 50% of the customers.
2. Player A lowers his price and Player B does not. Player A will attract more of the customers than Player B, and make more money
3. Player A keeps the price the same and Player B lowers it . Player A attracts less customers than Player B, and make less money
4. Both players lower their prices (assuming they both lower it by the same dollar amount). Both players get roughly 50% of the customers, but both their total profits are lower

Table 6 describes the "pain matrix" of this game. Note that the highlighted score goes with the left side or Player A, while the non-highlighted goes with the top or Player B. A negative score is more desirable. The goal is to have the least total pain.

Table 6: The pain matrix for each outcome of the "lemonade stand" game

	No change (B)	Lower price (B)
No change (A)	0, 0	+10, -10
Lower price (A)	-10,+10	+5, +5

The players were given the option to stop playing the game at any point. At the end of the game, each player was individually surveyed with the following prompt,

> Assume you could stop playing at any point during the game. If your score from the previous game were the value below, what is your probability of continuing to play? Remember, a more negative score is still more desirable.

The purpose of this survey was to create the perception function for each player, which enables the CGT solutions to be more accurate for each individual. Table 7 was presented to each player to fill out.

Table 7: After the final trial of the game, each player determines the probability of continuing to play the game after receiving a particular score

Your score of previous run	Probability of continuing to play (0-100%)
-15	
-10	
-5	
0	
5	
10	
15	

5.1 Running the Experiment

Three players are involved in this study: Player A, Player B, and Player C. The analysis applies specifically to Player A, who is learning from game to game. There are three trials that consist of 20 games each, indicating 60 games total are played. Player A and Player B play in the first trial. In the first trial, both players are able to see what their opponent played and their final score from each round. Figure 6 shows an example of how Player A enters his decision into the experiment spreadsheet and how he can view his score for the first trial. Player A enters "1" into the green cells to indicate keeping the same lemonade price or "'2" to indicate lowering the lemonade price.

Round 1
SUM (Games 1-20
120

	A	B	C	
1		1	2	-
2		1	2	-
3		2	2	-
4		1	2	-
5		1	2	-
6		2	2	-
7		1	2	-
8		1	2	-
9		2	2	-
10		1	2	-
11		1	2	-
12		2	2	-

Figure 6: Player A's perspective when playing the "lemonade game" for the first trial

For the next trial, Player A and Player B play the exact same game, however, they do not have access to the information of what their opponent's play was. Each player sees a note that reads "played" rather than an actual decision from his opponent. Additionally, the players are unable to see their final score from this trial. The point of this trial is to gather a general understanding of each player's strategy when they cannot learn or tell what their opponent's strategy is. Figure 7 shows how player A enters his decisions to the spreadsheet during the second trial and what he sees as he plays.

	A	B
21	2	played
22	2	played
23	1	played
24	2	played
25	2	played
26	1	played
27	2	played
28	2	played
29	1	played
30	2	played
31	2	played

Figure 7: Player A's perspective when playing the "lemonade game" for the second trial

For the third trial, Player A and Player C play each other. They are both able to see their opponent's strategy as they play the game, as well as their score. Figure 8 illustrates what Player A views for the third trial in the spreadsheet.

	A	B	C		Round 3
42	2	-		1	SUM(Games 41-60)
43	2	-		2	-45
44	2	-		2	
45	2	-		1	
46	2	-		1	
47	2	-		2	
48	2	-		1	
49	2	-		2	
50	1	-		2	
51	1	-		1	
52	1	-		1	

Figure 8: Player A's perspective when playing the "lemonade game" for the third trial

5.2 Analysis of Experimental Data

The first step of the analysis was to create a perception function for each individual player. This was possible given the data collected from the survey at the end of the experiment. Three separate perception functions were created for each respective player, allowing the Chemical Game Theory model to use an accurate representation of each player. With these perception functions, each player's "real" pain was assessed from the original values displayed in the pain matrix. These "real" pains are used in the CGT solver as the Gibbs Free Energies.

The next step of the analysis was to find the initial concentrations or pre-bias of each player. This is a component of the game that remains unknown and must be solved for. Since all other variables are defined, Excel Solver can be used to find the initial concentrations of each player. This a new finding for Chemical Game Theory and a method for determining someone's early thoughts entering a game. Figure 9 is a visual representation of how these concentrations were found.

