THE PENNSYLVANIA STATE UNIVERSITY
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DEPARTMENT OF RISK MANAGEMENT

PRICING JOINT-LIFE INSURANCE UNDER DEPENDENT ASSUMPTION

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ABSTRACT

Actuaries make multiple assumptions such as the mortality assumption when pricing insurance. Studying the mortality of the population is important since the price of the same insurance product can vary depending on the mortality assumption. This paper will first introduce different mortality assumptions and how they affect the price of insurance, especially joint-life insurance. Joint-life insurance is insurance with coverage of two or more people and its death benefit is paid when the first death occurs. The most popular form of joint-life insurance is insurance for couples, often married. Therefore, this paper will focus on joint-life insurance that covers couples. Actuaries use joint-life mortality when pricing joint-life insurance. Joint-life mortality is more complicated and less developed than single life mortality. To simplify the complex calculations of joint-life mortality, many actuaries today assume that the deaths of couples are independent. However, recent studies show that they may not be completely independent but somewhat dependent. A close examination of the correlation that exists between couples' deaths can help insurance companies to improve their pricing of joint-life insurance products.

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Chapter 1 – Introduction

Why Study Mortality Rates?

When selling life insurance, insurance companies are uncertain when they will need to pay out death benefits. They try to forecast when the deaths of the policyholders will occur by studying the mortality of the population. Accurate predictions give companies two advantages. First, they help calculate more accurate reserves – the money pooled in order to make future payments. While companies aim to have sufficient funds to make future payments, they also try to avoid excessive funds. Second, accurate predictions allow companies to price their insurance products appropriately. Pricing the products properly is important since underpricing or overpricing can put the company at a disadvantage. For example, if an insurance company underestimates the mortality rates of the policyholders and therefore underprices the insurance products, it may result in insufficient funds; there may be more deaths and therefore more benefits to pay out than expected. On the other hand, if the company overestimates the mortality rates and overprices its products, its products may become uncompetitive; people will buy cheaper products provided by different companies. In conclusion, it is critical to study the mortality rates so that the insurance company can forecast when the deaths of policyholders will occur. Accurate predictions let insurance companies not only calculate the reserves precisely but also price their insurance products appropriately.

Pricing Insurance

There are various factors that affect the price of insurance such as the type of insurance, benefit amount, interest rates, age of policyholder, and mortality assumption. Also, there are discrete insurance calculations and continuous insurance calculations. Discrete calculations assumes that death benefits are paid at the end of year of death while continuous calculations are based on paying out death benefits immediately upon death, which is the way it is done in the real world. Discrete calculations are simpler than continuous calculations. Therefore, we will focus on pricing discrete insurance for simplicity. The following is a general formula for calculating the price of insurance using the discrete calculation:

$$\sum_{t=1}^{\omega-x} \boldsymbol{Death\ Benefit\ at\ Time\ t} * \boldsymbol{Present\ Value\ Factor\ at\ Time\ t} * \boldsymbol{Probability\ of\ Death\ during\ Year\ t}$$

Second, the price of insurance depends on the benefit amount; insurance with larger benefits cost more. The amount of benefit is settled at the time of purchase. It may vary depending on when the death occurs. However, we assume that it is constant at $1 to allow simpler calculations.

Third, interest rates also affect the price of insurance since interest rates determine present value factors. The present value factor is used to calculate the present value of future benefits. Even though we assume that the benefit is constant, the present value of the benefit depends on when it is paid. Because of inflation, the values of a dollar from 2020, 2030, or 2040 would vary today; therefore, so would the value of the benefit. The present value factor for payment at the end of year *t* has a generalized formula:

$$PVF_t = \frac{1}{(1+i_1)} * \frac{1}{(1+i_2)} * \frac{1}{(1+i_3)} * \dots * \frac{1}{(1+i_t)}$$

where i_t represents an annual interest rate in year *t*. However, in this paper, in order to simplify the present value calculations, we assume that the annual interest rate is constant at 6%. Therefore, the present value factor for year t becomes:

$$\boldsymbol{PVF_t} = \frac{1}{(1+0.06)} * \frac{1}{(1+0.06)} * \frac{1}{(1+0.06)} * \dots * \frac{1}{(1+0.06)} = \boldsymbol{\frac{1}{(1+0.06)^t}}$$

Given that the death benefit is $1 and the interest rate is 6%, the pricing formula for discrete insurance can be written as:

$$\sum_{t=1}^{\omega-x} \$\boldsymbol{1} * \boldsymbol{\frac{1}{(1+0.06)^t}} * \boldsymbol{Probability\ that\ Death\ Occurs\ during\ Year\ t}$$

The price of insurance also depends on the age of a policyholder, denoted by *x*. The maximum possible age is often denoted by ω (the Greek letter omega) in the formula. Typically, older people have higher mortality rates and the benefits are paid sooner. Therefore, the price tends to be higher for older customers.

Lastly, the mortality assumption affects the price. There are multiple mortality assumptions such as the simple but stylized DeMoivre mortality table and the real world CSO mortality table. Different mortality assumptions result in different probabilities that death will occur during year *t*; therefore the price depends on the mortality assumption.

So far, we have only talked about pricing insurance when the insurance is for a single life. There is also joint-life insurance with coverage of multiple lives. The most popular joint-life insurance is for two people, most often spouses. Joint-life insurance pays the death benefit when the first death occurs. If both of them die at the same time, the death benefit will be made at the time of their deaths to the secondary beneficiary such as their children. The joint-life insurance lets the family financially maintain their lives even after a loss.

Another type of multi-lives insurance is last survivor insurance, which pays the benefit when the last death occurs. The family gets the benefit when the second death occurs regardless of when the first death occurs. Throughout this paper, we will focus on joint-life insurance where families get the benefits when the first death occurs.

With joint-life insurance, the mortality of multiple lives is often more complicated than that of single life insurance. Similar to the single life insurance pricing formula, the generic formula for pricing a joint-life insurance is as follows:

$$\sum_{t=1}^{\min(w-x,w-y)} \textit{DB at Time } t * \textit{PVF at Time } t * \textit{Probability that First Death Occurs during Year } t$$

The boundary for the summation has changed to minimum of (ω-x) and (ω-y), which represents the time when the first death occurs given the male age *x* and the female age *y* at the time of purchase. We will continue to assume that the death benefit is $1 and the interest rate is 6% each year. We assume that all the couples are heterosexual and both are at the age of 60 at the time of purchase. We can write the formula as:

$$\sum_{t=1}^{\min(100-60,100-60)} \$1 * \frac{1}{(1+0.06)^t} * \textit{Probability that First Death Occurs during Year t}$$

The probability that first death occurs during year *t* is denoted by ${}_{t-1|}q_{x:y}$ where *x* and *y* represent ages of male and female. The probability can be calculated by subtracting the probability that both male and female survive for *t* years from the probability that both survive for (t-1) years. Those probabilities are denoted by ${}_{t}p_{60:60}$ and ${}_{t-1}p_{60:60}$ respectively. Thus, we can rewrite the pricing formula using the notations:

$$\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{\mathrm{t-1|}}q_{60:60}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * [{}_{t-1}p_{60:60} - {}_{t}p_{60:60}]$$

We will use this formula to calculate the joint-life insurance under different assumptions.

Chapter 2 – Various Assumptions

Different Assumptions, Different Prices

The mortality assumption is one of many factors that affect the price of insurance. Among many potential mortality assumptions, we will study two assumptions: DeMoivre and the 2001 CSO mortality table. We will also consider three possible correlations between couple's lives: independence, direct dependence, and indirect dependence (inversely related).

First, under the DeMoivre mortality assumption, the deaths of policyholders are uniformly distributed; one person dies each year. The expected number of survivors each year under DeMoivre is plotted in Figure 2-1. We have 100 newborns today (year 0 on the graph). One person dies every year and eventually no one will be alive at year 100. The mortality rate or the probability of someone dying each year is one out of hundred or $\frac{1}{100}$ and is constant throughout years as shown in Figure 2-2.

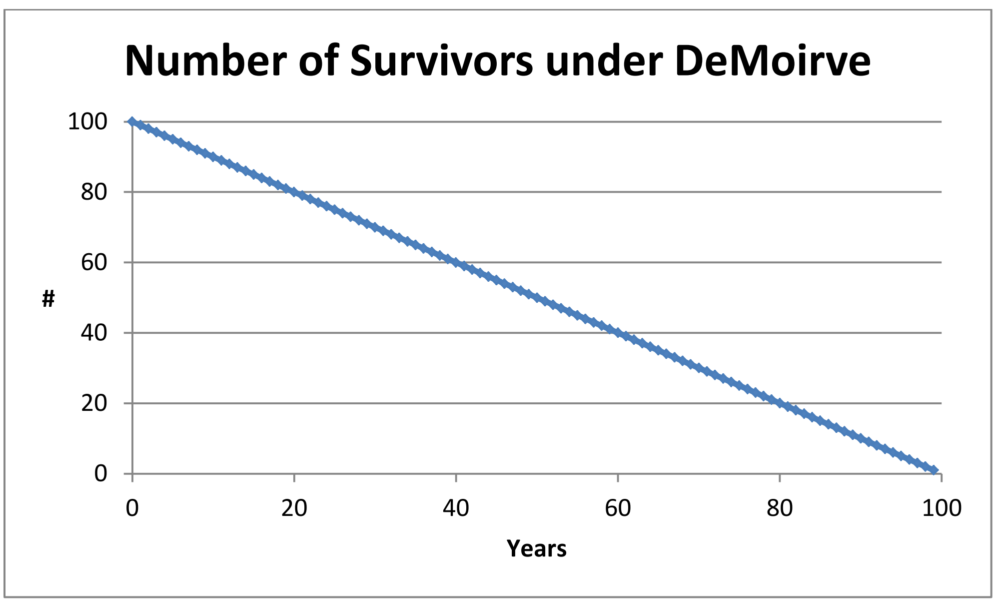

Figure 2-1

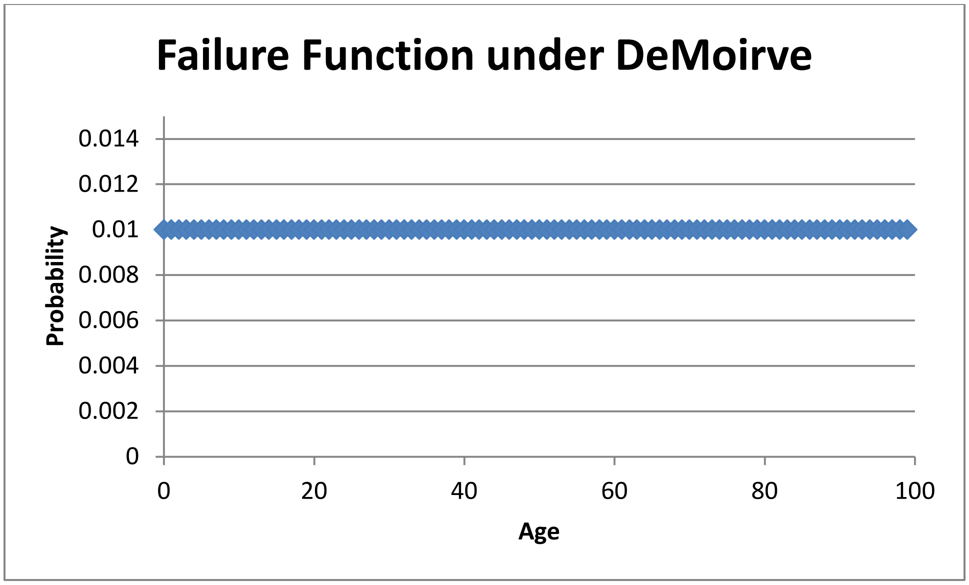

Figure 2-2

Under the DeMoivre mortality assumption, we will consider different correlations between a couple's deaths: independence, direct dependence, and indirect dependence. First, independent correlation assumes that the deaths of a couple are independent as seen in Figure 2-3. Many insurance companies today use this assumption because the independent assumption allows simpler calculations when pricing joint-life products.