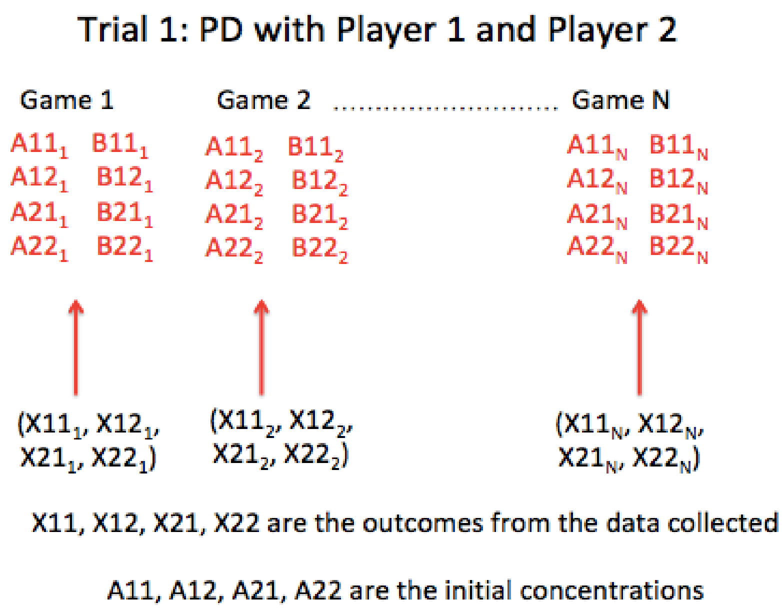

Figure 9: Representation of the analysis of experimental data for the first trial

This method of analysis illustrated in Figure 9 works backward instead of forward. The outcomes of the game are used to solve for the initial concentrations. This can be done from the first game to the last game. A change of initial concentration indicates that the player is learning or changing behavior based on the results of the previous game and their environment.

The analysis for the second trial is consistent with the first trial. The only difference between these trials is that the final outcomes are unknown to both players. This method of analysis is described in Figure 10.

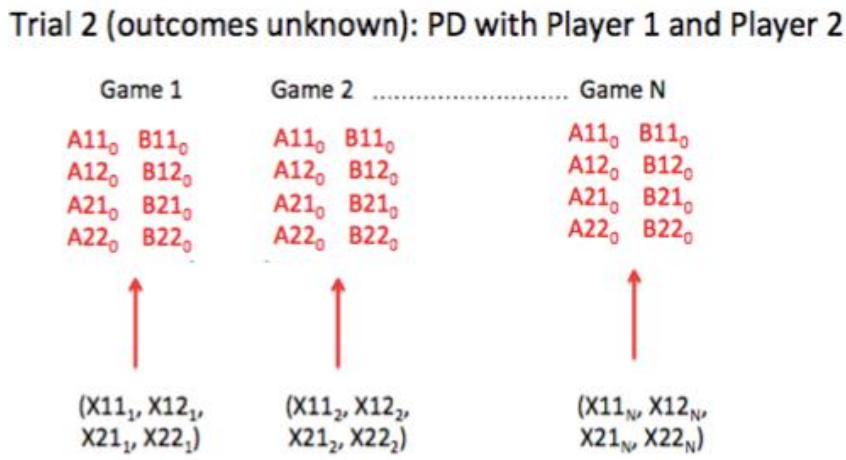

Figure 10: Representation of the analysis of experimental data for the second trial

Since player A and player B do not have any true interaction or effect on one another, it is hypothesized that the players will show no learning throughout this period. There initial concentrations should be similar or the same as the initial concentrations at the end of the first trial. The intention of this trial is to see what strategy Player A has after he or she has learned from the first trial. This hypothesis is visually represented in Figure 11**,** where each game in the second trial will lead to the same initial concentration when no learning occurred.

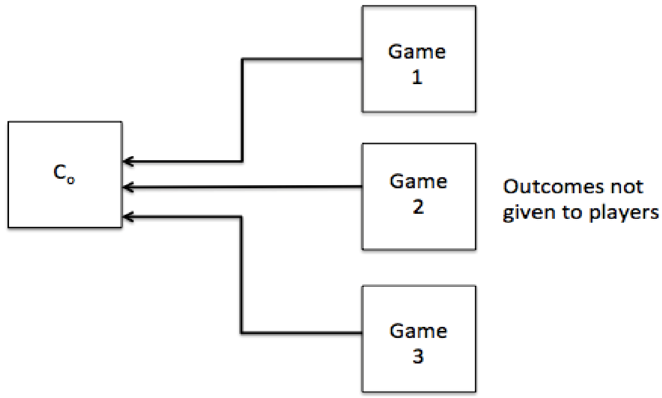

Figure 11: Representation of the hypothesis that the initial concentrations of a1, a2, b1, and b2 for Player A will not change when no learning occurs

The analysis used for the third trial differs slightly from the other two. Instead of working backwards for the initial concentrations, the final concentrations of decisions are solved for. This trial is between player A and player C. Player A is expected to have learned from the previous trials and have some insights about how the game works. This gives Player A an advantage over Player C. It is assumed that Player C goes into this trial with an initial concentration of 50-50 between lowering their price or keeping it the same. Therefore, the initial concentrations for Player A from the second trial are used in the CGT solver and the initial concentration of 50-50 is used for player C in the CGT solver. The CGT solver calculations can be seen in Appendix B. The results are then calculated and compared to the data collected from the experiment. The final probabilities, or normalized concentrations of D12, D21, D11, and D22 are expected to be similar to what is seen from the outcomes of the experimental data. This will determine the accuracy of the CGT model and the accuracy of the model's representation of learning. Figure 12 illustrates this analysis for the third trial.

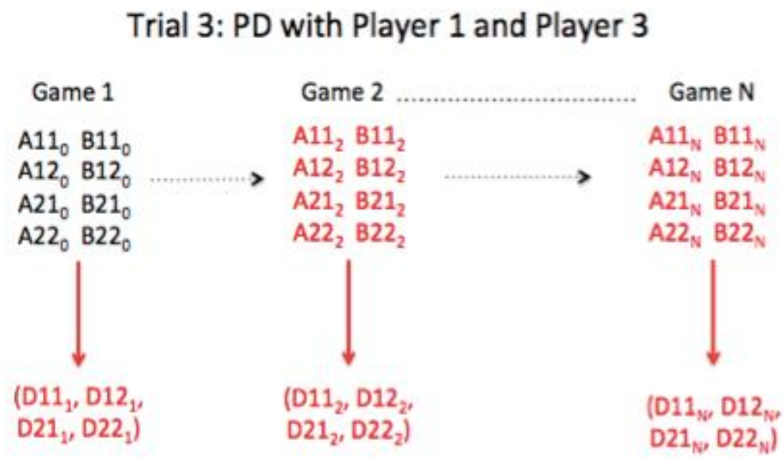

Figure 12: Representation of the analysis performed for the third trial, predicting the final outcomes of the player's decisions

5.3 Results and Discussions

A new "pain matrix" was made using the perception functions for each player. These new pain matrices show the "real" pain that each player feels when receiving a certain amount of money. Due to different personal experiences, these values can vary greatly. Table 8 and Table 9 show these differences in pain for each player.

Table 8: Pain matrix of the "real" pains for Player A and Player B for the first and second trials of the experiment

	Player B keeps the same price	Player B lowers the price
Player A keeps the same price	-1.01, -0.83	-0.99, -0.86
Player A lowers the price	-1.04, -0.82	-1.00, -0.82

Table 9: Pain matrix of the "real" pains for Player A and Player C for the third trial of the experiment

	Player C keeps the same price	Player C lowers the price
Player A keeps the same price	-1.01 -0.38	-0.99, -0.46
Player A lowers the price	-1.04, -0.33	-1.00, -0.35

This data indicates that Player A receives the greatest utility from winning and Player C receives the highest pain for losing.

Table 10 shows the initial concentrations that were calculated for Player A and Player B in the first trial, using the CGT solver and final outcomes of the experimental data.

Table 10: Initial concentrations solved for Player A and Player B for the first trial

Initial Concentrations	Player A	Player B
a1	0.56	0.50
a2	0.24	0.50
b1	0.10	0.00
b2	0.90	1.00

This data clearly shows that Player A has an initial pre-bias towards not changing the price and believes that Player B has a strong propensity towards lowering the price. Player B, however, believes that Player A does not have any pre-bias towards one decision and is very biased towards lowering the price of the lemonade. In the second trial, Player A reveals her new strategy after learning how Player B plays the game. The initial concentrations from the second trial were again solved for in the same method as the first trial. This is shown in Table 11.