Under the independence assumption, the joint-life survival rate can be calculated by simply multiplying two single life survival rates. The single life survival rate, often denoted by ${}_tp_{x}$, is the probability that someone survives for *t* years given that he is at age *x*. The joint-life survival rate, often denoted by ${}_tp_{xy}$, is the probability that the male (age x) lives for t years <u>and</u> the female (age y) survive for t years. Thus, under independence, ${}_tp_{xy} = {}_tp_x$ times ${}_tp_y$,

Therefore, the pricing formula for joint-life insurance can be rewritten as:

$$\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * [{}_{t-1}p_{60:60} - {}_tp_{60:60}]$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * \left[{}_{t-1}p_{60}^{m} * {}_{t-1}p_{60}^{f} - {}_{t}p_{60}^{m} * {}_{t}p_{60}^{f}\right] \quad \textit{under independence assumption}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * \left[\frac{(40-(t-1))}{(100-60)} * \frac{(40-(t-1))}{(100-60)} - \frac{(40-t)}{(100-60)} * \frac{(40-t)}{(100-60)}\right]$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * \left[\left(\frac{41-t}{40}\right)^2 - \left(\frac{40-t}{40}\right)^2\right] \quad \textit{under DeMorive}$$

$$= \frac{1}{1600} \sum_{t=1}^{40} \frac{1}{(1.06)^t} * [(41-\mathrm{t})^2 - (40-\mathrm{t})^2]$$

$$= \frac{1}{1600}\left[\frac{40^2-39^2}{1.06} + \frac{39^2-38^2}{1.06^2} + \cdots + \frac{1^2-0^2}{1.06^{40}}\right]$$

$$= \$\ 0.5105$$

Under DeMoivre, ${}_tp_x$ has a general formula of $\frac{w-x-t}{w-x}$ for each t. Thus, ${}_tp_{60}$ is $\frac{40-t}{100-60}$ or $\frac{40-t}{40}$ and ${}_{t-1}p_{60}$ is $\frac{41-t}{40}$. Since the male and female are at the same age, they have the same probabilities. Therefore, under DeMoivre assumption and independence, the price for discrete joint-life insurance with \$1 death benefit is \$0.5105.

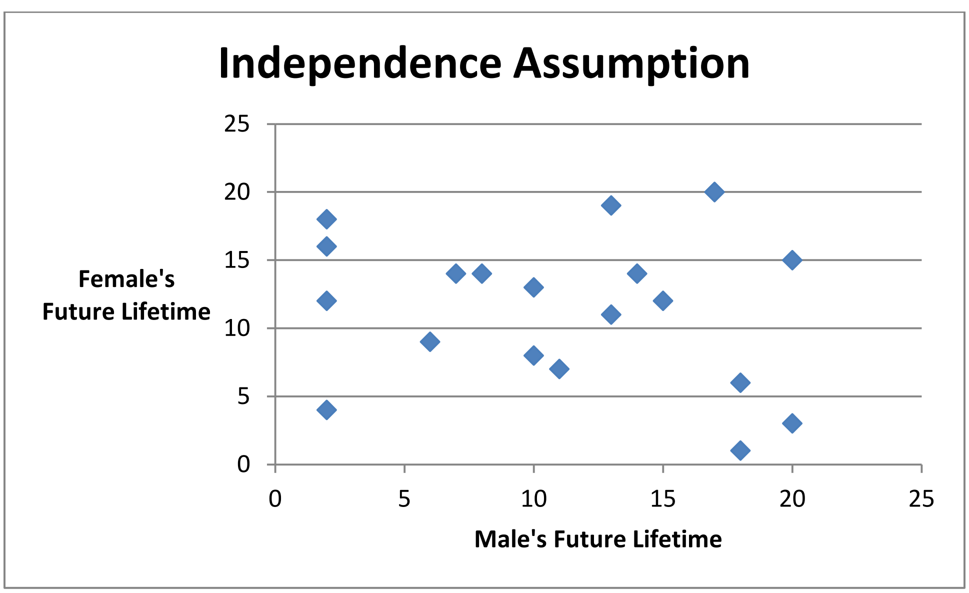

Figure 2-3

Under the direct dependence assumption, the death of a husband is directly dependent on the death of his wife and vice versa. In other words, they die at the same time. The husband and wife have the same future lifetimes and as shown in Figure 2-4 below, the future lifetime graph has a slope of 1. This theory can be explained by factors such as common disasters, similar lifestyles, and broken heart syndrome. The couple may have died in a common disaster such as car accident. They may have similar future lifetime since they are likely to have similar lifestyle such as diet. Someone who eats healthy is likely to share his or her healthy diet with the significant other. Therefore, both of them are more likely to live longer than someone who eats a lot of junk food, which can lead to health complications and consequently shorter lifetime. The broken-heart syndrome explains the event when one partner dies soon after the other due to depression and a broken heart from the other's death. Since the deaths are synchronous under the direct dependence assumption, ${}_{t-1|}q_{60:60} = {}_{t-1|}q_{60}$ and the pricing formula becomes:

$$\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * [{}_{t-1}p_{60} - {}_{t}p_{60}]$$

where ${}_{t}p_{60} = \frac{40-t}{100-60}$ or $\frac{40-t}{40}$ under DeMoivre. Thus, under DeMoivre and direct dependent correlation, the price for the discrete joint-life insurance of $1 death benefit is $0.3761 as shown in the calculations below.

$$\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * [{}_{t-1}p_{60} - {}_{t}p_{60}]$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * \left[\frac{40-(t-1)}{40} - \frac{40-t}{40}\right] \textit{under DeMoivre}$$

$$= \sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * \frac{1}{40}$$

$$= \frac{1}{40} \sum_{t=1}^{40} \frac{1}{1.06^t}$$

$$= \$0.3761$$

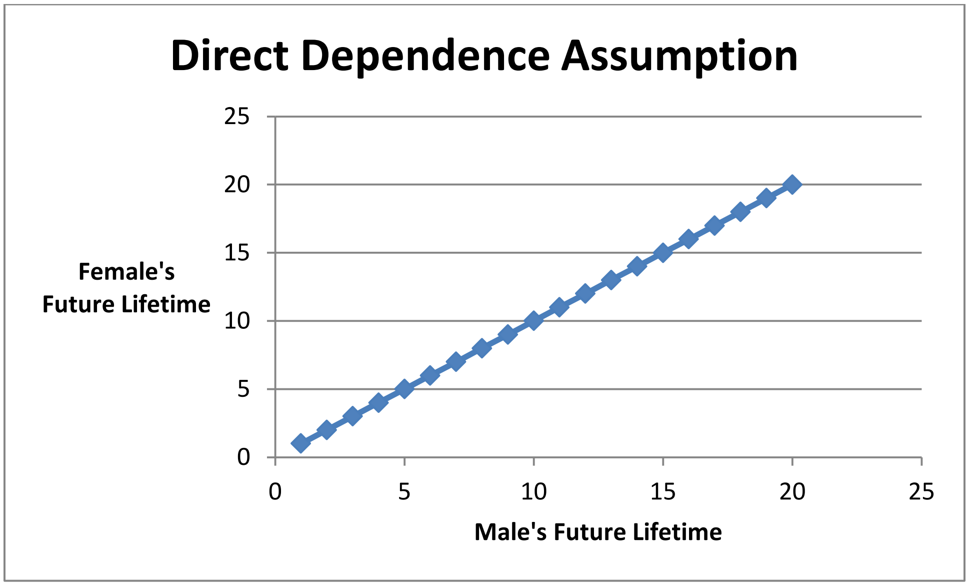

Figure 2-4

Lastly, under indirect dependent assumption, the deaths of couples are inversely related. As seen in Figure 2-5, the wife's future lifetime is longer when the husband's is shorter. This theory is suggested by a feminist, Gloria Steinem. She suggests that many widows live longer after their spouses die since they have more freedom and power. Under this assumption, the pricing formula becomes:

$$\sum_{t=0}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$$

$$= \sum_{t=0}^{20} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60} + \sum_{t=21}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$$

$$= \sum_{t=0}^{20} \$1 * \frac{1}{(1+0.06)^t} * 0.05 + \sum_{t=21}^{40} \$1 * \frac{1}{(1+0.06)^t} * 0 \quad under\ DeMoivre$$

$$= \sum_{t=21}^{40} \$1 * \frac{1}{(1+0.06)^t} * 0.05$$

$$= 0.05 * (\frac{1}{1.06} + \frac{2}{1.06^2} + \cdots + \frac{20}{1.06^{40}})$$

$$= \$0.5739$$

where ${}_{t-1|}q_{60:60} = 0$ for t>20 while ${}_{t-1|}q_{60:60} = 2 * \frac{1}{(w-x)} = \frac{1}{20}$ for t<20 under DeMoivre mortality. In conclusion, under DeMoivre and indirect dependent assumption, the price for the discrete joint-life insurance of $1 death benefit is $0.5739. The prices of the insurance calculated under different correlation assumptions are summarized below in the Table 2-1.

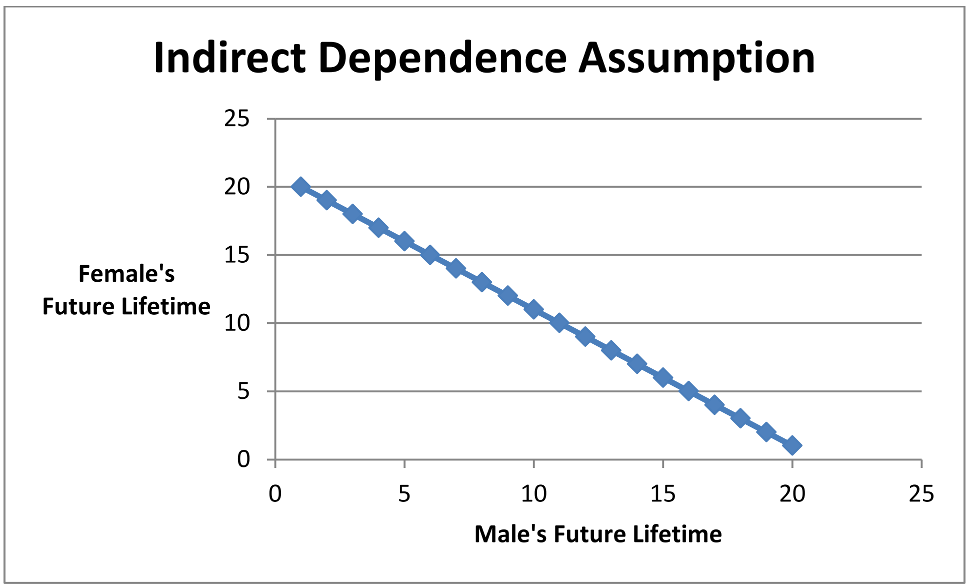

Figure 2-5

DeMoivre	Independent	Directly Dependent	Indirectly Dependent
Price for $1 Death Benefit	$0.5105	$0.3761	$0.5739

Table 2-1

Although DeMoivre mortality assumption simplifies probability calculations and still allows us to see the differences among various correlation assumptions, it is too theoretical. The 2001 CSO mortality table is a realistic example that is used in practice. The table was adopted in 2002 and was suggested for use by the National Association of Insurance Commissioners (NAIC) to calculate reserves until it was updated in 2006. The CSO table was developed by the American Academy of Actuaries CSO Task Force from the Valuation Basic Mortality Table developed by the Society of Actuaries Individual Life Insurance Valuation Mortality Task Force (DORA).

Using the 2001 CSO mortality table, the pricing formulas become:

- $\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$ under independence assumption
- $\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60}^{m}$ under direct dependence assumption[1]
- $\sum_{t=1}^{40} \$1 * \frac{1}{(1+0.06)^t} * {}_{t-1|}q_{60:60}$ under indirect dependence assumption

The prices for the discrete joint-life insurance of $1 death benefit paid were calculated under three different correlations using Microsoft Excel and can be found in Appendix A. The prices under different correlation assumptions are summarized below in Table 2-2. In both the DeMoivre and 2001 CSO tables, the price is the highest under the indirectly dependent assumption (correlation of -1), next highest under the independent assumption (correlation of 0), and lowest under the directly dependent assumption (correlation of +1). The price under directly dependent assumption is much further from the middle price than the price under indirectly dependent assumption is. The average of two prices under two dependent assumptions does not equal the independent price even though the sum of two distributions would have a correlation of 0, which implies independence.