Table 11: Initial Concentrations Solved for Player A and Player B for the second trial

Initial Concentrations	Player A	Player B
a1	0.24	0.50
a2	0.56	0.50
b1	0.00	0.00
b2	1.00	1.00

This data from Table 11shows learning. Player A illustrates that she learned Player B's strategy by changing her initial concentrations of b1 and b2. She now perceives that Player B will always lower the price of lemonade. She then responded to this learning by changing her own initial concentrations of a1 and a2, decreasing a1 and increasing a2. When Player A began playing Player C, she had more insight into the game than when she started. Table 12 shows the initial concentrations that were assumed for the analysis of the third trial.

Table 12: Assumed concentrations from the third trial between Player A and Player C for the CGT solver

Assumed Initial Concentrations	Player A	Player C
a1	0.24	0.50
a2	0.56	0.50
b1	0.00	0.50
b2	1.00	0.50

The assumption for Player A is that she keeps her initial concentrations from the second trial. Since there has been no further learning within the second trial, these concentrations are expected not to change. The assumption for Player C is that he does not have any preference towards keeping the lemonade price or lowering. He has not played the game yet and therefore should not have any specific intuitions into the game or how Player A makes decisions. With these assumptions, the initial concentrations were inserted into the CGT Solver and the final CGT results are given in Table 13.

Table 13: Final concentrations calculated using the CGT Solver between Player A and Player C

Strategies	Final Concentrations
D11	0.15
D12	0.15
D21	0.34
D22	0.35

These are the solutions from the CGT Solver. They are then compared to the results seen from experimental data, which is shown in Table 14.

Table 14: Final Concentrations from Experimental Data between Player A and Player C

Strategies	Final Concentrations
D11	0.10
D12	0.15
D21	0.45
D22	0.30

These results show that the CGT results can explain the experimental data results. Both CGT and experimental data show that both players will not choose to keep the same price very often. Additionally, both methods show that the outcome of Player A choosing to keep the price the same and Player C choosing to lower the price does not happen often. One discrepancy between the CGT solutions and the experimental data is that the CGT solutions predict that both players will lower the price most of the time. Contrary to this, the experimental data shows that the outcome of Player A lowering the price and Player C keeping the price the same happens most often. This difference could be contributed to the assumption that Player C has no pre-bias to one decision over the other. To make the CGT solutions more accurate, an analysis of Player C's initial concentration should be executed. This is something that can be continued in future CGT work. Table 15 is aimed to compare the results of this game from Classical Game Theory, Chemical Game Theory, and the experimental data.

Table 15: Comparison between the solutions of Classical Game Theory, Chemical Game Theory, and the experimental data. RMS indicates root mean square

Final Concentrations	CGT Method	Grim Trigger	Tit-for-tat	Experimental Data
D11	0.15	0.00	0.25	0.10
D12	0.15	0.00	0.30	0.15
D21	0.34	0.00	0.25	0.45
D22	0.35	1.00	0.20	0.30
RMS of Error	**13.1%**	**85.1%**	**30.8%**	**0.00%**

Table 15 indicates that the CGT solutions are most similar to the experimental data with a 13% error. This indicates that the accuracy of Chemical Game Theory is more precise than the solutions from Classical Game Theory in terms of learning. This can be contributed to the inclusion of pains and priors within the CGT model, which does not exist in the Classical Game Theory. This experiment will be run multiple times to ensure that these results and conclusions are consistent and reproducible.

Chapter 6

Conclusions

The findings in this thesis indicate that Chemical Game Theory yields solutions that reflect experimental data. In comparison to Classical Game Theory, Chemical Game Theory showed the smallest error in comparison to the experimental data. The total error of CGT solutions for the “lemonade game” was 13%. Tit-for-tat strategy proved to be a more accurate strategy than grim trigger for this experiment with an error of approximately 31%. The process control learning model of Chemical Game Theory explained experimental data better than Classical Game Theory learning models for this particular game.

Chemical Game Theory continues to solve various types of games that apply beyond just game theory. Future work for this model includes solving problems such as investing in the stock market, creating public health policies, negotiations with employers, etc. Since Chemical Game Theory measures various parameters of each game or problem, it is versatile and adaptable to almost any situation. Future work will be to determine how many pieces of information humans can retain within a short time frame. Bounded rationality will be incorporated into the Chemical Game Theory model. Additionally, temperature will be studied, and its affects within the CGT reactors. The game is also intended to be extended to n-number of possibilities with n-number of players. With the approval of the IRB application, the experimental data will be expanded. There will be approximately 125 players and 40 games to analyze. With this analysis, the accuracy of Chemical Game Theory will be further tested and improved.