2001 CSO Table	Independent	Directly Dependent	Indirectly Dependent
Price for $1 Death Benefit	$0.4147	$0.3377	$0.4530

Table 2-2

In addition to three correlations, mixed correlations were studied and the graphs of different combinations of correlations are illustrated in Figure 2-6 and 2-7. If half of the population (lifetimes of

[1] Since the percentage of males dying by a certain time is always greater than the percentage of females who are projected to die by that time, the joint life probability of death equals the probability that the male has died.

heterosexual couples) have independent correlation while the other half are directly dependent,[2] then the price for the joint-life insurance of $1 benefit paid at the end of year of the first death can be calculated as follows:

$$\frac{1}{2} * Price\ When\ Independent + \frac{1}{2} * Price\ When\ Directly\ Dependent$$

which results in $0.3762 under the CSO mortality table.

If a third of the population were independent while another third had direct dependence and the other third had indirect dependence, the price of the insurance can then be calculated as following:

$$\frac{1}{3} * Price\ When\ Independent + \frac{1}{3} * Price\ When\ Directly\ Dependent + \frac{1}{3} * Price\ When\ Indirectly\ Dependent$$

which results in $0.4018 under the CSO mortality table. The prices of the insurance calculated under different mortality and correlation assumptions are summarized in Table 2-3.

	Independent	Directly Dependent	Indirectly Dependent	Half Indep, Half DD	Third Each
CSO Table	0.4147	0.3377	0.4530	0.3762	0.4018

Table 2-3

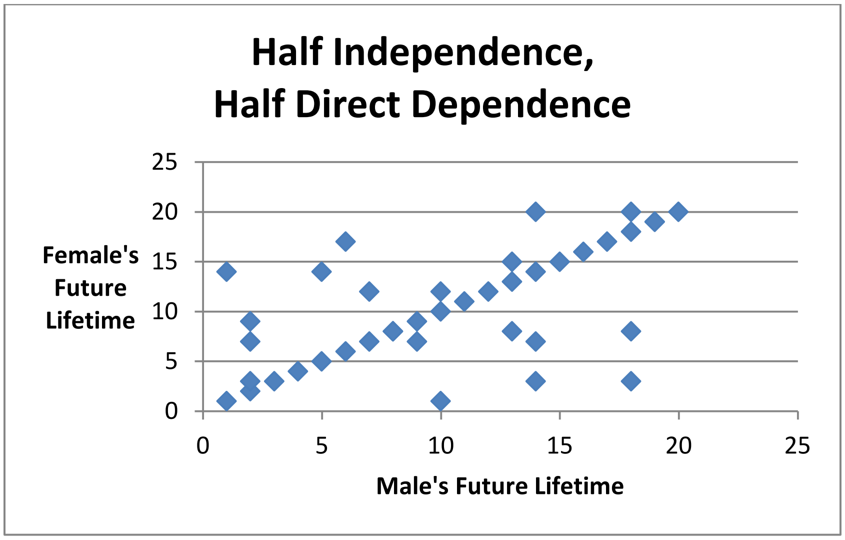

Figure 2-6

[2] This mixed correlation was seen in the data studied in this paper.

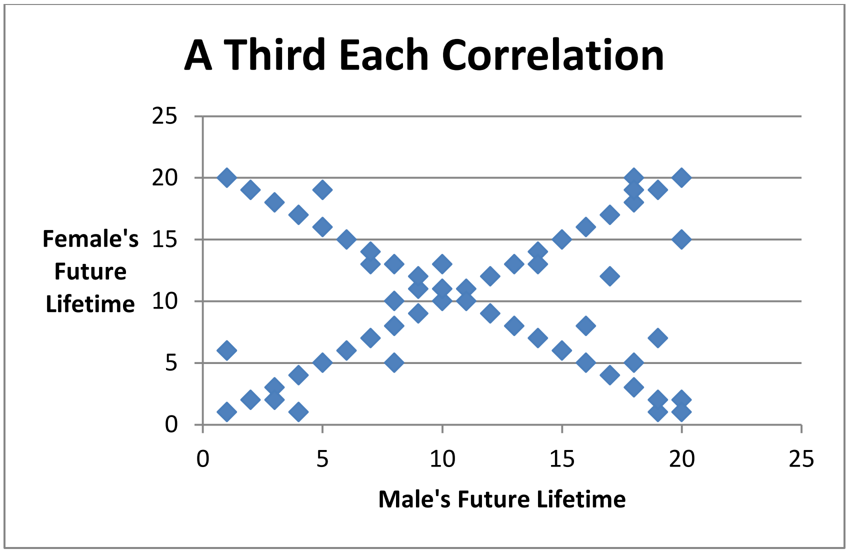

Figure 2-7

In conclusion, different correlations result in different prices for the joint-life insurance. As shown in the table 2-3 above, the price of joint-life insurance is the highest under indirectly dependent assumption, next highest under independent assumption, and lowest under directly dependent assumption. Many companies today price their products under the independent assumption since it simplifies joint-life mortality rate calculations. Next section discusses the possible financial outcomes in case this assumption is incorrect.

The Magnitude of Financial Impact of a Wrong Assumption

Some insurance companies may assume that the deaths of couples are independent when pricing their joint-life insurance products. While the area of pricing joint-life products needs further studying and improvement, the area of pricing single-life products has been thoroughly developed for the past half-century. Pricing joint-life products can be complicated under various assumptions while under the independence assumption, pricing joint-life products becomes as simple as pricing single-life products. For this reason, some insurance companies may price their joint-life products under the independence assumption. However, some recent studies on joint-life mortality show that the independence assumption may be incorrect. Many actuaries who studied joint-life mortality believe that the couples' deaths are somewhat dependent.

If the couple's deaths are indirectly dependent, companies are undercharging the policyholders for their joint-life insurance products. For instance, our earlier example of the 2001 CSO mortality table assumption shows that under the independent assumption, for a couple who are both at age 60, the price of discrete joint-life insurance with death benefit of $1 is $0.4147 while under the indirect dependence assumption, the price is $0.4530. In other words, for couples whose behavior follows $\rho = -1$ (for example, maybe some unhappy couples) the companies should be charging the couple $0.4530, but they are charging them only $0.4147 (or 9% too little). If the benefit were $100,000, the prices would become $45,300 and $41,470 and the difference becomes significant, $3,830. This difference accounts for one policy. The difference becomes even bigger when an insurance company sells hundreds and thousands of the insurance policies. For example, a company sells this insurance policy to 100,000 couples. The company has collected $41,470*100,000 = $4,147,000,000 when it should have collected $45,300*100,000=$4,530,000,000. In conclusion, if the company assumes the independence but indirect dependence exists, then the company will not have enough funds to make all payments in the long run.

What if there is direct dependence? If there is a direct dependency (for example, for couples who are very dependent on each other) but the company is pricing their joint-life insurance products under the independence assumption, the company is overcharging them. Again, using our CSO mortality table example, under the independent assumption, for a couple who are both at age 60, the price of discrete joint-life insurance with death benefit of $1 is $0.4147; however, under the direct dependence assumption, the price is $0.3377, which is 22% less than the independent price. The difference becomes much more significant when we consider $100,000 death benefit. The difference between the prices becomes $41,470 - $33,770 = $7,700. For instance, an insurance company A uses the independent assumption to price their joint-life insurance products and sells the insurance policy for $41,470. But another company B recognizes that couple's dependency and prices the same insurance product at $33,770. As a customer, $7,700 is enough difference that he or she will get the insurance product from company B. The company

A will lose the customers as a result of using the wrong assumption and mispricing their products. It once again reminds us how important it is to study the joint-life mortality and make an appropriate assumption when pricing insurance.

It is difficult for an insurer to tell whether a couple's mortality will fit the independence, direct dependence or indirect dependence model when selling insurance. One possible way to determine which model a couple's mortality will fit the best is to train sales agents so that they could decide if the couple is happily married. Happily married couples are more likely to be dependent on each other and spend more time together (increase the chance that a couple experiences a common disaster). Thus, insurance companies can use the information from the sales agents to determine the price accordingly. However, determining prices based on a couple's apparent happiness or unhappiness could create conflicts of interest for the sales agents. They may find that their clients get a better price when they inform the home office of the insurer that the couple was happily married. They would say all their clients were happily married so that they could sell the insurance at lower prices and attract more clients. In addition, if customers got wind of this pricing technique they would try to appear happily married to get the best price on the insurance. In that case, insurers might just have to price their insurance using one average price based on their understanding of the makeup of their customers.

For example, recent studies suggest that half of their customers will follow the independence while the other half will show direct dependence. If they assume that half of population follows the independence model and the other half follows the direct dependence model, then the price of joint-life insurance with $100,000 death benefit would be $37,620. The price under this assumption is less than the independent price of $41,470. If insurance companies recognized the recent studies and priced their joint-life insurance accordingly, then the companies would be at an advantage with more competitive products than the others that assume only independence.

However, data used in many recent studies were not observed for a long enough time to assess whether there are any couples that show indirect dependence. If there were some indirect dependence, the real price should be slightly higher than the independent price and the price under half-half assumption. If they assumed that one-third their customers are in each of the three categories, then the real price should be $40,180. Thus, if companies priced their joint-life insurance products under the half indirect dependence and half independent assumption as the recent studies suggest but there were some indirect dependence, the companies would be undercharging their customers. They may have insufficient funds in the long run.

In conclusion, making a wrong assumption on mortality may result in undercharging the customers and insufficient funds. It may also result in overcharging the customers but losing some when there are cheaper products provided by other companies. Thus, it is important to study the mortality and make an appropriate assumption. Many studies suggest that both independence and direct dependence exist between couples' deaths. However, data used in those studies are often observed over such a short period that it is difficult to assess whether there exists indirect dependence.

Chapter 3 – Dependence

Several actuaries have already done studies on the correlations between couples' deaths, and many believe that there is a positive correlation or direct dependence. This paper will discuss multiple studies that show the dependency between couples' deaths.

1) Danish Twin Study by Hougarrd, Harvald and Holm:

Direct dependence between couple's deaths can be explained by factors such as common disasters, common lifestyles, and a broken heart syndrome. Common disasters such as car accidents result in simultaneous deaths of the couple. Similar lifestyles such as healthy dieting also affect the lifetimes of the couples. Couples with a healthy diet are more likely to live longer than those with unhealthy diets. The third factor is the broken heart syndrome. The Danish twin study, done by Hougarrd, Harvald and Holm, shows the effect of the broken heart syndrome on the deaths of Danish twins and therefore possibly on the deaths of married couples.

The data consists of same-sex twins who were born between 1881 and 1930 in Denmark and were observed until 1980. The study only included pairs who were both alive by the age of 15. The main purpose of the study was to compare various models and measure the dependence between the lifetimes of adult Danish Twins. We will focus on a couple of their findings. The twin study examined "the hazard for a twin conditional on the present status of the partner." The higher the hazard means the higher probability that a twin will die. According to one of the models they compared in the study, "the hazard of a male twin based on the available knowledge at that time increases by 56% if the partner dies at age 20." In other words, a twin is about twice more likely to die when the partner or the other twin is dead at age 20 than when the other twin is still alive. Another model also showed that there was a large increase in mortality of a twin if the partner died young. However, for older twins, the status of the partner did not have a significant effect on mortality. In addition, the other model showed that after the death of the partner, the ratio of the observed to the expected number of deaths first increased, then later decreased. Thus, shortly after the death of the other twin, a twin dies more frequently than expected, which suggests a broken heart syndrome. However, sometime after the partner's death, the broken heart syndrome disappears as the emotional suffering diminishes with time. From the twin study, we can conclude that because a broken heart syndrome exists, there will sometimes be dependence between the deaths of pairs.

2) Parkes, Benjamin, Fitzgerald

The broken heart syndrome can be further explored through the study done by Parkes, Benjamin, and Fitzgerald in 1969. This study consisted of a total of 4,486 widowers who were 55 years of age or older. The widowers were observed for nine years after their wives' deaths. Information such as their age, occupation, and the certified cause of death were provided from the death certificates of the widowers. In the study, Parkes, Benjamin, and Firzgerald focused on whether a correlation exists between the cause of death of widowers and their wives. However, we will concentrate on one of their findings, the increased mortality rate for widowers. The study showed that of the 4,486 widowers that were studied, 213 (5%) died during the first six months of their wives' deaths. The numbers show that the mortality rate of widowers was 40% greater than the mortality rate of married men for the first six months. The percentage difference between mortality rate of widowers and that of married men is shown below in Figure 3-1. The percentage difference is positive for the first four years after the wives' deaths and this statistic suggests the broken heart syndrome and therefore dependence between the couples' deaths.

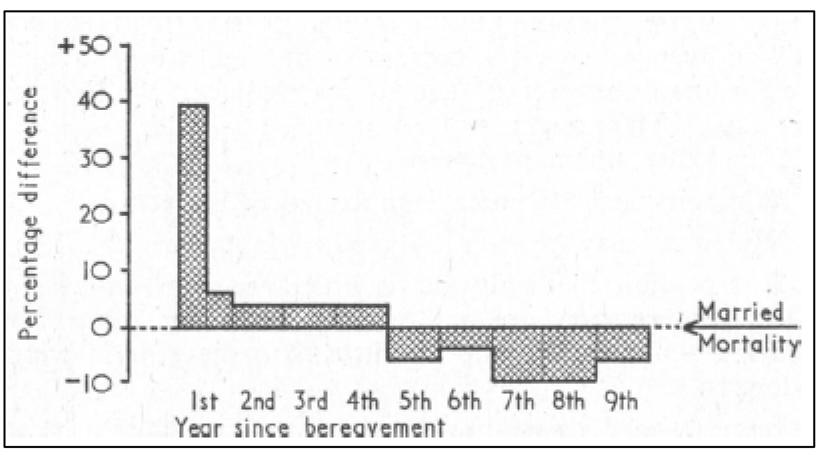

Figure 3-1

3) Study Done by Youn and Shemyakin:

Youn and Shemyakin used data that consisted of 14,947 joint and last-survivor annuity contracts of a large Canadian insurer. The contracts were observed from December 29, 1988 to December 31, 1993. They excluded contracts of same sex annuitants and contracts that were owned by duplicate couples so that each couple was in the data only once. After some elimination, their study was funneled down to 11,457 married heterosexual couples.

	All Data	D<0	D=0	D=1
Observed (Total)	11457	2133	1037	1290
Female Deaths (FD)	449	103	43	70
Male Deaths (MD)	1245	201	102	149
Both Died (BD)	194	39	19	30
Ratio $(BD \cdot Total)/(FD \cdot MD)$	3.98	4.02	4.49	3.71

	D=2	D=3	D=4	D>4
Observed (Total)	1246	1166	1025	3560
Female Deaths (FD)	47	44	39	103
Male Deaths (MD)	144	131	119	399
Both Died (BD)	20	20	19	47
Ratio $(BD \cdot Total)/(FD \cdot MD)$	3.68	4.05	4.20	4.07

Table 3-1

Table 3-1 above summarizes the number of deaths that occurred during the observation period. The observed number of deaths of the couples where both the male and female died was recorded as BD. The expected number of deaths of the couples where both the male and female die under the independence assumption can be calculated by:

$$\frac{The\ Number\ of\ Female\ Deaths}{Total} * \frac{The\ Number\ of\ Male\ Deaths}{Total} * Total$$

In the table, D denotes the age difference between the male and female. We are interested in the ratio between the observed number and expected number of deaths of couples during the observation period. The ratio for the entire data is 3.98 as shown in the table below. The actual number of deaths is about 4 times bigger than the expected number of deaths under the independence assumption. Thus, it shows that the independence assumption is not appropriate.

4) Different Mortality Rates for Single, Married, and Widowed

The Belgian National Institute of Statistics (NIS) also carried out a study in 1995 to determine whether marital status (married or widowed) had significant effects on an individual’s mortality. The Belgian NIS studied the mortality rates for males and females, married couples, and widowed males and females. It observed the mortality rates for each group and the result is shown in the graphs below. The probabilities that someone age x will die before age x+1 were observed and plotted as a function of the age x. The mortality rates for males can be seen in Figure 3-2 while Figure 3-3 shows the mortality rates for females. From these graphs, we can see that the mortality rates for widows and widowers are higher than the mortality rates for all males and females. Meanwhile, the mortality rates for married men and women are lower than that for all men and women, especially married men. This shows that the mortality rates differ depending on the marital status. The higher mortality rates of the widows and widowers can be explained by factors such as a common disaster, a common lifestyle, and a broken heart syndrome. A couple spends a lot of time together and is exposed to the same risks of experiencing a disaster such as a car accident. Both may suffer from medical complications after the accident and one dies shortly after the other, at which point they would be labeled as widows or widowers. Thus, the mortality rates for widows and widowers increase. In other studies, the widowers are labeled as non-married men and the mortality rates of non-married men increase when they die due to a common disaster. Similarly, a couple could share similar lifestyles such as workout habits and healthy diet. The workout habits and healthy diet can affect the length of lifetimes of a couple and the couple is may die around the same time. In addition, some people die from a broken-heart shortly after loss of their significant others. All three factors explain the higher mortality rates of the widows and widowers and imply that there is dependence between the pair’s deaths.

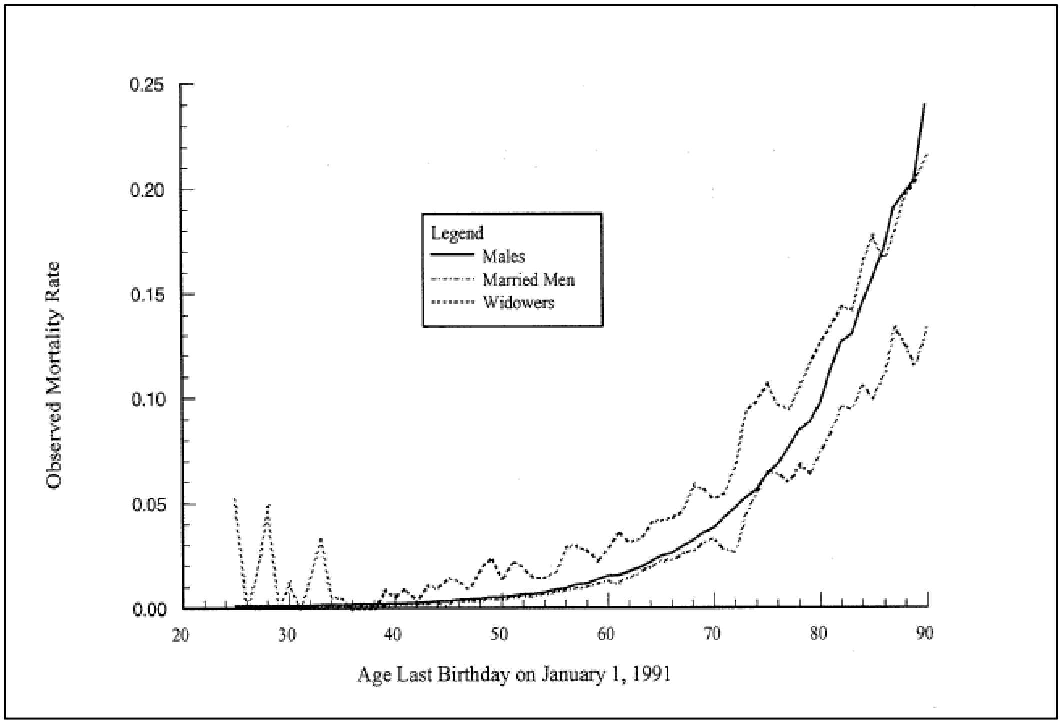

Figure 3-2

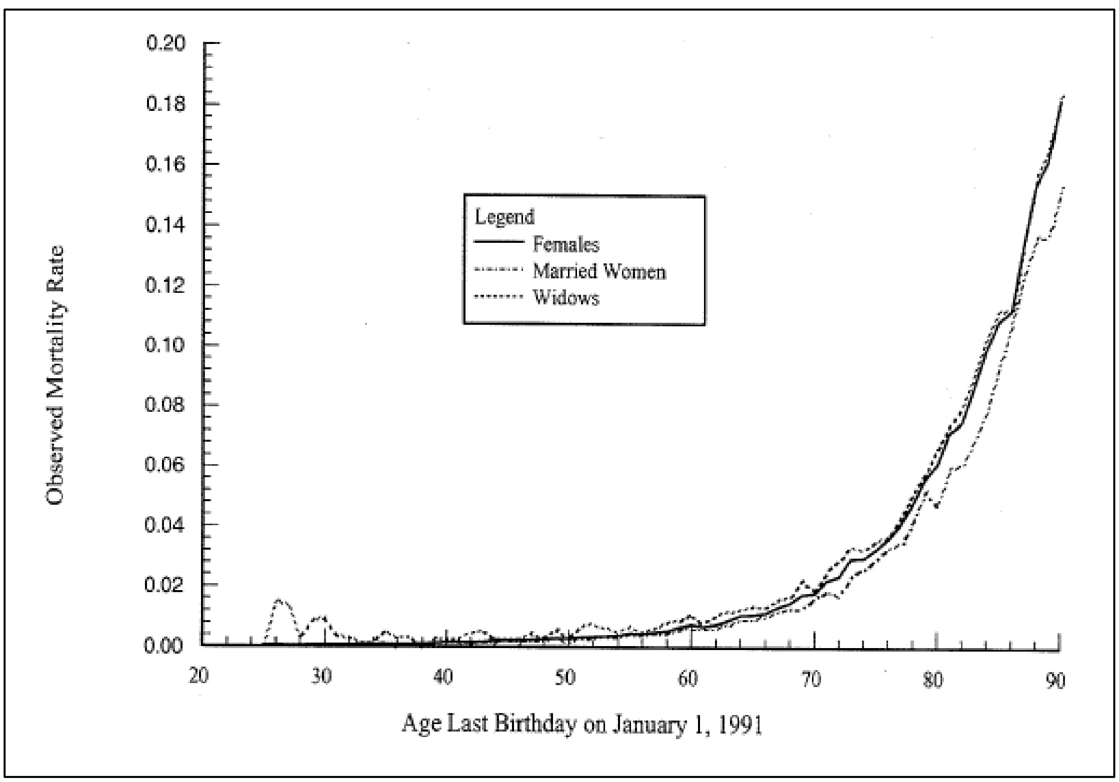

Figure 3-3

5) Data Held by the SOA

The data used in this paper was provided by a large Canadian company that consists of 12,183 joint-life annuity contracts. The contracts were observed from 1985 to 1989. Each contract contained information such as the date of birth for both annuitants, date of death if applicable, contract issue date, and sex of primary and contingent annuitants. From the information provided, the policyholders' ages at issue and their ages at death could be calculated. There were 190 couples where both died during the observation period. For each couple, we calculated the length of time between the issue date and the male's death, T_x. Similarly, we calculated T_y, the length of time between the issue date and the female's death. Then T_x and T_y were plotted as shown in Figure 3-4. The plot shows some dependent correlation between couple's deaths as well as some independent correlation. In addition, the time between the two deaths were calculated. Of those 190 couples, 60 (almost a 3rd) of them died within 7 days. Both findings support that the couples' deaths are directly dependent.

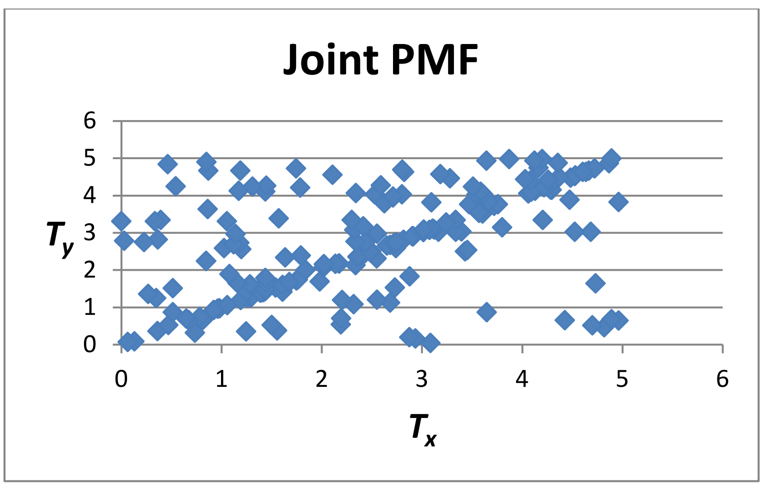

Figure 3-4

However, no conclusion can be made about whether indirect dependence exists. Since our observation period is only 5 years, we could not have seen the indirect dependence in the plot even if it existed. As shown in Figure 3-5, out of the entire graph, we are only able to tell what is in the red box which indicates the first 5-year-span. Thus, we don't have enough information to tell if any of the data would exhibit negative correlation. We would need a much longer observation period to determine that.