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Appendix A
Perception Function Calculation

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.9491244294
R Square	0.9008371824
Adjusted R S	0.881004618
Standard Err	0.110760359
Observation	7

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.55723214	0.55723214	45.4221251	0.00109090
Residual	5	0.06133928	0.01226785		
Total	6	0.61857142			

	Coefficients	*tandard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 90%*	*Upper 90%*
Intercept	-1.0875	0.07547046	-14.4096107	0.00002903	-1.2815030	-0.8934969	-1.23957663	-0.9354233
X Variable 1	0.028214285	0.00418634	6.73959384	0.00109090	0.01745293	0.03897563	0.01977859	0.03664997

Figure 13: Linear regression used to find coefficients for perception function

p = a + b ln s		
	value	variation
a	-1.0875	0.15207663
b	0.02821428	0.00843569

Figure 14: Values of coefficients with variation for perception function

p	s	ln s	p predict
1	0.001	-6.90775527	-1.282397381
0.95	5	1.60943791:	-1.042090859
0.8	10	2.30258509:	-1.022534206
0.75	15	2.70805020:	-1.011094298
0.7	20	2.99573227	-1.002977554
0.3	25	3.21887582!	-0.996681717
0.15	30	3.40119738:	-0.991537645
	5	1.60943791:	-1.042090859
	15	2.70805020:	-1.011094298
	20	2.99573227	-1.002977554
	25	3.21887582!	-0.996681717

Figure 15: Perception function used to find the "real" pain or "p predict"

Appendix B

REACTOR A						
	extents	e1	e2	e3	e4	sum |err|
	linear	-5.5E-11	0.10045697	-1.8E-11	0.22826	5.56988
	add	0		mult	1	

stoichiometric coefficients

M's	rxn 1	rxn 2	rxn 3	rxn 4
a1	-1	-1	0	0
a2	0	0	-1	-1
b1	-1	0	-1	0
b2	0	-1	0	-1
A11	1	0	0	0
A12	0	1	0	0
A21	0	0	1	0
A22	0	0	0	1
inert	0	0	0	0
$\Sigma\nu_i$ =	-1	-1	-1	-1

Δg^0 / RT	-1.01109	-0.9966817	-1.04209	-1.00298
Δh^0 / RT	-1.01109	-0.9966817	-1.04209	-1.00298
K_{t0} =	2.74861	2.70927675	2.83514	2.72639
K_{tT} =	2.74861	2.70927675	2.83514	2.72639
K_{MB} =	6.25049	1.48902859	4.98781	1.49533
|err Kt / Kmb|=	0.56026	0.81949277	0.43159	0.82327
|err Kmb / Kt| =	1.27406	0.45039628	0.75928	0.45154

SP	Initial	Change	End	Y frxn	y check
a1	0.24999	-0.10046	0.14953303	0.1005	0
a2	0.5666	-0.22826	0.33834022	0.2274	0
b1	0	7.2E-11	7.2401E-11	4.9E-11	0
b2	1	-0.32872	0.67128325	0.45117	0
A11	1E-10	-5.5E-11	4.5481E-11	3.1E-11	0
A12	1E-10	0.10046	0.10045697	0.06752	0
A21	1E-10	-1.8E-11	8.2118E-11	5.5E-11	0
A22	1E-10	0.22826	0.22825978	0.15341	0
inert	1E-10	0	1E-10	6.7E-11	0

Figure 16: CGT Excel solver used for Reactor A

REACTOR B						
	extents	e1	e2	e3	e4	sum |err
	linear	0.0734887	0.07828	0.07285	0.07254	2.92483
	add	0		mult	1	

stoichiometric coefficients

M's	rxn 1	rxn 2	rxn 3	rxn 4
a1	-1	-1	0	0
a2	0	0	-1	-1
b1	-1	0	-1	0
b2	0	-1	0	-1
B11	1	0	0	0
B12	0	1	0	0
B21	0	0	1	0
B22	0	0	0	1
inert	0	0	0	0
$\Sigma\nu_i$ =	-1	-1	-1	-1