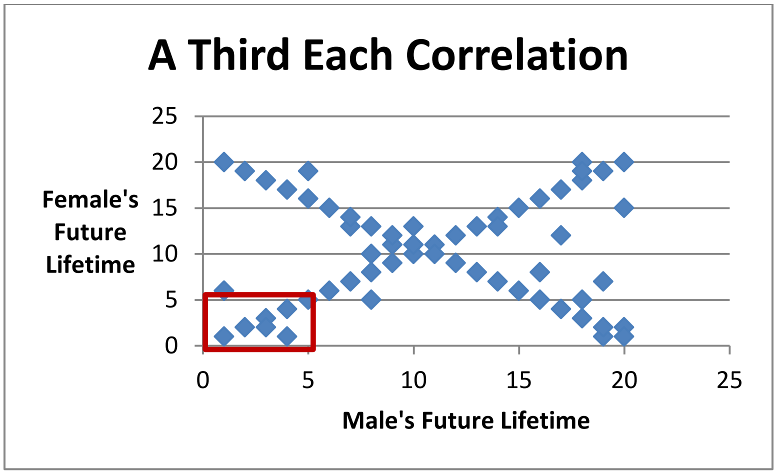

Figure 3-5

In conclusion, there are several studies done for the past decades that show that there is a positive correlation or direct dependence between the deaths of husband and wife. However, some insurance companies assume independence when pricing their joint-life products. As a result, joint-life insurance products could be overpriced. Some companies may reflect some direct dependence in the population, but in this case may underprice the product, if they don't reflect any couples where the spouses die far apart (indirect dependence). If an insurance company is able to come up with a model that has an appropriate weight for all three correlations, it will be able to better set the price of joint-life insurance products that is not too low and not too high. With more competitive but adequately priced products, the company will be able to attract more customers. In the next section, possible dependence models are discussed.

Chapter 4 – Possible Models

Some studies suggest using copula functions to construct joint-survival functions. The copula, $C(u,v)$, is defined as a bivariate distribution "function with special properties, mixing up the marginal [functions] with a certain association parameter." Copulas include parameters that control the strength of dependence between random variables, lifetimes of a husband and wife in this study. One type of copula, called Frank's copula, is shown below:

$$C(u,v) = u + v - 1 - \frac{1}{\alpha}\ln 1 + \left(\frac{\left(e^{-\alpha(1-u)} - 1\right)\left(e^{-\alpha(1-v)} - 1\right)}{e^{-\alpha} - 1}\right)$$

where u represents the marginal survival function for males and v represents the marginal survival function for females. The marginal survival function, $S(x)$, gives the probability that someone's age at death is greater than or equal to x. The other variable, α, is an association parameter that controls the degree of association between two random variables, two lifetimes. The association parameter less than 1 indicates that there is direct dependence, values greater than 1 indicate indirect dependence, and 1 indicates independence between two variables.

A Hougaard copula is another copula used to model dependence between lifetimes of a couple and is shown below:

$$C(u,v) = \exp -\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{1/\theta}$$

where θ is a parameter that measures a degree of dependence. Values of θ are always greater than or equal to 1. When $\theta=1$ indicates independence, large values of θ indicate perfect correlation.

There are many more copulas that can be used to model dependence between two random variables. The copula approach requires relatively few parameters, often five parameters. However, by using copulas, we assume the static dependence structure over time, which may not hold true in reality. In addition, it is difficult to tell whether a copula adequately fits the dependence structure and thus hard to choose which copula is the most suitable.

In addition to copulas, there are other models that can be used to model dependence between the lifetimes of a husband and wife, such as Markov or semi-Markov models. In those Markov models, four states are defined:

State 0: Both husband (*x*) and wife (*y*) are alive;

State 1: Husband (*x*) is dead and wife (*y*) is alive;

State 2: Husband (*x*) is alive and wife (*y*) is dead;

State 3: Both husband (*x*) and wife (*y*) are dead.

At each time t, a couple can go from state i to state j, where i,j=0,1,2,3 and $t=0,1,2,3,\ldots$.

Markov models study the probabilities that a couple will remain in the same state or move to another state at different times. Thorough descriptions of the models can be found in Ji, Hardy, and Li (2012). While Markov models assume that the future state only depends on the current state, semi-Markov models assume that the future state depends on the current state and the elapse time since the last transition. Both Markov models require as many as nine parameters. However, the Markovian approaches have multiple advantages over copulas. One of them is that the Markov models use the force of mortality rather than the distribution functions. The use of the force of mortality makes the Markovian models transparent; it is easy to see the impact of the partner's death on the mortality of the survivor. Another advantage is that the semi-Markov model captures the impact of the broken heart syndrome while copulas do not.

In conclusion, there are different models that can reflect dependence between the lifetimes of a husband and wife. Through these models, insurance companies are able to price their insurance products under the assumption that both independence and direct dependence exist between couples' deaths; this assumption was suggested by many recent studies. However, many studies on dependence between couples' deaths were based on the same data which was provided by a large Canadian insurance company. However, the data only consisted of the observations made over 5 years. However, such a short observation period does not allow us to see all correlations between the couples' deaths. It only shows us the small part of the graph as seen in Figure 3-5. Even if indirect dependence existed between couples' deaths, we would not be able to tell from the data provided. However, the financial impact of making a wrong assumption can be catastrophic to insurance companies. Therefore, for a more accurate analysis of dependence of couples' deaths, we need data with a longer observation period, ideally over 30-40 years. With a longer observation period, we will be able to assess whether indirect dependence exists along with direct dependence and independence. We will also be able to weigh each correlation and therefore build models that can describe the joint-life mortality of married couples better.

Chapter 4 – Conclusion

There are multiple factors determine the price of insurance including the mortality correlation assumptions. Some insurance companies assume that the deaths of couples are independent for simplicity when pricing joint-life insurance. However, recent studies suggest that there exists some direct dependence between couples' deaths. Some companies may reflect some dependence when pricing insurance. The price of insurance under the assumption that there are mixed correlations between deaths of pairs becomes slightly lower than the price under the only independent assumption. Insurance companies are able to provide more affordable joint-life insurance under the mixed correlation assumption. However, data used in those studies have a short observation period and it is difficult to see whether there are inversely correlated deaths. The price of insurance differs depending on whether there is indirect dependence between couples' deaths. The price under the assumption that all three correlations exist will be higher than the half independent and half directly dependent price. Making a wrong assumption can put insurance companies at a disadvantage. If the companies underprice their products, they will have insufficient funds. If the companies overprice their products, they may lose customers to other competitors. Depending on whether all three correlations exist or only two correlations (independent and direct dependent), the insurance companies may be underpricing their products. In order to see if all three correlations exist between couples' deaths, data with a much longer observation period (over 30-40 years) is required. A longer observation period will allow insurance companies to assess all three correlations between pairs' deaths and price their joint-life products appropriately.

APPENDIX A.1 - DeMoivre

			A_{xy}	ρ=0 error			$A_{x\bar{y}}$	ρ=0 error			
x	60	ρ=1	0.3762	26%	0%	ρ=1	0.38	26%	0.5	ρ=1	Direct Dependence
y	60	ρ=0	0.5105	0%	50%	ρ=0	0.24	53%	0.5	ρ=0	Independence
ω	100	ρ=-1	0.5735	-12%	50%	ρ=-1	0.18	65%	0	ρ=-1	Indirect Dependence
i	6.00%	mixed	0.0282	26%		mixed	0.3090037	39%			
d	5.66%										

							ρ=0					ρ=-1			ρ=1	ρ=0	ρ=-1	ρ=0			ρ=-1			ρ=0	ρ=-1
k	v^k	$v^k/(w-x)$	qx	$_kp_x$	lx	$_kp_y$	$_kp_{xy}$	$_{k-1}\mid q_x$	$_{k-1}\mid q_{xy}$	$v^k * {}_{k-1}\mid q_x$	$v^k * {}_{k-1}\mid q_{xy}$	$_kq_{xy}$	$_{k-1}\mid q_{xy}$	$v^k * {}_{k-1}\mid q_{xy}$	$v^k * {}_kp_x$	$v^k * {}_kp_{xy}$	$v^k * {}_kp_{xy}$	$_kp_{x\bar{y}}$	$_{k-1}\mid q_{x\bar{y}}$	$v^k * {}_{k-1}\mid q_{x\bar{y}}$	$_kq_{x\bar{y}}$	$_{k-1}\mid q_{x\bar{y}}$	$v^k * {}_{k-1}\mid q_{x\bar{y}}$	$v^k * {}_kp_{x\bar{y}}$	$v^k * {}_kp_{x\bar{y}}$
0	1	0.025	0.025	1	100	1	1					0			1	1	1	1			0			1	1
1	0.943396	0.023585	0.025	0.975	97.5	0.975	0.950625	0.025	0.049375	0.023585	0.04658019	0.05	0.05	0.0471698	0.9198113	0.896816	0.896226	0.999375	0.000625	0.00059	0	0	0	0.942807	0.943396
2	0.889996	0.02225	0.025	0.95	95	0.95	0.9025	0.025	0.048125	0.02225	0.04283108	0.1	0.05	0.0444998	0.8454966	0.803222	0.800997	0.9975	0.001875	0.001669	0	0	0	0.887771	0.889996
3	0.839619	0.02099	0.025	0.925	92.5	0.925	0.855625	0.025	0.046875	0.02099	0.03935715	0.15	0.05	0.041981	0.7766478	0.718399	0.713676	0.994375	0.003125	0.002624	0	0	0	0.834896	0.839619
4	0.792094	0.019802	0.025	0.9	90	0.9	0.81	0.025	0.045625	0.019802	0.03613927	0.2	0.05	0.0396047	0.7128843	0.641596	0.633675	0.99	0.004375	0.003465	0	0	0	0.784173	0.792094
5	0.747258	0.018681	0.025	0.875	87.5	0.875	0.765625	0.025	0.044375	0.018681	0.03315958	0.25	0.05	0.0373629	0.6538509	0.57212	0.560444	0.984375	0.005625	0.004203	0	0	0	0.735582	0.747258
6	0.704961	0.017624	0.025	0.85	85	0.85	0.7225	0.025	0.043125	0.017624	0.03040142	0.3	0.05	0.035248	0.5992165	0.509334	0.493472	0.9775	0.006875	0.004847	0	0	0	0.689099	0.704961
7	0.665057	0.016626	0.025	0.825	82.5	0.825	0.680625	0.025	0.041875	0.016626	0.02784927	0.35	0.05	0.0332529	0.5486721	0.452654	0.432287	0.969375	0.008125	0.005404	0	0	0	0.64469	0.665057
8	0.627412	0.015685	0.025	0.8	80	0.8	0.64	0.025	0.040625	0.015685	0.02548863	0.4	0.05	0.0313706	0.5019299	0.401544	0.376447	0.96	0.009375	0.005882	0	0	0	0.602316	0.627412
9	0.591898	0.014797	0.025	0.775	77.5	0.775	0.600625	0.025	0.039375	0.014797	0.023306	0.45	0.05	0.0295949	0.4587213	0.355509	0.325544	0.949375	0.010625	0.006289	0	0	0	0.561934	0.591898
10	0.558395	0.01396	0.025	0.75	75	0.75	0.5625	0.025	0.038125	0.01396	0.0212888	0.5	0.05	0.0279197	0.4187961	0.314097	0.279197	0.9375	0.011875	0.006631	0	0	0	0.523495	0.558395
11	0.526788	0.01317	0.025	0.725	72.5	0.725	0.525625	0.025	0.036875	0.01317	0.01942529	0.55	0.05	0.0263394	0.381921	0.276893	0.237054	0.924375	0.013125	0.006914	0	0	0	0.486949	0.526788
12	0.496969	0.012424	0.025	0.7	70	0.7	0.49	0.025	0.035625	0.012424	0.01770453	0.6	0.05	0.0248485	0.3478786	0.243515	0.198788	0.91	0.014375	0.007144	0	0	0	0.452242	0.496969
13	0.468839	0.011721	0.025	0.675	67.5	0.675	0.455625	0.025	0.034375	0.011721	0.01611634	0.65	0.05	0.023442	0.3164663	0.213615	0.164094	0.894375	0.015625	0.007326	0	0	0	0.419318	0.468839
14	0.442301	0.011058	0.025	0.65	65	0.65	0.4225	0.025	0.033125	0.011058	0.01465122	0.7	0.05	0.022115	0.2874956	0.186872	0.13269	0.8775	0.016875	0.007464	0	0	0	0.388119	0.442301
15	0.417265	0.010432	0.025	0.625	62.5	0.625	0.390625	0.025	0.031875	0.010432	0.01330032	0.75	0.05	0.0208633	0.2607907	0.162994	0.104316	0.859375	0.018125	0.007563	0	0	0	0.358587	0.417265
16	0.393646	0.009841	0.025	0.6	60	0.6	0.36	0.025	0.030625	0.009841	0.01205542	0.8	0.05	0.0196823	0.2361878	0.141713	0.078729	0.84	0.019375	0.007627	0	0	0	0.330663	0.393646
17	0.371364	0.009284	0.025	0.575	57.5	0.575	0.330625	0.025	0.029375	0.009284	0.01090883	0.85	0.05	0.0185682	0.2135345	0.122782	0.055705	0.819375	0.020625	0.007659	0	0	0	0.304287	0.371364
18	0.350344	0.008759	0.025	0.55	55	0.55	0.3025	0.025	0.028125	0.008759	0.00985342	0.9	0.05	0.0175172	0.1926891	0.105979	0.035034	0.7975	0.021875	0.007664	0	0	0	0.279399	0.350344
19	0.330513	0.008263	0.025	0.525	52.5	0.525	0.275625	0.025	0.026875	0.008263	0.00888254	0.95	0.05	0.0165257	0.1735193	0.091098	0.016526	0.774375	0.023125	0.007643	0	0	0	0.255941	0.330513
20	0.311805	0.007795	0.025	0.5	50	0.5	0.25	0.025	0.025625	0.007795	0.00799	1	0.05	0.0155902	0.1559024	0.077951	0	0.75	0.024375	0.0076	0	0	0	0.233854	0.311805
21	0.294155	0.007354	0.025	0.475	47.5	0.475	0.225625	0.025	0.024375	0.007354	0.00717004	1	0	0	0.1397238	0.066369	0	0.724375	0.025625	0.007538	0.05	0.05	0.014708	0.213079	0.279448
22	0.277505	0.006938	0.025	0.45	45	0.45	0.2025	0.025	0.023125	0.006938	0.00641731	1	0	0	0.1248773	0.056195	0	0.6975	0.026875	0.007458	0.1	0.05	0.013875	0.19356	0.249755
23	0.261797	0.006545	0.025	0.425	42.5	0.425	0.180625	0.025	0.021875	0.006545	0.00572682	1	0	0	0.1112638	0.047287	0	0.669375	0.028125	0.007363	0.15	0.05	0.01309	0.175241	0.222528
24	0.246979	0.006174	0.025	0.4	40	0.4	0.16	0.025	0.020625	0.006174	0.00509393	1	0	0	0.0987914	0.039517	0	0.64	0.029375	0.007255	0.2	0.05	0.012349	0.158066	0.197583
25	0.232999	0.005825	0.025	0.375	37.5	0.375	0.140625	0.025	0.019375	0.005825	0.00451435	1	0	0	0.0873745	0.032765	0	0.609375	0.030625	0.007136	0.25	0.05	0.01165	0.141984	0.174749
26	0.21981	0.005495	0.025	0.35	35	0.35	0.1225	0.025	0.018125	0.005495	0.00398406	1	0	0	0.0769335	0.026927	0	0.5775	0.031875	0.007006	0.3	0.05	0.010991	0.12694	0.153867
27	0.207368	0.005184	0.025	0.325	32.5	0.325	0.105625	0.025	0.016875	0.005184	0.00349933	1	0	0	0.0673946	0.021903	0	0.544375	0.033125	0.006869	0.35	0.05	0.010368	0.112886	0.134789
28	0.19563	0.004891	0.025	0.3	30	0.3	0.09	0.025	0.015625	0.004891	0.00305672	1	0	0	0.058689	0.017607	0	0.51	0.034375	0.006725	0.4	0.05	0.009782	0.099771	0.117378
29	0.184557	0.004614	0.025	0.275	27.5	0.275	0.075625	0.025	0.014375	0.004614	0.002653	1	0	0	0.0507531	0.013957	0	0.474375	0.035625	0.006575	0.45	0.05	0.009228	0.087549	0.101506
30	0.17411	0.004353	0.025	0.25	25	0.25	0.0625	0.025	0.013125	0.004353	0.0022852	1	0	0	0.0435275	0.010882	0	0.4375	0.036875	0.00642	0.5	0.05	0.008706	0.076173	0.087055
31	0.164255	0.004106	0.025	0.225	22.5	0.225	0.050625	0.025	0.011875	0.004106	0.00195053	1	0	0	0.0369573	0.008315	0	0.399375	0.038125	0.006262	0.55	0.05	0.008213	0.065599	0.073915
32	0.154957	0.003874	0.025	0.2	20	0.2	0.04	0.025	0.010625	0.003874	0.00164642	1	0	0	0.0309915	0.006198	0	0.36	0.039375	0.006101	0.6	0.05	0.007748	0.055785	0.061983
33	0.146186	0.003655	0.025	0.175	17.5	0.175	0.030625	0.025	0.009375	0.003655	0.0013705	1	0	0	0.0255826	0.004477	0	0.319375	0.040625	0.005939	0.65	0.05	0.007309	0.046688	0.051165
34	0.137912	0.003448	0.025	0.15	15	0.15	0.0225	0.025	0.008125	0.003448	0.00112053	1	0	0	0.0206867	0.003103	0	0.2775	0.041875	0.005775	0.7	0.05	0.006896	0.03827	0.041373
35	0.130105	0.003253	0.025	0.125	12.5	0.125	0.015625	0.025	0.006875	0.003253	0.00089447	1	0	0	0.0162632	0.002033	0	0.234375	0.043125	0.005611	0.75	0.05	0.006505	0.030493	0.032526
36	0.122741	0.003069	0.025	0.1	10	0.1	0.01	0.025	0.005625	0.003069	0.00069042	1	0	0	0.0122741	0.001227	0	0.19	0.044375	0.005447	0.8	0.05	0.006137	0.023321	0.024548
37	0.115793	0.002895	0.025	0.075	7.5	0.075	0.005625	0.025	0.004375	0.002895	0.0005066	1	0	0	0.0086845	0.000651	0	0.144375	0.045625	0.005283	0.85	0.05	0.00579	0.016718	0.017369
38	0.109239	0.002731	0.025	0.05	5	0.05	0.0025	0.025	0.003125	0.002731	0.00034137	1	0	0	0.0054619	0.000273	0	0.0975	0.046875	0.005121	0.9	0.05	0.005462	0.010651	0.010924
39	0.103056	0.002576	0.025	0.025	2.5	0.025	0.000625	0.025	0.001875	0.002576	0.00019323	1	0	0	0.0025764	6.44E-05	0	0.049375	0.048125	0.00496	0.95	0.05	0.005153	0.005088	0.005153
40	0.097222	0.002431	0	0	0	0	0	0.025	0.000625	0.002431	6.0764E-05	1	0	0	0	0	0	0	0.049375	0.0048	1	0.05	0.004861	0	0

APPENDIX A.2 – 2001 CSO Mortality Table (Independence)