Δg^0 / RT	-0.3756872	-0.4612	-0.33593	-0.3533
Δh^0 / RT	-0.3756872	-0.4612	-0.33593	-0.3533
K_{t0} =	1.4559916	1.58597	1.39924	1.42375
K_{tT} =	1.4559916	1.58597	1.39924	1.42375
K_{MB} =	1.0161062	1.09616	0.98922	0.99755
|err Kt / Kmb|=	0.4329128	0.44684	0.41449	0.42725
|err Kmb / Kt| =	0.3021208	0.30884	0.29303	0.29935

SP	Initial	Change	End	Y frxn	y check
a1	0.5	-0.1517647	0.34824	0.2045	0
a2	0.5	-0.1453916	0.35461	0.20824	0
b1	0.5	-0.1463421	0.35366	0.20769	0
b2	0.5	-0.1508142	0.34919	0.20506	0
B11	1E-10	0.0734887	0.07349	0.04316	0
B12	1E-10	0.0782761	0.07828	0.04597	0
B21	1E-10	0.0728535	0.07285	0.04278	0
B22	1E-10	0.0725381	0.07254	0.0426	0
inert	1E-10	0	1E-10	5.9E-11	0

Figure 17: CGT Excel solver used for Reactor B

REACTOR D					
	extents	e1	e2	e3	e4
	linear	0.02348893	0.02421	0.053371885	0.055
	normal	0.15050214	0.1551	0.341973148	0.35242
	add	0		mult	1

stoichiometric coefficients

M's	rxn 1	rxn 2	rxn 3	rxn 4
a1	-1	-1	0	0
a2	0	0	-1	-1
b1	-1	0	-1	0
b2	0	-1	0	-1
D11	1	0	0	0
D12	0	1	0	0
D21	0	0	1	0
D22	0	0	0	1
inert	0	0	0	0
$\Sigma\nu_i$ =	-1	-1	-1	-1
Δg^o / RT	-20	-20	-20	-20
Δh^o / RT	-20	-20	-20	-20

	rxn 1	rxn 2	rxn 3	rxn 4
K_{t0} =	485165195	4.9E+08	485165195.4	4.9E+08
K_{tT} =	485165195	4.9E+08	485165195.4	4.9E+08
K_{MB} =	1.01610622	1.09616	0.989219573	0.99755
|err Kt / Kmb|=	477474879	4.4E+08	490452481.5	4.9E+08
|err Kmb / Kt| =	1	1	0.999999998	1

SP	Initial	Change	End	Y frxn	y check
a1	0.100457	-0.04769565	0.05276	0.112305276	0
a2	0.2282598	-0.10837475	0.11989	0.255181669	0
b1	0.1463421	-0.07686081	0.06948	0.147894711	0
b2	0.1508142	-0.07920958	0.0716	0.15241419	0
D11	1E-10	0.02348893	0.02349	0.049997435	0
D12	1E-10	0.02420672	0.02421	0.05152529	0
D21	1E-10	0.05337189	0.05337	0.1136049	0
D22	1E-10	0.05500286	0.055	0.117076527	0
inert	1E-10	0	1E-10	2.12855E-10	0
S	0.6258731	-0.1560704	0.4698	1	0

Figure 18: CGT Excel solver used for Reactor D

Appendix C
Parts of IRB Application

1.0 Objectives

1.1 Study Objectives

Describe the purpose, specific aims or objectives. State the hypotheses to be tested.

There are two main purposes of this study. The first is to be able to improve the current Chemical Game Theory (CGT) model by including a learning function based on how people update their plays, or decisions, in repeated strategic games. The CGT model uses rigorous chemical engineering principles to predict strategies of players when they play games. Currently, the model assumes that the player does not update their strategy based on their payoffs, or "pains" from the previous rounds if a game is played multiple times. However, if a game is played more than once, it is likely that the players of the game will have updated the way they play the game based on the previous round(s). We wish to account for that updating process. The second is to calculate pre-bias, or a person's exogenous probability, which is the bias that they have toward or against one or more of the decisions that they can make in a strategic game. We would like to test the hypothesis that, in games without loaded words, or words with strong connotations, representing each possible decision, that players have a pre-bias equal to 100 divided by the number of possible decisions. This assumes that people do not have strong preferences toward decisions in games that they are completely unfamiliar with. Additionally, we would also like to test whether a player can learn how to reach the outcome of the game that is "best", or least painful for them. This "best" outcome is determined by a pain matrix, and can be calculated using CGT, as well as the Nash Equilibrium for comparison purposes.