						mixed					mixed
x	60		A_{xy}	ρ=1	0.337741893	0.4		$A_{\overline{xy}}$	ρ=1	0.1328737	0.50
y	60			ρ=0	0.414682753	0.4			ρ=0	0.2098247	0.50
ω	121			ρ=-1	0.452965128	0.2			ρ=-1	0.171542286	0.00
i	6.00%			mixed	0.391562884				mixed	0.171349182	
d	5.66%										
										ρ=0	
k	v^k	${}_kp_x$	${}_kp_y$	${}_kp_{xy}$	${}_{k-1\|}q_{xy}$	v^k*${}_{k-1\|}q_{xy}$	v^k*${}_kp_{xy}$	${}_kp_{\overline{xy}}$	${}_{k-1\|}q_{\overline{xy}}$	v^k*${}_{k-1\|}q_{\overline{xy}}$	v^k*${}_kp_{\overline{xy}}$
0	1	1	1	1			1	1			1
1	0.943396226	0.99014	0.99199	0.982208979	0.017791021	0.016783982	0.926612244	0.999921021	0.00007898	0.00007451	0.943321718
2	0.88999644	0.979307868	0.983379527	0.963031308	0.01917767	0.017068058	0.857094436	0.999656087	0.00026493	0.00023579	0.889690359
3	0.839619283	0.967311347	0.974145593	0.942302086	0.020729222	0.017404655	0.791175002	0.999154854	0.00050123	0.00042084	0.838909682
4	0.792093663	0.954049508	0.964267757	0.919959179	0.022342906	0.017697675	0.728693836	0.998358086	0.00079677	0.00063112	0.790793113
5	0.747258173	0.939509794	0.953699382	0.89600991	0.023949269	0.017896287	0.669550728	0.997199266	0.00115882	0.00086594	0.745165302
6	0.70496054	0.923679054	0.942398044	0.870473334	0.025536576	0.018002278	0.613649352	0.995603764	0.00159550	0.00112477	0.701861368
7	0.665057114	0.906618702	0.930316502	0.843442339	0.027030995	0.017977156	0.560937327	0.993492864	0.00211090	0.00140387	0.660729497
8	0.627412371	0.888404732	0.917394405	0.815017531	0.028424808	0.017834076	0.511352082	0.990781607	0.00271126	0.00170108	0.621628637
9	0.591898464	0.868993089	0.903569272	0.785195452	0.029822079	0.017651643	0.464755982	0.987366908	0.00341470	0.00202115	0.584420956
10	0.558394777	0.848450092	0.888786878	0.754091309	0.031104143	0.017368391	0.421080648	0.983145662	0.00422125	0.00235712	0.548983402
11	0.526787525	0.826585533	0.872957584	0.72157411	0.032517199	0.017129655	0.38011624	0.977969007	0.00517665	0.00272700	0.515181873
12	0.496969364	0.80331715	0.8559611	0.687608232	0.033965878	0.016880001	0.341720225	0.971670019	0.00629899	0.00313040	0.482890231
13	0.468839022	0.778157257	0.837729128	0.651885001	0.035723231	0.016748445	0.305629126	0.964001385	0.00766863	0.00359535	0.451961467
14	0.442300964	0.751217453	0.81821004	0.614653662	0.037231339	0.016467457	0.271861908	0.954773831	0.00922755	0.00408136	0.422297386
15	0.417265061	0.722611092	0.797345684	0.576170836	0.038482827	0.016057539	0.240415959	0.943785941	0.01098789	0.00458486	0.393808898
16	0.393646284	0.692326462	0.775099739	0.53662206	0.039548776	0.015568229	0.21123928	0.930804141	0.01298180	0.00511024	0.366407591
17	0.371364419	0.660424058	0.751435944	0.496266376	0.040355684	0.014986665	0.184295674	0.915593627	0.01521051	0.00564864	0.340018895
18	0.350343791	0.626795265	0.726330469	0.455260499	0.041005877	0.014366154	0.159497689	0.897865235	0.01772839	0.00621103	0.31456151
19	0.33051301	0.591343725	0.699761301	0.413799454	0.041461045	0.013703415	0.136766103	0.877305572	0.02055966	0.00679524	0.289960906
20	0.311804727	0.55405359	0.671735861	0.372177665	0.041621789	0.012977871	0.116046755	0.853611785	0.02369379	0.00738783	0.26616019
21	0.294155403	0.515192271	0.642273526	0.330894356	0.041283309	0.012143708	0.097334363	0.82657144	0.02704035	0.00795406	0.243140455
22	0.277505097	0.474909387	0.610731473	0.29004211	0.040852247	0.011336707	0.080488164	0.795598751	0.03097269	0.00859508	0.220782708
23	0.261797261	0.433810729	0.577171778	0.25038331	0.0396588	0.010382565	0.065549665	0.760599197	0.03499955	0.00916279	0.199122787
24	0.246978548	0.392377466	0.542073963	0.212697608	0.037685702	0.00930756	0.052531746	0.721753821	0.03884538	0.00959397	0.178257711
25	0.232998631	0.35100911	0.505608647	0.177473241	0.035224367	0.008207229	0.041351022	0.679144516	0.04260930	0.00992791	0.158239742
26	0.219810029	0.310091978	0.467966083	0.145112528	0.032360713	0.007113209	0.031897189	0.632945533	0.04619898	0.01015500	0.139127776
27	0.207367952	0.270118021	0.43006551	0.116168445	0.028944084	0.006002075	0.024089612	0.584015087	0.04893045	0.01014661	0.121106012
28	0.195630143	0.231666721	0.391019863	0.090586289	0.025582155	0.005004641	0.017721409	0.532100294	0.05191479	0.01015610	0.104094857
29	0.184556739	0.195357596	0.351499485	0.068668094	0.021918195	0.004045151	0.01267316	0.478188986	0.05391131	0.00994970	0.088253
30	0.174110131	0.161779532	0.312124513	0.050495358	0.018172737	0.003164058	0.008791753	0.423408687	0.05478030	0.00953781	0.073719742
31	0.16425484	0.131419985	0.274070292	0.036018314	0.014477044	0.002377925	0.005916182	0.369471964	0.05393672	0.00885937	0.060687558
32	0.154957397	0.104815323	0.239304476	0.025082776	0.010935538	0.001694542	0.003886762	0.319037023	0.05043494	0.00781527	0.049437147
33	0.146186223	0.081983401	0.206548479	0.016933547	0.008149229	0.001191305	0.002475451	0.271598334	0.04743869	0.00693488	0.039703935
34	0.137911531	0.062796006	0.175227468	0.011003585	0.005929962	0.00081781	0.001517521	0.227019889	0.04457844	0.00614788	0.031308661
35	0.130105218	0.047025417	0.145384478	0.006836766	0.004166819	0.000542125	0.000889499	0.185573129	0.04144676	0.00539244	0.024144032
36	0.122740772	0.034367586	0.11722932	0.004028889	0.002807877	0.000344641	0.000494509	0.147568017	0.03800511	0.00466478	0.018112612
37	0.115793181	0.024550828	0.091947645	0.002257391	0.001771498	0.000205127	0.00026139	0.114241082	0.03332693	0.00385903	0.013228338
38	0.10923885	0.017107508	0.07001997	0.001197867	0.001059524	0.000115741	0.000130854	0.085929611	0.02831147	0.00309271	0.009386852
39	0.103055519	0.011600944	0.053063934	0.000615592	0.000582276	6.00067E-05	6.34401E-05	0.064049286	0.02188033	0.00225489	0.006600632
40	0.097222188	0.007635161	0.039520426	0.000301745	0.000313847	3.05129E-05	2.93363E-05	0.046853843	0.01719544	0.00167178	0.004555233
41	0.091719045	0.004862147	0.028623459	0.000139171	0.000162573	1.49111E-05	1.27647E-05	0.033346435	0.01350741	0.00123889	0.003058503
42	0.086527401	0.003014142	0.020098248	6.0579E-05	7.85925E-05	6.8004E-06	5.24174E-06	0.023051811	0.01029462	0.00089077	0.001994613
43	0.081629624	0.001814333	0.013622392	2.47156E-05	3.58634E-05	2.92752E-06	2.01752E-06	0.015412009	0.00763980	0.00062363	0.001258076
44	0.077009079	0.001057393	0.00886736	9.37628E-06	1.53393E-05	1.18126E-06	7.22059E-07	0.009915376	0.00549663	0.00042329	0.000763574
45	0.072650074	0.00059472	0.005510089	3.27696E-06	6.09932E-06	4.43116E-07	2.38071E-07	0.006101532	0.00381384	0.00027708	0.000443277
46	0.068537806	0.000321619	0.003247812	1.04456E-06	2.2324E-06	1.53004E-07	7.15916E-08	0.003568386	0.00253315	0.00017362	0.000244569
47	0.064658308	0.000166528	0.001807959	3.01075E-07	7.43482E-07	4.80723E-08	1.9467E-08	0.001974186	0.00159420	0.00010308	0.000127648
48	0.060998403	8.21498E-05	0.000945762	7.76941E-08	2.23381E-07	1.36259E-08	4.73922E-09	0.001027834	0.00094635	0.00005773	6.26962E-05
49	0.057545664	3.83894E-05	0.000462808	1.77669E-08	5.99272E-08	3.44855E-09	1.02241E-09	0.00050118	0.00052665	0.00003031	2.88407E-05
50	0.054288362	1.68794E-05	0.000210203	3.54811E-09	1.42188E-08	7.71917E-10	1.92621E-10	0.000227079	0.00027410	0.00001488	1.23277E-05
51	0.051215436	6.92665E-06	8.79132E-05	6.08944E-10	2.93917E-09	1.50531E-10	3.11873E-11	9.48392E-05	0.00013224	0.00000677	4.85723E-06
52	0.048316449	2.62665E-06	3.37297E-05	8.85961E-11	5.20348E-10	2.51413E-11	4.28065E-12	3.63562E-05	0.00005848	0.00000283	1.7566E-06
53	0.045581555	9.09243E-07	1.18104E-05	1.07386E-11	7.78576E-11	3.54887E-12	4.8948E-13	1.27197E-05	0.00002364	0.00000108	5.79782E-07
54	0.043001467	2.82829E-07	3.77497E-06	1.06767E-12	9.67088E-12	4.15862E-13	4.59114E-14	4.0578E-06	0.00000866	0.00000037	1.74491E-07
55	0.040567422	7.74443E-08	1.04419E-06	8.08669E-14	9.86804E-13	4.00321E-14	3.28056E-15	1.12164E-06	0.00000294	0.00000012	4.5502E-08
56	0.038271153	1.81452E-08	2.47046E-07	4.4827E-15	7.63842E-14	2.92331E-15	1.71558E-16	2.65191E-07	0.00000086	0.00000003	1.01492E-08
57	0.036104861	3.49095E-09	4.81913E-08	1.68233E-16	4.31446E-15	1.55773E-16	6.07404E-18	5.16822E-08	0.00000021	0.00000001	1.86598E-09
58	0.03406119	5.16417E-10	7.20748E-09	3.72206E-18	1.64511E-16	5.60345E-18	1.26778E-19	7.7239E-09	0.00000004	0.00000000	2.63085E-10
59	0.032133198	5.20393E-11	7.75237E-10	4.03428E-20	3.68172E-18	1.18306E-19	1.29634E-21	8.27276E-10	0.00000001	0.00000000	2.6583E-11
60	0.030314338	2.64256E-12	5.03051E-11	1.32934E-22	4.02099E-20	1.21894E-21	4.02981E-24	5.29476E-11	0.00000000	0.00000000	1.60507E-12
61	0.028598432	0	0	0	1.32934E-22	3.80171E-24	0	0	0.00000000	0.00000000	0

APPENDIX A.3 – 2001 CSO Mortality Table (Direct Dependence)

pmf for ρ =				1.00		E(Tx) =	20.14	E(Ty) =	23.58		E(Kx · Ky) =	569.0	σ_x =	9.23	σ_y =	10.19
	k-1|qx	K														
	0.0000	61		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	60		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	59		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	58		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	57		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	56		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	55		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	54		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	53		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	52		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	51		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	50		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	49		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0001	48		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0002	47		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0003	46		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0005	45		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0008	44		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0012	43		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0018	42		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0028	41		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0040	40		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0055	39		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0074	38		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0098	37		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0127	36		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0158	35		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0192	34		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0228	33		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0266	32		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0304	31		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0336	30		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0363	29		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0385	28		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0400	27		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0409	26		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0414	25		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0414	24		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0411	23		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0403	22		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0389	21		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0373	20		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0355	19		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0336	18		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0319	17		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0303	16		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0286	15		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0269	14		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0252	13		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0233	12		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0219	11		-	-	-	-	-	-	-	-	-	-	-	-	0.0107
	0.0205	10		-	-	-	-	-	-	-	-	-	-	-	0.0130	0.0075
	0.0194	9		-	-	-	-	-	-	-	-	-	-	0.0154	0.0040	-
	0.0182	8		-	-	-	-	-	-	-	-	0.0030	0.0148	0.0004	-	-
	0.0171	7		-	-	-	-	-	-	-	0.0063	0.0108	-	-	-	-
	0.0158	6		-	-	-	-	-	-	0.0092	0.0066	-	-	-	-	-
	0.0145	5		-	-	-	-	0.0004	0.0113	0.0029	-	-	-	-	-	-
	0.0133	4		-	-	-	0.0030	0.0102	-	-	-	-	-	-	-	-
	0.0120	3		-	-	0.0052	0.0068	-	-	-	-	-	-	-	-	-
	0.0108	2		-	0.0068	0.0041	-	-	-	-	-	-	-	-	-	-
	0.0099	1		0.0080	0.0019	-	-	-	-	-	-	-	-	-	-	-
			k	1	2	3	4	5	6	7	8	9	10	11	12	13
			k-1|qx	0.0080	0.0086	0.0092	0.0099	0.0106	0.0113	0.0121	0.0129	0.0138	0.0148	0.0158	0.0170	0.0182

k-1|qx	K																					
0.0000	61	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	59	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	58	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	57	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	56	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	54	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	53	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	52	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	51	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	49	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0001	48	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0002	47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0003	46	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0005	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0008	44	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0012	43	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0018	42	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0028	41	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0040	40	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0055	39	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0074	38	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0098	37	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0127	36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0192	34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0228	33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0266	32	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0304	31	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0201
0.0363	29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0251	0.0112
0.0385	28	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0308	0.0076	-
0.0400	27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0360	0.0040	-	-
0.0409	26	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0389	0.0020	-	-	-
0.0414	25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0014	0.0395	0.0005	-	-	-	-
0.0414	24	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0037	0.0377	-	-	-	-	-	-
0.0411	23	-	-	-	-	-	-	-	-	-	-	-	-	0.0069	0.0342	-	-	-	-	-	-	-
0.0403	22	-	-	-	-	-	-	-	-	-	-	-	0.0096	0.0307	-	-	-	-	-	-	-	-
0.0389	21	-	-	-	-	-	-	-	-	-	-	0.0120	0.0269	-	-	-	-	-	-	-	-	-
0.0373	20	-	-	-	-	-	-	-	-	-	0.0142	0.0231	-	-	-	-	-	-	-	-	-	-
0.0355	19	-	-	-	-	-	-	-	-	0.0161	0.0194	-	-	-	-	-	-	-	-	-	-	-
0.0336	18	-	-	-	-	-	-	-	0.0182	0.0155	-	-	-	-	-	-	-	-	-	-	-	-
0.0319	17	-	-	-	-	-	-	0.0206	0.0113	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0303	16	-	-	-	-	-	0.0228	0.0074	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0286	15	-	-	-	-	0.0249	0.0037	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0269	14	-	-	0.0031	0.0237	0.0002	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0252	13	-	0.0060	0.0192	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0233	12	0.0084	0.0149	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0219	11	0.0111	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0205	10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0194	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0182	8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0171	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0145	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0133	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0120	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0108	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0099	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	k	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
k-1|qy		0.0195	0.0209	0.0222	0.0237	0.0251	0.0266	0.0280	0.0295	0.0315	0.0336	0.0351	0.0365	0.0376	0.0379	0.0390	0.0395	0.0394	0.0381	0.0348	0.0328	0.0313

k-1|qx	K																				
0.0000	61	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	59	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	58	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	57	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	56	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	54	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	53	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	52	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0000
0.0000	51	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0000	0.0000
0.0000	50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0000	0.0000	-
0.0000	49	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0000	-	-
0.0001	48	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0001	0.0000	-	-
0.0002	47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0001	0.0000	-	-	-
0.0003	46	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0001	0.0001	-	-	-	-
0.0005	45	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0001	0.0004	-	-	-	-	-
0.0008	44	-	-	-	-	-	-	-	-	-	-	-	-	0.0000	0.0008	-	-	-	-	-	-
0.0012	43	-	-	-	-	-	-	-	-	-	-	-	-	0.0012	-	-	-	-	-	-	-
0.0018	42	-	-	-	-	-	-	-	-	-	-	-	0.0016	0.0002	-	-	-	-	-	-	-
0.0028	41	-	-	-	-	-	-	-	-	-	-	0.0021	0.0006	-	-	-	-	-	-	-	-
0.0040	40	-	-	-	-	-	-	-	-	-	0.0027	0.0012	-	-	-	-	-	-	-	-	-
0.0055	39	-	-	-	-	-	-	-	-	0.0035	0.0020	-	-	-	-	-	-	-	-	-	-
0.0074	38	-	-	-	-	-	-	-	0.0045	0.0030	-	-	-	-	-	-	-	-	-	-	-
0.0098	37	-	-	-	-	-	-	0.0057	0.0041	-	-	-	-	-	-	-	-	-	-	-	-
0.0127	36	-	-	-	-	-	0.0075	0.0052	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	35	-	-	-	-	0.0097	0.0060	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0192	34	-	-	-	0.0120	0.0072	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0228	33	-	-	0.0129	0.0100	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0266	32	-	0.0142	0.0124	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0304	31	0.0164	0.0140	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	30	0.0134	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0363	29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0385	28	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0400	27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0409	26	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	24	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0411	23	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0403	22	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0389	21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0373	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0355	19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0319	17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0303	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0286	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0269	14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0252	13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0233	12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0219	11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0205	10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0194	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0182	8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0171	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0145	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0133	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0120	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0108	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0099	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	k	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
	k-1|qy	0.0298	0.0282	0.0253	0.0219	0.0170	0.0135	0.0109	0.0085	0.0065	0.0048	0.0034	0.0023	0.0014	0.0009	0.0005	0.0003	0.0001	0.0001	0.0000	0.0000

APPENDIX A.4 – 2001 CSO Mortality Table (Indirect Dependence)

pmf for ρ =				-0.99		E(Tx) =	20.14	E(Ty) =	23.58		E(Kx · Ky) =	382.4	σ_x =	9.23	σ_y =	10.19
	$_{k-1\vert}q_x$	K														
	0.0000	61		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	60		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	59		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	58		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	57		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	56		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	55		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	54		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	53		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	52		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	51		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	50		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0000	49		0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	0.0001	48		0.0001	-	-	-	-	-	-	-	-	-	-	-	-
	0.0002	47		0.0002	-	-	-	-	-	-	-	-	-	-	-	-
	0.0003	46		0.0003	-	-	-	-	-	-	-	-	-	-	-	-
	0.0005	45		0.0005	-	-	-	-	-	-	-	-	-	-	-	-
	0.0008	44		0.0008	-	-	-	-	-	-	-	-	-	-	-	-
	0.0012	43		0.0012	-	-	-	-	-	-	-	-	-	-	-	-
	0.0018	42		0.0018	-	-	-	-	-	-	-	-	-	-	-	-
	0.0028	41		0.0028	-	-	-	-	-	-	-	-	-	-	-	-
	0.0040	40		0.0004	0.0036	-	-	-	-	-	-	-	-	-	-	-
	0.0055	39		-	0.0050	0.0005	-	-	-	-	-	-	-	-	-	-
	0.0074	38		-	-	0.0074	-	-	-	-	-	-	-	-	-	-
	0.0098	37		-	-	0.0013	0.0085	-	-	-	-	-	-	-	-	-
	0.0127	36		-	-	-	0.0014	0.0106	0.0007	-	-	-	-	-	-	-
	0.0158	35		-	-	-	-	-	0.0106	0.0052	-	-	-	-	-	-
	0.0192	34		-	-	-	-	-	-	0.0069	0.0123	-	-	-	-	-
	0.0228	33		-	-	-	-	-	-	-	0.0006	0.0138	0.0084	-	-	-
	0.0266	32		-	-	-	-	-	-	-	-	-	0.0064	0.0158	0.0044	-
	0.0304	31		-	-	-	-	-	-	-	-	-	-	-	0.0126	0.0177
	0.0336	30		-	-	-	-	-	-	-	-	-	-	-	-	0.0005
	0.0363	29		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0385	28		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0400	27		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0409	26		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0414	25		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0414	24		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0411	23		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0403	22		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0389	21		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0373	20		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0355	19		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0336	18		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0319	17		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0303	16		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0286	15		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0269	14		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0252	13		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0233	12		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0219	11		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0205	10		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0194	9		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0182	8		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0171	7		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0158	6		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0145	5		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0133	4		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0120	3		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0108	2		-	-	-	-	-	-	-	-	-	-	-	-	-
	0.0099	1		-	-	-	-	-	-	-	-	-	-	-	-	-
			k	1	2	3	4	5	6	7	8	9	10	11	12	13
		$_{k-1\vert}q_y$		0.0080	0.0086	0.0092	0.0099	0.0106	0.0113	0.0121	0.0129	0.0138	0.0148	0.0158	0.0170	0.0182

k-1|qx	K																		
0.0000	61		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	60		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	59		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	58		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	57		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	56		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	55		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	54		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	53		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	52		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	51		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	50		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	49		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0001	48		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0002	47		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0003	46		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0005	45		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0008	44		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0012	43		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0018	42		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0028	41		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0040	40		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0055	39		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0074	38		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0098	37		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0127	36		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	35		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0192	34		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0228	33		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0266	32		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0304	31		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	30		0.0195	0.0136	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0363	29		-	0.0073	0.0222	0.0068	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0385	28		-	-	-	0.0169	0.0216	-	-	-	-	-	-	-	-	-	-	-	-
0.0400	27		-	-	-	-	0.0036	0.0266	0.0099	-	-	-	-	-	-	-	-	-	-
0.0409	26		-	-	-	-	-	-	0.0182	0.0227	-	-	-	-	-	-	-	-	-
0.0414	25		-	-	-	-	-	-	-	0.0067	0.0315	0.0031	-	-	-	-	-	-	-
0.0414	24		-	-	-	-	-	-	-	-	-	0.0305	0.0110	-	-	-	-	-	-
0.0411	23		-	-	-	-	-	-	-	-	-	-	0.0241	0.0170	-	-	-	-	-
0.0403	22		-	-	-	-	-	-	-	-	-	-	-	0.0195	0.0208	-	-	-	-
0.0389	21		-	-	-	-	-	-	-	-	-	-	-	-	0.0168	0.0220	-	-	-
0.0373	20		-	-	-	-	-	-	-	-	-	-	-	-	-	0.0159	0.0214	-	-
0.0355	19		-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0176	0.0178	-
0.0336	18		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0217	0.0119
0.0319	17		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0275
0.0303	16		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0286	15		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0269	14		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0252	13		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0233	12		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0219	11		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0205	10		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0194	9		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0182	8		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0171	7		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	6		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0145	5		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0133	4		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0120	3		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0108	2		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0099	1		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		k	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	k-1|qy		0.0195	0.0209	0.0222	0.0237	0.0251	0.0266	0.0280	0.0295	0.0315	0.0336	0.0351	0.0365	0.0376	0.0379	0.0390	0.0395	0.0394

k-1|qx	K																
0.0000	61		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	60		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	59		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	58		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	57		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	56		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	55		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	54		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	53		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	52		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	51		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	50		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	49		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0001	48		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0002	47		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0003	46		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0005	45		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0008	44		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0012	43		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0018	42		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0028	41		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0040	40		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0055	39		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0074	38		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0098	37		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0127	36		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	35		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0192	34		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0228	33		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0266	32		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0304	31		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	30		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0363	29		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0385	28		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0400	27		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0409	26		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	25		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	24		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0411	23		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0403	22		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0389	21		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0373	20		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0355	19		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	18		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0319	17		0.0045	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0303	16		0.0303	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0286	15		0.0033	0.0253	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0269	14		-	0.0095	0.0175	-	-	-	-	-	-	-	-	-	-	-	-
0.0252	13		-	-	0.0153	0.0099	-	-	-	-	-	-	-	-	-	-	-
0.0233	12		-	-	-	0.0215	0.0018	-	-	-	-	-	-	-	-	-	-
0.0219	11		-	-	-	-	0.0219	-	-	-	-	-	-	-	-	-	-
0.0205	10		-	-	-	-	0.0062	0.0144	-	-	-	-	-	-	-	-	-
0.0194	9		-	-	-	-	-	0.0138	0.0056	-	-	-	-	-	-	-	-
0.0182	8		-	-	-	-	-	-	0.0182	-	-	-	-	-	-	-	-
0.0171	7		-	-	-	-	-	-	0.0014	0.0156	-	-	-	-	-	-	-
0.0158	6		-	-	-	-	-	-	-	0.0063	0.0095	-	-	-	-	-	-
0.0145	5		-	-	-	-	-	-	-	-	0.0074	0.0071	-	-	-	-	-
0.0133	4		-	-	-	-	-	-	-	-	-	0.0064	0.0068	-	-	-	-
0.0120	3		-	-	-	-	-	-	-	-	-	-	0.0041	0.0079	-	-	-
0.0108	2		-	-	-	-	-	-	-	-	-	-	-	0.0006	0.0065	0.0038	-
0.0099	1		-	-	-	-	-	-	-	-	-	-	-	-	-	0.0010	0.0034
		k	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
k-1|qy			0.0381	0.0348	0.0328	0.0313	0.0298	0.0282	0.0253	0.0219	0.0170	0.0135	0.0109	0.0085	0.0065	0.0048	0.0034

k-1|qx	K																	
0.0000	61		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	60		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	59		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	58		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	57		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	56		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	55		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	54		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	53		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	52		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	51		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	50		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0000	49		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0001	48		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0002	47		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0003	46		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0005	45		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0008	44		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0012	43		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0018	42		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0028	41		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0040	40		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0055	39		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0074	38		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0098	37		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0127	36		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	35		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0192	34		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0228	33		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0266	32		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0304	31		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	30		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0363	29		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0385	28		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0400	27		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0409	26		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	25		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0414	24		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0411	23		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0403	22		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0389	21		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0373	20		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0355	19		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0336	18		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0319	17		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0303	16		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0286	15		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0269	14		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0252	13		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0233	12		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0219	11		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0205	10		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0194	9		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0182	8		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0171	7		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0158	6		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0145	5		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0133	4		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0120	3		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0108	2		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.0099	1		0.0023	0.0014	0.0009	0.0005	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-
		k	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61
	k-1|qy		0.0023	0.0014	0.0009	0.0005	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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VITA

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EDUCATION

Pennsylvania State University, *Schreyer Honors College Scholar* — University Park, PA
B.S. in Mathematics with Actuarial Science Option | Minor in Statistics — August 2009 – May 2012
Honors Thesis Title: Pricing Joint-life Insurance under Dependent Assumption
5/5 Dean's List | President's Freshman Award – Recognized for academic excellence

Actuarial Exams:	1/P Exam	Probability	Passed, May 2010
	2/FM Exam	Financial Mathematics	Passed, December 2010
	VEE Requirements Met Upon Graduation: Economics, Finance, Statistics		

WORK EXPERIENCE

John Hancock — Boston, MA
Actuarial Intern, Variable Annuities — May 2011 – August 2011

- Collaborated with other actuaries in spot-checking the variable annuity model, reserves, and capital and was exposed to various aspects of the VA business from product types to government regulations
- Gained knowledge of diverse businesses such as long term care, annuity, and actuarial auditing through a number of presentations and panels
- Improved business writing skills and technological skills such as GGY AXIS, Microsoft Excel and Access

Penn State University — University Park, PA
Teaching Assistant, Financial Mathematics — August 2010 – December 2010

- Worked directly with the professor and provided him with feedback about the class pace and materials, and relay suggestions from other students
- Developed teamwork and communication skills through working with other teaching assistants to discuss and grade homework for a class of over 50 students

PSU Know How — State College, PA
Tutor, Mathematics and Statistics — January 2010 – December 2011

- Enhanced communication skills while helping college students with all levels of statistics and mathematics
- Acquired leadership skills through organizing and conducting review sessions for a group of 10+ college students for an introductory statistics course

PROFESSIONAL DEVELOPMENT

Korean Undergraduate Student Association — State College, PA
Treasurer — September 2010 – May 2012

- Host and promote the screening of “Hiding”, a documentary of North Korean refugees in China
- Balanced the budget and expenditures of a 40+ member student organization

Study Abroad Experience in Reggio Calabria, Italy — Summer, 2010

- Gained understanding about a different culture and adapted to a new environment while experiencing the Southern Italian culture and interacting with the local people