Figure 19: Main study objectives for experiment

2.0 Background

2.1 Scientific Background and Gaps

Describe the scientific background and gaps in current knowledge.

For clinical research studies being conducted at Penn State Health/Penn State College of Medicine, and for other non-PSH locations as applicable, describe the treatment/procedure that is considered standard of care (i.e., indicate how patients would be treated in non-investigational setting); and if applicable, indicate if the study procedure is available to patient without taking part in the study.

A current gap in our knowledge includes determining a player's initial pre bias before they begin a game. A player may be more inclined to choose a specific decision based on past experiences or current circumstances. It is important that each individual player's pre bias is accurately determined before the game because it will have a significant impact on the way the individual plays. Additionally, a gap in our current knowledge is how players learn or update their decisions during the game. This is essentially how players learn throughout the game. Each player can change their strategy from game to game, which will enable the creation of a learning function for each player. The learning function for each individual will specifically depend on how the player updates their concentrations during the game.

Figure 20: Background and gaps in current CGT model

2.2 Previous Data

Describe any relevant preliminary data.

Preliminary data has been collected based on the Prisoner's Dilemma game, as well as some data that we've collected on the perception functions. This data was used to compare the accuracy between traditional game theory and chemical game theory.

2.3 Study Rationale

Provide the scientific rationale for the research.

Currently, there is no way to determine initial pre bias and the learning function in Chemical Game Theory. This study will enable the addition of both these parts to the current model. Additionally, current economic models mainly utilize "Tit for Tat" and "Grim Trigger" strategies for repetitive games. Unlike Chemical Game Theory, these methods do not quantify pre bias or learning. This is a distinguishing factor of Chemical Game Theory that will create more accurate predictions of final decisions. It is hypothesized that these predictions from Chemical Game Theory method will be more representative of human decision-making than both "Tit for Tat" and "Grim Trigger" methods after finding the pre bias and learning function.

Figure 21: Previous data collected and study rationale

ACADEMIC VITA

The Pennsylvania State University, Schreyer Honors College
Bachelor of Science in Chemical Engineering
Graduation Date: Spring 2020

PROFESSIONAL EXPERIENCE:

Position: **ExxonMobil Upstream Process Engineering Intern** ***May 2019- August 2019***

- Created hydraulic model of the gas gathering system to identify sites with shut-in oil production
- Created AFE deliverables to install a line loop that will recover 60 barrels of oil per day
- Collaborated with IT group to design and develop an interface that enables the detection of leaks in the oil pipeline

Position**: Linium Consulting Intern** ***Summer of 2017 and 2018***

- Conducted market research on Linium's customer base to improve efficacy of marketing strategies
- Created PowerPoint slides and other marketing material to assist in sales meetings
- Collaborated with cross functional teams across the organization to help increase efficiency with operations and sales

LEADERSHIP AND INVOLVEMENT:

American Institute of Chemical Engineers

Community Outreach Chair ***August 2018-May 2020***

- Organize and schedule all service events for AICHE with local schools and community businesses
- Maintain and develop relationships with schools and businesses
- Spearhead Haunted University and Exploration University for middle school STEM students

PSU Red Cross Club

Official Onsite Coordinator and Corporate Sponsorship Representative ***August 2016- May 2020***

- Oversee all operations of blood drive including delegating tasks to volunteers, coordinating with the American Red Cross, setting up, enforcing the appropriate processes, and managing the schedule

RESEARCH: ***January 2018- May 2020***

Team member of the research group focusing on Chemical Game Theory under the supervision of Penn State Professor

- Utilize Chemical Game Theory as a decision-making algorithm to solve problems in differentiation
- Collaborate with fellow researchers to enhance team building skills

THE BUSINESS EXPERIENCE PROGRAM FOR UNDERGRADUATES:

One of ten engineering students chosen through a selective application process to participate in the BEU program. I created business plans geared toward technology-based products and gained a working knowledge of traditional and non-traditional ways for identifying new business opportunities.

AWARDS AND HONORS:

The Endowment for the Chemical Engineering Department Undergraduate Award 2019 and 2020

- This scholarship is anonymously given to recognize academically talented students and help defray the costs of attending college